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# A Neutrosophic Offset Logic and Statistical Framework for Analyzing International Chinese Communication in Integrated Media Environments

Bing Li*

School of International Education,South China University of Technology,Guangzhou, Guangdong, 510006, China

*Corresponding author, E-mail: scutbing@163.com

**Abstract:** This paper proposes a novel mathematical model that applies Neutrosophic Offset Logic and Neutrosophic Offset Statistics to analyze and interpret the complexities of International Chinese Communication (ICC) in the context of integrated media. Unlike traditional logic or fuzzy systems, our model allows truth (T), indeterminacy (I), and falsehood (F) values to extend beyond the conventional interval [0,1], enabling accurate representation of over-assertive, under-informative, and ambiguous narratives common in geopolitical discourse. We introduce a robust algorithm and apply it to a case study of international Chinese news media to demonstrate the effectiveness of our framework.

**Keywords:** Neutrosophic Logic, Offset Logic, Neutrosophic Statistics, International Chinese Communication, Integrated Media, Misinformation Detection, Strategic Communication, Political Messaging, Fuzzy Logic Limitations, Truth-Indeterminacy-Falsehood Modeling

## 1. Introduction

In the era of integrated media where print, television, social platforms, and mobile devices converge the nature of communication, particularly state-sponsored international media, has evolved into a multidimensional space characterized by information overload, conflicting narratives, and subjective perceptions.

The ICC apparatus plays a strategic role in framing China's global narrative. However, its messages often exhibit overconfidence, partial misinformation, and strategic ambiguity, challenging classical analytic models. This paper addresses this by extending neutrosophic logic into the offset domain, allowing representations beyond conventional boundaries.

The concept of neutrosophic sets, introduced by Florentin Smarandache in 1995, represents a significant advancement in handling uncertainty and indeterminacy in various scientific and real-world applications. Unlike classical fuzzy sets and intuitionistic fuzzy sets, which confine membership values to the interval [0, 1], neutrosophic sets incorporate three independent components: T, I, and F. These components allow for a more nuanced representation of information, where the sum of T, I, and F can exceed 1 or even extend to 3 in cases of independent sources [1].

In 2007, the uncertain Set was extended by Smarandache to the uncertain OverSet (when some component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1, compared to an employee who only works regular full-time and whose degree of membership = 1. He also introduced the uncertain UnderSet (when some neutrosophic component is < 0), since, for example, an employee causing more damage than benefit to their company deserves a degree of membership < 0, compared to an employee who contributes positively to the company and has a degree of membership > 0. Additionally, he proposed the uncertain OffSet (when some neutrosophic components fall outside the interval [0, 1], i.e., some components > 1 and some < 0).

Similarly, uncertain Logic, Measure, Probability, Statistics, etc., were extended to uncertain Over-/Under-/Off-Logic, Measure, Probability, Statistics, etc., respectively. By "uncertain," we refer to all types of fuzzy and fuzzy-extensions (intuitionistic fuzzy, neutrosophic, spherical fuzzy, plithogenic, etc.) [6-9].

Building upon this foundation, Smarandache's seminal work, Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset (2016), extends the neutrosophic framework to accommodate membership degrees beyond the conventional boundaries, introducing the notions of overset (T, I, or F > 1), underset (T, I, or F < 0), and offset (combinations of components > 1 and < 0) [2]. This extension challenges the orthodox constraints of classical and fuzzy logic, offering a robust mathematical tool to model real-world scenarios where over- and under-membership degrees are prevalent, such as in workforce management, educational enrollment systems, and competitive dynamics.

The motivation for this extension stems from practical observations, as illustrated by Smarandache's example of employees in a company. A full-time employee working 40 hours per week has a membership degree of 1, while an employee working overtime exceeds this norm, warranting a membership degree greater than 1. Conversely, an employee causing damage, such as through negligence, may be assigned a negative membership degree to reflect their detrimental impact [2]. These examples underscore the necessity of a framework that transcends the [0, 1] interval to capture the complexities of real-world systems. The book further generalizes these concepts to neutrosophic logic, probability, statistics, graphs, matrices, and topologies, introducing novel constructs such as neutrosophic over topology, under topology, and off-topology, as well as bipolar and tripolar structures [2]. This work not only enriches the theoretical landscape of neutrosophic theory but also provides practical tools for applications in fields like information fusion, decision-making, and system modeling.

## 2. Literature Review

The development of neutrosophic sets and their extensions has been a subject of growing interest since their inception. Smarandache's initial formulation of neutrosophic logic and sets in 1995 laid the groundwork for a paradigm that integrates truth, indeterminacy, and falsehood as independent components, challenging the limitations of fuzzy and intuitionistic fuzzy frameworks [1]. Published in 2007, A Unifying Field in Logics: Neutrosophic Logic formalized these ideas, introducing neutrosophic probability and statistics as generalizations of classical and imprecise probabilities [3]. The work was further expanded in subsequent publications, including Neutrosophic Set - A Generalization of the Intuitionistic Fuzzy Set, which explored geometric interpretations and applications in granular computing [4]. These foundational texts established neutrosophic sets as a versatile tool for handling uncertainty in diverse domains.

The specific concepts of neutrosophic overset, underset, and offset were first presented by Smarandache at various international conferences between 1995 and 2016, with formal publication in 2016 [2]. This work builds on earlier explorations of neutrosophic measure, integral, and probability, as detailed in Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability [5]. The introduction of overset, underset, and offset addresses a critical gap in classical and fuzzy set theories, which do not account for membership degrees outside [0, 1]. Smarandache's framework has been applied in contexts such as physics, where neutrosophic oversets model phenomena exceeding standard limits, and in information fusion, where neutrosophic logic enhances decision-making under uncertainty [6, 7]. Additionally, Symbolic Neutrosophic Theory (2015) introduced symbolic representations of neutrosophic components, facilitating the development of neutrosophic offlogic and offprobability [8].

Recent studies have further explored the applications of neutrosophic extensions. For instance, the integration of neutrosophic sets with bipolar and tripolar structures, as discussed in Smarandache's 2016 work, enables the modeling of complex relationships in graphs and matrices, with applications in network analysis and system dynamics [2]. The concept of complex neutrosophic sets, introduced by Ali and Smarandache in 2015, incorporates amplitude and phase components, broadening the scope of neutrosophic theory to complex-valued systems [9]. Moreover, the development of neutrosophic statistics, as outlined in Introduction to Neutrosophic Statistics (2014), provides a framework for analyzing data with inherent indeterminacy, offering advantages over classical statistical methods in handling ambiguous datasets . These advancements highlight the growing relevance of neutrosophic theory in addressing the limitations of traditional mathematical models.

## 3. Theoretical Foundations

### 3.1 Neutrosophic Offset Representation

Let $\mathcal{M} = \{m_1, m_2, \ldots, m_n\}$ be a finite set of messages. Each message $m_i$ is associated with a neutrosophic vector:

$$m_i = \langle T(m_i), I(m_i), F(m_i) \rangle$$

Where:

$T(m_i) \in [\Psi_T, \Omega_T]$ : Degree of truth

$I(m_i) \in [\Psi_I, \Omega_I]$ : Degree of indeterminacy

$F(m_i) \in [\Psi_F, \Omega_F]$ : Degree of falsehood

And $\Psi_T, \Psi_I, \Psi_F < 0, \Omega_T, \Omega_I, \Omega_F > 1$. This allows undertruth (< 0) and overtruth (> 1).

A message is considered neutrosophically offset if:

$$\exists i: \min\{T(m_i), I(m_i), F(m_i)\} < 0 \ \text{ and } \ \max\{T(m_i), I(m_i), F(m_i)\} > 1$$

### 3.2 Neutrosophic Offlogic Operators

Let $A = \langle T_A, I_A, F_A \rangle, B = \langle T_B, I_B, F_B \rangle$.

Offnegation:

$$\neg A = \langle F_A, \Psi + \Omega - I_A, T_A \rangle$$

OffAND (intersection):

$$A \wedge B = \langle \min(T_A, T_B), \max(I_A, I_B), \max(F_A, F_B) \rangle$$

OffOR (union):

$$A \vee B = \langle \max(T_A, T_B), \min(I_A, I_B), \min(F_A, F_B) \rangle$$

These operations support message fusion in contradictory or noisy data environments.

## 4. Neutrosophic Offset Analysis for ICC

We now define the Neutrosophic Offset Analytical Algorithm (NOAA) designed for ICC message modeling.

Step 4.1: Neutrosophic Vector Construction

Each message $m_i \in \mathcal{M}$ is processed using natural language processing (NLP) and domain knowledge to assign:

$$m_i = \langle T_i, I_i, F_i \rangle$$

Where:

$T_i$ : Linguistic truth detection (e.g., factual alignment)

$I_i$ : Ambiguity score (e.g., contradiction or vagueness)

$F_i$ : Misinformation detection (e.g., false claims)

Step 4.2: Offset Extension

We define an extended scale using tunable parameters $\lambda_T, \lambda_I, \lambda_F$ :

$$T_i^* = \lambda_T \cdot T_i, I_i^* = \lambda_I \cdot I_i, F_i^* = \lambda_F \cdot F_i$$

Allowing $T^*, I^*, F^* \notin [0,1]$.

Step 4.3: Statistical Aggregation

Given n messages, calculate:

Offset Mean (central tendency):

$$\mu_T = \frac{1}{n}\sum_{i=1}^{n} T_i^*, \mu_I = \frac{1}{n}\sum_{i=1}^{n} I_i^*, \mu_F = \frac{1}{n}\sum_{i=1}^{n} F_i^*$$

Offset Entropy (uncertainty):

$$H_i = -[T_i^* \log_2 (|T_i^*|) + I_i^* \log_2 (|I_i^*|) + F_i^* \log_2 (|F_i^*|)]$$

Step 4.4: Message Quality Scoring

Define a weighted scoring function:

$$\mathrm{NMQI}(m_i) = \alpha T_i^* - \beta I_i^* - \gamma F_i^*$$

Where $\alpha, \beta, \gamma > 0$ reflect domain-specific weights (e.g., importance of clarity over assertiveness).

NMQI $> 1 \Rightarrow$ Highly assertive

NMQI $< 0 \Rightarrow$ Misleading/negative

$|\mu_I| > 0.7 \Rightarrow$ Highly ambiguous message

**5. ICC Messaging Simulation**

Consider five messages:

$\mathrm{m}_1 = \langle 1.3, 0.2, -0.2 \rangle$
$\mathrm{m}_2 = \langle 0.9, -0.3, 0.6 \rangle$
$\mathrm{m}_3 = \langle 1.1, 1.4, -0.4 \rangle$
$\mathrm{m}_4 = \langle 0.6, 0.1, 1.5 \rangle$
$\mathrm{m}_5 = \langle -0.2, 0.5, 1.2 \rangle$

Offset Means:

$$\mu_T = \frac{1.3 + 0.9 + 1.1 + 0.6 - 0.2}{5} = 0.74, \mu_I = \frac{0.2 - 0.3 + 1.4 + 0.1 + 0.5}{5} = 0.38, \mu_F = \frac{-0.2 + 0.6 - 0.4 + 1.5 + 1.2}{5} = 0.54$$

The analyzed messages show an above-average truth score, reflecting a strong and confident narrative tone. However, moderate indeterminacy indicates the presence of some ambiguity or unclear framing in communication. Additionally, the moderate falsehood levels suggest mixed reliability, with certain elements potentially leading to misleading or biased interpretations.

**5. Case Study**

**5.1 Problem Statement**

In international discourse, Chinese state-affiliated media aim to shape narratives abroad, blending factual accuracy, strategic ambiguity, and selective misinformation. Conventional logic models are inadequate to evaluate such mixed-content messages because:

I. Some messages overstate reality (economic success is exaggerated).
II. Others contain partial or conflicting data (diplomatic ambiguity).
III. Some may be misled (underreporting crises).

We propose a Neutrosophic Offset Fusion Algorithm (NOFA) that models and evaluates such content using offset logic.

**5.2 Data Context**

We simulate a week-long ICC campaign by a major Chinese state broadcaster targeting Western audiences. We extract 5 representative headlines/messages, preprocessed using NLP tools to derive approximate T (truth), I (indeterminacy), F (falsehood) values via semantic analysis as shown in Table 1.

Table 1. Message Dataset M

| ID | Message | T | I | F |
|---|---|---|---|---|
| $M_1$ | "China lifts 20 million out of poverty in 1 year" | 1.2 | 0.1 | -0.1 |
| $M_2$ | "Global experts praise China's vaccine leadership" | 0.8 | 0.3 | 0.6 |
| $M_3$ | "Zero-COVID policy hailed as human rights model" | 0.6 | 0.7 | 1.3 |
| $M_4$ | "China's economic growth second only to miracles" | 1.4 | 0.2 | -0.3 |
| $M_5$ | "West distorts China's humanitarian aid efforts" | 0.9 | 1.1 | 0.4 |

where: T, I, F $\in [-0.5, 1.5]$; values >1 are overstatements, <0 are undermining.

**5.3 Proposed Method - NOFA**

Step 5.1: Normalize Offset Space

Let:

$\mathrm{T}_\mathrm{i}^* = \mathrm{T}_\mathrm{i}$

$\mathrm{I}_\mathrm{i}^* = \mathrm{I}_\mathrm{i}$

$\mathrm{F}_\mathrm{i}^* = \mathrm{F}_\mathrm{i}$

(No rescaling here messages already expressed in offset domain.)

Step 5.2: Compute Offset Means

$$\mu_\mathrm{T} = \frac{1.2 + 0.8 + 0.6 + 1.4 + 0.9}{5} = \frac{4.9}{5} = 0.98$$
$$\mu_\mathrm{I} = \frac{0.1 + 0.3 + 0.7 + 0.2 + 1.1}{5} = \frac{2.4}{5} = 0.48$$
$$\mu_\mathrm{F} = \frac{-0.1 + 0.6 + 1.3 - 0.3 + 0.4}{5} = \frac{1.9}{5} = 0.38$$

Step 5.3: Compute Offset Entropy for Each Message

Using:

$$\mathrm{H}_\mathrm{i} = -[\mathrm{T}_\mathrm{i}^*\log_2(|\mathrm{T}_\mathrm{i}^*| + \epsilon) + \mathrm{I}_\mathrm{i}^*\log_2(|\mathrm{I}_\mathrm{i}^*| + \epsilon) + \mathrm{F}_\mathrm{i}^*\log_2(|\mathrm{F}_\mathrm{i}^*| + \epsilon)]$$

Where $\epsilon = 0.01$ avoids $\log(0)$.

Example: $\mathrm{H}_1$

$$\begin{gathered}\mathrm{H}_1 = -[1.2\log_2(1.2) + 0.1\log_2(0.1) + (-0.1)\log_2(0.1)] \\ = -[1.2 \cdot 0.263 + 0.1 \cdot (-3.321) + (-0.1) \cdot (-3.321)] \\ = -[0.315 - 0.332 + 0.332] = -0.315\end{gathered}$$

Repeat for $\mathrm{H}_2$ to $\mathrm{H}_5$.

Step 5.4: Compute Neutrosophic Quality Score (NMQI)

We define:

$$\mathrm{NMQI_i} = \alpha \mathrm{T_i^*} - \beta \mathrm{I_i^*} - \gamma \mathrm{F_i^*}$$

Let:

$\alpha = 1.0$

$\beta = 0.8$

$\gamma = 1.2$

Example: $\mathrm{NMQI_1}$

$$= 1.0 \cdot 1.2 - 0.8 \cdot 0.1 - 1.2 \cdot (-0.1) = 1.2 - 0.08 + 0.12 = 1.24$$

Table 2 illustrates the Neutrosophic NMQI scores for each message, reflecting the overall communication quality based on truth, indeterminacy, and falsehood components.

Table 2. NMQIs

| ID | T | I | F | NMQI |
|---|---|---|---|---|
| $\mathbf{M_1}$ | 1.2 | 0.1 | -0.1 | 1.24 |
| $\mathbf{M_2}$ | 0.8 | 0.3 | 0.6 | 0.44 |
| $\mathbf{M_3}$ | 0.6 | 0.7 | 1.3 | -1.02 |
| $\mathbf{M_4}$ | 1.4 | 0.2 | -0.3 | 1.64 |
| $\mathbf{M_5}$ | 0.9 | 1.1 | 0.4 | -0.02 |

Step 5.5: Based on NMQI:

NMQI > 1.0: Overconfident narrative (possible propaganda)

$0 <$ NMQI $\leq$ 1.0: Balanced message

NMQI $< 0$: Misleading or ambiguous

Table 3 shows the final classification of ICC messages based on NOFA analysis, showing the dominant communication characteristics identified in each case.

Table 3: Classification of ICC Messages by Communication Type Using NOFA

| ID | Message | Label |
|---|---|---|
| $M_1$ | Poverty alleviation | Overconfident |
| $M_2$ | Vaccine praise | Balanced |
| $M_3$ | Zero-COVID as human rights model | Misleading |

| | | |
|---|---|---|
| $M_4$ | Economic miracle claim | Overconfident |
| $M_5$ | Western distortion narrative | Misleading |

The analysis shows a high average truth score (0.98), indicating a strong narrative push across the messages. Indeterminacy is moderate (0.48), reflecting the presence of complex or unclear messaging. Messages such as $M_3$ and $M_5$ are polarizing, combining positive statements with high levels of falsehood or ambiguity. The NOFA model successfully identifies hidden overstatements in $M_1$ and $M_4$ and highlights $M_3$ as a case of rhetorical misdirection.

## 6. Comparative Evaluation: NOFA vs. Classical Logic Models

To assess the effectiveness of the proposed NOFA, we compare it to two mainstream frameworks often used in communication and media analysis.

### 6.1 Classical Binary Logic Model

In binary logic, a message is either:

1. True (1) or False (0).
2. No recognition of indeterminacy.
3. No gradation of truth or falsehood.
4. No allowance for contradictory or exaggerated claims.

The ICC message "China lifts 20 million out of poverty" is factually true, but possibly exaggerated. Binary logic treats it as 1 (true), ignoring the overstatement. Binary logic cannot:

1. Flag propaganda (truth score > 1)
2. Flag misleading positivity (truth + falsehood simultaneously)

### 6.2 Classical Fuzzy Set Model

In fuzzy logic:

1. Membership functions $\mu(x)\in[0,1$
2. Allows partial truth: e.g., $\mu(x)=0.7$
3. Can model ambiguity, but not contradiction or misinformation

Limitations of Fuzzy Logic:

1. Cannot assign values above 1 (overconfidence) or below 0 (counterproductive)
2. Cannot model ambiguous but manipulative messages with both high truth and high falsehood

Table 4 summarizes the key advantages of the NOFA model in comparison with classical binary logic and fuzzy logic, highlighting its ability to capture overstatements, contradictions, and ambiguity in ICC messages.

Table 4. NOFA Advantages

| Feature | Binary Logic | Fuzzy Logic | NOFA |
|---|---|---|---|
| Truth granularity | X | √ | √ |
| Ambiguity handling (indeterminacy) | X | Partial | √ |
| Detects exaggeration (T > 1) | X | X | √ |
| Detects contradiction (T high, F high) | X | X | √ |
| Handles misinformation (F > 1) | X | X | √ |
| Offset entropy for uncertainty | X | X | √ |

**6.3 Comparative of NOFA vs. Classical Logic Models with Solved Case**

Suppose Message (M): "China's COVID response sets a new global standard of human rights and scientific success."

This is a strategic communication message that mixes elements of:

1. Confidence (claiming global standard)
2. Ideological positioning (human rights narrative)
3. Scientific framing (success in COVID management)

Step 6.1: Manual Neutrosophic Evaluation

Based on content analysis, expert review, and semantic scores:

Truth (T): The claim may have partial factual support but is overconfident

→ $T=1.2$

Indeterminacy (I): The phrase "global standard" is ideologically loaded and vague.

→ $I=0.6$

Falsehood (F): There is international criticism and counter-narratives, but not total misinformation

→ $F=0.4$

Neutrosophic vector:

$M=\langle 1.2,0.6,0.4\rangle$

Step 6.2: Classical Binary Logic Evaluation

Binary logic forces the message into a binary judgment:

If the expert agrees with its factuality → 1

If not → 0

Let's assume a fact-checker says "Partially true". Binary logic cannot express this nuance:

$T_{binary} = 1$ (Accepted without qualification)

Limitation:

Cannot capture ambiguity ($I = 0.6$ ignored)

Cannot flag potential falsehood ($F = 0.4$ ignored)

Cannot distinguish overstatement ($T = 1.2 \rightarrow$ clipped to 1)

Step 6.3: Classical Fuzzy Logic Evaluation

Fuzzy systems allow a degree of truth:

Based on linguistic mapping:

"somewhat true" → $\mu=0.8$

But fuzzy logic does not accommodate multiple dimensions (truth, ambiguity, falsity) simultaneously unless extended intuitionistic fuzzy logic is used, and even then:

No room for $T > 1$ or $F < 0$

Ambiguity and falsity may be lumped together

Let's assume fuzzy membership:

$T_{fuzzy}=0.8$

Step 6.4: NOFA Evaluation (Step-by-Step)

We use the proposed NOFA model:

1. Message:

M=⟨T=1.2, I=0.6, F=0.4⟩

2. NMQI Scoring:

Let:

$\alpha$=1.0 (truth weight)

β=0.8 (indeterminacy penalty)

γ=1.2 (falsehood penalty)

$$\begin{aligned} \mathrm{NMQI} &= 1.0 \cdot 1.2 - 0.8 \cdot 0.6 - 1.2 \cdot 0.4 \\ &= 1.2 - 0.48 - 0.48 = 0.24 \end{aligned}$$

3. Offset Entropy:

$$\begin{aligned} \mathrm{H} &= -[\mathrm{T} \cdot \log_2(\mathrm{T}) + \mathrm{I} \cdot \log_2(\mathrm{I}) + \mathrm{F} \cdot \log_2(\mathrm{F})] \\ &= -[1.2 \cdot \log_2(1.2) + 0.6 \cdot \log_2(0.6) + 0.4 \cdot \log_2(0.4)] \\ &\approx -[1.2 \cdot 0.263 + 0.6 \cdot (-0.737) + 0.4 \cdot (-1.322)] \\ &\approx -[0.3156 - 0.4422 - 0.5288] = -(-0.6554) = 0.6554 \end{aligned}$$

The results show moderate entropy, indicating that the message is structured but carries an element of bias. The NMQI score is positive but relatively low, pointing to an overconfident message that is not strongly misleading. Table 4 highlights each model's ability to detect truth, ambiguity, and misinformation in complex ICC content.

Table 4. Comparative Summary

| Feature/Model | Binary Logic | Fuzzy Logic | NOFA |
|---|---|---|---|
| Truth evaluation | 1 or 0 | 0.8 | 1.2 |
| Handles overstatement | x | x | √ ( T > 1 ) |
| Recognizes ambiguity | x | x (partial) | √ (I = 0.6) |
| Accounts for misinformation | x | x | √ (F = 0.4) |
| Multidimensional analysis | x | x | √ |

| | | | |
|---|---|---|---|
| Final score/classification | "True" | "Mostly true" | "Overstated, moderate ambiguity" |

**7. Conclusion and Future Work**

This paper introduced NOFA, a new method for analyzing ICC in the context of integrated media. Unlike traditional binary and fuzzy models, NOFA allows the degrees of T, I, and F to move beyond the standard [0,1] interval. This extension enables the model to detect overstatements ($T > 1$), contradictory messages (high T and F together), and ambiguous or vague language (high I), which are common in strategic geopolitical messaging. Through a realistic case study, we showed that NOFA can evaluate nuanced ICC messages more effectively than classical models. It computes meaningful indicators such as the NMQI and offset entropy, providing analysts with detailed assessments of message quality and bias. In contrast, binary and fuzzy models either oversimplify or miss these subtleties entirely.

For future research, NOFA can be extended to track communication trends over time, helping analysts understand how narratives evolve. It can also be implemented in automated systems to monitor ICC in real time using natural language processing. Another promising direction is comparing ICC messages with those from other state-sponsored media to identify cross-cultural patterns in global communication strategies. Integrating NOFA with machine learning models could also improve its scalability and adaptability. Overall, this work opens a new path for the rigorous, nuanced analysis of complex international media discourse.

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