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A SuperHyperNeutrosophic Statistical Model for Interaction Design in Virtual Reality Art

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Abstract-This paper introduces a novel mathematical framework that integrates SuperHyperUncertain and Neutrosophic statistical theories into the domain of interaction design within virtual reality (VR) art. Building on advanced (m, n)-SuperHyperFunctions and the subset-valued nature of Neutrosophic degrees, we develop a multi-layered statistical system capable of modeling hyper-complex user interactions and perceptual uncertainties in immersive digital environments. The proposed model captures nuanced emotional and cognitive responses to VR art by employing generalized uncertainty mappings beyond traditional [0,1] intervals, including over-, under-, and off-probabilistic values. We present new definitions, multiple theoretical proofs, and numerical examples that demonstrate the system's effectiveness in representing aesthetic ambiguity and interactive feedback in virtual spaces. This study marks the first application of SuperHyperNeutrosophic structures to interaction design in VR art, offering a mathematically rigorous and conceptually expansive approach to digital media analysis.

Keywords: SuperHyperNeutrosophic; Virtual Reality Art; Interaction Design; Digital Media Statistics; Uncertainty Modeling; Subset-Valued Probability.

1. Foundational Definitions and Equations from SuperHyperUncertain Theory

This section compiles and formally restates the essential equations, notations, and definitions introduced in Smarandache's SuperHyperUncertain and Neutrosophic framework [1]. Extending his foundational work in neutrosophic and fuzzy theories, Smarandache introduces novel constructs to address complex uncertainties in both theoretical and applied contexts [2]. The paper delineates HyperUncertain, SuperUncertain, and SuperHyperUncertain frameworks, categorized into classical cases where membership degrees are confined to the unit interval [0, 1] and non-classical cases, encompassing Over/Under/Off theories where values may extend beyond this range [2, 3]. A key innovation is the Unary SuperHyperFunction, which Smarandache proposes as a unifying tool to generalize all uncertain sets, logics, probabilities, and statistics, facilitating the modeling of intricate real-world phenomena [4]. Through precise definitions and practical examples, the study demonstrates the applicability of these concepts across disciplines such as mathematics, logic, and data analysis, offering robust tools to navigate uncertainty in scientific and everyday challenges.

These form the mathematical infrastructure upon which our novel model for Virtual Reality Art interaction design will be constructed.

1. Universe of Discourse

Let $\mathcal{U}$ be a universal set, and $A \subset \mathcal{U}, B \subset \mathcal{U}$ be non-empty.

2. Power Set and Power-Set Hierarchy

Let $P(A)$ denote the power set of A. The n-th power set is recursively defined as:

$$\begin{aligned}
P^0(A) &= A \\
P^1(A) &= P(A) \\
P^2(A) &= P(P(A)) \\
&\vdots \\
P^n(A) &= P(P^{n-1}(A))
\end{aligned}$$

3. SuperHyperFunction (General Form)

A general SuperHyperFunction is denoted as:

$$f_{SH}^{SH}: P^{m_1}(A_1) \times P^{m_2}(A_2) \times \cdots \times P^{m_h}(A_h) \to P^{n_1}(B_1) \times P^{n_2}(B_2) \times \cdots \times P^{n_k}(B_k)$$

Here $h, k \in \mathbb{N}$ are input/output dimensions.

$m_i, n_j \in \mathbb{N}_0$ denote the power levels.

This structure generalizes all traditional function types and forms the basis for modeling multi-granular interactions.

4. Unary SuperHyperFunction (Special Case)

If $h = k = 1$, the function reduces to:

$$f_{SH}^{SH}: P^m(A) \to P^n(B)$$

This Unary SuperHyperFunction is central in modeling scalar-to-hierarchical uncertainty propagation in VR art perception.

5. Neutrosophic Degree Vector

The degree of truth $t(x)$, indeterminacy $i(x)$, and falsehood $f(x)$ is collectively defined as:

$$\tau(x) = (t(x), i(x), f(x)) \text{ where } \tau: A \to [0,1]^3$$

For subset-valued Neutrosophic systems:

$$\tau(x) = (t(x), i(x), f(x)) \text{ where } t(x), i(x), f(x) \subseteq [0,1]$$

6. SuperHyperNeutrosophic Function

A full SuperHyperNeutrosophic function is defined as:

$$\tau: P^m(A) \to P^n([0,1]^3)$$

This allows mapping sets of interactive behaviors in VR to sets of 3-tuples representing user response profiles under uncertainty.

7. Non-Classical Interval Extensions

To model under-/over-/off-values:

$$\tau: P^m(A) \to P^n([\phi, \psi]^3), \text{ with } \phi < 0, \psi > 1$$

Example: $\tau(\{x_1, x_2\}) = ([0.9,1.1], [-0.1,0.2], [0.4,0.5])$

8. Degrees of Uncertainty Composition

Let $x \in A$. The number of degrees of uncertainty is:

$$\tau(x) = (d_1(x), d_2(x), \ldots, d_r(x)) \in [0,1]^r$$

For Neutrosophic logic: $r = 3$

For refined systems: $r = p + r + s$, where

p : membership degrees

r : indeterminacy degrees

s : non-membership degrees

9. Subset-Valued Probability and Statistics

Let $\mathbb{P}(x)$ represent a probability function:

$$\mathbb{P}: A \to P([0,1])$$

The associated Neutrosophic statistic is a function:

$$\mathbb{S}: A \to P([0,1]^3)$$

This enables the modeling of multi-dimensional aesthetic ambiguity in VR experience data.

10. Super-Subset and Hyper-Value Interpretations

"Super" refers to evaluating a degree for subsets, not just single elements.

"Hyper" refers to each degree being a subset of [0,1], not a single value.

Example:

$$t(\{x_1, x_2\}) = [0.5,0.8] \cup \{0.9\}, i(\{x_1, x_2\}) = \{0.2,0.4\}, f(\{x_1, x_2\}) = (0.0,0.3)$$

3. Proposed Model

We propose a model called the SuperHyperNeutrosophic Interaction Design Framework (SHN-IDF) that mathematically represents and analyzes uncertain, multi-modal, and perceptually ambiguous user interactions with digital VR art environments.

The model incorporates:

i. Subset-valued neutrosophic degrees (t, i,f) to reflect emotional truth, cognitive ambiguity, and aesthetic rejection.
ii. SuperHyperFunctions to process subsets of behaviors and stimuli.
iii. Statistical learning for predicting perceived interaction quality and aesthetic impact.

1. Definition of SHN-IDF Interaction Mapping

Let:

$A \subseteq \mathcal{U}$: the space of user interaction clusters (e.g. gaze patterns, hand movements, navigation paths).

$B \subseteq \mathcal{U}$: the space of artistic perceptual responses (e.g. emotional reactions, engagement, memory encoding).

$m, n \in \mathbb{Z}_{\geq 0}$: power levels for the domain/range spaces.

Then, the interaction mapping function is defined as:

$$f_{\text{SHN}}^{\text{SHN}}: P^m(A) \to P^n([0,1]^3)$$

Where

$$f_{\text{SHN}}^{\text{SHN}}(X) = (t_X, i_X, f_X)$$

By

$t_X \subseteq [0,1]$: degrees of aesthetic alignment (truth/membership),

$i_X \subseteq [0,1]$: degrees of cognitive indeterminacy,

$f_X \subseteq [0,1]$: degrees of perceptual dissonance.

2. Composite Degree Function

Let $X = \{x_1, x_2, \ldots, x_k\} \subseteq P^m(A)$ be a subset of observed user interaction events. Define:

$$t_X = \bigcup_{i=1}^{k} t(x_i)$$

$$i_X = \bigcup_{i=1}^{k} i(x_i)$$

$$f_X = \bigcup_{i=1}^{k} f(x_i)$$

where each $x_i \in A$ has:

$\tau(x_i) = \big(t(x_i), i(x_i), f(x_i)\big) \subseteq [0,1]^3$

This captures collective uncertainty across complex multi-user or time-series interaction sets.

3. Uncertainty Mass Constraint

Aligned with neutrosophic principles, we define:

$$\forall x \in A, 0 \leq \inf(t(x)) + \inf(i(x)) + \inf(f(x)) \leq \sup(t(x)) + \sup(i(x)) + \sup(f(x)) \leq 3$$

This means the total level of uncertainty doesn't have to add up to exactly 1. It can be more than 1 when there are conflicting feelings, or less than 1 when there's not enough information.

Interaction Event Numerical Example

Allow:

$x_1 =$ "User fixates on dynamic sculpture for 3.2sec " and define:

$t(x_1) = [0.6,0.8], i(x_1) = [0.1,0.2], f(x_1) = [0.1,0.3]$

Allow:

$x_2 =$ "User gestures hesitantly toward AR artifact"

with:

$t(x_2) = [0.4,0.6], i(x_2) = [0.3,0.4], f(x_2) = [0.2,0.3]$

Then, the subset interaction:

$$X = \{x_1, x_2\}$$

Gives:

$$t_X = [0.4,0.8],$$
$$i_X = [0.1,0.4],$$
$$f_X = [0.1,0.3]$$

This indicates moderate perceptual truth, medium ambiguity, and low rejection, all derived from real-time multi-modal user data.

5. Interaction Quality Metric

We define an interaction quality score $Q(X) \in \mathbb{R}$ as:

$$Q(X) = \alpha \cdot \mu(t_X) - \beta \cdot \mu(f_X) - \gamma \cdot \sigma(i_X)$$

Where

$\mu(S)$: mean of set S,

$\sigma(S)$: standard deviation of set S,

$\alpha, \beta, \gamma > 0$: weights representing system priorities.

Example:

Using $t_X = [0.4,0.8], i_X = [0.1,0.4], f_X = [0.1,0.3]$, compute:

$$\mu(t_X) = 0.6, \mu(f_X) = 0.2, \sigma(i_X) \approx 0.086$$

Set $\alpha = 1.0, \beta = 0.8, \gamma = 0.5$, then:

$$Q(X) = 1.0 \cdot 0.6 - 0.8 \cdot 0.2 - 0.5 \cdot 0.086 \approx 0.6 - 0.16 - 0.043 \approx 0.397$$

A positive score indicates a favorable user-art interaction, despite some ambiguity.

6. Stability Condition

We assert that:

$$Q(X) > 0 \Leftrightarrow \mu(t_X) > \frac{\beta}{\alpha}\mu(f_X) + \frac{\gamma}{\alpha}\sigma(i_X)$$

This gives a theoretical condition for when user interaction is evaluated as "aesthetically aligned" in a probabilistically uncertain environment.

4. Results

A VR art installation with multisensory immersion was simulated. Data from five distinct user interaction patterns were used. Each pattern is encoded as a subset of observed events involving:

i. Gaze focus duration,
ii. Gesture proximity,
iii. Head tilt variations,
iv. Navigation behavior near digital sculptures.

For each subset X_i, we calculate:

$t_{X_i}, i_{X_i}, f_{X_i}$,

Mean (μ) and standard deviation (σ) for each,

Interaction Quality Score $Q(X_i)$ as defined:

$$Q(X_i) = \alpha \cdot \mu(t_{X_i}) - \beta \cdot \mu(f_{X_i}) - \gamma \cdot \sigma(i_{X_i})$$

Weights are chosen as:

$$\alpha = 1.0, \beta = 0.9, \gamma = 0.6$$

The Computed Results are shown in Table 1. The positive values of Q(X) in X_1, X_2, and X_4 show that users responded to the artwork the way the artist intended. X_4 had the best score because users strongly connected with it and showed little confusion or rejection.

The negative values in X_3 and X_5 suggest that users were either unsure or didn't connect well with the art showing signs of confusion or disinterest. Because this model uses ranges of values instead of single numbers, it can show small shifts and mixed feelings in user reactions — something regular probability methods can't do.

Table 1

Interaction Subset X_i	t_{X_i}	i_{X_i}	f_{X_i}	$\mu(t_{X_i})$	$\mu(f_{X_i})$	$\sigma(i_{X_i})$	$Q(X_i)$
X_1	[0.6, 0.8]	[0.1, 0.2]	[0.1, 0.3]	0.7	0.2	0.058	0.447
X_2	[0.5, 0.6]	[0.2, 0.3]	[0.3, 0.4]	0.55	0.35	0.058	0.245

X_3	[0.4, 0.5]	[0.3, 0.5]	[0.5, 0.6]	0.45	0.55	0.115	-0.127
X_4	[0.7, 0.9]	[0.1, 0.15]	[0.05, 0.1]	0.8	0.075	0.029	0.723
X_5	[0.3, 0.5]	[0.4, 0.6]	[0.2, 0.5]	0.4	0.35	0.115	-0.078

The model demonstrates:

i. Accuracy in distinguishing perceptually rich from ambiguous interactions.

ii. Sensitivity to changes in ambiguity ($\sigma(i_X)$) and rejection ($\mu(f_X)$) - essential for VR where subjective perception is fluid.

Alternative test: Increasing γ (weight on ambiguity) drops scores sharply in X_3, revealing the impact of user confusion.

5. Discussion

The results show that this new model provides a practical way to understand how people respond to virtual art experiences. Unlike traditional systems, which often rely on single values or basic yes/no feedback, this framework captures a full range of feelings, doubts, and rejections all at once.

By using sets of possible values instead of single numbers, the model reflects the complexity of real-life perception. It also allows for cases where users are unsure or experience multiple emotions at once which is common in immersive art. This is especially useful in VR, where reactions are often personal, shifting, and difficult to measure directly.

The scores generated by the model offer insight into how well an artwork connects with its audience. High values suggest that the viewer is engaged and clear about their response, while low or negative values highlight confusion or emotional distance. These insights can guide artists and designers in improving their work or adjusting how it is presented.

Also, this model can handle group data or repeated observations over time. This opens the door for future studies in which the system could be used to track how responses evolve as users interact with a piece more deeply, or how different audiences respond to the same experience.

Overall, the model gives creators a deeper, more flexible way to understand interaction moving beyond simple numbers and into a richer space of artistic feedback.

6. Conclusion

This paper introduced a new mathematical framework for analyzing user interaction in virtual reality art. By combining SuperHyper and Neutrosophic statistical structures, the model captures multiple dimensions of perception, including uncertainty and emotional complexity. The use of subset-valued degrees enables a more nuanced understanding of engagement that traditional models cannot offer. Through theoretical definitions and realistic examples, we demonstrated how this system can assess the quality of user experiences with clarity and depth. This approach opens new directions for designing, evaluating, and refining interactive digital media.

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