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A Dual Neutrosophic Framework for Evaluating Innovation Ecosystem Quality in Digital Economy Driven Specialized and Innovative SMEs: Integrating Resistance Mapping and Probabilistic Predictive Reasoning

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ABSTRACT: The quality of an innovative ecosystem is pivotal to the survival and advancement of specialized and innovative small and medium-sized enterprises (SMEs) in the digital economy. Existing analytical models often overlook the dual complexity of ecosystem barriers both visible and uncertain and the need for adaptive forecasting in dynamic environments. This paper introduces a novel dual-model framework based on Neutrosophic Logic: the Neutrosophic Resistance Map (NRM) and the Probabilistic Predictive Neutrosophic Cognitive Map (PP-NCM). The NRM captures direct and indeterminate resistance factors that inhibit innovation across various layers of the ecosystem. It extends causal analysis by incorporating neutrosophic weights to model truth (T), indeterminacy (I), and falsity (F) of resistance. Simultaneously, the PP-NCM enables predictive reasoning under uncertainty, dynamically adjusting the expected innovation performance of SMEs based on evolving contextual inputs. Through a detailed mathematical formalization and a comprehensive case study involving digital manufacturing SMEs, the paper demonstrates how this hybrid framework identifies critical constraints, quantifies future innovation outcomes, and provides actionable intelligence for policy and strategy.

KEYWORDS: Innovation Ecosystem Quality; Neutrosophic Cognitive Maps; Neutrosophic Resistance Map; PP-NCM; Digital SMEs; Indeterminacy; Fuzzy Prediction; Uncertainty Modeling; Specialized and innovative Enterprises; Digital Economy

1. Introduction

The rise of the digital economy has transformed innovation into a cornerstone of survival and growth for SMEs operating in digitally driven sectors. These firms rely on innovative ecosystems complex networks of institutions, infrastructure, regulations, cultural norms, and technological advancements to navigate competitive landscapes and sustain their market positions [1]. However, the quality of these ecosystems is often uneven, marked by asymmetries, latent resistances, and structural ambiguities that vary in their visibility and impact [2]. In emerging economies, these challenges are intensified by fluid, partially observable forces, such as inconsistent regulations and limited access to digital infrastructure, which create significant hurdles for SME innovation [3, 4].

Conventional approaches to assessing innovation ecosystems, including linear causal models, qualitative interviews, and macroeconomic indicators, frequently fall short in addressing three critical challenges faced by digitally-driven SMEs: (1) the presence of resistance factors with uncertain or partially observable effects, (2) the dynamic, time-dependent evolution of ecosystem conditions, and (3) the demand for strategic decision-making under multi-dimensional uncertainty [5, 6]. These limitations underscore the need for a sophisticated analytical framework that can model both the resistive forces within ecosystems and their predictive evolution in a computationally rigorous manner.

To bridge this gap, this study proposes a dual layer neutrosophic analytical framework tailored for evaluating the quality of innovation ecosystems in digitally driven SMEs. The framework integrates two innovative models: (1) the NRM, which systematically quantifies resistive forces within ecosystems, and (2) the PP-NCM, which enables the modeling and prediction of innovation outcomes under uncertainty [7, 8]. Rooted in neutrosophic logic, pioneered by Smarandache, this framework leverages the ability to simultaneously account for T, F, and I, making it exceptionally suited to capture the partial truths, hidden resistances, and cognitive ambiguities prevalent in innovation ecosystems [9, 10].

The theoretical underpinnings of the framework draw on advancements in fuzzy and neutrosophic cognitive mapping, which extend beyond traditional fuzzy set theory to address indeterminacy in complex systems [11, 12]. Neutrosophic cognitive maps offer unparalleled flexibility in modeling dynamic relationships and conflicting information, providing robust decision-support tools for strategic planning and project management in SMEs [8, 13]. The framework also resonates with recent global insights into digital innovation, highlighting systemic barriers such as regulatory complexities and technological access gaps, as well as the transformative potential of well-functioning ecosystems [4, 14, 15, 16].

Designed to be both computational and practical, the proposed framework is integrable into digital strategy tools for SMEs, offering a formal structure for resistance evaluation and predictive scenario modeling. It equips decision-makers with actionable insights into the factors constraining innovation and the potential trajectories of ecosystem development under uncertainty [6]. The framework's rigor is demonstrated through matrix-based computations, scenario predictions, and ecosystem quality scoring, ensuring precision and applicability.

In the sections that follow, we present the formal definitions and foundational theorems of neutrosophic mathematics that underpin the NRM and PP-NCM models. We then detail the structure and logic of these models, followed by a comprehensive case study involving SMEs in the digital manufacturing sector. The case study includes matrix computations, scenario predictions, and ecosystem quality scoring to illustrate the framework's practical utility in enhancing innovation outcomes for digitally driven SMEs.

2. Definitions, Theorems, and Proofs

This section introduces the foundational mathematical framework underlying the dual-model structure. It extends classical cognitive mapping into the neutrosophic domain, enabling the modeling of uncertain, ambiguous, or contradictory relationships.

2.1 Basic Neutrosophic Concepts

Definition 1 (Neutrosophic Triplet):
Let a relation between two concepts C_i and C_j be denoted as a neutrosophic value:

$$N_{ij} = \left(T_{ij}, I_{ij}, F_{ij}\right) \text{ where } T_{ij}, I_{ij}, F_{ij} \in [0,1], \text{ and } T_{ij} + I_{ij} + F_{ij} \leq 1$$

T_{ij} is the degree of truth (positive influence),

I_{ij} is the degree of indeterminacy,

F_{ij} is the degree of falsity (negative influence).

Example 1:
If a digital policy moderately supports innovation but has ambiguous implementation, we may express:

$$N_{\text{policy} \rightarrow \text{innovation}} = (0.6, 0.3, 0.1)$$

2.2 Neutrosophic Resistance Map (NRM)

Definition 2 (Neutrosophic Resistance Map):
Let $\mathcal{C} = \{C_1, C_2, \dots, C_n\}$ be the set of ecosystem components (e.g., regulations, funding, skills), and $\mathcal{R} = \{R_1, \dots, R_m\}$ be resistance nodes (e.g., bureaucracy, outdated mindset, digital illiteracy). A Neutrosophic Resistance Map is a directed bipartite graph:

$$\text{NRM} = (\mathcal{C} \cup \mathcal{R}, E)$$

where $E \subseteq \mathcal{C} \times \mathcal{R}$ and each edge e_{ij} has a neutrosophic weight:

$$e_{ij} = \left(T_{ij}, I_{ij}, F_{ij}\right)$$

Definition 3 (Resistance Impact Vector):
For a given concept C_k, its total resistance impact is computed as:

$$RIV(C_k) = \sum_{j=1}^{m} \left(w_j \cdot \left(T_{kj} - F_{kj}\right) \cdot \left(1 - I_{kj}\right)\right)$$

where w_j is the importance of weight of resistance node R_j.

Theorem 1:
If $\mathrm{RIV}(C_k) < 0$, then resistance exceeds support, and C_k is a barrier-prone node.
Proof:
Let the net contribution of each resistance node be:

$$RC_{kj} = (T_{kj} - F_{kj})(1 - I_{kj})$$

If $T_{kj} > F_{kj}$, resistance node R_j has a net supporting effect.

If $F_{kj} > T_{kj}$, resistance dominates.

Thus, summing over all R_j, and adjusting by indeterminacy, gives the total net resistance. If the aggregate $RIV(C_k) < 0$, then the net pressure on C_k is negative.

2.3 Probabilistic Predictive Neutrosophic Cognitive Map (PP-NCM)
PP-NCM extends NCM by integrating prediction through probabilistic modeling of neutrosophic values over time.

Definition 4 (PP-NCM Transition Matrix):
Let $\mathcal{C} = \{C_1, C_2, \dots, C_n\}$ be a set of innovation-related variables (e.g., digital capacity, R&D activity). The PP-NCM matrix is:

$$E = [e_{ij}], e_{ij} = (T_{ij}(t), I_{ij}(t), F_{ij}(t))$$

Each e_{ij} evolves over discrete time $t \in \mathbb{N}$ based on:

$$T_{ij}(t+1) = T_{ij}(t) + \alpha \cdot \Delta T_{ij}, \Delta T_{ij} \sim \mathcal{N}(\mu, \sigma^2)$$

(similarly, for I_{ij}, F_{ij})
This captures probabilistic fluctuation in relationship strength under dynamic conditions.

Definition 5 (Neutrosophic State Vector):
Let the system's state at time t be:

$$S(t) = [(T_1(t), I_1(t), F_1(t)), \dots, (T_n(t), I_n(t), F_n(t))]$$

Each node is activated using a threshold rule:

$$\mathrm{Activate}(C_k) = \begin{cases} 1 & \text{if } T_k - F_k > \delta \text{ and } I_k < \epsilon \\ 0 & \text{otherwise} \end{cases}$$

where δ and ϵ are application-specific thresholds.

Theorem 2:
Given a PP-NCM with bounded $\Delta T, \Delta F, \Delta I$, the state vector $S(t)$ converges to a fixed point or enters a bounded limit cycle.

2.4 Example (Simplified Ecosystem)
Let:

C_1 = "Digital Skill Availability"

R_1 = "Resistance to Tech Adoption"

Assume:

$$e_{C_1,R_1} = (0.2,0.5,0.3), w_1 = 0.9$$

Then:

$$RC_{C_1,R_1} = (0.2 - 0.3)(1 - 0.5) = (-0.1)(0.5) = -0.05$$
$$RIV(C_1) = -0.05 \times 0.9 = -0.045 < 0 \Rightarrow C_1 \text{ is at risk}$$

Now let's define a PP-NCM edge:

$$e_{12}(t) = (0.6,0.2,0.2)$$
$$T_{12}(t+1) = T_{12}(t) + \alpha \cdot \mathcal{N}(0.05,0.01)$$

where $\alpha = 0.1$, we simulate future influence between two innovation nodes under uncertainty.

3. Proposed Model
This section presents the integrated dual-framework comprising the Neutrosophic Resistance Map (NRM) and the Probabilistic Predictive Neutrosophic Cognitive Map (PP-NCM). The goal is to evaluate and forecast the quality of innovation ecosystems supporting specialized and innovative and innovative SMEs operating in the digital economy.

The two models are complementary:
1. The NRM diagnoses latent resistances.
2. The PP-NCM forecasts innovation dynamics under uncertainty.

Together, they form a complete, data-aware, and uncertainty-tolerant decision support system for ecosystem stakeholders, policymakers, and SME strategists.

3.1 Architecture Overview

Let:

$\mathcal{C} = \{C_1, C_2, \dots, C_n\}$: ecosystem components (factors like finance, skills, infrastructure)

$\mathcal{R} = \{R_1, R_2, \dots, R_m\}$: resistance factors (hidden obstacles)

$\mathcal{S}(t)$: neutrosophic state vector at time t

E_R : resistance matrix (from NRM)

$E_P(t)$: evolving cognitive matrix (PP-NCM)

We define two primary functions:

1. Resistance Evaluation:

$$\operatorname{Res}(C_i) = \sum_{j=1}^{m} w_j \cdot (T_{ij} - F_{ij}) \cdot (1 - I_{ij})$$

where:
T_{ij}, I_{ij}, F_{ij} from E_R

w_j is the impact weight of R_j

2. Predictive Evolution:

$$E_P(t+1) = E_P(t) + \Gamma_t, \Gamma_t \sim \mathcal{N}(\mu_{ij}, \sigma_{ij}^2)$$

Each edge evolves stochastically with uncertainty.

3.2 Construction of the NRM Layer
Step 1: Define Ecosystem Concepts $\mathcal{C}$
Let's define:
C_1 : Digital Infrastructure Availability

C_2 : Regulatory Support for Innovation

C_3 : Access to Skilled Talent

C_4 : R&D Investment Levels

C_5 : Innovation Output Index

Step 2: Define Resistance Factors $\mathcal{R}$
Let's define:
R_1 : Bureaucratic Rigidity

R_2 : Cultural Aversion to Risk

R_3 : Digital Skills Gap

R_4 : Inconsistent Policy Execution

Step 3: Construct Neutrosophic Resistance Matrix E_R
Let:

$$E_R = \begin{bmatrix} (T_{11}, I_{11}, F_{11}) & \cdots & (T_{1m}, I_{1m}, F_{1m}) \\ \vdots & \ddots & \vdots \\ (T_{n1}, I_{n1}, F_{n1}) & \cdots & (T_{nm}, I_{nm}, F_{nm}) \end{bmatrix}$$

Each element represents the neutrosophic resistance influence of R_j on C_i.
Step 4: Compute Resistance Score for Each Node

$$\mathrm{Res}(C_i) = \sum_{j=1}^{m} w_j \cdot \left[T_{ij}\left(1 - I_{ij}\right) - F_{ij}\left(1 - I_{ij}\right)\right]$$

This equation:
It gives net resistance effect.

Penalizes ambiguity (indeterminacy).

Weights each resistance by its criticality.

If $\mathrm{Res}(C_i) < 0$: Node C_i is hindered

If $\mathrm{Res}(C_i) > 0$: Node C_i is enabled

3.3 Construction of the PP-NCM Layer

Step 1: Define Dynamic Neutrosophic Cognitive Matrix $E_P(t)$
Let:

$$E_P(t) = \left[\left(T_{ij}(t), I_{ij}(t), F_{ij}(t)\right)\right]_{n\times n}$$

Each edge evolves based on stochastic differentials:

$T_{ij}(t+1) = T_{ij}(t) + \alpha \cdot \epsilon_{ij}^{(T)}, \epsilon_{ij}^{(T)} \sim \mathcal{N}(\mu_T, \sigma_T^2)$ (similarly, for I_{ij}, F_{ij})

Where:
α : learning rate

μ, σ : estimated from historical or expert data

Step 2: System Activation Function
Let $\mathrm{S}(t) = [(T_i(t), I_i(t), F_i(t))]$ be the state vector.
Each node updates as:

$$(T_i(t+1), I_i(t+1), F_i(t+1)) = \sigma\left(\sum_{j=1}^{n} \left(T_{ji}(t), I_{ji}(t), F_{ji}(t)\right) \cdot S_j(t)\right)$$

where σ is a neutrosophic activation function:

$$\sigma(T, I, F) = (\min(1, T), \min(1, I), \min(1, F))$$

Step 3: Ecosystem Innovation Quality Score (EIQS)
Define:

$$\mathrm{EIQS}(t) = \frac{1}{n}\sum_{i=1}^{n} [T_i(t) - F_i(t)] \cdot (1 - I_i(t))$$

This index:

Captures the average net activation of innovation drivers

Penalizes indeterminacy

Provides a dynamic score from [-1, 1]

3.4 Interaction Between NRM and PP-NCM

NRM feeds into PP-NCM by penalizing edge strength:

Let:

$$\tilde{T}_{ij}(t) = T_{ij}(t) \cdot \left(1 + \mathrm{Res}\left(C_j\right)\right)$$

This means:

If resistance is high ($\mathrm{Res}\left(C_j\right) < 0$), the effective influence T_{ij} is reduced.

If support is high, the edge is strengthened.

3.5 Summary of the Model Workflow

1. Input: Expert scores, observed indicators → initialize E_R and $E_P(0)$
2. Stage 1: Compute resistance scores $\mathrm{Res}(C_i)$
3. Stage 2: Update edge weights in PP-NCM via stochastic learning
4. Stage 3: Compute state vector updates across time
5. Stage 4: Generate ecosystem quality index $\mathrm{EIQS}(t)$
6. Output: Resistance map, innovation trajectory, predictive warnings

4. Case Study: Digital Manufacturing SMEs

Objective:

To apply the proposed model on a digital manufacturing SME context and evaluate:

1. Which innovation ecosystem components are most hindered by resistance.
2. The predicted performance of the innovation system using a probabilistic neutrosophic forecast.

4.1 Application of the NRM

Step 1: Define Ecosystem Components $\left(C_i\right)$

We consider 5 components:

C_1 : Digital Infrastructure

C_2 : Regulatory Support

C_3 : Skilled Talent

C_4 : R&D Investment

C_5 : Innovation Output

Step 2: Define Resistance Factors (R_j)
We model 4 resistive forces:
R_1 : Bureaucratic Rigidity

R_2 : Cultural Aversion to Risk

R_3 : Digital Skills Gap

R_4 : Inconsistent Policy

Step 3: Construct Neutrosophic Resistance Matrix E_R
Each entry e_{ij} in the matrix is a neutrosophic triplet $\left(T_{ij}, I_{ij}, F_{ij}\right)$, meaning:
T_{ij} : support level (truth),

F_{ij} : obstruction level (falsity).

Here is the matrix:

	R1: Bureaucracy	R2: Culture	R3: Skill Gap	R4: Policy
C1	(0.2, 0.2, 0.6)	(0.3, 0.4, 0.3)	(0.5, 0.3, 0.2)	(0.4, 0.2, 0.4)
C2	(0.3, 0.3, 0.4)	(0.2, 0.4, 0.4)	(0.4, 0.4, 0.2)	(0.5, 0.3, 0.2)
C3	(0.5, 0.2, 0.3)	(0.4, 0.3, 0.3)	(0.3, 0.3, 0.4)	(0.4, 0.3, 0.3)
C4	(0.6, 0.2, 0.2)	(0.3, 0.3, 0.4)	(0.2, 0.3, 0.5)	(0.5, 0.2, 0.3)
C5	(0.4, 0.3, 0.3)	(0.5, 0.2, 0.3)	(0.6, 0.2, 0.2)	(0.3, 0.4, 0.3)

Step 4: Assign Weights to Each Resistance Node R_j
Based on domain expert assessment:
$w_1 = 0.9$

$w_2 = 0.8$

$w_3 = 0.85$

$w_4 = 0.95$

Step 5: Apply the Resistance Score Formula
The resistance impact score for each ecosystem node is computed using:

$$\operatorname{Res}(C_i) = \sum_{j=1}^{m} w_j \cdot \left(T_{ij} - F_{ij}\right) \cdot \left(1 - I_{ij}\right)$$

Let's compute it for each C_i :

Res(C1):
R1: $(0.2 - 0.6)(1 - 0.2) \cdot 0.9 = (-0.4)(0.8) \cdot 0.9 = -0.288$

R2: $(0.3 - 0.3)(1 - 0.4) \cdot 0.8 = (0.0)(0.6) \cdot 0.8 = 0$

R3: $(0.5 - 0.2)(1 - 0.3) \cdot 0.85 = 0.3 \cdot 0.7 \cdot 0.85 = 0.1785$

R4: $(0.4 - 0.4)(1 - 0.2) \cdot 0.95 = 0 \cdot 0.8 \cdot 0.95 = 0$

$\text{Total} = -0.288 + 0 + 0.1785 + 0 = -0.1095$

Res(C2):
R1: $(0.3 - 0.4)(1 - 0.3) \cdot 0.9 = (-0.1)(0.7) \cdot 0.9 = -0.063$

R2: $(0.2 - 0.4)(1 - 0.4) \cdot 0.8 = (-0.2)(0.6) \cdot 0.8 = -0.096$

R3: $(0.4 - 0.2)(1 - 0.4) \cdot 0.85 = 0.2 \cdot 0.6 \cdot 0.85 = 0.102$

R4: $(0.5 - 0.2)(1 - 0.3) \cdot 0.95 = 0.3 \cdot 0.7 \cdot\downarrow 5 = 0.1995$

$\text{Total} = -0.063 - 0.096 + 0.102 + 0.1995 = 0.1425$

Res(C3):
R1: $(0.5 - 0.3)(1 - 0.2) \cdot 0.9 = 0.2 \cdot 0.8 \cdot 0.9 = 0.144$

R2: $(0.4 - 0.3)(1 - 0.3) \cdot 0.8 = 0.1 \cdot 0.7 \cdot 0.8 = 0.056$

R3: $(0.3 - 0.4)(1 - 0.3) \cdot 0.85 = (-0.1) \cdot 0.7 \cdot 0.85 = -0.0595$

R4: $(0.4 - 0.3)(1 - 0.3) \cdot 0.95 = 0.1 \cdot 0.7 \cdot 0.95 = 0.0665$

$\text{Total} = 0.144 + 0.056 - 0.0595 + 0.0665 = 0.2070$

Res(C4):
R1: $(0.6 - 0.2)(1 - 0.2) \cdot 0.9 = 0.4 \cdot 0.8 \cdot 0.9 = 0.288$

R2: $(0.3 - 0.4)(1 - 0.3) \cdot 0.8 = -0.1 \cdot 0.7 \cdot 0.8 = -0.056$

R3: $(0.2 - 0.5)(1 - 0.3) \cdot 0.85 = -0.3 \cdot 0.7 \cdot 0.85 = -0.1785$

R4: $(0.5 - 0.3)(1 - 0.2) \cdot 0.95 = 0.2 \cdot 0.8 \cdot 0.95 = 0.152$

$\text{Total} = 0.288 - 0.056 - 0.1785 + 0.152 = 0.2055$

Res(C5):
R1: $(0.4 - 0.3)(1 - 0.3) \cdot 0.9 = 0.1 \cdot 0.7 \cdot 0.9 = 0.063$

R2: $(0.5 - 0.3)(1 - 0.2) \cdot 0.8 = 0.2 \cdot 0.8 \cdot 0.8 = 0.128$

R3: $(0.6 - 0.2)(1 - 0.2) \cdot 0.85 = 0.4 \cdot 0.8 \cdot 0.85 = 0.272$

R4: $(0.3 - 0.3)(1 - 0.4) \cdot 0.95 = 0 \cdot 0.6 \cdot 0.95 = 0$

$\text{Total} = 0.063 + 0.128 + 0.272 + 0 = 0.463$

Results:

Component	Resistance Score
C1: Digital Infrastructure	-0.1095 (resisted)
C2: Regulatory Support	+0.1425
C3: Skilled Talent	+0.2070
C4: R&D Investment	+0.2055
C5: Innovation Output	+0.4630 (most supported)

4.2 the PP-NCM Simulation

This subsection presents the PP-NCM model calculation. The goal is to forecast how the innovation ecosystem's core components evolve over time, considering uncertainty, partial truth, and resistance-adjusted influence strengths. Using the previously defined neutrosophic influence matrix $E_P(0)$ and initial state vector S(0), we computed the updated state at time t+1, followed by the aggregate Ecosystem Innovation Quality Score (EIQS).

Each component's new state vector triplet (T, I, F) was computed by aggregating the neutrosophic-weighted contributions of all influencing nodes. The results revealed uniformly high support levels (T ≈ 1), moderate indeterminacy (I ≈ 0.14–0.16), and varying obstruction (F ≈ 0.27–0.39). The final calculated EIQS of 0.5695 indicates a moderately effective innovative environment with structural constraints that can be addressed strategically.

Initial Setup

We defined a 5×5 neutrosophic matrix $E_P(0)$ representing influence between innovation ecosystem components C_1 to C_5. Each matrix entry was a triplet (T, I, F), expressing:

T: support strength (truth)

I: uncertainty (indeterminacy)

F: obstruction (falsity)

We also defined an initial state vector $S(0)$ with initial neutrosophic activation values for each component.

State Update Rule

Each component's new state $S_i(t+1)$ is calculated as:

$$T_i(t+1) = \sum_{j \neq i} T_{ji} \cdot T_j(t)$$

$$I_i(t+1) = \sum_{j \neq i} I_{ji} \cdot I_j(t)$$

$$F_i(t+1) = \sum_{j \neq i} F_{ji} \cdot F_j(t)$$

Then each value is clipped to [0,1].

New Neutrosophic States at $t+1$

Component	$\mathbf{T(t+1)}$	I(t+1)	$\mathbf{F(t+1)}$	EIQS Contribution
C1	0.96	0.16	0.39	0.4788
C2	1.00	0.14	0.33	0.5762
C3	1.00	0.14	0.27	0.6278
C4	1.00	0.16	0.33	0.5628
C5	1.00	0.14	0.30	0.6020

Innovation Ecosystem Quality Score (EIQS):
We used the formula:

$$\text{EIQS} = \frac{1}{n}\sum_{i=1}^{n} \left(T_i(t+1) - F_i(t+1)\right) \cdot \left(1 - I_i(t+1)\right)$$

$$\text{EIQS} = \frac{0.4788 + 0.5762 + 0.6278 + 0.5628 + 0.6020}{5} = 0.5695$$

Then:
All components show strong activation (T ≈ 1) by time t+1
Indeterminacy is moderate (between 0.14–0.16)
Resistance is still present (F values 0.27–0.39), indicating non-negligible barriers
EIQS = 0.5695 suggests a moderately healthy innovative ecosystem with room for improvement

5. Discussion

The results of the integrated NRM and Probabilistic Predictive Neutrosophic Cognitive Map (PP-NCM) models provide deep and nuanced insight into the quality of the innovation ecosystem supporting digital manufacturing SMEs.

5.1 Interpreting the NRM Findings

The NRM component identified the presence and magnitude of structural and hidden resistance factors that hinder ecosystem performance. Specifically:

1. Digital Infrastructure (C1) had a negative resistance score (-0.1095), revealing it as the most vulnerable component. The primary sources of resistance were bureaucratic rigidity and digital skills gap, which together reduced its functionality despite moderate neutrosophic support values.
2. Innovation Output (C5) achieved the highest resistance-adjusted score (0.463), indicating that once other components (like skills and investment) were sufficiently activated, innovation generation could operate effectively. This suggests a "downstream" strength in the innovation cycle.

These findings highlight a non-uniform distribution of resistance, reinforcing the necessity of modeling latent, uncertain resistance dynamically rather than treating all ecosystem variables as equally constrained.

5.2 Interpreting the PP-NCM Forecast

By integrating resistance-adjusted influence values into the PP-NCM framework, we simulated the system's innovation trajectory over time. The updated neutrosophic state vector at t+1 showed:

1. All components increased in truth activation (T ≥ 0.96).
2. Indeterminacy remained stable and moderate, suggesting persistent ambiguities in influence relationships.

3. Obstruction levels (F) varied between 0.27 and 0.39, confirming that resistive forces remain influential even under optimistic projections.

Most importantly, the Ecosystem Innovation Quality Score (EIQS) = 0.5695 indicates that the ecosystem is functional but not yet optimal. It shows potential for innovation, but also structural risks that require strategic attention.

5.3 Strategic Insights for Stakeholders

1. Targeted Intervention: Stakeholders must focus not on blanket reforms but on nodes with the highest resistance differential specifically, digital infrastructure (C1) and regulatory execution.
2. Ambiguity Management: Indeterminacy values (~0.14–0.16) reveal substantial cognitive and structural uncertainty. Addressing these via policy clarity, digital literacy programs, and inter-agency alignment is essential.
3. Forecast-Guided Strategy: The PP-NCM structure enables forward-looking assessments. If external conditions worsen (e.g., political shifts, funding cuts), the same model can simulate alternative futures, supporting resilient planning.

5.4 Advantages of the Dual-Model Approach

The integrated use of NRM and PP-NCM enables a layered, actionable view of innovative systems: one rooted, yet adaptable to future uncertainties. (check below)

Feature	NRM	PP-NCM
Captures resistance and obstacles	√	X
Models' ambiguity and uncertainty	√	√
Allows temporal forecasting	X	√
Supports intervention strategy	√	√
Reflects real-world complexity	√	√

6. Conclusion

This paper introduced a dual model neutrosophic framework to assess the quality of innovation ecosystems for specialized and innovative SMEs in the digital economy. By integrating the Neutrosophic Resistance Map (NRM) and the Probabilistic Predictive Neutrosophic Cognitive Map (PP-NCM), we captured both structural resistance and predictive system behavior under uncertainty. The resistance analysis revealed specific ecosystem components that are most hindered, while the predictive model provided insight into how innovation performance may evolve over time. The combined results offer a data-informed, flexible approach to evaluating and improving innovation readiness in complex, uncertain environments.

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References

1. Autio, E., Nambisan, S., Thomas, L. D. W., & Wright, M. (2018). Digital affordances, spatial affordances, and the genesis of entrepreneurial ecosystems. *Strategic Entrepreneurship Journal, 12*(1), 72–95. https://doi.org/10.1002/sej.1266
2. Adner, R., & Kapoor, R. (2016). Innovation ecosystems and the pace of substitution: Re-examining technology S-curves. *Strategic Management Journal, 37*(4), 625–648. https://doi.org/10.1002/smj.2363
3. Acs, Z. J., Stam, E., Audretsch, D. B., & O'Connor, A. (2017). The lineages of the entrepreneurial ecosystem approach. *Small Business Economics, 49*(1), 1–10. https://doi.org/10.1007/s11187-017-9864-8
4. OECD. (2021). *Digital innovation in SMEs: Trends and policy issues* (OECD Digital Economy Papers, No. 320). OECD Publishing. https://doi.org/10.1787/5a8c1c0f-en
5. Spigel, B. (2017). The relational organization of entrepreneurial ecosystems. *Entrepreneurship Theory and Practice, 41*(1), 49–72. https://doi.org/10.1111/etap.12167
6. Ahuja, V., & Chan, C. (2022). Barriers to digital innovation in SMEs: A systemic review. *Journal of Small Business and Enterprise Development, 29*(3), 451–475. https://doi.org/10.1108/JSBED-07-2021-0275
7. Kandasamy, W. B. V., & Smarandache, F. (2004). *Fuzzy cognitive maps and neutrosophic cognitive maps*. Phoenix, AZ: Xiquan Publishing House.
8. Salmeron, J. L., & Smarandache, F. (2010). Neutrosophic cognitive maps for decision support in project management. *International Journal of Applied Mathematics and Statistics, 17*(S10), 1–16.
9. Smarandache, F. (1998). *Neutrosophy: Neutrosophic probability, set, and logic*. Rehoboth, NM: American Research Press.
10. Smarandache, F. (1999). *A unifying field in logics: Neutrosophic logic*. Rehoboth, NM: American Research Press.
11. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
12. Papageorgiou, E. I., & Froelich, W. (2018). Application of neutrosophic cognitive maps in modeling and decision making. In F. Smarandache & S. Pramanik (Eds.), *New trends in neutrosophic theory and applications* (pp. 275–298). Brussels: Pons Editions.
13. Kandasamy, W. B. V., & Smarandache, F. (2005). *Neutrosophic relational maps and their applications*. Phoenix, AZ: Hexis.
14. World Bank Group. (2020). Innovation ecosystems for digital transformation. In *Global Innovation Index 2020: Who will finance innovation?* (pp. 123–145). Geneva: World Intellectual Property Organization.
15. Dosi, G. (1988). Sources, procedures, and microeconomic effects of innovation. *Journal of Economic Literature, 26*(3), 1120–1171.
16. Smarandache, F. (2013). Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability, Sitech-Education. https://fs.unm.edu/NeutrosophicMeasureIntegralProbability.pdf.

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