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Linguistic Pythagorean Hypersoft Set with Application: Decision Making in Medical Diagnosis and Treatment

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Abstract

This study explores the role of Linguistic Pythagorean Hypersoft set (LPHS) in addressing the inherent uncertainty and vagueness present in medical diagnosis and treatment due to the limitations of natural language. The purpose of the study is to develop a structured decision-making framework using LPHS that can model linguistic imprecision and manage complex, multidimensional medical information more effectively. By categorizing medical conditions and treatment parameters such as disease severity and treatment effectiveness into linguistic terms and assigning them Pythagorean values, the framework supports a more detailed analysis of patient data. The findings suggest that LPHS improves the accuracy and uniformity of medical decisions, “reflecting its potential to standardize diagnostic outcomes." aids in identifying more accurate diagnoses, and supports more tailored and effective treatment strategies. These outcomes have the potential to improve patient care, assist healthcare professionals in managing uncertainty, and contribute to more informed clinical and policy decisions in the medical field.

Keywords: Linguistic set, Pythagorean set, soft set, hypersoft set, Aggregate operators, multi-criteria decision-making (MCDM).

1.Introduction

Language is essential for expressing and sharing information, but its inherent uncertainty means it can lead to multiple interpretations and ambiguity. Linguistic expressions often carry imprecision and vagueness, which can hinder effective understanding and decision-making. Interpretation of language depends heavily on context, individual perspectives, and subjective experiences, resulting in multiple, and sometimes conflicting, interpretations. These linguistic characteristics generate significant challenges in domains where precision and clarity are essential, such as healthcare.

Frameworks like fuzzy logic and Pythagorean set theory have emerged as powerful tools for modeling and handling linguistic uncertainty. These approaches allow for the representation of partial membership and non-membership, thereby offering a more flexible and realistic means of analyzing vague or imprecise information.

Motivation and Research Gap: Despite the development of intelligent decision-support systems, current models often struggle to fully integrate the linguistic and contextual uncertainty present in real-world medical settings. Traditional fuzzy systems lack the capacity to handle multidimensional and indeterminate linguistic data simultaneously, while conventional decision-making frameworks do not adequately capture the subjective and evolving nature of medical information. This reveals a clear research gap in developing a comprehensive, context-aware, and linguistically robust decision-support framework for healthcare applications.

Research Questions

1. How can linguistic uncertainty in medical diagnosis and treatment be effectively modeled and managed?
2. Can Linguistic Pythagorean Hypersoft set (LPHS) provide a more accurate and flexible framework for decision-making in healthcare settings compared to existing approaches?
3. What impact does the proposed LPHS framework have on the consistency, accuracy, and reliability of medical diagnosis and treatment planning?

Contributions and Novelty: The LPHS based framework tailored specifically for the medical domain, capable of addressing complex, uncertain, and multidimensional linguistic information. Integration of Pythagorean logic with linguistic terms to model the membership and non-membership in both patient symptom descriptions and clinical data. Development of a structured decision-support methodology that enhances diagnostic accuracy and treatment selection in scenarios characterized by linguistic vagueness and limited data. Empirical validation showing the potential of the proposed approach to improve diagnostic consistency and support healthcare professionals in making more informed and precise decisions.

Even more reason for this study is to address existing research gaps highlighted in the literature review.

Literature Review: A review of existing literature reveals that current methodologies lack the capacity to effectively handle the uncertainty associated with further-bifurcated attributes of linguistic variables. These methods often do not incorporate standard frameworks, aggregate operators or similarity measures are required for accurately assigning Pythagorean values in decision-making problems. Put differently, they translate vague assessments into structured numerical representations suitable for analysis. Consequently, there is a critical research gap in developing a comprehensive and structured approach that can address these limitations in complex, language-dependent domains such as medical diagnostics.

In 1960, Zadeh [1] presented a fuzzy set allows partial membership, with elements assigned values between 0 and 1. Operations like union, intersection and complement extend classical set theory. A separation theorem for convex fuzzy set is established without requiring disjointness, making fuzzy sets useful in decision-making and control systems. Atanassov [2] proposed an intuitionistic fuzzy set (IFS) extends fuzzy sets by incorporating both membership and non-membership degrees, allowing for uncertainty in decision making. Yager [3] proposed the idea of (IFS) to Pythagorean Fuzzy Set (PFS) to overcome the above discussed difficulties by amending MD + NMD ≤ 1 to $MD^2 + NMD^2 \leq 1$. Harish Garg [4] developed a Linguistic Pythagorean Fuzzy set (LPFS) merge Pythagorean and linguistic fuzzy sets for better handling of uncertainty in decision-making, with an example proving its effectiveness. Florentin Smarandache [5] introduced hypersoft sets with multi-attribute functions and introduces their hybrids with various uncertainty models.

Saqlain, et. al [6] proposed Neutrosophic-linguistic valued hypersoft sets (N-LVHS) help manage linguistic uncertainty in medical diagnosis by assigning neutrosophic values to vague terms, improving accuracy in treatment and decision-making.

Rana Muhammad Zulqarnain, et, al. [7] explored correlation coefficients (CC and WCC) for Interval-valued Pythagorean fuzzy hypersoft sets (IVPFHSS) to optimize hospital bed allocation during the COVID-19 pandemic. Using the TOPSIS model and a dynamic algorithm, it enhances decision-making efficiency, outperforming existing methods. Imran Siddique, et, al. [8] developed decision-making with Pythagorean fuzzy soft sets (PFSS), introducing key operations and a score-based technique that outperforms existing methods. Palanikumaran, et, al. [9] explored possibility Pythagorean bipolar fuzzy soft sets and defines key operations. It proves essential laws and proposes an algorithm for decision-making based on this model. Muhammad Naveed Jafar, et, al. [10] presents PFHSS matrices and explores future uses in fields like image processing and medical diagnosis, aiming to improve MCDM techniques. Saraj Khan, et, al. [11] the study presents present Pythagorean fuzzy hypersoft sets (PFHSSs) improve parameter classification and decision-making, applied to mobile selection and wastewater treatment. They are versatile for fields like image processing and medical diagnosis, with future focus on enhancing MCDM techniques. Muhammad Akram, et, al. [12] developed Pythagorean fuzzy aggregation operators and presents a MCDM technique, demonstrated through a textile industry selection example.

Khalid Naeem, et, al. [13] they presented TOPSIS for multi-criteria decision-making to identify effective COVID-19 treatments using Pythagorean m-polar fuzzy topology. Jing Tang, et, al. [14] developed the concept of Type-2 fuzzy Total interpretive structural modelling (TISM) and a Bayesian network to assess COVID-19 medical waste transportation risks, with a case study showing its superiority over traditional models. Zulqarnain, et, al. [15-16] developed the concept of PFSEOWG operator for Pythagorean fuzzy soft sets in multi-attribute decision-making, with a real-life business investment case showing its effectiveness over existing methods.

The motivation for this study stems from the limitations observed in existing decision-making frameworks, which struggle to effectively address the uncertainty inherent in linguistically expressed information especially in high-stakes domains like medical diagnosis. Current methods often lack standard aggregate operators, similarity measures, and systematic approaches for assigning Pythagorean values to complex, multidimensional linguistic attributes. To fill this research gap, this study introduces the concept of Linguistic Pythagorean Hypersoft set (LPHS) as a novel framework that enhances decision-making by accommodating uncertainty and subjectivity in medical data. The proposed model not only provides formal definitions and mathematical tools for processing linguistic variables but also standardizes medical terminology to reduce inconsistencies across diagnoses. By applying LPHS to the Acute Coronary Syndrome case study, the framework demonstrates its practical capability to handle overlapping symptoms and ambiguous language, ultimately supporting more precise and personalized healthcare decisions.

This research paper explores Linguistic Pythagorean Hypersoft sets (LPHS), starting with their fundamental principles and properties. It introduces key operational laws and two mathematical operators, LPHSOWGAO and LPHSWGAO, explaining their significance. A framework for Multi-Criteria Decision-Making (MCDM) is presented using an LPHS algorithm, demonstrated through a case study. The paper concludes with findings and future research directions.

Acronyms

PFS - Pythagorean Fuzzy Sets

PFN - Pythagorean Fuzzy Numbers

LPFS - Linguistic Pythagorean Fuzzy Sets

LPFV - Linguistic Pythagorean fuzzy Values

LPFN - Linguistic Pythagorean Fuzzy numbers

LPHS - Linguistic Pythagorean Hypersoft Sets

ELPHS - Empty Linguistic Pythagorean Hypersot sets

LPHSWGAO - Linguistic Pythagorean Hypersoft set Weighted geometric averaging Operator

LPHSOWGAO - Linguistic Pythagorean Hypersoft Set Ordered Weighted Geometric Averaging Operator.

ACS - Acute Coronary syndrome

Preliminary

2.1 Linguistic Set [5]

Let A = $\{A_1, A_2, A_3, \ldots A_t\}$ where t = 2m + 1: m ≥ 1 and m ∈ $\mathbb{R}^+$ (finite and positive real numbers), a finite, strictly increasing sequence. For example, if m = 1 then,

A = $\{A_1, A_2, A_3\}$ = {low, medium, high}

For the linguistic set under study, each element A_t is associated with a subscript t, which follows a strictly increasing order. To define the continuity this set is extended to A = $\{A_\varphi : \varphi \in \mathbb{R}\}$ where φ is also strictly increasing.

2.2 Pythagorean Fuzzy Set [4]

PFS F is defined as a set of ordered pairs over a universal set U given by

$$F = \{< w, \alpha_F(w), \beta_F(w) > | \, w \in U\}$$

where the function α_F: U → [0, 1] and β_F: U → [0, 1] defines the degrees of membership and non-membership of the element w ∈ U to F, respectively and for every w ∈ U, it holds that $(\alpha_F(w))^2 + (\beta_F(w))^2 \leq 1$. Corresponding to its membership degrees, the degree of indeterminacy is given by $\pi_F(w) = \sqrt{1 - (\alpha_F(w))^2 - (\beta_F(w))^2}$. For convenience, we denote this pair as F = (α_F, β_F), where α_F, $\beta_F \in [0, 1]$, $(\alpha_F(w))^2 + (\beta_F(w))^2 \leq 1$ and called as PFN.

2.3 Linguistic Pythagorean Fuzzy Set [4]

Let U be a universal set and A = $\{a_\varphi : a_0 \leq a_\varphi \leq a_t, \ \varphi \in [0, t]\}$ be a continuous linguistic term set. A LPFS is defined in the finite universe of discourse U mathematically with the form

$$Ł = \{(w, a_\alpha(w), a_\beta(w)) \mid w \in U\}$$

where $a_\alpha(w)$, $a_\beta(w)$ ∈ A stand for the linguistic membership degree and linguistic non-membership degree of the element w to Ł. We shall denote the pairs of ($a_\alpha(w)$, $a_\beta(w)$) as Ł = (a_α, a_β) and called as linguistic Pythagorean fuzzy value (LPFV) or linguistic Pythagorean fuzzy number (LPFN).

2.4 Soft Set [9]

Let U be a universe set and let E = $\{e_1, e_2, \ldots e_t\}$ be a finite set of Parameters. Let Ᵽ (U) denote the collection of all subsets of U. For any S ⊆ E, a pair (F, S) is called soft Set over U, where the mapping F is given by F: S → Ᵽ (U).

2.5 Hypersoft Set [5]

Let E = $\{E_1, E_2, E_3, \ldots E_t\}$ for each t ≥1, consider t distinct attributes whose associated attribute values are the respective sets $h_1, h_2, h_3, \ldots h_t$ with $h_x \cap h_y = \emptyset$, for x ≠ y and x, y ∈ {1, 2, 3, . . . t}. Then the pair (F, λ) where λ is defined as the finite Cartesian product $h_1 \times h_2 \times h_3 \times \ldots \times h_t$, t ∈ Ɽ which is referred to as a hypersoft set over ṻ with the associated mapping F: $h_1 \times h_2 \times h_3 \times \ldots \times h_t$ = λ → Ᵽ (ṻ).

2.6 Linguistic Hypersoft set [5]

Let, ξ = ($\xi_1, \xi_2, \xi_3, \ldots \xi_t$) for each t ≥1, consider t distinct attributes whose associated attribute values are the respective sets $\theta_1, \theta_2, \theta_3, \ldots \theta_t$ with $\theta_x \cap \theta_y = \emptyset$, where x ≠ y for each t ≥ 1 and x, y ∈ {1, 2, 3, . . . t}. Then the pair (∂, λ) where λ is defined as the finite Cartesian product $\theta_1 \times \theta_2 \times \theta_3 \times \ldots \times \theta_t$, t ∈ Ɽ, which is referred to as a hypersoft set over ṻ with the associated mapping ∂ : ($\theta_1 \times \theta_2 \times \theta_3 \times \ldots \times \theta_t$) = λ → Ᵽ (ṻ).

Then the linguistic hypersoft set will be,

∂({β(ṻ)(x)}): β ⊆ λ & x ∈ A_φ = $\{A_1, A_2, A_3, \ldots A_t\}$ where t = 2m + 1: m ≥ 1 and m ∈ $Ɽ^+$}

3. Linguistic Pythagorean Hypersoft Set (LPHS)

Let ʊ̆ be a universe of discourse Ᵽ(ʊ̆) be a power set of ʊ̆. Take $\xi = (\xi_1, \xi_2, \xi_3, \ldots. \xi_t)$ for t ≥1. where $(\xi_1, \xi_2, \xi_3, \ldots. \xi_t)$ are attributes, whose attribute values are respectively the sets $\theta_1, \theta_2, \theta_3, \ldots. \theta_t$ with $\theta_x \cap \theta_y = \emptyset$, where x ≠ y for each t ≥ 1 and x, y ∈ {1, 2, 3, . . . t}. Let λ = $\{\theta_1 \times \theta_2 \times \theta_3 \times \ldots. \times \theta_t$ where t is finite and real valued} and $\partial : (\theta_1 \times \theta_2 \times \theta_3 \times \ldots. \times \theta_t)$ = λ → Ᵽ (ʊ̆). Now the pair ((∂, λ) is known as the Linguistic Pythagorean Hypersoft Set (LPHS) can be defined as

$\partial(\xi(A)) = \{\beta(\xi\,(M, N)) \mid \beta \subseteq \lambda\ \&\ M, N \in A_\varphi = \{A_1, A_2, A_3, \ldots A_t\}\}$

Where A_φ is the set of Linguistic Parameters and M, N represent the Pythagorean membership and Pythagorean non-membership values in Linguistic Parameters with the condition

M, N ∈ [0, 1], $0 \leq (M)^2 + (N)^2 \leq 1$.

Example 3.1.1

Let U = $\{w_1, w_2, w_3\}$ be a universal of discourse, consisting of a set of three cars, describes the attraction of the car $\beta(\xi\,(M, N) = \{w_1, w_2\}$. consider the attributes be ξ_1 = Model, ξ_2 = Brand and their corresponding sub-attributes values are:

Model = θ_1 = {2020, 2022, 2024}

Brand = θ_2 = {Honda, Hyundai, Skoda}

and set $\beta(\xi\,(A)) = \{w_1, w_2\} \subset U$. Then the function $\partial : \theta_1 \times \theta_2 \rightarrow$ Ᵽ (ʊ̆) and we have three Linguistic Parameters $A_\varphi = \{A_1, A_2, A_3\}$ = {low, medium, high}, each linguistic Parameter corresponds a Pythagorean value: A_1 = 0.2 for low, A_2 = 0.5 for medium and A_3 = 0.7 for high.

Define the attraction of cars in Linguistic Pythagorean Hypersoft Set (LPHS)

$(\partial, \lambda) = \partial(\xi(A)) = \{\beta(\xi\,(M, N)) \mid \beta \subseteq \lambda\ \&\ M, N \in A_\varphi = \{A_1, A_2, A_3, \ldots A_t\}\}$

$\partial(\{2020, \text{Honda}\}) = \{w_1, w_2\}$ = {(w_1(medium, low)), (w_2 (high, medium))} = X

Similarly,

$\partial_1(\{2022, \text{Skoda}\}) = \{w_1, w_2\}$ = {(w_1 (high, low)), (w_2(low, medium))} = X_1

$\partial_2(\{2024, \text{Hyundai}\}) = \{w_1, w_3\}$ = {(w_1(medium, low))), w_3(high, medium))} = X_2

Definition 3.2: Let $(\partial_1, \lambda_1) = X_1$ be a LPHS, then the subset X_y can be represented as. (∂, λ) = $\partial(\xi(A)) = \{\beta(\xi\,(M, N)) \mid \beta \subseteq \lambda\ \&\ M, N \in A_\varphi = \{A_1, A_2, A_3, \ldots A_t\}\}$

1. $X_y \subseteq X_1$;
2. $\forall A_\varphi \in X_y, \partial_2(A_\varphi) \subseteq \partial_1(A_\varphi)$.

The statement holds true provided that the linguistic variables A_φ fulfill the specified property i.e., each A_φ of $(\partial_y, \lambda_y) \leq A_\varphi$ of (∂_1, λ_1). Where A_φ represents Linguistic variables associated with Pythagorean evaluation.

Example 3.2.1: Example Recap 3.1.1. Let's consider LPHS for compact car Evaluation based on Linguistic variables $(\partial_1, \lambda_1) = X_1$. Then the function ∂_1: $\lambda_y = \theta_1 \times \theta_2 \rightarrow$ Ᵽ(ʊ̆) and assume the hypersoft set $\partial_1(\{2022, \text{Skoda}\}) = \{w_1, w_2\} = \{(w_1(\text{high, low})), w_2(2022 \text{ (low, medium)})\} = X_y$. where $\lambda_y \subseteq \lambda_1$ and $X_y \subseteq X_1$.

Definition 3.3: Null Linguistic Pythagorean Hypersoft Set (NLPHS) can be represented as. $\partial_1: \lambda_E = \theta_1 \times \theta_2 \times \theta_3 \times \ldots \times \theta_t \rightarrow Ᵽ\,(\breve{\upsilon})$

Such that each θ_x ($x \leq t$) is empty. $\partial_1(\{X_E(\breve{\upsilon})\})$

1. $(\partial_1, \lambda_E)(\emptyset) = X_E$ if $\forall\, \partial_1(A_\varphi) = \emptyset$: $\forall A_\varphi \in \lambda_E$.

Example 3.3.1: Example Recap 3.1.1. The function $\partial_1: \lambda_E = \theta_1 \times \theta_2 \rightarrow Ᵽ\,(\breve{\upsilon})$, where θ_1, θ_2 are all empty sets ($\theta_1 = \theta_2 = \emptyset$). Assume the Hypersoft set, $\partial_1(\emptyset) = \emptyset = X_E$, where $\lambda_E \subseteq \lambda_1$.

Definition 3.4: The AND operation on two $(\partial_1, \lambda_1) = X_1$ and $(\partial_2, \lambda_2) = X_2$ Linguistic Pythagorean Hypersoft set (LPHS) can be represented by

1. $X_1 \wedge X_2 = (\partial_{1\wedge 2}, \lambda_{1\wedge 2}) = X_{1\wedge 2}$
2. $(A_x, A_y) = A_\varphi = X_{1\wedge 2}$ where $A_x \in X_1$ and $A_y \in X_2$ with $x \neq y$ & $x, y \in \{1, 2, 3, \ldots t\}$
3. $\partial_{1\wedge 2}(A_x, A_y) = \partial_1(A_x) \cup \partial_2(A_y)$.

Definition 3.5: The OR operation on two $(\partial_1, \lambda_1) = X_1$ and $(\partial_2, \lambda_2) = X_2$ Linguistic Pythagorean Hypersoft set (LPHS) can be represented by

1. $X_1 \vee X_2 = (\partial_{1\vee 2}, \lambda_{1\vee 2}) = X_{1\vee 2}$
2. $(A_x, A_y) = A_\varphi = X_{1\vee 2}$ where $A_x \in X_1$ and $A_y \in X_2$ with $x \neq y$ & $x, y \in \{1, 2, 3, \ldots t\}$
3. $\partial_{1\vee 2}(A_x, A_y) = \partial_1(A_x) \cap \partial_2(A_y)$.

Definition 3.6: The NOT operation on $(\partial, \lambda) = X$ Linguistic Pythagorean Hypersoft set (LPHS) can be represented by;

1. $\sim X = \sim(\partial, \lambda) = \sim\theta_1 \times \sim\theta_2 \times \sim\theta_3 \times \ldots \times \sim\theta_t$;
2. $\sim X = \sim\Pi A_x : x = 1, 2, 3, \ldots t$

Definition 3.7: The Complement on $(\partial, \lambda) = X$ Linguistic Pythagorean Hypersoft set (LPHS) can be represented by

1. $(\partial, \lambda)^{\sim} = (\partial^{\sim}, \sim \lambda)$; $\partial^{\sim}: \sim \lambda \rightarrow Ᵽ\,(\breve{\upsilon})$
2. $\partial^{\sim}(\sim A_\varphi) = \breve{\upsilon} \setminus \partial(A_\varphi)$; $\forall A_\varphi \in X$

Proposition 3.8: Let $(\partial, \lambda) = X$, $(\partial_1, \lambda_1) = X_1$ and $(\partial_2, \lambda_2) = X_2$ be Linguistic Pythagorean Hypersoft set (LPHS) then following holds.

1. $(\partial_1, \lambda_1) \subseteq (\partial_1, \lambda_1)$
2. $(\partial_1, \lambda_E)(\emptyset) \subseteq (\partial_1, \lambda_1)$

3. $\sim(\sim X) = X$
4. $\sim(\partial_1, \lambda_E)(\emptyset) = \breve{\mho}$
5. If $(\partial_1, \lambda_1) \subseteq (\partial_2, \lambda_2)$ and $(\partial_2, \lambda_2) \subseteq (\partial_1, \lambda_1)$ then $(\partial_1, \lambda_1) = (\partial_2, \lambda_2)$ iff each A_φ of $(\partial_1, \lambda_1) = w_A$ of (∂_2, λ_2).

This property holds only when Linguistic variables satisfy the property i.e., each A_φ of (∂_1, λ_1) = A_φ of (∂_2, λ_2).

Proof

1. ∂_1 contains Linguistic variables $A_1, A_2, A_3, \ldots A_t$

For each $A_\varphi \in \lambda_1$, We have a mapping $\partial_1(A_\varphi) \subseteq \partial_1(A_\varphi)$ each Linguistic variable map to itself. Thus, the set (∂_1, λ_1) is a subset of itself by the Definition of 3.2, as the mappings are trivially reflexive.

$\therefore (\partial_1, \lambda_1) \subseteq (\partial_1, \lambda_1)$.

2. λ_E refers to an empty domain, (i.e., no Linguistic variable) the complement

operation on LPHS with empty domain means there are no variables to map to Pythagorean values.

$\partial_1(A_\varphi) = \emptyset$ for all $A_\varphi \in \lambda_E$ the function maps nothing to the power set $\breve{\mho}$. Since the empty set is a subset of any set, it follows that $(\partial_1, \lambda_E)(\emptyset) \subseteq (\partial_1, \lambda_1)$ because the empty set maps to no Pythagorean values, making it trivially a subset of any non-empty LPHS.

3. Let's assume $\sim(\partial_1, \lambda_1)$ represents the complement of the LPHS, which is (∂_1, λ_1)

with each Linguistic variable A_φ replaced by its complement. Now, apply the complement again would return the original Pythagorean values, as: $\sim(\sim(A_\varphi)) = A_\varphi$ for each A_φ Thus, $\sim(\sim X) = X$, Confirming the double complementation property.

4. When the LPHS is complemented and the domain is the empty set, the result is

the complement of the empty set, which is the entire universal set $\breve{\mho}$ Therefore, $\sim(\partial_1, \lambda_E)(\emptyset) = \breve{\mho}$ holds.

5. Let's assume that $(\partial_1, \lambda_1) \subseteq (\partial_2, \lambda_2)$ and $(\partial_2, \lambda_2) \subseteq (\partial_1, \lambda_1)$. This means that for all

$A_\varphi \in \partial_1$, there exists a corresponding $A_\varphi \in \partial_2$ such that: $\partial_1(A_\varphi) \subseteq \partial_2(A_\varphi)$

Similarly, $(\partial_2, \lambda_2) \subseteq (\partial_1, \lambda_1)$ implies: $\partial_2(A_\varphi) \subseteq \partial_1(A_\varphi)$

Since $\partial_1(A_\varphi) \subseteq \partial_2(A_\varphi)$ and $\partial_2(A_\varphi) \subseteq \partial_1(A_\varphi)$,

we conclude: $\partial_1(A_\varphi) = \partial_2(A_\varphi)$

Thus, $(\partial_1, \lambda_1) = (\partial_2, \lambda_2)$, provided that the Linguistic mappings and variables A_φ from both sets match exactly.

4.Operational Laws for Linguistic Pythagorean Hypersoft Set (LPHS)

In this section, we discuss the importance of operational laws, theorems and propose for LPHS. Let $(\partial_1, \lambda_1) = X_1$ and $(\partial_2, \lambda_2) = X_2$ be two LPHS, where $\lambda_1 = \theta_1 \times \theta_2 \times \theta_3 \times \ldots \times \theta_a$: a is finite and real valued} over U with mapping $\partial: \lambda_1 = \theta_1 \times \theta_2 \times \theta_3 \times \ldots \times \theta_a \rightarrow$ Ᵽ (ΰ) and $\lambda_2 = \theta_1 \times \theta_2 \times \theta_3 \times \ldots \times \theta_b$: b is finite and real valued} over ΰ with mapping $\partial: \lambda_2 = \theta_1 \times \theta_2 \times \theta_3 \times \ldots \times \theta_b \rightarrow$ Ᵽ (ΰ)

Such that.

$$(\partial, \lambda) = \partial(\xi(A)) = \{\beta(\xi(M, N)) \mid \beta \subseteq \lambda \ \&\ M, N \in A_\varphi = \{A_1, A_2, A_3, \ldots A_t\}\}$$

Where A_φ is the set of Linguistic terms and M, N represent the Pythagorean membership and Pythagorean non-membership values in Linguistic terms are ordered increasingly, from low to high. Accordingly, the operational laws on LPHS can be formulated subject to specific conditions.

Definition 4.1 Union of LPHS

The union of two LPHS, $(\partial_1, \lambda_1) = X_1$ and $(\partial_2, \lambda_2) = X_2$, can be represented as $X_1 \cup X_2$. Depending on the relationship between their Linguistic variables and their domains, the union is defined in two cases

Case 1: Let $(\partial_1, \lambda_1) = X_1$ and $(\partial_2, \lambda_2) = X_2$ be two LPHS, then the union can be defined as

$$X_1 \cup X_2 = \{\Pi\, \xi_x(A_x) \times \Pi\, \xi_y(A_y) \in \prod_{x=1}^{t} \theta_x \times \prod_{y=1}^{t} \theta_y\}$$

Where, $\xi_x(A_x) \in \prod_{x=1}^{t} \theta_x$ and $\xi_y(A_y) \in \prod_{y=1}^{t} \theta_y$ should be distinct with $\theta_x \cap \theta_y = \emptyset$, for $x \neq y$ and $x, y \in \{1, 2, \ldots, t\}$ and $A_\varphi = \{A_1, A_2, a_3, \ldots A_t\}$.

Case 2: $X_1 \cup X_2 = \{\xi_x(A_x) \in \prod_{x=1}^{t} \theta_x \times \prod_{y=1}^{t} \theta_y\}$

with x = y and Linguistic variable A_x of θ_x should be same.

Example: example Recap 3.1.1

Case 1: $\partial_1(\{2022, \text{Skoda}\}) = \{w_1, w_2\} = \{w_1$ (high, low), w_2(low, medium)$\} = X_1$

$\partial_2(\{2020, \text{Hyundai}\}) = \{w_1, w_2\} = \{w_1$(low, high), w_2(medium, high)$\} = X_2$

Since $\theta_x \cap \theta_y = \emptyset$

$X_1 \cup X_2 = \{w_1(\text{high, low}), w_2(\text{low, medium}), w_1(\text{low, high}), w_2(\text{medium, high})\}$.

Case 2: $\partial_1(\{2024, \text{Honda}\}) = \{w_1, w_2\} = \{w_1$(high, low), w_2(medium, low)$\} = X_1$

$\partial_2(\{2022,\ \text{Hyundai}\}) = \{w_1, w_2\}$ = $\{w_1$(high, low), w_2(medium, low)$\}$ = X_2

Since $\theta_x \cap \theta_y \neq \emptyset$ with x = y

$X_1 \cup X_2 = \{w_1(\text{high, low}), w_2(\text{medium, low})\}$.

Case 3: (counter example)

$\partial_1(\{2024, \text{Honda}\}) = \{w_1, w_2\}$ = $\{w_1$(high, low), w_2(medium, low)$\}$

$\partial_2(\{2022,\ \text{Hyundai}\}) = \{w_1, w_2\}$ = $\{w_1$(high, medium), w_2(high, medium)$\}$

Since $\theta_x \cap \theta_y \neq \emptyset$ with x = y

Each Linguistic variable A_x of X_1 is less then the presence of the linguistic variable A_x in X_2 implies that the combined structure $X_1 \cup X_2$ must be defined with the constraint of choosing the highest A_x per parameter.

Example

X_1 = $\{w_1$(high, low), w_2(medium, low)$\}$

X_2 = $\{w_1$(high, medium), w_2(high, medium)$\}$

As, w_1(high, low) < w_1(high, medium) and w_2(medium, low) < w_2(high, medium)

Then $X_1 \cup X_2$ = $\{w_1$(high, medium), w_2(high, medium)$\}$.

Definition 4.2 Intersection of LPHS

Case 1: Let $(\partial_1, \gamma_1) = X_1$ and $(\partial_2, \gamma_2) = X_2$ be two LPHS, then the intersection can be defined as

$$X_1 \cap X_2 = \{\Pi\, \xi_x(A_x) \times \Pi\, \xi_y(A_y) \in \prod_{x=1}^{t} \theta_x \times \prod_{y=1}^{t} \theta_y\} = \emptyset$$

Where, $\xi_x(A_x) \in \prod_{x=1}^{t} \theta_x$ and $\xi_y(A_y) \in \prod_{y=1}^{t} \theta_y$ should be distinct with $\theta_x \cap \theta_y = \emptyset$, for x = y and x, y ∈ {1, 2, ….t}.

Case 2: $X_1 \cap X_2 = \{\Pi\, \xi_x(A_x) \in \prod_{x=1}^{t} \theta_x \times \prod_{y=1}^{t} \theta_y\}$

With x = y and Pythagorean variable A_x of θ_x, then $X_1 \cap X_2 = X_1$ or X_2

Example: Example Recap3.1.1

Case 1: $\partial_1(\{2022, \text{Skoda}\}) = \{w_1, w_2\}$ = $\{w_1$(high, medium), w_2 (low, medium)$\}$ = X_1

$\partial_2(\{2020,\ \text{Hyundai}\}) = \{w_1, w_2\}$ = $\{w_1$(medium, high), w_2 (high, low)$\}$ = X_2

Since $\theta_x \cap \theta_y = \emptyset$

$X_1 \cap X_2 = \{\emptyset\}$

Case 2: $\partial_1(\{2024, \text{ Honda}\}) = \{w_1, w_2\} = \{w_1(\text{high, low}), w_2 \text{ (low, high)}\} = X_1$

$\partial_2(\{2022, \text{ Hyundai}\}) = \{w_1, w_2\} = \{w_1(\text{high, low}), w_2(\text{low, high})\} = X_2$

$\therefore \theta_x \cap \theta_y \neq \emptyset$ with x = y

$X_1 \cap X_2 = \{w_1(\text{high, low}), w_2(\text{low, high})\}$.

Case 3: (counter example)

$\partial_1(\{2024, \text{ Honda}\}) = \{w_1, w_2\} = \{w_1(\text{medium, low}), w_2 \text{ (low, high)}\}$

$\partial_2(\{2022, \text{ Hyundai}\}) = \{w_1, w_2\} = \{w_1(\text{high, low}), w_2 \text{ (high, medium)}\}$

$\therefore \theta_x \cap \theta_y \neq \emptyset$ with x = y

Each Linguistic variable A_x of X_1 is less then the presence of the linguistic variable A_x in X_2 implies that the combined structure $X_1 \cup X_2$ must be defined with the constraint of choosing the highest A_x per parameter.

Example

$X_1 = \{w_1(\text{medium, low}), w_2(\text{low, high})\}$

$X_2 = \{w_1(\text{high, low}), w_2(\text{high, medium})\}$

As, $w_1(\text{medium, low}) < w_1(\text{high, low})$ and $w_2(\text{low, high}) < w_2(\text{high, medium})$

Then $X_1 \cap X_2 = \emptyset$.

Theorem 4.3: If X_1 and X_2 be two LPHS, then the following holds:

1. $X_1 \cup X_1 = X_1$
2. $X_1 \cup \emptyset = X_1$
3. $X_1 \cap X_1 = X_1$
4. $X_1 \cap \emptyset = \emptyset$
5. $X_1 \cup X_2 = X_2 \cup X_1$
6. $X_1 \cap X_2 = X_2 \cap X_1$
7. If $X_1 \subset X_2$ and $X_2 \subset X_1$ then $X_1 = X_2$.
8. $\delta\,(X_1) = \delta X_1$; $\delta \geq 0$.
9. $\delta\,(X_1 \cup X_2) = \delta\,(X_2 \cup X_1)$

Proof

Straight forwared.

Theorem 4.4

If X_1 and X_2 be two LPHS then the operations are given as follows:

1. $\delta \times X_1 = X_{\delta \times 1}$;

2. $X_1 \oplus X_2 = X_{1\oplus 2}$;
3. $X_1 \otimes X_2 = X_{1\otimes 2}$;
4. $(X_1)^{\delta} = X_{1^{\delta}}$.

Proof

Straight forwared.

5.Some Aggregation operators

Aggregate operators combine inputs like Pythagorean values or Linguistic terms to evaluate multiple parameters at once. They simplify decision – making by enabling comprehensive assessments, such as analysing quality, reliability and customer satisfaction.

Definition 5.1 LPHSWGAO

Consider $\xi_1, \xi_2, \xi_3, \ldots. \xi_t$ for each t ≥1, consider t distinct attributes whose associated attribute values are the respective sets $\theta_1, \theta_2, \theta_3, \ldots \theta_t$ with $\theta_x \cap \theta_y = \emptyset$, for $x \neq y$ and $x, y \in \{1,2,3,\ldots.t\}$.

$\rho^{\Psi} : \lambda = \theta_1 \times \theta_2 \times \theta_3 \times \ldots. \times \theta_t \rightarrow Ᵽ(ΰ) = (\partial, \lambda) = \partial(\xi(A)) = \{\beta(\xi(M, N)) \mid \beta \subseteq \lambda \ \& \ M, N \in A_{\varphi} = \{A_1, A_2, A_3, \ldots A_t\}\}$ (1)

Where A_{φ} is the set of Linguistic variables and (M, N) represent the Pythagorean membership and Pythagorean non-membership values in Linguistic variables.

if $\rho^{\Psi}(\xi_1, \xi_2, \xi_3, \ldots. \xi_t) = \prod_{t=1}^{n} \xi_t(M, N)^{\Psi_t}$

such that

$\rho^{\Psi}(\xi_1, \xi_2, \xi_3, \ldots. \xi_t) = {\xi_1}^{\Psi_1} \otimes {\xi_2}^{\Psi_2} \otimes {\xi_3}^{\Psi_3} \otimes \ldots.. \otimes {\xi_t}^{\Psi_t} = \{\beta(\xi(M, N)\}$

where $\Psi = (\Psi_1, \Psi_2, \Psi_3, \ldots.., \Psi_t)$ is the exponential weighting vector of the $\xi_t(M, N)^{\Psi_t} \in \{\beta(\xi(M, N)\}$ and $\Psi_t \in [0,1]$ with $\sum_{t=1}^{n} \Psi_t = 1$, then ρ^{Ψ} is called Linguistic Pythagorean hyper soft weighted geometric averaging operator (LPHSWGAO).

Example

Let $\Psi = (0.36, 0.16)^t$ then LPHSWGAO $\{w_1(2022, \text{skoda}), w_2(2020, \text{honda})\} = \{w_1(2022(\text{medium, low}), \text{Skoda (high, low)})\}$

$\therefore \rho^{\Psi}(\xi_1, \xi_2, \xi_3, \ldots. \xi_t) = \prod_{n=1}^{t}(\xi_t(M, N)^{\Psi_t}) = {\xi_1}^{\Psi_1} \otimes {\xi_2}^{\Psi_2} \otimes {\xi_3}^{\Psi_3} \otimes \ldots.. \otimes {\xi_t}^{\Psi_t}$

$= \{\beta(\xi(M, N)\}$

$= \{w_1(2022(\text{medium, low})^{0.36}), (\text{Skoda (high, low)}^{0.16})\}$

$= \{w_1(\text{medium, low})^{0.36} + (\text{high, low})^{0.16}\}$

$= \{w_1(\text{medium, low})\}$

Similarly, w_2(none, none).

Definition 5.2 LPHSOWGAO

Consider $\xi_1, \xi_2, \xi_3, \dots \xi_t$, for each t ≥1, consider t distinct attributes whose associated attribute values are the respective sets $\theta_1, \theta_2, \theta_3, \dots \theta_t$ with $\theta_x \cap \theta_y = \emptyset$, for x ≠ y and x, y ∈ {1,2,3,….t}.

$¥^{\Psi} : \lambda = \theta_1 \times \theta_2 \times \theta_3 \times \dots \times \theta_t \rightarrow Ᵽ(ΰ) = (\partial, \lambda) = \partial(\xi(A)) = \{\beta(\xi(M, N)) \mid \beta \subseteq \lambda \;\&\; M, N \in A_\varphi = \{A_1, A_2, A_3, \dots A_t\}\}$ (2)

Where A_φ is the set of Linguistic variables and M, N represent the Pythagorean membership and Pythagorean non-membership values in Linguistic variables.

if $¥^{\Psi}(\xi_1, \xi_2, \xi_3, \dots \xi_t) = \prod_{t=1}^{n} \xi_t(M, N)^{\Psi_t}$

such that

$¥^{\Psi}(\xi_1, \xi_2, \xi_3, \dots \xi_t) = \xi_1^{\Psi_1} \otimes \xi_2^{\Psi_2} \otimes \xi_3^{\Psi_3} \otimes \dots \otimes \xi_t^{\Psi_t} = \{\beta(\xi(M, N)\}$

where $\Psi = (\Psi_1, \Psi_2, \Psi_3, \dots, \Psi_t)$ is the exponential weighting vector of the $\{\xi_t(M, N)^{\Psi_t}\} \in \{\beta(\xi(M, N)\}$ and $\Psi_t \in [0,1]$ with $\sum_{t=1}^{n} \Psi_t = 1$. Then $¥^{\Psi}$ is called Linguistic Pythagorean hyper soft ordered weighted geometric averaging operator (LPHSOWGAO).

Example

Let $\Psi = (0.36, 0.16)^T$ then LPHSWGAO {w_1(2022, skoda), w_2(2020, honda)} = {w_1(2022(medium, low), Skoda (high, low))}

$\therefore ¥^{\Psi}(\xi_1, \xi_2, \xi_3, \dots \xi_t) = \prod_{n=1}^{t} \xi_t(M, N)^{\Psi_t} = \xi_1^{\Psi_1} \otimes \xi_2^{\Psi_2} \otimes \xi_3^{\Psi_3} \otimes \dots \otimes \xi_t^{\Psi_t} = \{\beta(\xi(M, N)\}$

= {(2022(medium, low)$^{0.36}$, skoda(high, v.low)$^{0.16}$} = {w_1(medium, low)$^{0.36}$ + (high, low)$^{0.16}$} = {w_1(medium, low)}

Similarly, w_2(none, none).

Theorem 5.1

1. $\min_x(\xi_x(M, N)) \leq \rho^{\Psi}(\xi_1, \xi_2, \xi_3, \dots \xi_t) \leq \max_x(\xi_x(M, N))$
2. $\min_x(\xi_t(M, N)) \leq ¥^{\Psi}(\xi_1, \xi_2, \xi_3, \dots \xi_t) \leq \max_x(\xi_x(M, N))$

Proof

Straight forwared.

Theorem 5.2

1. $¥^{\Psi}(\xi_x(M, N)) = ¥^{\Psi}(\xi_x(M, N))$, where $(\xi_x(M, N))$ is any permutation of $(\xi_x(M, N))$
2. If $\forall(\xi_x(M, N)) = (\xi_x(M, N))$, then $¥^{\Psi}(\xi_x(M, N)) = A_\varphi(\xi_x(M, N))$

Proof: straight forward.

6.LPHS Algorithm to solve MCDM problem

A decision-making technique based on Linguistic Pythagorean hypersoft weighted geometric averaging operator (LPHSWGAO) has been used to construct an algorithm known as Linguistic Pythagorean hypersoft set (LPHS) based multi criteria group decision-making algorithm. The graphical representation of the proposed LPHS algorithm is presented in figure1.

Step 1

Consider, $\xi_1, \xi_2, \xi_3, \ldots . \xi_t$ for each t ≥1, consider t distinct attributes whose associated attribute values are the respective sets $\theta_1, \theta_2, \theta_3, \ldots \theta_t$ with $\theta_x \cap \theta_y = \emptyset$, for x ≠ y and x, y ∈ {1,2,3,….t}. let $\Psi = (\Psi_1, \Psi_2, \Psi_3, \ldots\ldots, \Psi_t)$ be the exponential weighting vector, where $\Psi_t \leq [0,1]$ with $\sum_{t=1}^{n} \Psi_t = 1$.

$\rho^{\Psi} : \lambda = \theta_1 \times \theta_2 \times \theta_3 \times \ldots . \times \theta_t \rightarrow$ Ᵽ $(U) = (\partial, \lambda) = \{(\xi_x, \theta_x, \partial(\theta_x), (M, N)) \mid \xi_x \in \xi, \theta_x \subseteq \lambda,$ $\partial(\theta_x) \subseteq$ Ᵽ $(\breve{u})$, $M, N \in A_{\varphi} = \{A_1, A_2, A_3, \ldots A_t\}\}$

Where A_{φ} is the set of Linguistic Parameters and M, N represent the Pythagorean membership and Pythagorean non-membership values in Linguistic Parameters

The decision-maker Đ assign the values with assign Linguistic variable to each alternative as $A_{\varphi} = \{\xi_x(M, N)\}$ and $A \in \breve{u}$ and construct a Pythagorean preference table for $(\xi_x(M, N))^{(\Psi_t)}$.

Step 2

Construct a matrix $[\xi_{xy}]_{x \times y}$ for Đ using Linguistic Pythagorean hypersoft weighted geometric averaging operator (LPHSWGAO),

$\{\xi_x(M, N)\} = \xi_1^{\Psi_1} \otimes \xi_2^{\Psi_2} \otimes \xi_3^{\Psi_3} \otimes \ldots\ldots \otimes \xi_t^{\Psi_t}$

Step 3

Tabulate the aggregated values of the respective alternatives $\{\xi_x(M, N)\}$.

Step 4

The alternative achieving the highest membership value (M) is deemed the positive ideal alternative in the decision-making process.

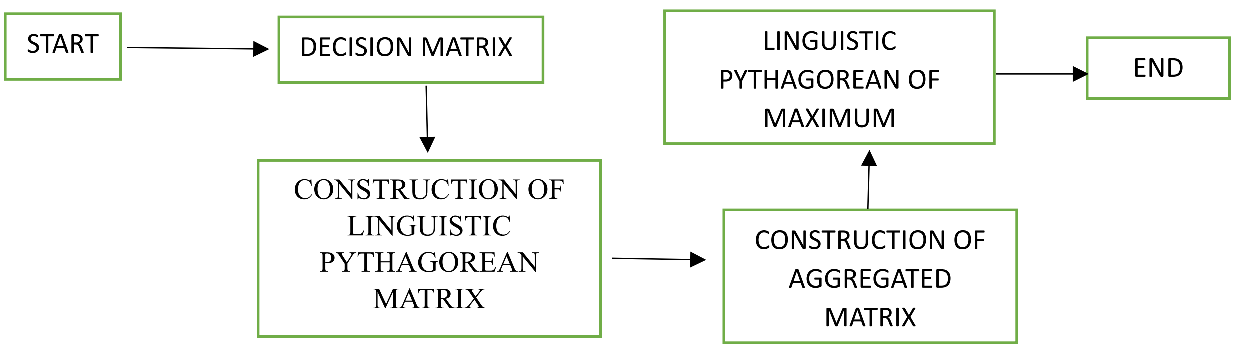

Figure 1. Graphical representation of proposed LPHS algorithm

6.1 Illustrative Example

When seven patients visit a cardiology doctor with symptoms. Like Chest pain, Palpitations, Diaphoresis, Giddiness and Shortness of Breath. These symptoms are making a doubt to affect Acute Coronary Syndrome (ACS) and also making the diagnosis questionable even if they are symptomatic of several medical diseases, including Acute Coronary Syndrome. To evaluate their symptoms more precisely, the doctor uses LPHS, and data presented in table 1.

Consider C = $\{C^1, C^2, C^3, \ldots\ldots C^7\}$ be seven patients as alternatives and doctor want to diagnose them. The medical diagnose system should be to identify Acute Coronary Syndrome (ACS) patients, while minimizing heart damage and improve outcomes.

Consider the attributes ξ_1 = Chest pain, ξ_2 = Palpitations, ξ_3 = Diaphoresis, ξ_4 = Giddiness and ξ_5 = Shortness of Breath.

Then the function $\partial: \lambda = \theta^1 \times \theta^2 \times \theta^3 \times \theta^4 \times \theta^5 \rightarrow$ Ᵽ (U) and assume the hyper soft set C = $\{C^1, C^2, C^3, \ldots . C^7\} \subset$ ΰ where ΰ = $\{C^1, C^2, C^3, \ldots\ldots C^7\}$ be the universal set.

Step1: Construction of Pythagorean preference table for alternatives

This table organizes the system of each patient as fallows

Table1: Linguistic Pythagorean Representation of Clinical Interactions and Patient Feedback.

Patient No./ Symptoms	**Chest Pain**	**Palpitations**	**Diaphoresis**	**Giddiness**	**Shortness of Breath**
C1	(high, low)	(high, low)	(medium, low)	(high, low)	(low, low)
C2	(medium, low)	(high, low)	(medium, low)	(low, low)	(medium, low)
C3	(high, low)	(high, medium)	(low, high)	(high, low)	(medium, low)
C4	(high, medium)	(medium, low)	(low, medium)	(high, low)	(medium, low)
C5	(low, medium)	(high, medium)	(low, high)	(low, low)	(high, low)
C6	(medium, low)	(high, low)	(high, low)	(high, low)	(high, medium)
C7	(high, low)	(high, medium)	(high, low)	(high, medium)	(medium, low)

Step 2: The LPHSWGAO is designed to aggregate these values across different symptoms (attributes) for a single patient.

PATIENTS LPHSWGAO VALUES

$$\begin{matrix} C1 \\ C2 \\ C3 \\ C4 \\ C5 \\ C6 \\ C7 \end{matrix} = \begin{bmatrix} (\text{high}, \ \text{low}) \\ (\text{medium}, \ \text{low}) \\ (\text{high}, \ \text{low}) \\ (\text{medium}, \text{low}) \\ (\text{low}, \text{low}) \\ (\text{high}, \text{low}) \\ (\text{high}, \ \text{low}) \end{bmatrix}$$

Step 3

Next, the doctor uses this operator to aggregate the Pythagorean values for each Patients. This aggregation takes into account the Membership and Non membership of all symptoms to calculate an overall score for each Patients.

Patients **Aggregated Values**

$$\begin{matrix} C1 \\ C2 \\ C3 \\ C4 \\ C5 \\ C6 \\ C7 \end{matrix} = \begin{bmatrix} \text{high} \\ \text{medium} \\ \text{high} \\ \text{medium} \\ \text{low} \\ \text{high} \\ \text{high} \end{bmatrix}$$

Step 4

The alternative achieving the highest membership value (M) is deemed the positive ideal alternative in the decision-making process.

Alternative **Result**

$$\begin{matrix} C1 \\ C2 \\ C3 \\ C4 \\ C5 \\ C6 \\ C7 \end{matrix} = \begin{bmatrix} \text{Positive} \\ \text{Negative} \\ \text{Positive} \\ \text{Negative} \\ \text{Negative} \\ \text{Positive} \\ \text{Positive} \end{bmatrix}$$

In this study, the difficulty doctors face when diagnosing patients with vague and common symptoms, such as Chest Pain, palpitations, Diaphoresis, Giddiness and shortness of breath, is highlighted. These symptoms are common to several different illnesses, including Acute Coronary Syndrome, making diagnosis challenging. The doctor in this scenario utilized the LPHS (a hypothetical algorithm) to address this diagnostic challenge.

The LPHS algorithm leverages advanced language models to analyse and interpret patient information more effectively. By meticulously examining the symptoms of the 7 patients and comparing them with a broad database of medical data, the LPHS was able to deliver a more accurate and data-driven assessment. This approach significantly reduced diagnostic ambiguity, making it easier to distinguish between disorders with overlapping symptoms.

The relationship between the symptoms and the resulting diagnosis has been visually represented in Table 2, which presumably illustrates how the LPHS algorithm correlates specific symptoms with potential diagnosis, enhancing the accuracy and confidence in the medical assessment.

Table 2: symptoms with diagnosis

Patients/ symptoms	**Chest Pain**	**Palpitations**	**Diaphoresis**	**Giddiness**	**Shortness of Breath**
C1					
C2					
C3					
C4					
C5					
C6					
C7					

6.3 Result Discussion Comparison and Future Direction

The LPHS algorithm outperforms traditional diagnostic methods by using advanced language models and comprehensive medical data to analyse symptoms more accurately. Unlike traditional techniques that rely on clinical judgment, LPHS considers a vast range of medical data, including the latest research, which leads to more precise diagnoses. It also adapts quickly to new information, making it especially effective in rapidly evolving situations like the Acute Coronary Syndrome. Table 3 compares these approaches, showing the significant advantages of LPHS in terms of more accuracy.

Table 3: Comparing Research Result with Existing Studies.

METHOD	POSITIVE	NEGATIVE
LPHS	C1, C6, C7	C2, C3, C4, C5
FPHS	C1, C2, C4, C5	C3, C6, C7

This comparison highlights the potential of LPHS to enhance healthcare outcomes and complement traditional diagnostic methods. While LPHS method cannot replace the expertise and experience of healthcare professionals, their integration can significantly boost diagnostic accuracy, particularly in cases with complex and hard-to-identify symptoms. With ongoing research and collaboration between technology and the medical field, we are on the brink of a new era in healthcare-one that is more precise, efficient and patient-centered.

To evaluate the stability and robustness of the proposed Linguistic Pythagorean Hypersoft Set (LPHS) framework, a sensitivity analysis was conducted by systematically varying key input parameters and linguistic weights within the decision matrix. This analysis assesses how fluctuations in symptom severity levels, diagnostic importance, and treatment response ratings affect the final decision rankings. The results demonstrate that the proposed method maintains consistent output patterns despite moderate changes in the Pythagorean values, indicating strong resilience to input uncertainty. In cases where linguistic terms such as "low" or "high" were interchanged within a controlled range, the ranking of diagnostic or treatment alternatives exhibited minimal variation, highlighting the robustness of the similarity measures and aggregation operators used in the model. These findings confirm that the LPHS framework provides a stable decision-making structure under varying degrees of linguistic ambiguity, further validating its suitability for complex and dynamic environments like medical diagnosis and treatment planning.

Limitations of the study: While the proposed Linguistic Pythagorean Hypersoft Sets (LPHS) framework demonstrates promising capabilities in managing linguistic uncertainty in medical diagnosis and treatment, several limitations should be acknowledged. First, the assignment of Linguistic Pythagorean values still depends on expert judgment and domain knowledge, which may introduce subjectivity despite the formal modeling. Second, the model's effectiveness may be influenced by the quality and granularity of linguistic data, which can vary significantly across medical institutions and patient populations. Third, computational complexity can increase with the addition of more bifurcated attributes or multidimensional linguistic variables, potentially affecting scalability in large-scale implementations. Fourth, while the Acute Coronary Syndrome (ACS) case study illustrates practical utility, broader validation across diverse medical conditions and datasets is needed to generalize findings. Finally, the integration of real-time clinical data remains a challenge, particularly in dynamically changing environments such as intensive care or emergency settings. These limitations provide important directions for future research aimed at refining and expanding the applicability of the LPHS framework in real-world healthcare systems.

7.Conclusion

In conclusion, this study underscores the significance of language and the challenges it poses in medical diagnosis and treatment. By presenting Linguistic Pythagorean Hypersoft Sets (LPHS), the study presents a powerful tool designed to effectively manage Linguistic Pythagorean uncertainty offering a promising approach to enhancing healthcare decision-making. By extending conventional soft set and Pythagorean theories, this work proposes new definitions, aggregate operators, and similarity measures tailored for multidimensional linguistic information. The quality of the presented study lies in its methodological rigor, mathematical formulation, and practical demonstration through an Acute Coronary Syndrome (ACS) case study, where the model proved capable of handling vague, overlapping, and ambiguous medical data.

From a practical and managerial perspective, the proposed framework offers a reliable decision-support tool for healthcare professionals by standardizing subjective language, enhancing the precision of symptom interpretation, and supporting more personalized treatment strategies. Hospital administrators and policymakers can also benefit from this approach to optimize diagnostic protocols, reduce variability in decision-making, and improve the quality of care.

In comparison to previous studies, this research stands out by directly tackling the limitations related to the bifurcation of linguistic attributes, an area that has often been overlooked in earlier models. Unlike traditional fuzzy, soft, or basic Pythagorean sets, the LPHS framework enables a deeper, more nuanced representation of linguistic inputs, providing more robust and context-sensitive outputs. Moreover, while earlier methodologies lacked comprehensive tools to assign Pythagorean values with precision, this study fills that gap by incorporating structured similarity and aggregation techniques.

For future research, several directions are suggested. First, further empirical validation across various medical datasets and diagnostic conditions would enhance the generalizability of the model. Second, integrating machine learning algorithms with the LPHS framework could automate Pythagorean value assignment, improving efficiency and objectivity. Third, developing software tools or clinical decision support systems based on this methodology could facilitate its real-time adoption in healthcare environments. Finally, extending the application of LPHS to other domains such as sentiment analysis, intelligent tutoring systems or psychological diagnostics could reveal its broader interdisciplinary potential.

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