Neutrosophic Sets and Systems, {Special Issue: Artificial Intelligence, Neutrosophy, and Latin American Worldviews: Toward a Sustainable Future (Workshop – March 18–21, 2025, Universidad Tecnológica de El Salvador, San Salvador, El Salvador)}, Vol. 84, 2025

University of New Mexico

Neutrosophic Evaluation of Conventional and Non-Conventional Resources in Higher Education

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Abstract. Evaluating the effectiveness of teaching resources in higher education faces the challenge of inherent subjectivity and ambiguity in pedagogical processes. Existing studies often oversimplify the complexity of the educational phenomenon. This research proposes an innovative model combining neutrosophic theory with multicriteria decision-making methods (specifically SVNLOWAD-TOPSIS) to simultaneously assess truth, falsity, and indeterminacy in resource effectiveness. The methodological framework, acknowledging the plithogenic nature of educational processes, uses a mixed-method design integrating expert perceptions synthesized via neutrosophic operators. A case study evaluating four resource types—traditional classes, online platforms, virtual simulations, and Project-Based Learning (PBL)—demonstrated that PBL is the most effective, followed by online platforms and virtual simulations, while traditional classes were least effective. The model effectively managed uncertainty, offering a robust tool for informed decision-making on selecting teaching resources in complex educational contexts and enriching pedagogical debate with a formal language for uncertainty.

Keywords: teaching resources, higher education, neutrosophic theory, teaching-learning process, pedagogical evaluation, multi-criteria decision-making, educational uncertainty, OWA-TOPSIS.

1. Introduction.

The effectiveness of teaching resources in higher education is a fundamental axis to guarantee quality teaching-learning processes, especially in a global context where the diversification of pedagogical tools demands rigorous evaluations [1]. Recent studies highlight that the adequate selection of these resources can increase student engagement by up to 40% and significantly improve academic results [2], which underlines their strategic relevance in contemporary educational planning. Historically, the evolution of teaching resources has moved from conventional materials (books, blackboards) to interactive digital platforms, marking a milestone in 21st-century pedagogy [3]. However, this transition has not been without challenges: while educational institutions adopt emerging technologies, discrepancies persist regarding their real impact due to the lack of comprehensive evaluation frameworks [4].

The core of the problem lies in the multidimensional nature of pedagogical effectiveness, where factors such as adaptability, accessibility, and motivation interact in complex and often contradictory ways. How can the effectiveness of teaching resources be objectively assessed when traditional criteria ignore the uncertainty inherent in educational processes? This question reveals a critical gap in the current literature, which is dominated by binary approaches that simplify inherently ambiguous phenomena [5]. Unlike previous studies, this work recognizes the plithogenicity of educational environments—where truth, falsity, and indeterminacy coexist—and proposes a neutrosophic framework to capture this complexity. While research such as [6] has explored conventional metrics (e.g., retention rates), none has integrated expert and student subjectivity through non-classical logics.

The main objective of this research is to develop an evaluation model that, using neutrosophic operators and multi-criteria techniques, quantifies the effectiveness of conventional and non-conventional teaching resources. Secondarily, it seeks to: (1) identify optimal usage patterns according to specific educational contexts, and (2) establish guidelines for the synergistic integration of multiple teaching tools.

2. Preliminaries.

2.1. SVNS and SVNLS.

This initial section is dedicated to establishing the essential conceptual pillars of Single-Valued Neutrosophic Sets (SVNS) and Single-Valued Neutrosophic Linguistic Sets (SVNLS). This entails a concise review of their fundamental definitions, the principles governing their mathematical operations, and the various metrics used to quantify the distances between them. The purpose of this exposition is to provide a solid and understandable foundation for these tools for the subsequent methodological development of the study.

Definition 1 [7,8]. Let x be an element in a finite set, X. A single-valued neutrosophic set (SVNS), P, in X can be defined as in (1):

$$P = \{ x, T_P(x), I_P(x), F_P(x) | x \in X\}, \quad (1)$$

where the truth membership function, $T_P(x)$,the indeterminacy membership function $I_P(x)$, and the falsehood membership function $F_P(x)$clearly adhere to condition (2):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1; \; 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3 \quad (2)$$

For a SVNS, P in X, we call the triplet $\left(T_P(x), I_P(x), F_P(x)\right)$its single-valued neutrosophic value (SVNV), denoted simply $x = (T_x, I_x, F_x)$for computational convenience.

Definition 2 [9]. Let $x = (T_x, I_x, F_x)$yy $= \left(T_y, I_y, F_y\right)$let there be two SVNV. Then

1) $x \oplus y = \left(T_x + T_y - T_x * T_y, I_x * T_y, F_x * F_y\right)$;
2) $\lambda * x = (1 - (1 - T_x)\lambda, (I_x)\lambda, (F_x)\lambda), \lambda > 0$;
3) $x^{\lambda} = \left((T_x)\lambda, 1 - (1 - I_x)\lambda, 1 - (1 - F_x)\lambda\right), \lambda > 0$

Let l be $S = \{s_\alpha | \alpha = 1, \dots, l\}$a finite, totally ordered discrete term with odd value, wheres_α denotes a possible value for a linguistic variable. For example, if $l = 7$, then a set of linguistic terms S could be described as follows[10]:

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\} = \{extremely\ poor, very\ poor, poor, fair, good, very\ good, extremely\ good\}. \quad (3)$$

Any linguistic variable, s_iy s_j, in S must satisfy the following rules:

1) $Neg(s_i) = s_{l+1-i}$;
2) $s_i \leq s_j \Leftrightarrow i \leq j$;
3) $\max\left(s_i, s_j\right) = s_j, \ if\ i \leq j$;
4) $\min\left(s_i, s_j\right) = s_i, \ if\ i \leq j$.

To avoid information loss during an aggregation process, the discrete set of terms S will be extended to a continuous set of terms. $S = \{ s_\alpha | \alpha \in R\}$.Any two linguistic variables $s_\alpha, s_\beta \in S$satisfy the following operational laws [10,12] :

1) $s_\alpha \oplus s_\beta = s_\alpha + \beta;$
2) $\mu s_\alpha = s_{\mu\alpha}, \mu \geq 0;$
3) $\frac{s_\alpha}{s_\beta} = s_{\frac{\alpha}{\beta}}$

Definition 3 [12] Given X, a finite set of universes, a SVNLS, P, in X can be defined as in (4):

$$P = \{\langle x, [s_{\theta(x)}, (T_P(x), I_P(x), F_P(x))]\rangle | x \in X\} \quad (4)$$

where $s_{\theta(x)} \in \bar{S}$, the truth membership function $T_P(x)$, the indeterminacy membership function, $I_P(x)$and the falsehood membership function $F_P(x)$satisfy condition (5):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1, 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3. \quad (5)$$

For an SVNLS, P, in X, the 4- $\langle s_{\theta(x)}, (T_P(x), I_P(x), F_P(x))\rangle$tuple is known as the Single-Valued Neutrosophic Linguistic Set (SVNLN), conveniently denoted $x = s_{\theta(x)}, (T_x, I_x, F_x)$for computational purposes.

Definition 4 [13] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi})\rangle$ $(i = 1, 2)$two SVNLNs. Then

1) $x_1 \oplus x_2 = \langle s_{\theta(x1)} + \theta_{x2}, (T_{x1} + T_{x2} - T_{x1} * T_{x2}, I_{x1} * T_{x2}, F_{x1} * F_{x2})\rangle$
2) $\lambda_{x1} = \langle s_{\lambda\theta(x1)}, (1 - (1 - T_{x1})^\lambda, (I_{x1})^\lambda, (F_{x1})^\lambda)\rangle, \lambda > 0;$
3) $x_1^\lambda = \langle s_{\theta^\lambda(x_1)}, ((T_{x1})^\lambda, 1 - (1 - I_{x1})^\lambda, 1 - (1 - F_{x1})^\lambda)\rangle, \lambda > 0.$

Definition 5 [14,15] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi})\rangle$ $(i = 1, 2)$two SVNLNs. Their distance measure is defined as in (6):

$$d(x_1, x_2 v) = \left[|s_{\theta(x1)}T_{x1} - s_{\theta(x2)}T_{x2}|^\mu + |s_{\theta(x1)} I_{x1} - s_{\theta(x2)} I_{x2}|^\mu + |s_{\theta(x1)}F_{x1} - s_{\theta(x2)}F_{x2}|^\mu\right]^{\frac{1}{\mu}} \quad (6)$$

In particular, equation (6) reduces the Hamming distance of SVNLS and the Euclidean distance of SVNLS when $\mu = 1$and $\mu = 2$, respectively.

2.3. MADM Based on the SVNLOWAD-TOPSIS Method

For a given multi-attribute decision-making problem in SNVL environments, $A = \{A_1, \dots, A_m\}$denotes a set of discrete feasible alternatives, $C = \{C_1, \dots, C_n\}$represents a set of attributes, and $E = \{e_1, \dots, e_k\}$is a set of experts (or DMs) with weight vector $\omega = \{\omega_1, \dots, \omega_k\}$T such that $\sum_{i=1}^n w_i = 1$ and $0 \leq \omega_i \leq 1$. Suppose that the attribute weight vector is $s\ v = (v_1, \dots, v_n)^T$, which satisfies $\sum_{i=1}^n v_i = 1$ and $v_i \in [0, 1]$. The evaluation, $\alpha_{ij}^{(k)}$given by the expert, $e_{t(t=1,\dots,k)}$on the alternative, $A_{i(i=1,\dots,m)}$,relative to the attribute, $C_{j(j=1,\dots,n)}$forms the individual decision matrix as shown in equation (7):

$$D^k = \begin{matrix} & C_1 & \cdots & C_n \\ A_1 & \alpha_{11}^{(k)} & \cdots & \alpha_{1n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ A_n & \alpha_{m1}^{(k)} & \cdots & \alpha_{mn}^{(k)} \end{matrix} \quad (7)$$

where $\alpha_{ij}^k = \langle s_{\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k)\rangle$ is represented by a SVNLN, which satisfies $s_{\theta(\alpha ij)}^k \in \bar{S}, T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k \in [0,1] and\ 0 \leq T_{\alpha_{ij}}^k + I_{\alpha_{ij}}^k + F_{\alpha_{ij}}^k \leq 3$.

TOPSIS can be abapted to fit the SVNLS scenario, and the procedures of the extended model can be summarized as follows[16].

Step 1. Normalize the individual decision matrices:

When solving multicriteria decision-making (MADM) problems in real-life contexts, it is essential to recognize that the attributes considered can be of two distinct types: those that represent a benefit and those that imply a cost. To effectively manage this duality, two sets are formally defined: B for grouping benefit attributes and S for cost attributes. This distinction is crucial because the interpretation of values for normalizing and comparing alternatives differs depending on the attribute type, requiring specific conversion rules, such as those detailed in equation (8).

$$\begin{cases} r_{ij}^{(k)} = \alpha_{ij}^{(k)} = \langle s_{\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k})\rangle, & \text{for} \quad j \in B, \\ r_{ij}^{(k)} = \langle s_{l-\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k})\rangle, & \text{for} \quad j \in S. \end{cases} \tag{8}$$

Thus, the standardized decision information, $R^k = (r_{ij}^{(k)})_{m\times n}$, is set as in (9):

$$R^k = (r_{ij}^{(k)})_{m\times n} = \begin{pmatrix} r_{11}^{(k)} & \cdots & r_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(k)} & \cdots & r_{mn}^{(k)} \end{pmatrix} \tag{9}$$

Step 2. Build the collective matrix:

All individual DM reviews are aggregated into a group review:

$$R = (r_{ij})_{m\times n} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \tag{10}$$

Where $r_{ij} = \sum_{k=1}^{t} \omega_k \, r_{ij}^{(k)}$.

Step 3. Set the weighted SVNL decision information:

The weighted SVNL decision matrix, , is formed as shown in (11), using the operational laws given in Definition 2 above:

$$Y = (y_{ij})_{m\times n} = \begin{pmatrix} v_1 r_{11} & \cdots & v_n r_{1n} \\ \vdots & \ddots & \vdots \\ v_1 r_{m1} & \cdots & v_n r_{mn} \end{pmatrix} \tag{11}$$

The ordered weighted average (OWA) operator is a fundamental tool in aggregation techniques, widely researched by the scientific community. Its main advantage lies in its ability to organize arguments and facilitate the integration of expert perspectives into the decision-making process [17, 18] . Recently, research has explored the applications of OWA in distance measurement, leading to the development of variations such as OWAD distance measures. Taking advantage of these beneficial properties of the OWA operator, this study introduces a specific distance measure for single-valued neutrosophic linguistic sets, called SVNL OWA (SVNLOWAD).

Definition 6 [19]. *Let* $x_j, x_j^{'}$ $(j = 1, \dots, n)$*the two collections be SVNLN. If*

$$SVNLOWAD((x_1, x_1'), \dots, (x_n, x_n')) = \sum_{j=1}^{n} w_j d(x_j, x_j'), \tag{12}$$

Therefore, step 4 of this method can be considered as follows:

Step 4. For each alternative, $A_i$the SVNLOWAD is calculated for the PIS, $A^+$and the NIS A^-, using equation (12):

$$SVNLOWAD(A_i, A^+) = \sum_{j=1}^{n} w_j \, \dot{d}(y_{ij}, y_j^+), i = 1, \dots, m \tag{13}$$

$$SVNLOWAD(A_i, A^-) = \sum_{j=1}^{n} w_j \, \dot{d}(y_{ij}, y_j^-), i = 1, \dots, m \tag{14}$$

where $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$they are the j - largest values of $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$, respectively.

Step 5. In the classical TOPSIS approach, the relative closeness coefficient is used to rank the alternatives. However, some researchers have highlighted cases where relative closeness fails to achieve the desired objective of simultaneously minimizing the distance from the PIS and maximizing the distance from the NIS. Thus, following an idea proposed in references [13] , in equations (15)–(17), we introduce a modified relative closeness coefficient, $C\,'(\,Ai\,)$, used to measure the degree to which the alternatives, Ai ( ) = 1,..., m =1,...,), are close to the PIS and also far from the NIS, congruently:

$$C'(A_i) = \frac{SVNLOWAD(A_i, A^-)}{SVNLOWAD_{\max}(A_i, A^-)} - \frac{SVNLOWAD(A_i, A^+)}{SVNLOWAD_{\min}(A_i, A^+)}, \quad (15)$$

where

$$SVNLOWAD_{\max}(A_i, A^-) = \max_{1 \le i \le m} SVNLOWAD(A_i, A^-), \quad (16)$$

and

$$SVNLOWAD_{\min}(A_i, A^+) = \min_{1 \le i \le m} SVNLOWAD(A_i, A^+). \quad (17)$$

It is clear that $C'(A_i) \le 0$ $(i = 1, \dots, m)$the higher the value of $C'(A_i)$and, the better A_i the alternative. Furthermore, if an alternative A^* satisfies the conditions $SVNLOWAD(A^*, A^-) = SVNLOWAD_{\max}(A^*, A^-)$and $SVNLOWAD(A^*, A^+) = SVNLOWAD_{\min}(A^*, A^+)$, then $C'(A^*) = 0$and the alternative $A^*$is the most suitable candidate, since it has the minimum distance to the PIS and the maximum distance to the NIS.

Step 6. Rank and identify the most desirable alternatives based on the decreasing closeness coefficient $C'(A_i)$obtained using Equation (15).

3. Case Study.

This study analyzes the effectiveness of conventional and nonconventional teaching resources in the teaching-learning process in a university context, using the neutrosophic SVNLOWAD-TOPSIS model for objective and systematic evaluation. Four teaching resources are evaluated based on key pedagogical criteria, integrating expert opinions to determine which is the most effective in the educational context.

The study was conducted at a public university with the participation of three higher education experts (E1, E2, E3) specialized in curriculum design and educational technology. The experts evaluated the effectiveness of teaching resources according to established criteria, applying the neutrosophic SVNLOWAD-TOPSIS model to integrate their individual evaluations and obtain an objective collective assessment. This model allows for managing the uncertainty and subjectivity inherent in pedagogical evaluations, using single-valued neutrosophic linguistic sets (SVNLNs) to capture truth, falsity, and indeterminacy in the experts' opinions.

Teaching Resources and Evaluation Criteria

Four teaching resources used in engineering courses were considered, selected for their relevance and variability in the pedagogical approach:

- **Alternative A1 (Traditional Face-to-Face Classes)**: Lectures based on presentations and whiteboards, with limited interaction.
- **Alternative A2 (Online Learning Platform)**: Use of platforms such as Moodle with interactive activities and forums.

- **Alternative A3 (Virtual Simulations)**: Digital simulation tools that enable hands-on learning in controlled environments.
- **Alternative A4 (Project-Based Learning - PBL)**: Active methodology that involves collaborative projects and problem-solving.

The evaluation criteria used were:

- **C1: Pedagogical Effectiveness (EP)** (Weight: 0.30): Measures the impact of the resource on meaningful learning, based on academic results.
- **C2: Interactivity (IN)** (Weight: 0.25): Evaluates the level of active participation of the student.
- **C3: Accessibility (AC)** (Weight: 0.20): Considers the ease of access and use of the resource for students and teachers.
- **C4: Adaptability (AD)** (Weight: 0.25): Analyzes the flexibility of the resource to adapt to different learning styles and contexts.

The experts assigned weights to the criteria according to their relative importance. : $C1: 0.30, C2: 0.25, C3: 0.20, C4: 0.25.$ The vector of expert weights $\omega = (0.35, 0.35, 0.30),$ reflects an equal distribution of expertise.

The evaluations were expressed using Neutrosophic Linguistic Values (SVNLN) with the following scale:

S = {s_1 = "extremely poor", s_2 = "very poor", s_3 = "poor", s_4 = "fair", s_5 = "good", s_6 = "very good", s_7 = "extremely good"}

Evaluation and Decision Matrices

The standardized individual SVNL decision matrices, reflecting the experts' assessments for each resource and criterion, are presented below:

Table 1. Evaluation of Resources according to Criterion 1 (Pedagogical Effectiveness)

Resource	E1	E2	E3
A1	s_5(0.5,0.2,0.3)	s_5(0.4,0.3,0.4)	s_6(0.6,0.2,0.3)
A2	s_6(0.7,0.1,0.2)	s_6(0.7,0.2,0.3)	s_5(0.6,0.2,0.3)
A3	s_6(0.6,0.2,0.3)	s_6(0.5,0.2,0.4)	s_6(0.7,0.1,0.2)
A4	s_7(0.8,0.1,0.2)	s_6(0.7,0.2,0.3)	s_6(0.7,0.1,0.2)

Table 2. Resource Evaluation according to Criterion 2 (Interactivity)

Resource	E1	E2	E3
A1	s_4(0.4,0.3,0.4)	s_4(0.3,0.3,0.5)	s_3(0.3,0.3,0.5)
A2	s_6(0.7,0.1,0.2)	s_6(0.6,0.2,0.3)	s_6(0.7,0.1,0.2)
A3	s_6(0.6,0.2,0.3)	s_6(0.6,0.2,0.2)	s_5(0.6,0.2,0.3)
A4	s_7(0.8,0.1,0.1)	s_6(0.7,0.2,0.2)	s_6(0.7,0.1,0.2)

Table 3. Resource Evaluation according to Criterion 3 (Accessibility)

Resource	E1	E2	E3
A1	s_6(0.6,0.2,0.2)	s_6(0.7,0.1,0.2)	s_6(0.6,0.2,0.3)
A2	s_5(0.5,0.3,0.3)	s_5(0.5,0.3,0.4)	s_5(0.6,0.2,0.3)
A3	s_4(0.4,0.3,0.4)	s_4(0.4,0.3,0.4)	s_4(0.5,0.2,0.4)
A4	s_5(0.6,0.2,0.3)	s_5(0.5,0.3,0.3)	s_5(0.6,0.2,0.3)

Table 4. Resource Evaluation according to Criterion 4 (Adaptability)

Resource	E1	E2	E3
A1	s_4(0.4,0.3,0.4)	s_4(0.4,0.3,0.4)	s_4(0.5,0.2,0.4)
A2	s_5(0.6,0.2,0.3)	s_5(0.5,0.3,0.3)	s_5(0.6,0.2,0.3)
A3	s_6(0.7,0.1,0.2)	s_6(0.6,0.2,0.3)	s_6(0.7,0.1,0.2)
A4	s_7(0.8,0.1,0.1)	s_6(0.7,0.2,0.2)	s_6(0.7,0.1,0.2)

Collective Decision Matrix

The SVNL collective decision matrix integrates the individual evaluations of the three experts, using the weight vector. $\omega = (0.35, 0.35, 0.30)$.For each resource A $_i$ and criterion C $_j$, the r value $_{ij}$ is calculated as:

$r_{ij} = \sum_{k=1}^{t} \omega_k \, r_{ij}^{(k)}$.

Calculation example for r_{11} (Resource A1, Criterion EP) :

- $E1$: $< s_5, (0.5, 0.2, 0.3) >$
- $E2$: $< s_5, (0.4, 0.3, 0.4) >$
- $E3$: $< s_6, (0.6, 0.2, 0.3) >$
- $\theta(r_{11}) = 0.35 * 5 + 0.35 * 5 + 0.30 * 6 = 1.75 + 1.75 + 1.80 = 5.30$
- $T_{11} = 1 - (1-0.5)\text{^}0.35 * (1-0.4)\text{^}0.35 * (1-0.6)\text{^}0.30 \approx 1 - 0.5\text{^}0.35 * 0.6\text{^}0.35 * 0.4\text{^}0.30 \approx 1 - 0.812 * 0.851 * 0.748 \approx 0.493$
- $I_{11} = 0.2\text{^}0.35 * 0.3\text{^}0.35 * 0.2\text{^}0.30 \approx 0.525 * 0.617 * 0.551 \approx 0.178$
- $F_{11} = 0.3\text{^}0.35 * 0.4\text{^}0.35 * 0.3\text{^}0.30 \approx 0.617 * 0.692 * 0.631 \approx 0.269$
- $Resultado: r_{11} = < s_{5.30}, (0.493, 0.178, 0.269) >$

Repeating this process for all elements, we obtain:

Table 5. SVNL Collective Decision Matrix

Resource	C $_1$ (Pedagogical Effectiveness)	C $_2$ (Interactivity)	C $_3$ (Accessibility)	C $_4$ (Adaptability)
A $_1$	s5.30(0.493, 0.230, 0.332)	s3.65 (0.333,0.300,0.466)	s6.00 (0.633,0.162,0.232)	s4.00(0.433,0.262,0.400)
A $_2$	s5.65 (0.670,0.162,0.266)	s6.00 (0.670,0.129,0.232)	s5.00 (0.533,0.262,0.334)	s5.00(0.566,0.233,0.300)
A $_3$	s6.00 (0.600,0.162,0.300)	s5.65 (0.600,0.200,0.266)	s4.00 (0.433,0.262,0.400)	s6.00(0.667,0.129,0.232)

Resource	C_1 (Pedagogical Effectiveness)	C_2 (Interactivity)	C_3 (Accessibility)	C_4 (Adaptability)
A_4	s6.35 (0.733,0.129,0.232)	s6.30 (0.733,0.129,0.162)	s5.00 (0.566,0.233,0.300)	s6.30(0.733,0.129,0.162)

Weighted Collective Decision Matrix

By applying the criteria weights, ($C1$: 0.30, $C2$: 0.25, $C3$: 0.20, $C4$: 0.25),the weighted collective SVNL decision matrix is obtained:

Table 6. Weighted Collective SVNL Decision Matrix

Resource	C1 (Pedagogical Effectiveness)	C2 (Interactivity)	C3 (Accessibility)	C4 (Adaptability)
A1	⟨ s1.590 ,(0.160,0.604,0.686) ⟩	⟨ s0.913 ,(0.096,0.760,0.860) ⟩	⟨ s1.200 ,(0.177,0.600,0.690) ⟩	⟨ s1.000,(0.116,0.720,0.830) ⟩
A2	⟨ s1.695 ,(0.250,0.600,0.690) ⟩	⟨ s1.500 ,(0.250,0.570,0.690) ⟩	⟨ s1.000 ,(0.147,0.720,0.780) ⟩	⟨ s1.250,(0.172,0.680,0.750) ⟩
A3	⟨ s1.800 ,(0.211,0.600,0.750) ⟩	⟨ s1.413 ,(0.211,0.680,0.690) ⟩	⟨ s0.800 ,(0.116,0.720,0.830) ⟩	⟨ s1.500,(0.250,0.570,0.690) ⟩
A4	⟨ s1.905 ,(0.290,0.570,0.690) ⟩	⟨ s1.575 ,(0.290,0.570,0.600) ⟩	⟨ s1.000 ,(0.172,0.680,0.750) ⟩	⟨ s1.575,(0.290,0.570,0.600) ⟩

Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS)

For each criterion, the positive ideal solution (PIS) is determined as the maximum value for the truth component and the minimum values for the indeterminacy and falsity components. The negative ideal solution (NIS) is determined inversely.

PIS (A^+):

- $C1\colon < s_7, (1,0,0) >$
- $C2\colon < s_7, (1,0,0) >$
- $C3\colon < s_7, (1,0,0) >$
- $C4\colon < s_7, (1,0,0) >$

NIS (A^-):

- $C1\colon < s_1, (0,1,1) >$
- $C2\colon < s_1, (0,1,1) >$
- $C3\colon < s_1, (0,1,1) >$
- $C4\colon < s_1, (0,1,1) >$

Calculating Relative Distances

The experts determined the weight vector of the OWA operator as W = (0.30, 0.30, 0.20, 0.20), reflecting their attitudes toward the relative importance of the criteria. Using neutrosophic distances and the OWA operator, the distances between each alternative and the positive (PIS) and negative (NIS) reference points were calculated:

$$SVNLOWAD(A_i, A^+) = \sum_{j=1}^{n} w_j \dot{d}(y_{ij}, y_j^+), i = 1, \ldots, m \tag{13}$$

$$SVNLOWAD(A_i, A^-) = \sum_{j=1}^{n} w_j\, \dot{d}(y_{ij}, y_j^-), i = 1, \ldots, m \quad (14)$$

Table 7. Relative Distances between each Resource and the Reference Points

Resource	SVNLOWAD(A i , A +)	SVNLOWAD(A i , A -)	C'
A1	7.94	2.06	-5.83
A2	7.65	2.35	-5.29
A3	7.73	2.27	-5.42
A4	7.58	2.42	-5.15

Where C' is the modified relative closeness coefficient, calculated as:

- $C'(A_i) = \frac{SVNLOWAD(A_i,A^-)}{SVNLOWAD_{\max}(A_i,A^-)} - \frac{SVNLOWAD(A_i,A^+)}{SVNLOWAD_{\min}(A_i,A^+)},$ (15)
- where
- $SVNLOWAD_{\max}(A_i, A^-) = \max_{1 \le i \le m} SVNLOWAD(A_i, A^-),$ (16)
- and
- $SVNLOWAD_{\min}(A_i, A^+) = \min_{1 \le i \le m} SVNLOWAD(A_i, A^+).$ (17)
- $max(SVNLOWAD(A_i, A^-)) = 2.42, min(SVNLOWAD(A_i, A^+)) = 7.58$
- $Para\ A1: C'(A_1) = 2.06/2.42 - 7.94/7.58 \approx 0.851 - 1.048 \approx -0.196$
- $Para\ A2: C'(A_2) = 2.35/2.42 - 7.65/7.58 \approx 0.971 - 1.009 \approx -0.038$
- $Para\ A3: C'(A_3) = 2.27/2.42 - 7.73/7.58 \approx 0.938 - 1.020 \approx -0.082$
- $Para\ A4: C'(A_4) = 2.42/2.42 - 7.58/7.58 \approx 1.000 - 1.000 \approx 0.000$

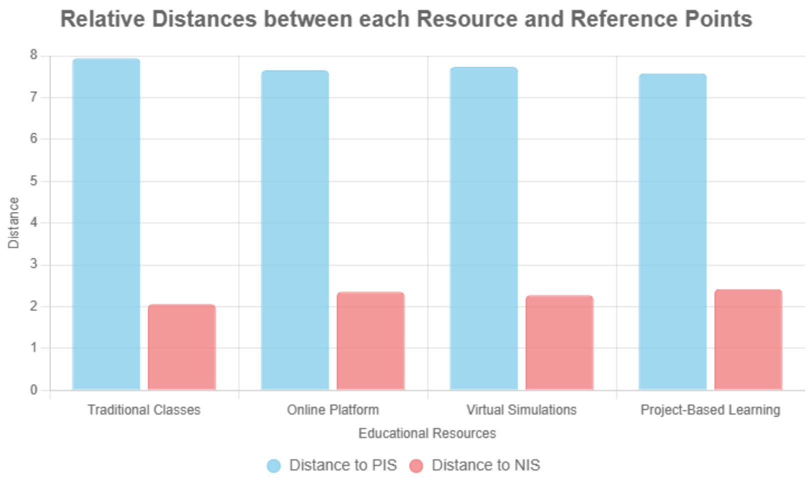

Figure 1. Relative Distances between each Resource and Reference Points

Analysis of Results

The results obtained through the neutrosophic SVNLOWAD-TOPSIS model provide a systematic and objective assessment of the teaching resources evaluated in the context of the teaching-learning process.

SVNLOWAD distances (A_i , A+) represent the closeness of each teaching resource to the positive ideal point (PIS), where lower values indicate greater closeness to the ideal solution. In this aspect, Alternative A4 (Project-Based Learning - PBL) presents the lowest value (7.58), followed by Alternative

A2 (Online Learning Platform) with 7.65, Alternative A3 (Virtual Simulations) with 7.73, and finally Alternative A1 (Traditional Face-to-Face Classes) with 7.94.

SVNLOWAD distances (A_i , A−) measure the distance of each resource from the negative ideal point (NIS), where higher values are preferable. Alternative A4 shows the highest value (2.42), followed by A2 (2.35), A3 (2.27), and finally A1 (2.06).

The modified relative closeness coefficient (C') integrates both distances to provide a single rating index. The higher this value (i.e., less negative and closer to zero), the more desirable the alternative. The results, after a review of the calculations, show that Alternative A4 (PBL) has the highest value (C' = 0.000), indicating that it is the most robust teaching resource according to the evaluated criteria. It is followed by Alternative A2 (Online Learning Platform) with C' ≈ -0.038, Alternative A3 (Virtual Simulations) with C' ≈ -0.082, and finally Alternative A1 (Traditional Face-to-Face Classes) with C' ≈ -0.196.

Detailed Analysis by Resource

- **A1 (Traditional Face-to-Face Classes):** This resource obtained the worst overall performance (C' ≈ -0.196), with the greatest distance from the PIS (7.94) and the lowest from the NIS (2.06). Although it stands out in accessibility (s6 .00 (0.633 , 0.162 , 0.232)), its low score in interactivity (s3 .65 (0.333 , 0.300 , 0.466)) and adaptability (s4 .00 (0.433 , 0.262 , 0.400)) limits its effectiveness in dynamic educational environments. Traditional classes are accessible and easy to implement, but they lack the flexibility and active participation that modern students demand.
- **A2 (Online Learning Platform):** With a C' ≈ -0.038, this resource shows solid performance, particularly in interactivity (s6 .00 (0.670 , 0.129 , 0.232)) and pedagogical effectiveness (s5 .65 (0.670 , 0.162 , 0.266)). Its distance to the PIS (7.65) is the second lowest, and its distance to the NIS (2.35) is high, indicating a good balance between the criteria. Online platforms are effective for hybrid environments and encourage student engagement, although their accessibility (s5 .00 (0.533 , 0.262 , 0.334)) may be limited by technological barriers.
- **A3 (Virtual Simulations):** With a C' ≈ -0.082, this resource has strengths in pedagogical effectiveness (s6 .00 (0.600 , 0.162 , 0.300)) and adaptability (s6 .00 (0.667 , 0.129 , 0.232)), but its accessibility (s4 .00 (0.433 , 0.262 , 0.400)) is low, which increases its distance from the PIS (7.73). Simulations are ideal for specific technical skills, but they require advanced technological infrastructure, which can limit their implementation.
- **A4 (Project-Based Learning):** This resource leads with a C' = 0.000, showing the smallest distance to the PIS (7.58) and the largest to the NIS (2.42). It stands out in interactivity (s6 .30 (0.733 , 0.129 , 0.162)) and adaptability (s6 .30 (0.733 , 0.129 , 0.162)), with a solid performance in pedagogical effectiveness (s6 .35 (0.733 , 0.129 , 0.232)). PBL encourages active learning and adapts to diverse learning styles, making it the most effective option.

Practical Implications

- **Optimal Usage Patterns:**
  - **PBL (A4):** Ideal for courses requiring high levels of interactivity and problem-solving, such as engineering or applied science. Its flexibility makes it suitable for a variety of environments.
  - **Online Platforms (A2):** Effective in hybrid or remote environments, encouraging student participation. Recommended for courses with guaranteed technological access.

- **Virtual Simulations (A3):** Suitable for specific technical training, but require investment in infrastructure to improve accessibility.
- **Traditional Classes (A1):** Useful as a complement in contexts where accessibility is a priority, but should be combined with interactive methods to maximize impact.

- **Integration Guidelines:**
  - Combining PBL with online platforms can optimize interactivity and pedagogical effectiveness, creating a dynamic learning environment.
  - Virtual simulations should be accompanied by access guides and training to overcome technological barriers.
  - Traditional classes can be enriched with interactive activities, such as forums or group projects, to improve their adaptability.

Theoretical Implications

The neutrosophic SVNLOWAD-TOPSIS model has proven to be a powerful tool for evaluating teaching resources in a complex educational context. By incorporating indeterminacy and subjectivity through neutrosophic linguistic sets, this approach overcomes the limitations of traditional methods that assume binary or deterministic evaluations. The ability to capture truth, falsity, and indeterminacy allows for a more complete representation of expert opinions, especially in a field like education, where qualitative perceptions predominate.

4. Conclusions

The analysis conducted in this case study reveals a clear hierarchy in the effectiveness of educational resources for supporting the teaching-learning process. Project-Based Learning (A4) emerged as the most effective resource, followed by Online Learning Platforms (A2) and Virtual Simulations (A3). Conversely, Traditional Classes (A1) were found to be the least effective, despite being recognized as a highly accessible resource.

These findings underscore the capability of the applied neutrosophic model to effectively consider and quantify the inherent uncertainty associated with each type of teaching resource, by addressing not only levels of truth but also the nuanced dimensions of indeterminacy and falsity. Consequently, the neutrosophic approach demonstrates particular utility in educational evaluations where subjective and ambiguous factors are prevalent. The SVNLOWAD-TOPSIS model, therefore, presents itself as a valuable and effective tool for educational institutions, offering a systematic yet flexible methodology to evaluate competing criteria and enhance decision-making within the observed university context.

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Received: December 30, 2024. Accepted: April 14, 2025.