Neutrosophic Sets and Systems, {Special Issue: Artificial Intelligence, Neutrosophy, and Latin American Worldviews: Toward a Sustainable Future (Workshop – March 18–21, 2025, Universidad Tecnológica de El Salvador, San Salvador, El Salvador)}, Vol. 84, 2025

University of New Mexico

Neutrosophic Cognitive Maps to Analyze Barriers and Opportunities in the Development of Critical Thinking through Inquiry Strategies in Public Universities

Betsy Maryetta Quispe Baca [1*], Adán Humberto Estela Estela [2], Vladímir Bendezú Pérez [3], Yessika Madelaine Abarca Arias [4], Manuel Cesar Vildoso Villegas [5], Pierre Sean Rojas Padilla [6], and Juvita Dina Soto Hilario [7]

[1] Universidad Nacional Mayor de San Marcos, Lima, Peru. b72897659@gmail.com
[2] Universidad Nacional Mayor de San Marcos, Lima, Peru. aestelae@unmsm.edu.pe
[3] Universidad Nacional Mayor de San Marcos, Lima, Peru. vbendezup@unmsm.edu.pe
[4] Universidad Nacional de San Agustín de Arequipa, C. San Agustín 106, Arequipa 04001, Peru. yabarca@unsa.edu.pe
[5] Universidad Nacional Jorge Basadre Grohmann, Tacna, Peru. mvildosov@unjbg.edu.pe
[6] Instituto Latinoamericano de Empresas y Negocios (ILEN), Lima, Peru. projas@ilen.edu.pe
[7] Universidad Nacional Hermilio Valdizán, Huánuco, Peru. jsoto@unheval.edu.pe

Abstract. The effective implementation of inquiry strategies in higher education is hampered by multiple interconnected factors, from institutional constraints to pedagogical barriers, creating a complex scenario that traditional analysis methods fail to fully capture. This problem takes on particular relevance in the current context, where the development of critical thinking has become a fundamental competency for facing the challenges of constantly changing societies. While numerous studies on active pedagogies exist, there remains a notable lack of approaches that can adequately manage the ambiguity, contradictions, and uncertainty inherent in real-life educational systems. To address this limitation, this research proposes an innovative analytical framework that combines cognitive maps with neutrosophic logic, allowing for the representation and evaluation of causal relationships through values of truth, falsity, and indeterminacy. Through a rigorous methodological design that includes systematic data collection and the use of specialized algorithms, the study seeks to uncover the hidden dynamics that affect the adoption of these pedagogical strategies. The implications of this work are both theoretical and practical, offering a new perspective for understanding complex educational systems, and providing managers and educators with more sophisticated tools for making informed decisions in contexts of high uncertainty.

Keywords: Inquiry Strategies, Critical Thinking, Higher Education, Neutrosophic Modeling, Cognitive Maps, Pedagogical Uncertainty, Qualitative-Quantitative Analysis

1. Introduction

Critical thinking training in higher education is a fundamental pillar for the development of professionals capable of analyzing, questioning, and transforming complex realities [1]. In a world where information is abundant but often contradictory, this competence has become indispensable, not only for academic success, but also for informed civic participation and ethical decision-making in professional contexts [2]. However, despite its recognized importance, numerous institutions face persistent difficulties in implementing pedagogical strategies that effectively foster this skill, particularly in public universities with limited resources [3]. Efforts to integrate active methodologies, such as inquiry-based learning, have a history dating back to the proposals of Dewey and Freire, but their contemporary application remains irregular [4]. While some universities have successfully adopted these innovations, others struggle with structural barriers—from rigid curricula to resistance to pedagogical change—that perpetuate traditional models focused on memorization [5]. This divergence highlights the need to analyze not only the strategies themselves, but also the systemic factors that facilitate or hinder their implementation.

The core of the problem lies in the multidimensional nature of these obstacles, where pedagogical, institutional, and sociocultural aspects interact in non-linear ways and with varying degrees of uncertainty. Previous studies have identified challenges such as lack of teacher training [3] or unequal access to technologies [6], but few have explored how these elements dynamically interrelate, limiting the effectiveness of isolated solutions. How can public universities diagnose and prioritize interventions when the barriers to implementing inquiry strategies are simultaneously technical, cultural, and contextual? This question gains urgency in light of recent findings showing that, without a systemic approach, even the best-designed pedagogical innovations fail when faced with complex institutional realities [7]. Traditional methods of analysis, based on linear causal relationships or binary data, are insufficient to capture this complexity, leaving educational managers without adequate tools for decision-making.

This is where the innovation of this study emerges: by integrating cognitive maps with neutrosophic logic, a framework is proposed capable of modeling causal relationships under uncertainty, representing not only the presence or absence of factors, but also their degree of influence and the contradictions perceived by different educational actors. This hybrid approach—which combines qualitative and quantitative analysis—allows for the visualization of interactions that conventional models overlook. The objectives of this research are threefold: (1) to develop a neutrosophic model to map the systemic barriers that affect the implementation of inquiry strategies; (2) to identify critical nodes (factors with the greatest influence) in networks of pedagogical relationships; and (3) to propose an intervention protocol adapted to contexts of high uncertainty. These contributions seek not only to enrich the theoretical debate on educational innovation but also to offer institutions practical tools to move toward more effective pedagogies.

2. Related Work.

2.1. Neutrosophic Cognitive Maps.

Neutrosophic Cognitive Maps represent a significant evolution in the field of complex data representation and analysis. This unconventional methodology not only seeks to capture the inherent complexity of human perceptions, but also integrates principles of neutrosophic theory, which deals with truth, falsity, and indeterminacy simultaneously. This innovative approach is especially relevant in contexts where ambiguity and uncertainty are key factors in decision-making and the understanding of complex phenomena [8].

From a conceptual point of view, Neutrosophic Cognitive Maps allow to visualize and structure relationships between concepts that may be ambiguous or contradictory according to different perspectives. This not only broadens the spectrum of analysis by including divergent opinions and perceptions, but also promotes a deeper and more holistic understanding of the problems investigated [9-11]. This ability to handle the inherent vagueness of human reality is crucial in disciplines such as philosophy, psychology, and sociology, where subjective interpretations play a central role in the construction of knowledge. In practical terms, Neutrosophic Cognitive Maps find application in a variety of fields, from scientific research to strategic planning and business decision-making. Their methodological flexibility allows researchers and practitioners to explore and analyze complex and multidimensional data in a structured and comprehensive manner. This methodology not only offers a visual representation of the inherent complexity of the systems and processes studied, but also facilitates the identification of hidden patterns and subtle connections that could be overlooked with more traditional approaches. However, like any emerging methodology, Neutrosophic Cognitive Maps face challenges and criticisms. One of the main concerns lies in the difficulty of quantifying and validating the indeterminacy and vagueness represented in these maps. Objectively assessing the quality and reliability of the data entered into the maps can be challenging, especially when dealing with subjective or qualitative information. Furthermore, the interpretation of the results can vary significantly depending on the theoretical framework and underlying assumptions of those using this methodology [12].

Despite these challenges, Neutrosophic Cognitive Maps offer considerable potential for advancing the understanding and modeling of complex systems in an increasingly interconnected and dynamic world. By integrating principles of neutrosophic theory, these maps not only address reality in all its complexity and ambiguity, but also promote an inclusive and multidimensional approach to research and decision-making. This is especially valuable in contexts where the diversity of opinions and perspectives enriches the analysis process and contributes to more robust and adaptive solutions. In conclusion, Neutrosophic Cognitive Maps represent a powerful and promising tool for researchers and practitioners seeking to navigate the complexity of the contemporary world. Their ability to represent and analyze vagueness and indeterminacy offers new opportunities for understanding and addressing complex problems in fields as diverse as business management, public policy, and social science. As research in this area advances, it is crucial to continue exploring and refining this methodology to maximize its usefulness and accuracy in the information and knowledge age [13].

This section contains the basic concepts of neutrosophic cognitive maps and the algorithms associated with them.

Definition 1: ([14]) Let X be a universe of discourse. A *neutrosophic set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow]^-_r0, 1^+[$ which satisfy the condition $^-_r0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ are the truth, indeterminacy, and falsity membership functions of x in A, respectively, and their images are standard or nonstandard subsets of $]^-_r0, 1^+[$.

Definition 2: ([14]) Let X be a universe of discourse. A *single-valued neutrosophic set* (SVNS) A in X is a set of the form:

$A = \{\langle x, u_A(x), r_A(x), v_A(x) \rangle : x \in X\}$(1)

Where $u_A, r_A, v_A : X \rightarrow [0,1]$, satisfies the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all $x \in X$. $u_A(x), r_A(x)$and $v_A(x)$denotes the truthfulness, indeterminacy, and falsity membership functions of x in A, respectively. For convenience, a *single-valued neutrosophic number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$and satisfy $0 \leq a + b + c \leq 3$.

Other important definitions are related to graphics.

Definition 3: ([15, 17-18]) A *neutrosophic graph* is a graph that contains at least one indeterminate edge, which is represented by dotted lines.

Definition 4: ([15, 17-18]) A *neutrosophic directed graph* is a directed graph that contains at least one indeterminate edge, which is represented by dotted lines.

Definition 5: ([15, 17-18]) A *neutrosophic cognitive map* (NCM) is a neutrosophic directed graph, whose nodes represent concepts and whose edges represent causal relationships between edges.

If $C_1, C_2, \ldots, C_k$ there are k nodes, each of them C_i $(i = 1, 2, \ldots, k)$ can be represented by a vector $(x_1, x_2, \ldots, x_k)$where $x_i \in \{0, 1, I\}$. $x_i = 0$ means that the node $C_i$is in the off state, , $x_i = 1$means that the node $C_i$is in the on state, and , $x_i = I$means that the node $C_i$is in an indeterminate state, at a specific time or in a specific situation.

If $C_m$and $C_n$are two nodes in the NCM, a directed edge from $C_m$to $C_n$is called *a connection* and represents causality from $C_m$to C_n. Each node in the NCM is associated with a weight within the set $\{-1, 0, 1, I\}$. If α_{mn}denotes the edge weight C_mC_n, $\alpha_{mn} \in \{-1, 0, 1, I\}$then we have the following:

$\alpha_{mn} = 0$ if C_m does not affect C_n,

$\alpha_{mn} = 1$if an increase (decrease) in $C_m$produces an increase (decrease) in C_n,

$\alpha_{mn} = -1$if an increase (decrease) in $C_m$produces a decrease (increase) in C_n,

$\alpha_{mn} = I$ if the effect ofC_m ignition $C_n$is indeterminate.

Definition 6: ([19]) An NCM that has edges with weights $\{-1, 0, 1, I\}$is called *a simple neutrosophic cognitive map*.

Definition 7: ([19]) If $C_1, C_2, \dots, C_k$are the nodes of an NCM. The *neutrosophic matrix* N(E) is defined as $N(E) = (\alpha_{mn})$, whereα_{mn} denotes the weight of the directed edge C_mC_n, such that $\alpha_{mn} \in \{-1, 0, 1, I\}$. N(E)is called *neutrosophic adjacency* NCM *matrix.*

Definition 8: ([19]) Let be $C_1, C_2, \dots, C_k$ the nodes of an NCM. Let $A = (a_1, a_2, \dots, a_k)$, where $a_m \in \{-1, 0, 1, I\}$. A is called *the instantaneous state. Neutrosophic vector* and means a position of the on -off-indeterminate state of the node at a given instant.

$a_m = 0$ if $C_m$is disabled (has no effect),

$a_m = 1$if $C_m$is activated (has an effect),

$a_m = I$ if C_m is indeterminate (its effect cannot be determined).

Definition 9: ([19]) Let, , ,…, be $C_1, C_2, \dots, C_k$the nodes of an NCM. Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…,$\overrightarrow{C_mC_n}$ be the edges of the NCM, then the edges constitute a *directed cycle*.

The NCM is called *cyclic* if it has a directed cycle. It is called *acyclic.* if you do not have a directed cycle.

Definition 10: ([19]) An NCM containing cycles is said to have *feedback*. When there is feedback in the NCM it is said to be a *dynamical system*.

Definition 11: ([19]) Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…,$\overrightarrow{C_{k-1}C_k}$ be a cycle. When $C_m$it is activated and its causality flows along the edges of the cycle and is then the cause of $C_m$itself, then the dynamical system circulates. This is true for each node $C_m$with $m = 1, 2, \dots, k$. The equilibrium state of this dynamical system is called the *hidden pattern*.

Definition 12: ([19]) If the equilibrium state of a dynamical system is a unique state, then it is called *a fixed point*.

An example of a fixed point is when a dynamical system starts being triggered by C_l. If the NCM is assumed to sit atC_l and C_k, that is, the state remains as $(1, 0, \dots, 0, 1)$, then this neutrosophic state vector is called a *fixed point*.

Definition 13: ([19]) If the NCM is established with a neutrosophic state vector that repeats in the form:

$A_1 \to A_2 \to \cdots \to A_m \to A_1$, then the equilibrium is called the NCM *limit cycle.*

<u>Method for determining hidden patterns</u>

Let be $C_1, C_2, \dots, C_k$the nodes of the feedback NCM. Let E be the associated adjacency matrix. A hidden pattern is found when is activated and C_la vector input is provided. $A_1 = (1, 0, 0, \dots, 0)$The data must pass through the neutrosophic matrix N(E), which is obtained by multiplyingA_1 by the matrix N(E).

Leave$A_1N(E) = (\alpha_1, \alpha_2, \dots, \alpha_k)$ with the threshold operation of replacing $\alpha_m$by 1 if $\alpha_m > p$andα_m for 0 if $\alpha_m < p$(p is a suitable positive integer) andα_m is replaced by I if it is not an integer. The resulting concept is updated; the vector $C_1$is included in the updated vector by transforming the first coordinate of the resulting vector to 1.

If $A_1N(E) \rightarrow A_2$ It is assumed, then $A_2N(E)$considered, and the same procedure is repeated until a limit cycle or fixed point is reached.

Definition 14 : ([20]) A *neutrosophic number* N is defined as a number as follows:

$N = d + I(2)$

Where d is called *determinate part* and call me the *indeterminate part*.

Given $N_1 = a_1 + b_1 I$and $N_2 = a_2 + b_2 I$are two neutrosophic numbers, some operations between them are defined as follows:

$N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$(Addition) ;

$N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$(Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$(Division).

3. Case Study.

The effective implementation of inquiry strategies in higher education is hampered by multiple interconnected factors, from institutional limitations to pedagogical barriers, creating a complex landscape that traditional analysis methods fail to fully capture. This problem takes on particular relevance in the current context, where the development of critical thinking has become a fundamental competency for addressing the challenges of constantly changing societies.

Neutrosophic Cognitive Maps represent an innovative tool that addresses the inherent complexity of educational systems, where elements of certainty, uncertainty, and indeterminacy coexist. This methodology is especially relevant for analyzing the implementation of pedagogical strategies that seek to develop critical thinking, as it allows us to capture the complex causal relationships between different factors that can act as barriers or opportunities.

Study Variables

For the present study, five critical variables related to the development of critical thinking through inquiry strategies in public universities were identified:

V1 - Teacher Training in Active Pedagogies : This variable encompasses ongoing teacher training in inquiry methodologies, active learning facilitation techniques, and skills to foster critical thinking. It includes both formal training and the development of skills to implement innovative pedagogical strategies.

V2 - Technological Infrastructure and Teaching Resources : Includes the material and technological resources necessary to implement effective inquiry strategies. This includes equipped laboratories, digital platforms, updated libraries, flexible learning spaces, and digital tools that facilitate research and critical analysis.

V3 - Institutional Policies for Educational Innovation : This variable covers the regulations, incentives, and regulatory frameworks that public universities establish to promote pedagogical innovation. It includes policies for teacher evaluation, recognition of educational innovation, curricular flexibility, and institutional support for experimental pedagogical projects.

V4 - Learning-Oriented Organizational Culture : Represents the institutional climate that favors or hinders the implementation of new methodologies. It includes openness to change, interdisciplinary collaboration, peer support, effective communication, and the appreciation of pedagogical experimentation as a factor in institutional growth.

V5 - Student Research Competencies: This variable considers students' prior and developed skills to actively participate in inquiry processes. It includes competencies in information searching, critical analysis, question formulation, basic research design, and synthesis and argumentation skills.

Data Collection

To obtain the weights and create the NCM, 35 specialists in higher education and university pedagogy were surveyed. The group of experts included:

- 12 researchers in university pedagogy with more than 15 years of experience
- 10 academic directors of public universities
- 8 teachers with proven experience in active methodologies
- 5 specialists in critical thinking development

Let R_ijk be $E = \{e_1, e_2, \ldots, e_{35}\}$the set of 35 experts. R_ijk represents the relationship between the jth and kth criteria according $(j, k \in \{1,2,\ldots,5\}, j \neq k)$ to the expert. e_i $(i = 1,2,\ldots,35)$ $tal\ que\ R\_ijk \in \{-5, -4, \ldots, -1,0,1,\ldots,4,5, I\}$.

The numerical values of R $_{ijk}$ are calculated as follows:

- If Rijk is numerical, then $\hat{R}_{ijk} = R_{ijk}$
- If $R_{ijk} = I$, then $\hat{R}_{ijk} = I$ holds

For each fixed pair, $j, k \in \{1,2,\ldots,5\}$,$\bar{R}_{jk}$ is calculated by applying the neutrosophic consensus algorithm:

- $\hat{R}_{ijk}$ mode $para\ i = 1,2,\ldots,35$is unimodal, it is taken$Rjk = modei(Rijk)\ y\ Rkj = 0$
- If the mode is not unimodal , $\hat{R}_{ikj}$ is evaluated and the same criterion is applied
- If both directions present multimodality , it is assigned$Rjk = Rkj = I$

Adjacency Matrix

The Adjacency Matrix obtained from the consensus of the 35 experts is presented in Table 1:

Table 1: Adjacency matrix for the analysis of barriers and opportunities in the development of critical thinking through inquiry strategies according to 35 experts.

Variable	V1	V2	V3	V4	V5
V1	0	1	I	1	1
V2	I	0	0	I	1
V3	1	1	0	1	I
V4	I	1	I	0	1
V5	0	I	-1	I	0

Neutrosophic Cognitive Map

The updated NCM based on the adjacency matrix is as follows:
Determined Positive Relationships:

V1 → V2: Teacher training positively influences the effective use of technological resources.

V1 → V4: Teacher training strengthens the organizational learning culture.
V1 → V5: Trained teachers better develop student research skills.
V2 → V5: Adequate resources facilitate the development of research skills.
V3 → V1: Institutional policies promote teacher training.
V3 → V2: Policies facilitate the acquisition of resources.
V3 → V4: Policies shape organizational culture.
V4 → V2: A positive culture optimizes the use of resources.
V4 → V5: The organizational environment favors student learning.

Negative Relationships:

V5 → V3: Deficiencies in student competencies can generate resistance to innovative policies.

Indeterminate Relationships:

V1 → V3: The influence of teacher training on institutional policies is undetermined.
V2 → V1: The impact of technological resources on teacher training is uncertain.
V2 → V4: The relationship between technological resources and organizational culture is ambiguous.
V4 → V1: The influence of organizational culture on teacher training is undetermined.
V4 → V3: The interaction between organizational culture and institutional policies shows uncertainty.
V5 → V2: The impact of student competencies on technological resources is ambiguous.
V5 → V4: The relationship between student competencies and organizational culture is complex.

Null Relationships:

V2 → V3: Technological resources do not directly influence institutional policies.
V5 → V1: Student competencies do not directly influence teacher training .

Convergence Analysis

All possible cases of convergence were analyzed when at least one variable was activated, totaling $2^5 - 1 = 31$the cases. Table 2 presents the results:

Table 2. Absolute and relative frequency of convergence of the system for each possible state.

Variable	State 0	%	State 1	%	State I	%
V1: Teacher training	7	22.58%	14	45.16%	10	32.26%
V2: Technological resources	10	32.26%	14	45.16%	7	22.58%
V3: Institutional policies	9	29.03%	12	38.71%	10	32.26%
V4: Organizational culture	6	19.35%	12	38.71%	13	41.94%
V5: Student research skills	9	29.03%	12	38.71%	10	32.26%

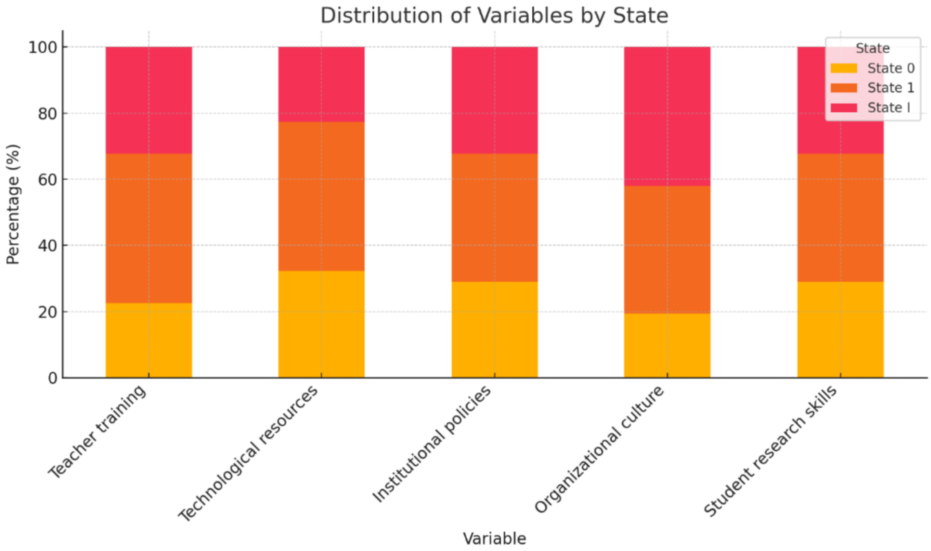

Figure 1: Distribution of educational variables across neutrosophic states (0, 1, I).

Interpretation of Results

The updated results highlight the complexity inherent in developing critical thinking through inquiry strategies in public universities:

- **V3 - Institutional Policies**: This variable is a key factor, activating in 38.71% of the initial conditions (State 1). Its positive influence on teacher training (V3 → V1: 1), technological resources (V3 → V2: 1), and organizational culture (V3 → V4: 1) confirms its central role as a system articulator. However, its considerable indeterminacy (32.26%) suggests that its effectiveness depends on contextual conditions.
- **V1 - Teacher Training**: This variable shows significant activation (45.16% in State 1), one of the highest in the system. This indicates that teacher training has a defined impact, positively influencing technological resources (V1 → V2: 1), organizational culture (V1 → V4: 1), and student competencies (V1 → V5: 1). Nevertheless, its indeterminacy (32.26%) reflects the need for specific conditions to maximize its effectiveness.
- **V5 - Student Research Competencies**: This variable exhibits a balanced behavior, with 29.03% in State 0, 38.71% in State 1, and 32.26% in State I (Indeterminate). This suggests that student competencies are an active and dynamic challenge. They are positively influenced by teacher training (V1), resources (V2), and culture (V4), but in turn, show a negative relationship toward policies (V5 → V3: -1), which could indicate potential student resistance to innovative policies.
- **V4 - Organizational Culture**: This variable exhibits the highest indeterminacy of all variables (41.94%), which underscores the complexity of cultural changes. Despite this uncertainty, it also shows significant activation (38.71% in State 1). Its direct positive influence on resources (V4 → V2: 1) and student competencies (V4 → V5: 1) confirms that, although it is a complex process, it can have a defined impact on the system.
- **V2 - Technological Resources**: This variable shows moderately indeterminate behavior (22.58%) and a high activation rate (45.16% in State 1). Its positive influence on student competencies (V2 → V5: 1) is direct. Its dependence on teacher training (V1), policies (V3), and culture (V4) indicates that, while resources are essential, their effective impact requires systemic support.

Optimal Configurations

The updated analysis identifies the following most efficient configurations for activating the system, based on positive relationships and activation frequencies:

- **V3 + V1 activated (x_0 = (1 ,0,1,0,0))**: The combination of strong institutional policies and effective teacher training remains a key configuration, supported by the V3 → V1: 1 relationships and the high activation of both variables (38.71% and 45.16% in State 1).
- **V3 + V4 activated (x_0 = (0 ,0,1,1,0))**: Integrating institutional policies with the development of organizational culture, supported by V3 → V4: 1 and V4 → V2, V5: 1, remains effective, with balanced activation frequencies.
- **V3 + V1 + V4 activated (x_0 = (1 ,0,1,1,0))** : This configuration, which combines policies, training and culture, remains the most complete, maximizing system activation through positive interactions between V3, V1 and V4.

Analysis of Relationships between Variables
Patterns of Interdependence

The updated neutrosophic analysis reveals more balanced patterns of interdependence:

- **Centrality of Institutional Policies (V3)**: V3 remains the key driver, with positive relationships with V1, V2, and V4 (all with a value of 1). Its activation (38.71%) and lower uncertainty (32.26%) reinforce its role as the main lever, although the negative relationship between V5 and V3 (-1) suggests that student competencies may limit its impact.
- **Teacher Training Cascade Effect (V1)**: V1, with high activation (45.16%), generates positive effects on V2, V4, and V5 (all with a value of 1). Its lower uncertainty (32.26%) indicates that training is more effective when supported by policies and resources.
- **Resource Paradox (V2)**: With 45.16% activation and 22.58% indeterminacy, V2 confirms that technological resources are necessary but insufficient without systemic support. Its positive relationship with V5 (V2 → V5: 1) highlights its importance for student competencies.
- **Cultural Complexity (V4)**: The high indeterminacy of V4 (41.94%) and its indeterminate bidirectional relationships (V4 ↔ V1, V4 ↔ V3) confirm the difficulty of transforming organizational culture, although its activation (38.71%) and positive relationships towards V2 and V5 position it as a key factor.
- **Student Challenge (V5)**: V5, with 38.71% activation and 32.26% indeterminacy, shows a more active dynamic than in the previous analysis. Its negative relationship with V3 (V5 → V3: -1) and indeterminate relationships with V2 and V4 highlight the complexity of developing research competencies.

Feedback Dynamics
Updated NCM reveals adjusted feedback loops:

- **Virtuous Loop**: V3 → V1 → V5 → (development of critical thinking) → reinforcement of V3. This loop is still key, but the negative relationship V5 → V3 (-1) introduces a potential resistance that needs to be addressed.

- **Indeterminacy Loop**: V4 ↔ V1, V4 ↔ V3, V1 ↔ V3. The indeterminacy in these relationships (I) persists, indicating that the interaction between culture, training and policies requires specific conditions to converge.
- **Dependency Loop**: V2 → V5 → V3 → V2. This loop continues, with V2 and V5 showing higher activation (45.16% and 38.71%), reinforcing the interdependence between resources, competencies, and policies.

Recommendations
Implementation Strategies

Based on the updated results, the following adjusted strategies are proposed:

1. **Prioritize the Development of Institutional Policies**:
   - Establish clear regulatory frameworks that encourage pedagogical innovation, considering the potential resistance of student competencies (V5 → V3: -1).
   - Create evaluation and recognition systems that value active methodologies.
   - Develop curricular flexibility policies to encourage experimentation.
2. **Implement Contextualized Teacher Training Programs**:
   - Design specific training in inquiry strategies, taking advantage of the high activation of V1 (45.16%).
   - Create communities of practice to reduce uncertainty (32.26%) by sharing experiences.
   - Establish mentoring between experienced and novice teachers.
3. **Strategically Manage Resources**:
   - Optimize the use of existing infrastructure, given the 45.16% activation of V2.
   - Prioritize resources that impact V5 (student competencies), such as collaborative learning technologies.
   - Develop internal capabilities to maximize technological leverage.
4. **Promote Gradual Cultural Changes**:
   - Start with groups of motivated teachers to create nuclei of change, considering the high indeterminacy of V4 (41.94%).
   - Celebrate and disseminate successful experiences to reinforce V4 → V2, V5.
   - Provide visible institutional support for innovative initiatives.
5. **Develop Student Competencies Systematically**:
   - Integrate research skills into the curriculum, taking advantage of V1 → V5, V2 → V5, V4 → V5 (all with a value of 1).
   - Create extracurricular spaces for the practice of inquiry.
   - Establish mentoring systems to mitigate policy resistance (V5 → V3: -1).

Adapted Intervention Protocol

- **Phase 1 - Neutrosophic Diagnosis (3-6 months)** :
  - Evaluate the current state of each variable using neutrosophic scales, with emphasis on the activation of V1 (45.16%) and V2 (45.16%).
  - Identify patterns of indeterminacy, especially in V4 (41.94%) and V3 (32.26%).
  - Map stakeholders and resistances, considering the negative relationship V5 → V3.
- **Phase 2 - Targeted Intervention (6-12 months)** :
  - Implement changes in V3 (policies) as the main lever, supporting V1 and V4.
  - Develop capabilities in V1 (training) to maximize their impact in V2, V4 and V5.
  - Monitor for indeterminacy in V4 ↔ V1, V4 ↔ V3, V1 ↔ V3.

- **Phase 3 - Systemic Consolidation (12-24 months)** :
    - Integrate all variables into a coherent strategy, prioritizing optimal configurations (V3 + V1, V3 + V4, V3 + V1 + V4).
    - Establish continuous feedback mechanisms to mitigate V5 resistance.
    - Evaluate long-term impacts on the development of critical thinking.

4. Conclusions

Neutrosophic Cognitive Maps have proven to be an essential tool for understanding the complexity inherent in developing critical thinking through inquiry strategies in public universities. The updated analysis, based on convergence relationships and frequencies, reveals that the success of these initiatives depends on the strategic articulation of multiple interconnected factors, with institutional policies (V3) emerging as a key element, albeit with slightly lower activation (38.71%) than in previous analyses. The indeterminacy observed in variables such as organizational culture (V4, 41.94%) and, to a lesser extent, teacher training (V1, 32.26%) underscores the importance of adopting adaptive approaches that address the uncertainty inherent in educational systems. This indeterminacy, far from being a limitation, is a fundamental characteristic that requires specific management strategies, especially for V4, which displays indeterminate bidirectional relationships with V1 and V3. The results confirm that there is no single solution for implementing effective inquiry strategies. The high activation of teacher training (V1, 45.16%) and technological resources (V2, 45.16%), together with the positive influence of V3 on V1, V2, and V4, highlight the need for a systemic approach that integrates institutional , pedagogical, and student factors. The neutrosophic methodology has made it possible to capture these complex dynamics, including the negative relationship of student competencies (V5) with institutional policies (V5 → V3: -1), which suggests potential resistances that need to be addressed.

The identified optimal configurations— $V3 + V1\ (x_0 = (1,0,1,0,0)), V3 + V4\ (x_0 = (0,0,1,1,0))$and V $3 + V1 + V4\ (x_0 = (1,0,1,1,0))$—provide clear pathways for action, leveraging the positive relationships between these variables. The adapted intervention protocol offers a practical framework for implementing these strategies, with a focus on mitigating indeterminacy and resistance, such as that observed in V5. This research demonstrates that developing critical thinking through inquiry strategies is an achievable goal, but it requires a sophisticated approach that recognizes and constructively engages with the complexity, indeterminacy, and interdependence that characterize contemporary education systems. The contributions of neutrosophic analysis not only enrich the theoretical debate on educational innovation but also provide practical tools for administrators and educators seeking to transform their pedagogical practices.

5. References

[1] Facione, P. A. (2020). Critical thinking: What it is and why it counts. Measured Reasons LLC. https://doi.org/10.13140/RG.2.2.19199.38561

[2] Barnett, R. (2018). The ecological university: A feasible utopia. Routledge. https://doi.org/10.4324/9781315146063

[3] Cangalaya Sevillano, L. M. (2020). Critical thinking skills in university students. Desde el Sur, 12(1), 141-153. https://doi.org/10.21142/DES-1201-2020-0010

[4] Lagemann, M. J. (2000). An elusive science: The troubling history of educational research. University of Chicago Press. https://doi.org/10.7208/chicago/9780226467766.001.0001

[5] Freeman, S., et al. (2014). Active learning increases student performance in science, engineering, and mathematics. Proceedings of the National Academy of Sciences, 111(23), 8410-8415. https://doi.org/10.1073/pnas.1319030111

[6] Selwyn, N. (2017). Education and technology: Key issues and debates. Bloomsbury. https://doi.org/10.5040/9781474235921

[7] Kezar, A. (2018). How colleges change: Understanding, leading, and enacting change. Routledge. https://doi.org/10.4324/9781315104346

[8] Vasantha, W. B., et al. (2023). NCMPy: A modeling software for neutrosophic cognitive maps based on Python package. Neutrosophic Sets and Systems, 13, 1-22.

[9] Lema Atiencie, A. E., Moscoso Abad, E., Quito, E., & Calle, D. (2024). Análisis y determinación de las principales causas de las infecciones y enfermedades periodontales mediante mapas cognitivos neutrosóficos (NCM). Neutrosophic Computing and Machine Learning, 33, 28-40. https://doi.org/10.5281/zenodo.13800076

[10] Villamar, C. M., Suarez, J., Coloma, L. D. L., Vera, C., & Leyva, M. (2019). Analysis of Technological Innovation Contribution to Gross Domestic Product Based on Neutrosophic Cognitive Maps and Neutrosophic Numbers. Neutrosophic Sets and Systems, 34.

[11] Bakhtavar, E., et al. (2021). Fuzzy cognitive maps in system risk analysis: A comprehensive review. Complex & Intelligent Systems, 7, 621-637.

[12] Peer, M., et al. (2021). Structuring knowledge with cognitive maps and cognitive graphs. Trends in Cognitive Sciences, 25, 37-54.

[13] Castillo, D. T. P., et al. (2022). Analyzing the impact of social factors on homelessness with neutrosophic cognitive maps. International Journal of Neutrosophic Science, 19(1), 260-266.

[14] Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2010). Single valued neutrosophic sets. Review of the Air Force Academy, 17(1).

[15] Mahapatra, R., et al. (2020). Link prediction in social networks using neutrosophic graph. International Journal of Computational Intelligence Systems, 13, 1699-1713.

[16] Francisco Infante Estrabao, Maikel Leyva Vázquez, Saimelyn Aileen Forteza Rojas and Rodolfo González Ortega. *Mapas cognitivos neutrosóficos para el análisis de la vulnerabilidad socioeconómica*, Neutrosophic Computing and Machine Learning, Vol. 15, pp. 11–16, 2021. DOI: https://zenodo.org/record/4455605

[17] Jácome-Mogro, E. J., Morales, P., Jiménez-Jácome, C., & Abidova, D. (2025). Development of Neutrosophic Cognitive Maps (NCM) for the Evaluation and Ranking of the Main Causes of the Appearance of Fruit Fly Pest. International Journal of Neutrosophic Science (IJNS), 25(1).

[18] Alsubhi, S. H., et al. (2021). Triangular neutrosophic cognitive map for multi-stage sequential decision-making problems. International Journal of Fuzzy Systems, 23, 657-679.

[19] Paraskevas, A., & Madas, M. (2025). Neutrosophic cognitive maps: Theoretical and mathematical formulations, literature review, and applications. Soft Computing and Machine Learning, 27-58.

[20] Ye, J. (2018). Neutrosophic number linear programming method and its application under neutrosophic number environments. Soft Computing, 22, 4639-4646.

Received: December 30, 2024. Accepted: April 13, 2025.