



Application of Neutrosophic Micro Binary Topological Space

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Abstract: In this work, we examine a few two-person games and the topological characteristics they define. Binary topological spaces and micro topology are the two literary constructs that give rise to the idea of Neutrosophic Micro Binary topology, which was discussed in this article. The purpose of this study is to develop a novel structure for Neutrosophic Micro Binary sets. The Neutrosophic Micro Binary topological spaces are defined here, and a number of associated characteristics and attributes are also looked at.

Keywords: Core; Neutrosophic Nano Binary lower approximation; Neutrosophic Nano Binary upper approximation; Neutrosophic Nano Binary Boundary; Neutrosophic Micro Binary topology; Neutrosophic Micro Binary topological space.

1. Introduction

A number of hypotheses have recently been put up to address ambiguity, imprecision, and uncertainty. Zadeh first proposed the idea of fuzzy sets ($\mathcal{FS}$) in 1965 [26]. In 1986, Atanassov created intuitionistic $\mathcal{FS}$ ($\mathcal{IFS}$) using these $\mathcal{FS}$ [4]. Gau and Buehrer initially introduced and developed the theory of ambiguous sets as an extension of $\mathcal{FS}$ theory [7].

The $\mathcal{N}$ elements are introduced by Smarandache [22], and they stand for truth function ($\mathcal{TF}$), indeterminacy function ($\mathcal{IF}$), and falsity function ($\mathcal{fF}$), respectively. Salama and Alblowi [20], in 2012 revealed and constructed the notion of neutrosophic set ($\mathcal{NS}$) and $\mathcal{N}$ topological spaces ($\mathcal{NTs}$). F. Smarandache [23] initially introduced the $\mathcal{N}$ Two Fold Algebra in 2024.

Rough set ($\mathcal{RS}$) theory was developed by pawlak [17] in 1982. Ramachandran [18] presented $\mathcal{TFNTs}$ in 2017. [3] Arokiarani et al. explored some new ideas in $\mathcal{NTs}$. The notion of binary $\mathcal{T}$ ($\mathcal{bT}$) was presented by the authors [14, 15], who also covered some of its fundamental characteristics. Smarandache et al. evaluated blockchain cybersecurity in 2024 using the Tree Soft and Opinion Weight Criteria Method in an uncertain environment [24].

The idea of Nano Topological Space ($\mathcal{NTs}$) was first proposed by Lellis Thivagar [11]. $\mathcal{NNT}$ was invented by Lellis Thivagar et.al,[12]. The idea of $\mathcal{NN}$ ideal $\mathcal{T}$ was first presented by Parimala et al

[16]. $\mathcal{N}$ -open sets were first proposed by Vadivel et al [25]. An Use of $\mathcal{N}$ - $\check{T}$ s in Medical Diagnosis by C. Janaki and A. Jayalakshmi [10]. $\mathcal{N}\check{T}$ s was invented by J. Jasmine Elizabeth and G. Hari Siva Annam [8]. Mary Margaret et.al [13] introduced the idea of application of $\mathcal{N}$ vague $\mathcal{N}$ - $\check{T}$ s ($\mathcal{N}\check{V}\mathcal{N}\check{T}$ s). Chandrasekar was the first to suggest Micro $\check{T}$ s ($M\check{T}$ s) [5]. Ganesan and Jafari [6] illustrate the topic Neutrosophic micro topological spaces. The authors of An Application of $M\check{T}$ s with Decision Making Problem in Medical Events were M. Josephine Rani, R. Bhavani, and Bharathi Ramesh Kumar [9]. Abdel-Basset et al. developed a number of multi-criteria decision-making (MCDM) techniques [1, 2]. In 2024, C. Sangeetha and G. Sindhu discovered the Micro Binary $\check{T}$ s ($M\check{B}\check{T}$ s) [19].

In this article, I have used the idea of $\mathcal{N}M\check{B}\check{T}$ s to find the MCDM problem to find the deciding factors for the game players.

2. Preliminaries

2.1 Definition [21] A $\mathcal{N}_b\check{T}$ is a b structure consisting of two universal sets G and I where

$\mathcal{N}^B \subseteq L(G) \times L(I)$ and it satisfies the following conditions:

1. $(0_G, 0_I) \in \mathcal{N}^B$ and $(1_G, 1_I) \in \mathcal{N}^B$.
2. $(g_{1b} \cap i_{1b}, g_{2b} \cap i_{2b}) \in \mathcal{N}^B$ whenever $(g_{1b} \cap i_{1b}) \in \mathcal{N}^B$ and $(g_{2b} \cap i_{2b}) \in \mathcal{N}^B$.
3. If $(g_{\theta b}, i_{\theta b})_{\theta \in \alpha}$ is a family of members of $\mathcal{N}^B$, then $(\bigcup_{\theta \in \alpha} g_{\theta b}, \bigcup_{\theta \in \alpha} i_{\theta b}) \in \mathcal{N}^B$. The triplet $(G, I, \mathcal{N}^B)$ is called $\mathcal{N}_b\check{T}$ s.

2.2 Definition [21] $(0_g, 0_i)$ can be defined as;

- 1) $0_g = \{ \langle G, 0, 0, 1 \rangle \}$, $0_i = \{ \langle I, 0, 0, 1 \rangle \}$
- 2) $0_g = \{ \langle G, 0, 1, 1 \rangle \}$, $0_i = \{ \langle I, 0, 1, 1 \rangle \}$
- 3) $0_g = \{ \langle G, 0, 1, 0 \rangle \}$, $0_i = \{ \langle I, 0, 1, 0 \rangle \}$
- 4) $0_g = \{ \langle G, 0, 0, 0 \rangle \}$, $0_i = \{ \langle I, 0, 0, 0 \rangle \}$

$(1_g, 1_i)$ can be defined as;

- 5) $1_g = \{ \langle G, 1, 1, 0 \rangle \}$, $1_i = \{ \langle I, 1, 1, 0 \rangle \}$
- 6) $1_g = \{ \langle G, 1, 0, 0 \rangle \}$, $1_i = \{ \langle I, 1, 0, 0 \rangle \}$
- 7) $1_g = \{ \langle G, 1, 0, 1 \rangle \}$, $1_i = \{ \langle I, 1, 0, 1 \rangle \}$
- 8) $1_g = \{ \langle G, 1, 1, 1 \rangle \}$, $1_i = \{ \langle I, 1, 1, 1 \rangle \}$

2.3 Definition Let Ω be a nonempty set and Υ be an equivalence relation on Ω . Let $\mathcal{H}$ be a $\mathcal{N}$ s in Ω with the $\mathcal{T}_F \zeta_{\mathcal{H}}$, the $\mathcal{T}_F \omega_{\mathcal{H}}$, and the $f_F \gamma_{\mathcal{H}}$. The $\mathcal{N}$ -lower ($\mathcal{N}\eta^{\text{Low}}$), $\mathcal{N}$ -upper ($\mathcal{N}\eta^{\text{upp}}$) and $\mathcal{N}$ -boundary ($\mathcal{N}\eta^{\text{bou}}$) of $\mathcal{H}$ in the approximation (Ω, Υ) , represented by $\mathcal{N}\eta^{\text{Low}}(\mathcal{H})$, $\mathcal{N}\eta^{\text{upp}}(\mathcal{H})$, $\mathcal{N}\eta^{\text{bou}}(\mathcal{H})$ as follows:

- a) $\mathcal{N}\eta^{\text{Low}}(\mathcal{H}) = \{ \langle p^*, \zeta_{\Upsilon(u)}^{\text{Low}}(p^*), \omega_{\Upsilon(u)}^{\text{Low}}(p^*), \gamma_{\Upsilon(u)}^{\text{Low}}(p^*) / a \in [u]_{\Upsilon}, p^* \in \Omega \rangle \}$
- b) $\mathcal{N}\eta^{\text{upp}}(\mathcal{H}) = \{ \langle p^*, \zeta_{\Upsilon(u)\text{upp}}(p^*), \omega_{\Upsilon(u)\text{upp}}(p^*), \gamma_{\Upsilon(u)\text{upp}}(p^*) / a \in [u]_{\Upsilon}, p^* \in \Omega \rangle \}$
- c) $\mathcal{N}\eta^{\text{bou}}(\mathcal{H}) = \mathcal{N}\eta^{\text{upp}}(\mathcal{H}) - \mathcal{N}\eta^{\text{Low}}(\mathcal{H})$

Where $\zeta_{\Upsilon(u)}^{\text{Low}}(p^*) = \wedge_{a \in [u]_{\Upsilon}} \zeta_{\mathcal{H}}(a)$, $\omega_{\Upsilon(u)}^{\text{Low}}(p^*) = \wedge_{a \in [u]_{\Upsilon}} \omega_{\mathcal{H}}(a)$, $\gamma_{\Upsilon(u)}^{\text{Low}}(p^*) = \vee_{a \in [u]_{\Upsilon}} \gamma_{\mathcal{H}}(a)$,
 $\zeta_{\Upsilon(u)\text{upp}}(p^*) = \vee_{a \in [u]_{\Upsilon}} \zeta_{\mathcal{H}}(a)$, $\omega_{\Upsilon(u)\text{upp}}(p^*) = \vee_{a \in [u]_{\Upsilon}} \omega_{\mathcal{H}}(a)$, $\gamma_{\Upsilon(u)\text{upp}}(p^*) = \wedge_{a \in [u]_{\Upsilon}} \gamma_{\mathcal{H}}(a)$.

2.4 Definition Let Ω be an full set, Υ be an equivalence relation on Ω and $\mathcal{H}$ a $\mathcal{N}$ s in Ω and if the collection $\tau_{\Upsilon}(\mathcal{H}) = \{0_{\Upsilon}, 1_{\Upsilon}, \mathcal{N}\eta^{\text{Low}}(\mathcal{H}), \mathcal{N}\eta^{\text{upp}}(\mathcal{H}), \mathcal{N}\eta^{\text{bou}}(\mathcal{H})\}$ forms a topology then it is known as $\mathcal{N}\eta\check{T}$ s. We call $(\Omega, \tau_{\Upsilon}(\mathcal{H}))$ as the $\mathcal{N}\eta\check{T}$ s. The elements of $\tau_{\Upsilon}(\mathcal{H})$ are indicated by $\mathcal{N}\eta$ open sets ($\mathcal{N}\eta\mathcal{O}_{\S}$).

2.5 Definition Let $(\Omega, \tau_i(\mathcal{H}))$ be a $\mathcal{N}\mathcal{H}\mathcal{T}$ s. Then $\Psi_i(\mathcal{H}) = \{W \cup (W' \cap \Psi) : W, W' \in \tau_i(\mathcal{H}) \text{ and } \Psi \text{ doesn't } \in \tau_i(\mathcal{H})\}$ is labeled as $\mathcal{N}\mathcal{M}\mathcal{T}$ s on with Ω respect to $\mathcal{H}$. The triplet $(\Omega, \tau_i(\mathcal{H}), \Psi_i(\mathcal{H}))$ is called $\mathcal{N}\mathcal{M}\mathcal{T}$ s.

2.6 Definition The $\mathcal{N}\mathcal{M}\mathcal{T}$ s $\Psi_i(\mathcal{H})$ meets the following criteria:

1. $\Omega, \phi \in \Psi_i(\mathcal{H})$.
2. The $\cup$ of the elements of any sub-collection of $\Psi_i(\mathcal{H})$ is in $\Psi_i(\mathcal{H})$.
3. The $\cap$ of the elements of any finite sub-collection of $\Psi_i(\mathcal{H})$ is in $\Psi_i(\mathcal{H})$.

Then $\Psi_i(\mathcal{H})$ is known as $\mathcal{N}\mathcal{M}\mathcal{T}$ on Ω with respect to $\mathcal{H}$. The triplet $(\Omega, \tau_i(\mathcal{H}), \Psi_i(\mathcal{H}))$ is named as $\mathcal{N}\mathcal{M}$ open sets ($\mathcal{N}\mathcal{M}\mathcal{O}_s$) and the opposite of a $\mathcal{N}\mathcal{M}\mathcal{O}_s$ is labeled as a $\mathcal{N}\mathcal{M}$ closed set ($\mathcal{N}\mathcal{M}\mathcal{C}_s$).

3. Neutrosophic Nano Binary Topological Space

3.1 Definition Let (U, V) be a absolute set and Υ be an equivalence relation on (U, V) . Let (H_{1a}, H_{1b}) be a $\mathcal{N}$ s in (U, V) with the $\mathcal{T}_F$ of U is $\Delta_{\mathcal{H}1a}$, the $\mathcal{T}_F$ of U is $\omega_{\mathcal{H}1a}$, and the f_F of U is $\gamma_{\mathcal{H}1a}$, then the $\mathcal{T}_F$ of V is $\Delta_{\mathcal{H}1b}$, the $\mathcal{T}_F$ of V is $\omega_{\mathcal{H}1b}$, and the f_F of V is $\gamma_{\mathcal{H}1b}$. The $\mathcal{N}\mathcal{H}\mathcal{B}^{low}$, $\mathcal{N}\mathcal{H}\mathcal{B}^{upp}$ and $\mathcal{N}\mathcal{H}\mathcal{B}^{bou}$ of $\mathcal{H}$ in the approximation $((U, V), \Upsilon)$, represented by $\mathcal{N}\mathcal{H}\mathcal{B}^{Low}(H_{1a}, H_{1b})$, $\mathcal{N}\mathcal{H}\mathcal{B}^{upp}(H_{1a}, H_{1b})$, $\mathcal{N}\mathcal{H}\mathcal{B}^{bou}(H_{1a}, H_{1b})$ as follows:

1. $\mathcal{N}\mathcal{H}\mathcal{B}^{Low}(H_{1a}, H_{1b}) = \{<(u, v), \Delta_{\Upsilon}(h_{1*}, h_{2*})^{Low}(u, v), \omega_{\Upsilon}(h_{1*}, h_{2*})^{Low}(u, v), \gamma_{\Upsilon}(h_{1*}, h_{2*})^{Low}(u, v) / (u, v) \in [h_{1*}, h_{2*}]_{\Upsilon}, (M, A) \in (u, v), (u, v) \in (U, V)>\}$
2. $\mathcal{N}\mathcal{H}\mathcal{B}^{upp}(H_{1a}, H_{1b}) = \{<(u, v), \Delta_{\Upsilon}(h_{1*}, h_{2*})^{upp}(u, v), \omega_{\Upsilon}(h_{1*}, h_{2*})^{upp}(u, v), \gamma_{\Upsilon}(h_{1*}, h_{2*})^{upp}(u, v) / (u, v) \in [h_{1*}, h_{2*}]_{\Upsilon}, (u, v) \in (U, V)>\}$
3. $\mathcal{N}\mathcal{H}\mathcal{B}^{bou}(H_{1a}, H_{1b}) = \mathcal{N}\mathcal{H}\mathcal{B}^{upp}(H_{1a}, H_{1b}) - \mathcal{N}\mathcal{H}\mathcal{B}^{Low}(H_{1a}, H_{1b})$

Where $\Delta_{\Upsilon}(h_{1*}, h_{2*})^{Low}(u, v) = \Delta_{\Upsilon(h_{1*})}(u) \wedge \Delta_{\Upsilon(h_{2*})}(v)$, $\omega_{\Upsilon}(h_{1*}, h_{2*})^{Low}(u, v) = \omega_{\Upsilon(h_{1*})}(u) \wedge \omega_{\Upsilon(h_{2*})}(v)$, $\gamma_{\Upsilon}(h_{1*}, h_{2*})^{Low}(u, v) = \gamma_{\Upsilon(h_{1*})}(u) \vee \gamma_{\Upsilon(h_{2*})}(v)$

$\Delta_{\Upsilon}(h_{1*}, h_{2*})^{upp}(u, v) = \Delta_{\Upsilon(h_{1*})}(u) \vee \Delta_{\Upsilon(h_{2*})}(v)$, $\omega_{\Upsilon}(h_{1*}, h_{2*})^{upp}(u, v) = \omega_{\Upsilon(h_{1*})}(u) \vee \omega_{\Upsilon(h_{2*})}(v)$, $\gamma_{\Upsilon}(h_{1*}, h_{2*})^{upp}(u, v) = \gamma_{\Upsilon(h_{1*})}(u) \wedge \gamma_{\Upsilon(h_{2*})}(v)$

$\Delta_{\Upsilon}(h_{1*}, h_{2*})^{bou}(u, v) = \Delta_{\Upsilon(h_{1*})}(u) \wedge \gamma_{\Upsilon(h_{2*})}(v)$, $\omega_{\Upsilon}(h_{1*}, h_{2*})^{bou}(u, v) = \omega_{\Upsilon(h_{1*})}(u) \wedge (1 - \omega_{\Upsilon(h_{2*})}(v))$, $\gamma_{\Upsilon}(h_{1*}, h_{2*})^{bou}(u, v) = \gamma_{\Upsilon(h_{1*})}(u) \vee \Delta_{\Upsilon(h_{2*})}(v)$.

3.2 Definition Let (X, Y) be the full set, $\check{R}$ be an equivalence relation on (X, Y) and $\tau_{\check{R}}(H_{1a}, H_{1b}) = \{(\phi, \phi), (X, Y), \mathcal{N}\mathcal{H}\mathcal{B}^{Low}(H_{1a}, H_{1b}), \mathcal{N}\mathcal{H}\mathcal{B}^{upp}(H_{1a}, H_{1b}), \mathcal{N}\mathcal{H}\mathcal{B}^{bou}(H_{1a}, H_{1b})\}$ where $(H_{1a}, H_{1b}) \subseteq (X, Y)$. Then by the property $\tau_{\check{R}}(H_{1a}, H_{1b})$ meets the follwing criteria:

1. $(X, Y), (\phi, \phi) \in \tau_{\check{R}}(H_{1a}, H_{1b})$.
2. The $\cup$ of the elements of any sub-collection of $\tau_{\check{R}}(H_{1a}, H_{1b})$ is in $\tau_{\check{R}}(H_{1a}, H_{1b})$.
3. The $\cap$ of the elements of any finite sub-collection of $\tau_{\check{R}}(H_{1a}, H_{1b})$ is in $\tau_{\check{R}}(H_{1a}, H_{1b})$.

ie., $\tau_{\check{R}}(H_{1a}, H_{1b})$ is a topology on (X, Y) is labeled as $\mathcal{N}\mathcal{H}\mathcal{B}$ on (X, Y) with respect to (H_{1a}, H_{1b}) . We call $((X, Y), (\phi, \phi), \tau_{\check{R}}(H_{1a}, H_{1b}))$ as the $\mathcal{N}\mathcal{H}\mathcal{B}\mathcal{T}$ s. The elements of $\tau_{\check{R}}(H_{1a}, H_{1b})$ are named as $\mathcal{N}\mathcal{H}\mathcal{B}\mathcal{O}_s$.

3.3 Example

Let $X = \{e_1, e_2, e_3\}$ and $Y = \{f_1, f_2, f_3\}$ be the absolute set. Let $(X, Y)/\check{R} = \{(\{e_1\}, \{f_2\}), (\{e_2\}, \{f_3\}), (\{e_3\}, \{f_1\})\}$ be an equivalence relation on (X, Y) and $(m, n) = \{(<\{e_1\}, (0.6, 0.7, 0.8)>, <\{e_2\}, (0.1, 0.3, 0.5)>, <\{e_3\}, (0.2, 0.4, 0.9)>), (<\{f_1\}, (0.5, 0.6, 0.7)>, <\{f_2\}, (0.2, 0.4, 0.6)>, <\{f_3\}, (0.1, 0.3, 0.8)>)\}$ be a subset of (X, Y) then $\mathcal{N}\mathcal{H}\mathcal{B}^{Low}(m, n) = \{(<\{e_1\}, (0.2, 0.4, 0.8)>, <\{e_2\}, (0.1, 0.3, 0.8)>, <\{e_3\}, (0.2, 0.4, 0.9)>), (<\{f_1\}, (0.2, 0.4, 0.9)>, <\{f_2\}, (0.2, 0.4, 0.6)>, <\{f_3\}, (0.1, 0.3, 0.8)>)\}$

$\langle \{f_2\}, (0.2, 0.4, 0.8) \rangle, \langle \{f_3\}, (0.1, 0.3, 0.8) \rangle \rangle$, $\mathcal{N}\mathcal{H}\mathcal{B}_{\text{supp}}(m, n) = \{ \langle \{e_1\}, (0.6, 0.7, 0.6) \rangle, \langle \{e_2\}, (0.1, 0.3, 0.5) \rangle, \langle \{e_3\}, (0.5, 0.6, 0.7) \rangle, \langle \{f_1\}, (0.5, 0.6, 0.7) \rangle, \langle \{f_2\}, (0.6, 0.7, 0.6) \rangle, \langle \{f_3\}, (0.1, 0.3, 0.5) \rangle \rangle$, $\mathcal{N}\mathcal{H}\mathcal{B}^{\text{bou}}(m, n) = \{ \langle \{e_1\}, (0.6, 0.6, 0.6) \rangle, \langle \{e_2\}, (0.1, 0.3, 0.5) \rangle, \langle \{e_3\}, (0.5, 0.6, 0.7) \rangle, \langle \{f_1\}, (0.5, 0.6, 0.7) \rangle, \langle \{f_2\}, (0.2, 0.4, 0.8) \rangle, \langle \{f_3\}, (0.1, 0.3, 0.8) \rangle \rangle$. Then the collection $\psi_{\mathcal{R}}(m, n) = \{ (0_X, 0_Y), (1_X, 1_Y), \{ \langle \{e_1\}, (0.2, 0.4, 0.8) \rangle, \langle \{e_2\}, (0.1, 0.3, 0.8) \rangle, \langle \{e_3\}, (0.2, 0.4, 0.9) \rangle, \langle \{f_1\}, (0.2, 0.4, 0.9) \rangle, \langle \{f_2\}, (0.2, 0.4, 0.8) \rangle, \langle \{f_3\}, (0.1, 0.3, 0.8) \rangle \rangle, \{ \langle \{e_1\}, (0.6, 0.7, 0.6) \rangle, \langle \{e_2\}, (0.1, 0.3, 0.5) \rangle, \langle \{e_3\}, (0.5, 0.6, 0.7) \rangle, \langle \{f_1\}, (0.5, 0.6, 0.7) \rangle, \langle \{f_2\}, (0.6, 0.7, 0.6) \rangle, \langle \{f_3\}, (0.1, 0.3, 0.5) \rangle \rangle, \{ \langle \{e_1\}, (0.6, 0.6, 0.6) \rangle, \langle \{e_2\}, (0.1, 0.3, 0.5) \rangle, \langle \{e_3\}, (0.5, 0.6, 0.7) \rangle, \langle \{f_1\}, (0.5, 0.6, 0.7) \rangle, \langle \{f_2\}, (0.2, 0.4, 0.8) \rangle, \langle \{f_3\}, (0.1, 0.3, 0.8) \rangle \rangle \}$ is a $\mathcal{N}\mathcal{H}\mathcal{B}\check{\mathcal{T}}$ on (X, Y) .

4. Neutrosophic Micro Binary Topological Space

4.1 Definition Let $((U, V), (\phi, \psi), \psi_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*}))$ be a $\mathcal{N}\mathcal{H}\mathcal{B}\check{\mathcal{T}}$ s. Then, $\mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*}) = \{W \cup (W' \cap \mathcal{B}) : W, W' \in \psi_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*}) \text{ and } \mathcal{B} \text{ doesn't } \in \psi_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})\}$ is called the $\mathcal{N}\mathcal{M}\mathcal{B}\check{\mathcal{T}}$ on with (U, V) respect to $(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$. Then, $((U, V), (\phi, \psi), \psi_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*}), \mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*}))$ is called $\mathcal{N}\mathcal{M}\mathcal{B}\check{\mathcal{T}}$ s.

4.2 Definition The neutrosophic micro binary topology $\mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$ fulfills the following requirements.

1. $(U, V), (\phi, \psi) \in \mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$.
2. The $\cup$ of the elements of any sub-collection of $\mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$ is in $\mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$.
3. The $\cap$ of the elements of any finite sub-collection of $\mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$ is in $\mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$.

Then $\mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$ is known as $\mathcal{N}\mathcal{M}\mathcal{B}\check{\mathcal{T}}_{\text{on}}(U, V)$ with respect to $(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$. Then $((0_U, 0_V), (1_U, 1_V), \psi_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*}), \mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*}))$ is $\mathcal{N}\mathcal{M}\mathcal{B}\check{\mathcal{T}}$ s and the elements of $\mathcal{B}_{\mathcal{R}}(\mathcal{H}_{1a^*}, \mathcal{H}_{1b^*})$ is denoted by $\mathcal{N}\mathcal{M}\mathcal{B}\mathcal{O}_{\mathcal{S}}$ and the opposite of a $\mathcal{N}\mathcal{M}\mathcal{B}\mathcal{O}_{\mathcal{S}}$ is named as $\mathcal{N}\mathcal{M}\mathcal{B}\mathcal{C}_{\mathcal{S}}$.

4.3 Definition Let $\hat{A}$ and $\mathcal{S}$ the universe. Then the following statements hold:

$(\mathcal{L}, h)$ and $(\mathcal{B}, \mathcal{E})$ in the form $(\mathcal{L}, h) = \{ \langle (m, n) : \mathcal{C}_{(\mathcal{L}, h)}(m, n), \mathfrak{I}_{(\mathcal{L}, h)}(m, n), \mathcal{E}_{(\mathcal{L}, h)}(m, n) \rangle, (m, n) \in (\hat{A}, \mathcal{S}) \}$ and $(\mathcal{B}, \mathcal{E}) = \{ \langle (m, n) : \mathcal{C}_{(\mathcal{B}, \mathcal{E})}(m, n), \mathfrak{I}_{(\mathcal{B}, \mathcal{E})}(m, n), \mathcal{E}_{(\mathcal{B}, \mathcal{E})}(m, n) \rangle \}$.

- $0_{\hat{A}} = \{ \langle m, 0, 0, 1 \rangle : (m) \in \hat{A} \}$ and $0_{\mathcal{S}} = \{ \langle n, 0, 0, 1 \rangle : n \in \mathcal{S} \}$
- $1_{\hat{A}} = \{ \langle m, 1, 1, 0 \rangle : (m) \in \hat{A} \}$ and $1_{\mathcal{S}} = \{ \langle n, 1, 1, 0 \rangle : n \in \mathcal{S} \}$
- $(\mathcal{L}, h) \subseteq (\mathcal{B}, \mathcal{E})$ iff $\mathcal{C}_{(\mathcal{L})}(m) \leq \mathcal{C}_{(\mathcal{B})}(m), \mathfrak{I}_{(\mathcal{L})}(m) \leq \mathfrak{I}_{(\mathcal{B})}(m), \mathcal{E}_{(\mathcal{L})}(m) \geq \mathcal{E}_{(\mathcal{B})}(m)$
 $\mathcal{C}_{(h)}(n) \leq \mathcal{C}_{(\mathcal{E})}(n), \mathfrak{I}_{(h)}(n) \leq \mathfrak{I}_{(\mathcal{E})}(n), \mathcal{E}_{(h)}(n) \geq \mathcal{E}_{(\mathcal{E})}(n)$
- $(\hat{A}, \mathcal{S})^{\mathcal{E}} = \{ \langle m, \mathcal{E}_{\hat{A}}(m), 1 - \mathfrak{I}_{\hat{A}}(m), \mathcal{C}_{\hat{A}}(m) \rangle : m \in \hat{A}, \langle n, \mathcal{E}_{\mathcal{S}}(n), 1 - \mathfrak{I}_{\mathcal{S}}(n), \mathcal{C}_{\mathcal{S}}(n) \rangle : n \in \mathcal{S} \}$
- $(\mathcal{L}, h) \cap (\mathcal{B}, \mathcal{E}) = \{ \langle \mathcal{C}_{(\mathcal{L})}(m) \wedge \mathcal{C}_{(\mathcal{B})}(m), \mathfrak{I}_{(\mathcal{L})}(m) \wedge \mathfrak{I}_{(\mathcal{B})}(m), \mathcal{E}_{(\mathcal{L})}(m) \vee \mathcal{E}_{(\mathcal{B})}(m) \rangle, \langle \mathcal{C}_{(h)}(n) \wedge \mathcal{C}_{(\mathcal{E})}(n), \mathfrak{I}_{(h)}(n) \wedge \mathfrak{I}_{(\mathcal{E})}(n), \mathcal{E}_{(h)}(n) \vee \mathcal{E}_{(\mathcal{E})}(n) \rangle : (m, n) \in (\hat{A}, \mathcal{S}) \}$
- $(\mathcal{L}, h) \cup (\mathcal{B}, \mathcal{E}) = \{ \langle \mathcal{C}_{(\mathcal{L})}(m) \vee \mathcal{C}_{(\mathcal{B})}(m), \mathfrak{I}_{(\mathcal{L})}(m) \vee \mathfrak{I}_{(\mathcal{B})}(m), \mathcal{E}_{(\mathcal{L})}(m) \wedge \mathcal{E}_{(\mathcal{B})}(m) \rangle, \langle \mathcal{C}_{(h)}(n) \vee \mathcal{C}_{(\mathcal{E})}(n), \mathfrak{I}_{(h)}(n) \vee \mathfrak{I}_{(\mathcal{E})}(n), \mathcal{E}_{(h)}(n) \wedge \mathcal{E}_{(\mathcal{E})}(n) \rangle : (m, n) \in (\hat{A}, \mathcal{S}) \}$

4.4 Example Let $\hat{A} = \{b, m, r\}$ and $\mathcal{S} = \{l, \mathcal{F}, \lambda\}$ be the two universal sets. Then, the equivalence relation between the two sets are $(\hat{A}, \mathcal{S})/\mathcal{R} = \{ \{ \{b\}, \{\lambda\} \}, \{ \{m\}, \{\mathcal{F}\} \}, \{ \{r\}, \{l\} \} \}$. Let $(\mathcal{E}, \mathcal{P}) = \{ \{ \langle b, (0.6, 0.7, 0.8) \rangle, \langle m, (0.1, 0.3, 0.5) \rangle, \langle r, (0.1, 0.3, 0.8) \rangle \}, \{ \langle l, (0.2, 0.4, 0.5) \rangle, \langle \mathcal{F}, (0.2, 0.4, 0.9) \rangle, \langle \lambda, (0.5, 0.6, 0.7) \rangle \} \}$ be a subset of $(\hat{A}, \mathcal{S})$. Now, $\mathcal{N}\mathcal{H}\mathcal{B}^{\text{Low}}(\mathcal{E}, \mathcal{P}) = \{ \langle \{ \{b\}, \{\lambda\} \}, (0.5, 0.6, 0.8) \rangle, \langle \{ \{m\}, \{\mathcal{F}\} \}, (0.1, 0.3, 0.9) \rangle, \langle \{ \{r\}, \{l\} \}, (0.1, 0.3, 0.8) \rangle \}$, $\mathcal{N}\mathcal{H}\mathcal{B}_{\text{supp}}(\mathcal{E}, \mathcal{P}) = \{ \langle \{ \{b\}, \{\lambda\} \}, (0.6, 0.7, 0.7) \rangle, \langle \{ \{m\}, \{\mathcal{F}\} \}, (0.2, 0.4, 0.5) \rangle, \langle \{ \{r\}, \{l\} \}, (0.2, 0.4, 0.5) \rangle \}$, $\mathcal{N}\mathcal{H}\mathcal{B}^{\text{bou}} = \{ \langle \{ \{b\}, \{\lambda\} \}, (0.6, 0.4, 0.7) \rangle, \langle \{ \{m\}, \{\mathcal{F}\} \}, (0.2, 0.4, 0.5) \rangle, \langle \{ \{r\}, \{l\} \}, (0.2, 0.4, 0.5) \rangle \}$. Then the $\mathcal{N}\mathcal{H}\mathcal{B}\check{\mathcal{T}}$ $\psi_{\mathcal{R}}(\mathcal{E}, \mathcal{P}) = \{ (0_{\hat{A}}, 0_{\mathcal{S}}), (1_{\hat{A}}, 1_{\mathcal{S}}), \{ \langle \{ \{b\}, \{\lambda\} \}, (0.5, 0.6, 0.8) \rangle, \langle \{ \{m\}, \{\mathcal{F}\} \}, (0.1, 0.3, 0.9) \rangle, \langle \{ \{r\}, \{l\} \}, (0.1, 0.3, 0.8) \rangle \}, \{ \langle \{ \{b\}, \{\lambda\} \}, (0.6, 0.7, 0.7) \rangle, \langle \{ \{m\}, \{\mathcal{F}\} \}, (0.2, 0.4, 0.5) \rangle, \langle \{ \{r\}, \{l\} \}, (0.2, 0.4, 0.5) \rangle \}$.

$(0.2, 0.4, 0.5)>\}, \{<(\{6\}, \{\lambda\}), (0.6, 0.4, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.2, 0.4, 0.5)>, <(\{x\}, \{\ell\}), (0.2, 0.4, 0.5)>\}$ is a $\mathcal{N}\mathcal{M}\mathcal{B}\mathcal{T}$ on $(\hat{A}, \mathfrak{B})$. Let $\mathcal{B} = \{<(\{6\}, \{\lambda\}), (0.1, 0.5, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.6, 0.7, 0.2)>, <(\{x\}, \{\ell\}), (0.5, 0.6, 0.1)>\}$. Then the $\mathcal{N}\mathcal{M}\mathcal{B}\mathcal{T}$ $\mathcal{B}_3(\mathfrak{B}, \mathfrak{p}) = \{(0.4, 0.8), (1.4, 1.8), \{<(\{6\}, \{\lambda\}), (0.5, 0.6, 0.8)>, <(\{m\}, \{\mathcal{P}\}), (0.1, 0.3, 0.9)>, <(\{x\}, \{\ell\}), (0.1, 0.3, 0.8)>\}, \{<(\{6\}, \{\lambda\}), (0.6, 0.7, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.2, 0.4, 0.5)>, <(\{x\}, \{\ell\}), (0.2, 0.4, 0.5)>\}, \{<(\{6\}, \{\lambda\}), (0.1, 0.5, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.6, 0.7, 0.2)>, <(\{x\}, \{\ell\}), (0.5, 0.6, 0.1)>\}, \{<(\{6\}, \{\lambda\}), (0.1, 0.5, 0.8)>, <(\{m\}, \{\mathcal{P}\}), (0.1, 0.3, 0.9)>, <(\{x\}, \{\ell\}), (0.1, 0.3, 0.8)>\}, \{<(\{6\}, \{\lambda\}), (0.1, 0.5, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.2, 0.4, 0.5)>, <(\{x\}, \{\ell\}), (0.2, 0.4, 0.5)>\}, \{<(\{6\}, \{\lambda\}), (0.5, 0.6, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.2, 0.4, 0.5)>, <(\{x\}, \{\ell\}), (0.2, 0.4, 0.5)>\}, \{<(\{6\}, \{\lambda\}), (0.1, 0.4, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.2, 0.4, 0.5)>, <(\{x\}, \{\ell\}), (0.2, 0.4, 0.5)>\}, \{<(\{6\}, \{\lambda\}), (0.5, 0.6, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.6, 0.7, 0.2)>, <(\{x\}, \{\ell\}), (0.5, 0.6, 0.1)>\}, \{<(\{6\}, \{\lambda\}), (0.6, 0.7, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.2, 0.4, 0.5)>, <(\{x\}, \{\ell\}), (0.2, 0.4, 0.5)>\}, \{<(\{6\}, \{\lambda\}), (0.6, 0.7, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.6, 0.7, 0.2)>, <(\{x\}, \{\ell\}), (0.5, 0.6, 0.1)>\}, \{<(\{6\}, \{\lambda\}), (0.6, 0.5, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.2, 0.4, 0.5)>, <(\{x\}, \{\ell\}), (0.2, 0.4, 0.5)>\}, \{<(\{6\}, \{\lambda\}), (0.6, 0.4, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.2, 0.4, 0.5)>, <(\{x\}, \{\ell\}), (0.2, 0.4, 0.5)>\}, \{<(\{6\}, \{\lambda\}), (0.6, 0.5, 0.7)>, <(\{m\}, \{\mathcal{P}\}), (0.6, 0.7, 0.2)>, <(\{x\}, \{\ell\}), (0.5, 0.6, 0.1)>\}\}.$

5. Two Player Game Algorithm

CASE I : Players Won the Game

Step 1: Find the equivalence relation of $(\mathfrak{C}, \mathfrak{P})$ relative to the attributes in $\mathfrak{B}$.

Step 2: Evaluate the Lower approximation, Upper approximation and we get the boundary region to frame the Micro Binary Topological Space.

Step 3: Subtract the features of attribute A from $\mathfrak{B}$ and get a low, high and boundary region from $\mathfrak{B}$ -A obtain the Micro Binary Topological Space.

Step 4: Compare the topologies in step 2 and step 3. If both are not equal then that corresponding attribute A is included as a key factor and proceed to steps. If the topologies are not equal then the corresponding attribute A cannot be considered as a key factor to proceed to step 5.

Step 5: Repeat step 2 through step 4 to get the key factors.

Step 6: Find the source from step 1 to step 5.

Do the same for **CASE II** "Players Not Won the Game".

6. Application of Neutrosophic Micro Binary Topological Space

In this example, I utilize the $\mathcal{N}\mathcal{M}\mathcal{B}\mathcal{T}$ s to analyze the topological reaction of qualities in the data set to determine the key factors of "players losing the game".

I have collected the data from the Google form, and the selected players filled in the details through the Google form. The maximum number of reasons, like Lack of Effort (LoE), Reaction Time (RT), Lack of Practice (LoP), Self-Confidence Level (SCL), and Fear of Failure (FoF), are chosen from the players. Then the collected data is given below.

I look at the following table on the different traits of players A, including Lack of Effort (LoE), the data set is based on Reaction Time (RT), Lack of Practice (LoP), Self-Confidence Level (SCL) and

Fear of Failure (FoF). I can determine the crucial element that prevented players from winning the match from this data set.

Here $\mathcal{O} = \{Ps_1, Ps_2, Ps_3\}$ and $\mathcal{P} = \{N_{q1}, N_{q2}\}$ be the set of players where

$(\mathcal{O}, \mathcal{P}) = (\{\mathcal{O}_i\}, \{\mathcal{P}_j\})_{i=1,2,3, j=1,2} (\{Ps_i\}, \{N_{qj}\})$ and $\mathcal{F} = \{LoE, RT, LoP, SCL, FoF\}$ the set of factors that may lead to not won the game. Table-1 gives the information of the set of players, Table-2 the combination of players and Table -3 the players are denoted using the $\mathcal{NMBs}$.

PLAYERS	Lack of Effort	Reaction Time	Lack of Practice	Self-Confidence Level	Fear of Failure	Decision
$(\{Ps_1\}, \{N_{q1}\})$	Very High	Yes	No	No	High	LOSE
$(\{Ps_1\}, \{N_{q2}\})$	High	No	Yes	Yes	Very High	WIN
$(\{Ps_2\}, \{N_{q1}\})$	Very High	Yes	No	No	High	LOSE
$(\{Ps_2\}, \{N_{q2}\})$	High	No	No	Yes	Very High	WIN
$(\{Ps_3\}, \{N_{q1}\})$	Very High	Yes	Yes	No	High	WIN
$(\{Ps_3\}, \{N_{q2}\})$	High	No	No	Yes	Very High	LOSE

Table - 1 : Player's possible attributes

$(\{Ps_i\} \times \{N_{qj}\})$	N_{q1}	N_{q2}
Ps_1	$(\{Ps_1\}, \{N_{q1}\})$	$(\{Ps_1\}, \{N_{q2}\})$
Ps_2	$(\{Ps_2\}, \{N_{q1}\})$	$(\{Ps_2\}, \{N_{q2}\})$
Ps_3	$(\{Ps_3\}, \{N_{q1}\})$	$(\{Ps_3\}, \{N_{q2}\})$

Table - 2 : Combination of Players

Pair of players	Neutrosophic Micro Binary Sets
$(\{Ps_1\}, \{N_{q1}\})$	(0.6, 0.5, 0.2)
$(\{Ps_1\}, \{N_{q2}\})$	(0.4, 0.7, 0.9)
$(\{Ps_2\}, \{N_{q1}\})$	(0.1, 0.3, 0.4)
$(\{Ps_2\}, \{N_{q2}\})$	(0.5, 0.1, 0.1)
$(\{Ps_3\}, \{N_{q1}\})$	(0.3, 0.9, 0.3)
$(\{Ps_3\}, \{N_{q2}\})$	(0.7, 0.2, 0.6)

Table - 3 : Neutrosophic Micro Binary Sets

Here $\mathfrak{O} = \{Ps_1, Ps_2, Ps_3\}$ $\mathfrak{P} = \{N_{q1}, N_{q2}\}$ be the set of players and $\mathfrak{B} = \{\text{Lack of Effort, Reaction Time, Lack of Practice, Self-confidence Level, Fear of Failure}\}$ the set of factors that may lead to not won the game. Briefly, the set is identified by $\mathfrak{B} = \{\text{LoE, RT, LoP, SCL, FoF}\}$.

CASE - 1 : PLAYERS WON THE GAME

Let $\mathfrak{O} = \{Ps_1, Ps_2, Ps_3\}$ and $\mathfrak{P} = \{N_{q1}, N_{q2}\}$ be the set of players. Let $(\mathfrak{O}, \mathfrak{P})/\gamma(\mathfrak{B}) = \{(\{Ps_1, Ps_2\}, \{N_{q1}\}), (\{Ps_1\}, \{N_{q2}\}), (\{Ps_3\}, \{N_{q1}\}), (\{Ps_2, Ps_3\}, \{N_{q2}\})\}$ be an equivalence relation on $(\mathfrak{O}, \mathfrak{P})$ and $(\mathfrak{k}, \mathfrak{z}) = \{(\{Ps_1\}, \{N_{q2}\}), (\{Ps_2\}, \{N_{q2}\}), (\{Ps_3\}, \{N_{q1}\})\}$ be the set of players won the game. Here, $N_{\mathfrak{B}}^{\text{low}}(\mathfrak{B})(\mathfrak{k}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}$.

$N_{\mathfrak{B}}^{\text{upp}}(\mathfrak{B})(\mathfrak{k}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}$.

$N_{\mathfrak{B}}^{\text{ts}}(\mathfrak{B})(\mathfrak{k}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)>\}$. Then the $N_{\mathfrak{B}}^{\text{ts}}$ is given by

$\tau_{\gamma(\mathfrak{B})}(\mathfrak{k}, \mathfrak{z}) = \{(0_{\mathfrak{O}}, 0_{\mathfrak{P}}), (1_{\mathfrak{O}}, 1_{\mathfrak{P}}), \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)>\}\}$.

$\partial = \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.1, 0.1)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)>\}$.

Therefore, the $N_{\mathfrak{B}}^{\text{ts}}$ is given by

$\partial_{\gamma(\mathfrak{B})}(\mathfrak{k}, \mathfrak{z}) = \{(0_{\mathfrak{O}}, 0_{\mathfrak{P}}), (1_{\mathfrak{O}}, 1_{\mathfrak{P}}), \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.1, 0.1)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.1, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.1, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.2)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.2)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.1, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.7, 0.1)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.9, 0.2)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.2)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.7, 0.1)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)>, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.9, 0.2)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.5)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.2)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.3, 0.1)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.5)>, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)>\}\} \longrightarrow (1).$

Step 1. When $\mathfrak{B}$'s "Lack of Effort (LoE)" is eliminated, we have

$(\mathfrak{O}, \mathfrak{P})/\gamma(\mathfrak{B} - \text{LoE})(\mathfrak{k}, \mathfrak{z}) = \{(\{Ps_1, Ps_2\}, \{N_{q1}\}), (\{Ps_1\}, \{N_{q2}\}), (\{Ps_3\}, \{N_{q1}\}), (\{Ps_2, Ps_3\}, \{N_{q2}\})\}$ and $(\mathfrak{k}, \mathfrak{z}) = \{(\{Ps_1\}, \{N_{q2}\}), (\{Ps_2\}, \{N_{q2}\}), (\{Ps_3\}, \{N_{q1}\})\}$, here $N_{\mathfrak{B}}^{\text{low}}(\mathfrak{B} - \text{LoE})(\mathfrak{k}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}$.

$N_{\mathfrak{B}}^{\text{upp}}(\mathfrak{B} - \text{LoE})(\mathfrak{k}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}$.

$N_{\mathfrak{B}}^{\text{ts}}(\mathfrak{B} - \text{LoE})(\mathfrak{k}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)>\}$. Then the $N_{\mathfrak{B}}^{\text{ts}}$ is given by

$\tau_{\gamma(\mathfrak{B} - \text{LoE})}(\mathfrak{k}, \mathfrak{z}) = \{(0_{\mathfrak{O}}, 0_{\mathfrak{P}}), (1_{\mathfrak{O}}, 1_{\mathfrak{P}}), \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)>\}\}$.

$\partial = \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.1, 0.1)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)>\}$.

Therefore, the $N_{\mathfrak{B}}^{\text{ts}}$ is given by

$\partial_{\gamma(\mathfrak{B} - \text{LoE})}(\mathfrak{k}, \mathfrak{z}) = \{(0_{\mathfrak{O}}, 0_{\mathfrak{P}}), (1_{\mathfrak{O}}, 1_{\mathfrak{P}}), \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)>\}, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)>, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)>, <(\{Ps_3\}, \{N_{q1}\}), (0.3,$

$\{0.9, 0.3\}\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)\rangle, \\ \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.1, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)\rangle, \{<(\{Ps_1\}, \\ \{N_{q2}\}), (0.4, 0.1, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.3)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, \\ 0.1, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, \\ <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.1, 0.9)\rangle, <(\{Ps_2\}, \\ \{N_{q2}\}), (0.6, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.6, \\ 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.7, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, \\ <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.9, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)\rangle, <(\{Ps_3\}, \\ \{N_{q1}\}), (0.3, 0.9, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.7, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.6, \\ 0.9, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.3)\rangle, \\ \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.2)\rangle, \{<(\{Ps_1\}, \\ \{N_{q2}\}), (0.5, 0.3, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)\rangle\}.$

Hence $\partial_{\Upsilon(\mathfrak{B}-LoE)}(\mathfrak{t}, \mathfrak{z}) = \partial_{\Upsilon(\mathfrak{B})}(\mathfrak{t}, \mathfrak{z})$.

Step 2. After removing the characteristic "Reaction Time (RT)" from $\mathfrak{B}$ then

$(\mathfrak{O}, \mathfrak{P})/\Upsilon(\mathfrak{B}-RT) = \{(\{Ps_1, Ps_2\}, \{N_{q1}\}), (\{Ps_1\}, \{N_{q2}\}), (\{Ps_3\}, \{N_{q1}\}), (\{Ps_2, Ps_3\}, \{N_{q2}\})\}$ be an equivalence relation on $(\mathfrak{O}, \mathfrak{P})$ and $(\mathfrak{t}, \mathfrak{z}) = \{(\{Ps_1\}, \{N_{q2}\}), (\{Ps_2\}, \{N_{q2}\}), (\{Ps_3\}, \{N_{q1}\})\}$ be the set of players won the game. Here, $N_{\mathfrak{H}}\check{\mathfrak{T}}s^{low}(\mathfrak{B}-RT)(\mathfrak{t}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)\rangle\}$.

$N_{\mathfrak{H}}\check{\mathfrak{T}}s^{upp}(\mathfrak{B}-RT)(\mathfrak{t}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)\rangle\}$.

$N_{\mathfrak{H}}\check{\mathfrak{T}}s^{bou}(\mathfrak{B}-RT)(\mathfrak{t}, \mathfrak{z}) = \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)\rangle\}$. Then the $N_{\mathfrak{H}}\check{\mathfrak{T}}s$ is given by

$\Upsilon_{(\mathfrak{B}-RT)}(\mathfrak{t}, \mathfrak{z}) = \{(0\mathfrak{O}, 0\mathfrak{P}), (1\mathfrak{O}, 1\mathfrak{P}), \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)\rangle, \\ \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)\rangle, \\ \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)\rangle\}$.

$\partial = \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.1, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)\rangle\}$, and the $N_{\mathfrak{M}}\check{\mathfrak{T}}s$ is given by

$\partial_{\Upsilon(\mathfrak{B}-RT)}(\mathfrak{t}, \mathfrak{z}) = \{(0\mathfrak{O}, 0\mathfrak{P}), (1\mathfrak{O}, 1\mathfrak{P}), \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)\rangle, \\ \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)\rangle, \\ \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)\rangle, \\ \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.1, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)\rangle, \{<(\{Ps_1\}, \\ \{N_{q2}\}), (0.4, 0.1, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.5, 0.1, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.3)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, \\ 0.1, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, \\ <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.1, 0.9)\rangle, <(\{Ps_2\}, \\ \{N_{q2}\}), (0.6, 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.1, 0.3)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.6, \\ 0.2, 0.6)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.9, 0.3)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.7, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.6)\rangle, \\ <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.9, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.7, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)\rangle, <(\{Ps_3\}, \\ \{N_{q1}\}), (0.3, 0.9, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.5, 0.7, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.1)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.6, \\ 0.9, 0.2)\rangle, \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.6, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.3)\rangle, \\ \{<(\{Ps_1\}, \{N_{q2}\}), (0.4, 0.3, 0.9)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.3, 0.5, 0.2)\rangle, \{<(\{Ps_1\}, \\ \{N_{q2}\}), (0.5, 0.3, 0.1)\rangle, <(\{Ps_2\}, \{N_{q2}\}), (0.7, 0.2, 0.5)\rangle, <(\{Ps_3\}, \{N_{q1}\}), (0.6, 0.5, 0.2)\rangle\}.$

Hence $\partial_{\Upsilon(\mathfrak{B}-RT)}(\mathfrak{t}, \mathfrak{z}) = \partial_{\Upsilon(\mathfrak{B})}(\mathfrak{t}, \mathfrak{z})$.

$\langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.1, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.1, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.5, 0.7, 0.1) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.6, 0.9, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.1) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.5, 0.7, 0.1) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.1) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.6, 0.9, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.3, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.5, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.3, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.5, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.5, 0.3, 0.1) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.6, 0.5, 0.2) \rangle\}.$

Hence $\partial_{Y(\mathfrak{B}-SCL)}(\mathfrak{t}, \mathfrak{z}) = \partial_{Y(\mathfrak{B})}(\mathfrak{t}, \mathfrak{z})$.

Step 5. When the feature “Fear of Failure (FoF)” is separated from $\mathfrak{B}$ then

$(\mathfrak{O}, \mathfrak{P})/_{Y(\mathfrak{B}-FoF)} = \{(\{Ps_1, Ps_2\}, \{N_{q_1}\}), (\{Ps_1\}, \{N_{q_2}\}), (\{Ps_3\}, \{N_{q_1}\}), (\{Ps_2, Ps_3\}, \{N_{q_2}\})\}$ be an equivalence relation on $(\mathfrak{O}, \mathfrak{P})$ and $(\mathfrak{t}, \mathfrak{z}) = \{(\{Ps_1\}, \{N_{q_1}\}), (\{Ps_2\}, \{N_{q_2}\}), (\{Ps_3\}, \{N_{q_1}\})\}$ be the set of students won the game. Here, $N_{\mathfrak{H}}\check{T}^{low}(\mathfrak{B} - FoF)(\mathfrak{t}, \mathfrak{z}) = \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.5, 0.1, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.3) \rangle$.

$N_{\mathfrak{H}}\check{T}^{upp}(\mathfrak{B} - FoF)(\mathfrak{t}, \mathfrak{z}) = \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.1) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.3) \rangle$.

$N_{\mathfrak{H}}\check{T}^{bou}(\mathfrak{B} - FoF)(\mathfrak{t}, \mathfrak{z}) = \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.3, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.1, 0.3) \rangle$, and the $N_{\mathfrak{H}}\check{T}$ s is given by

$T_{Y(\mathfrak{B}-FoF)}(\mathfrak{t}, \mathfrak{z}) = \{(0\mathfrak{O}, 0\mathfrak{P}), (1\mathfrak{O}, 1\mathfrak{P}), \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.5, 0.1, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.1) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.3, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.1, 0.3) \rangle\}$.

$\partial = \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.5, 0.1, 0.1) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.6, 0.5, 0.2) \rangle$ and the $N_{\mathfrak{M}}\check{T}$ s is given by

$\partial_{Y(\mathfrak{B}-FoF)}(\mathfrak{t}, \mathfrak{z}) = \{(0\mathfrak{O}, 0\mathfrak{P}), (1\mathfrak{O}, 1\mathfrak{P}), \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.5, 0.1, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.1) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.3, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.1, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.5, 0.1, 0.1) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.6, 0.5, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.1, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.5, 0.1, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.5, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.1, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.5, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.1, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.1, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.5, 0.7, 0.1) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.6) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.6, 0.9, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.7, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.1) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.9, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.5, 0.7, 0.1) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.1) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.6, 0.9, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.3, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.6, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.5, 0.3) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.4, 0.3, 0.9) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.3, 0.5, 0.2) \rangle, \langle \{Ps_1\}, \{N_{q_2}\} \rangle, (0.5, 0.3, 0.1) \rangle, \langle \{Ps_2\}, \{N_{q_2}\} \rangle, (0.7, 0.2, 0.5) \rangle, \langle \{Ps_3\}, \{N_{q_1}\} \rangle, (0.6, 0.5, 0.2) \rangle\}$.

Hence $\partial_{Y(\mathfrak{B}-FoF)}(\mathfrak{t}, \mathfrak{z}) = \partial_{Y(\mathfrak{B})}(\mathfrak{t}, \mathfrak{z})$

CASE - 2 : PLAYERS NOT WON THE GAME

Let $\mathfrak{O} = \{Ps_1, Ps_2, Ps_3\}$, $\mathfrak{P} = \{N_{q_1}, N_{q_2}\}$ be the set of players. Let $(\mathfrak{O}, \mathfrak{P})/_{Y(\mathfrak{B})} = \{(\{Ps_1, Ps_2\}, \{N_{q_1}\}), (\{Ps_1\}, \{N_{q_2}\}), (\{Ps_3\}, \{N_{q_1}\}), (\{Ps_2, Ps_3\}, \{N_{q_2}\})\}$ be an equivalence relation on $(\mathfrak{O}, \mathfrak{P})$ and $(\mathfrak{t}, \mathfrak{z}) = \{(\{Ps_1\}, \{N_{q_1}\}), (\{Ps_2\}, \{N_{q_1}\}), (\{Ps_3\}, \{N_{q_2}\})\}$ be the set of players not won the game. Here, $N_{\mathfrak{H}}\check{T}^{low}(\mathfrak{B})(\mathfrak{t}, \mathfrak{z}) = \langle \{Ps_1\}, \{N_{q_1}\} \rangle, (0.1, 0.3, 0.4) \rangle, \langle \{Ps_2\}, \{N_{q_1}\} \rangle, (0.1, 0.3, 0.4) \rangle, \langle \{Ps_3\}, \{N_{q_2}\} \rangle, (0.3, 0.1, 0.9) \rangle$.

Hence $\partial_{Y(\mathfrak{B}-\text{FoF})}(\mathfrak{L}, \mathfrak{Z}) = \partial_{Y(\mathfrak{B})}(\mathfrak{L}, \mathfrak{Z})$.

7. Observation: The two aforementioned examples demonstrate that "Lack of Practice (LoP)" are the main reasons why players lose games.

8. Conclusions

Through the application of $\mathfrak{NM}\mathfrak{b}\mathfrak{T}$ s to the sports industry, I have determined that "Lack of Practice (LoP)" are the primary causes of players' defeats. When playing games, gamers encounter both wins and losses, with losing being an essential component of the whole experience. Losing can be disheartening and discouraging, but it also presents invaluable opportunities for growth. Players might find areas where they might improve their abilities, strategies, or decision-making processes by researching the reasons behind a loss. You can succeed in the future by changing your perspective and seeing failures as opportunities for growth rather than as setbacks. To succeed in overcoming barriers and working toward game fulfillment, athletes may develop resilience, adaptability, and tenacity. It can be applied in many different kinds of domains, including advertising, commerce, health care, and beyond.

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References

1. M. Abdel-Basset, A. Gamal, R.K. Chakraborty, A new hybrid multi-criteria decision-making approach for location selection of sustainable offshore wind energy stations. A case study. *Journal of Cleaner Production*, 280 (2021), 124462.
2. M. Abdel-Basset, G. Manogaran, and M. Mohamed, A neutrosophic theory based security approach for fog and mobile-edge computing. *Computer Networks*, 157 (2019), pp. 122 - 132.
3. R. Arokiarani, S. Dhavaseelan, S. Jafari, M. Parimal, On Some New Notions and Functions in Neutrosophic Topological Spaces, *Neutrosophic Sets and System*, 16 (2017), pp. 16 - 19.
4. K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986), pp. 87 - 96.
5. Chandrasekar, On Micro Topological Spaces, *Journal of New Theory*, 26 (2019), pp. 23 - 31.
6. S. Ganesan and S. Jafari, NEUTROSOPHIC MICRO TOPOLOGICAL SPACES, *Annals of Communication in Mathematics*, 5 (3) (2022), pp. 153 - 160.
7. W.L. Gau, D.J. Buehrer, Vague sets, *IEEE Trans. Systems Man and Cybernet*, 23 (2) (1993), pp. 610 - 614.
8. G. Hari Siva Annam, J. Jasmine Elizabeth, Cognition of Nano Binary Topological Spaces, *Global Journal of Pure and Applied Mathematics*. 15 (6) (2019), pp. 1045 - 1054.
9. M. Josephine et.al., An Application of Micro Topological Spaces with Decision Making Problem in Medical Events, 21 (11) (2022), pp. 1790 - 1805.
10. A. Jayalakshmi and C. Janaki, *Int.Journal of engineering Research*, New class sets in nano topological fields and application in Medical Diagnosis, 12 (16) (2017), pp. 5894 - 5899.
11. M. Lellis Thivagar and Carmel Richard, In nano-forms of weak open sets, *International journal for the establishment of statistics*, *Journal of New Theory*, 26 (2013), pp. 31 - 37.
12. M. Lellis Thivagar, S. Jafari, V. Sudha Devi and V. Antonysamy, A novel approach to nano topology via neutrosophic sets, *Neutrosophic Sets and System*, 20 (2018), pp. 86 - 94.
13. A. Mary Margaret and M. Trinita Pricilla, Application of Neutrosophic Vague Nano Topological Spaces, *Neutrosophic Sets and System*, 39 (2021), pp. 54 - 69.
14. S. Nithyanandha Jothi and P. Thangavelu, Topology between two sets, *Journal of Mathematical Sciences and Computer Applications*, 1 (3) (2011), pp. 95 - 107.

15. S. Nithyanandha Jothi and P. Thangavelu, On binary topological spaces, *Pacific-Asian Journal of Mathematics*, 5 (2) (2011), pp. 133 - 138.
16. M. Parimala, M. Karthika, S. Jafari, F. Smarandache and R. Udhayakumar, Neutrosophic Nano ideal structure, *Neutrosophic Sets and System*, 24 (2019), pp. 70 - 76.
17. Z. Pawlak, Rough Sets, *International Journal of Computer and Information Sciences*, (11) (1982), pp. 341 - 356.
18. M. Ramachandran and A. Stephan Antony Raj was developed by Intuitionistic Fuzzy Nano Topological Space: Theory and Applications, *International Journal of Research in Science*, 4 (1) (2017), pp. 1 - 6.
19. C. Sangeetha and G. Sindhu, An Introduction to Micro Binary Topological Spaces, *International Journal of Creative Research Thoughts (IJCRT)*, 12 (3) (2024), pp. f885 - f890.
20. A. A. Salama, S. A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, *IOSR Journal of Mathematics*, 3 (4) (2012), pp. 31 - 35.
21. S.S. Surekha, J. Elekhia and G.Sindhu, A study on Neutrosophic Binary Topological space, *Stochastic Modelling and Applications*, 26 (3) (2022), pp. 479 - 486.
22. F. Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth: American Research Press, (1998).
23. F. Smarandache, Neutrosophic TwoFold Algebra, Plithogenic Logic and Computation, 1 (2024), pp. 11 - 15.
24. F. Smarandache, M. Mohamed and M. Voskoglou, Evaluating Blockchain Cybersecurity Based on Tree Soft and Opinion Weight Criteria Method under Uncertainty Climate, *HyperSoft Set Methods in Engineering*, 1 (2024), pp. 1 - 10.
25. A. Vadivel, C. John Sundar, K. Kirubadevi, S. Tamilselvan, More on Neutrosophic Nano Open Sets, *International Journal of Neutrosophic Science*, 18 (4) (2022), pp. 204 - 222.
26. L.A. Zadeh, Fuzzy sets. *Information and Control*, 8 (1965), pp. 338 - 353.

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