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NeutroGeometry and NeutroAlgebra: A Mathematical Framework for the Sustainable Development of College Music Education in the New Era

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Abstract-This paper develops a precise and rigorous mathematical framework for NeutroGeometry and NeutroAlgebra, aiming to support the sustainable development of college music education in the new era. These theories generalize classical algebraic and geometric structures by introducing partial truth, indeterminacy, and partial falsehood, reflecting the dynamic and flexible nature of musical learning and performance. We define the fundamental laws, establish key equations, and provide clear examples. Additionally, we propose innovative extensions to these concepts, showing how they can transform the evaluation and growth of college music education to be more adaptive, creative, and sustainable.

Keywords-Sustainable development, college music education, NeutroGeometry, NeutroAlgebra, neutrosophic triplet, partial truth, indeterminacy, innovative teaching

1. Introduction

College music education often follows strict rules, focusing on whether students perform correctly according to musical standards. However, music in the real world is more complex, involving creativity, improvisation, and personal expression that don't always fit into a simple "right or wrong" framework. To support sustainable development in music education, we need a mathematical model that can capture these complexities clearly and quantitatively.

This paper uses the NeutroGeometry and NeutroAlgebra, concepts introduced by Florentin Smarandache, as a new way to understand and improve college music education. Unlike traditional geometry and algebra, where everything is assumed to be completely true, NeutroGeometry and NeutroAlgebra consider three aspects for any idea or operation: Truth (T), Indeterminacy (I), and Falsehood (F). These are numbers between 0 and 1, and they always add up to 3 ($T + I + F \leq 3$) [1]. This approach reflects the real world, where musical choices, like a student's improvisation, might be partly correct, partly uncertain, and partly a creative "mistake" made on purpose.

NeutroGeometry and NeutroAlgebra are perfect for music education because they allow us to model situations where teaching methods, student performances, or creative decisions are not fully true or false but exist in between. For example, a student's unique interpretation of a piece might follow some rules (truth), leave room for debate due to style

(indeterminacy), and break other rules for artistic effect (falsehood). By using these tools, educators can better understand and encourage creativity and critical thinking.
The goals of this paper are:
To explain the mathematical definitions and equations of NeutroGeometry and NeutroAlgebra clearly.
To show how these concepts apply to music education with practical examples.
To suggest new ways to use these frameworks to make music education more sustainable, creative, and suited to modern needs.

Traditional geometry and algebra assume everything is 100% true, which doesn't work well for creative fields like music that involve uncertainty and partial truths [2]. NeutroGeometry and NeutroAlgebra were created to fill this gap, offering a flexible way to handle situations where ideas or rules are only partially true or uncertain [1, 3]. While these ideas have been used in other fields, they have not been widely applied to college music education. This paper aims to change that by showing how NeutroGeometry and NeutroAlgebra can help build a more innovative and sustainable approach to teaching music.

Smarandache invented in general the NeutroStructure and AntiStructure [4], and as particular cases the NeutroGeometry and AntiGeometry to think about geometry differently [1]. Unlike regular geometry, where everything is completely true, NeutroGeometry says some rules are only partly true, partly unclear, and partly false. This fits real life, like when a music student plays some notes right but changes others to be creative. It helps teachers give fair feedback, balancing skill and imagination in music classes.

Smarandache also made neutrosophic triplets, using truth (T), uncertainty (I), and falsehood (F), where T + I + F ≤ 3 [1, 5]. In music, this helps teachers judge a student's work not just as right or wrong but as a mix of good skills, creative ideas, and mistakes to fix. For example, they can say which parts are strong and which need practice, making lessons more fun and helpful for students.

2. Mathematical Framework for NeutroGeometry and NeutroAlgebra

A neutrosophic triplet describes any mathematical statement, operation, or axiom:

$$\langle T, I, F\rangle, T, I, F \in [0,1], T + I + F \leq 3$$

where:
T is the degree of truth (certainty)
I is the degree of indeterminacy (uncertainty)
F is the degree of falsehood (falsity)
These triplets generalize the binary logic (0 or 1) by adding continuous degrees of each property, reflecting partial truth and partial falsehood that arise in real-world structures, including music education.

2.1. NeutroAlgebra

Let $(S, \star)$ be an algebraic structure.

Classical axiom

$$\langle 1,0,0\rangle$$

NeutroAxiom

$$\langle T, I, F\rangle, 0 < T, I, F < 1$$

AntiAxiom

$$\langle 0,0,1\rangle$$

2.1.1. NeutroClosure

The operation $\star$ is neutrosophically closed on S if:

$$\forall a, b \in S, a \star b \in S \text{ with degree } \langle T_{\text{closure}}, I_{\text{closure}}, F_{\text{closure}}\rangle$$

Example in music: combining two rhythm patterns a, b.

$T_{\text{closure}} = 0.9$ (mostly results in an acceptable pattern)

$I_{\text{closure}} = 0.05$ (some rhythmic ambiguity)

$F_{\text{closure}} = 0.05$ (few combinations yield off-beat results)

2.1.2. NeutroAssociativity

For associativity,

$$(a \star b) \star c = a \star (b \star c)$$

The degree of neutro-associativity:

$$\langle T_{\text{assoc}}, I_{\text{assoc}}, F_{\text{assoc}}\rangle$$

Illustrations:

$T_{\text{assoc}} = 0.7$: mostly true for classical chord progressions.

$I_{\text{assoc}} = 0.2$: in jazz improvisation, associativity of harmonies is ambiguous.

$F_{\text{assoc}} = 0.1$: in experimental music, associativity breaks completely.

2.1.3. Neutroldentity and Inverse Elements

Neutroldentity:

$$\exists e \in S: a \star e = e \star a = a, \text{ degree } \langle T_{\text{id}}, I_{\text{id}}, F_{\text{id}}\rangle$$

Neutrolnverse:

$$\exists b \in S: a \star b = b \star a = e, \text{ degree } \langle T_{\text{inv}}, I_{\text{inv}}, F_{\text{inv}}\rangle$$

Illustration in music:

Identity: silence (rest note) in rhythmic structures.

Inverse: retrograde inversion of a motif.

2.2. NeutroGeometry

In geometry, consider points P, Q, lines L, etc.

Neutrolncidence Axiom

$$\forall P \neq Q, \exists! L \text{ incident with } P, Q, \text{ degree } \langle T_{\text{inc}}, I_{\text{inc}}, F_{\text{inc}}\rangle$$

NeutroParallel Postulate

$$\forall P \notin L, \exists! M \| L \text{ through } P, \text{ degree } \left\langle T_{\text{par}}, I_{\text{par}}, F_{\text{par}}\right\rangle$$

2.3. Structure-Wide Degree Vectors

For a structure with n axioms/properties $E_1, \ldots, E_n$:

$$\bar{T} = \frac{1}{n}\sum_{j=1}^{n} T_j$$
$$\bar{I} = \frac{1}{n}\sum_{j=1}^{n} I_j$$
$$\bar{F} = \frac{1}{n}\sum_{j=1}^{n} F_j$$

The overall neutrosophic vector:
StructureVector $= \langle \bar{T}, \bar{I}, \bar{F} \rangle$

2.4. NeutroSustainability Index for Music Education
A measure for the sustainability of music education performance:

$$\text{SustainabilityIndex } = \bar{T} \cdot (1 - \bar{F}) + \lambda \bar{I}$$

where λ is a weight factor (to reward creative indeterminacy in music performance).

2.5. Explicit Calculation Illustration
Consider 3 statements , harmonic accuracy, rhythmic stability, and expressiveness in a music performance:

Property	T	I	F
Harmonic accuracy	0.8	0.1	0.1
Rhythmic stability	0.6	0.3	0.1
Expressiveness	0.5	0.4	0.1

Calculate:

$$\bar{T} = \frac{0.8 + 0.6 + 0.5}{3} = 0.63$$
$$\bar{I} = \frac{0.1 + 0.3 + 0.4}{3} = 0.27$$
$$\bar{F} = \frac{0.1 + 0.1 + 0.1}{3} = 0.1$$

If $\lambda = 0.5$:
SustainabilityIndex $= 0.63 \cdot (1 - 0.1) + 0.5 \cdot 0.27 = 0.63 \cdot 0.9 + 0.135 = 0.567 + 0.135 = 0.702$
This explicit calculation shows how the framework can be quantitatively implemented in real music education contexts.

2.6. Extensions: NeutroVector Spaces and Transformations
NeutroVector Space V over a field $\mathbb{F}$:
Each vector $v \in V$ has a neutrosophic degree $\langle T_v, I_v, F_v \rangle$.
NeutroLinear Transformation:
A map $T: V \to W$ that preserves linear structure up to neutrosophic degrees:

$$T(\alpha u + \beta v) = \alpha T(u) + \beta T(v),\ \text{degree}\ \langle T_T, I_T, F_T \rangle$$

So, Average degree of truth=

$$\bar{T} = \frac{0.8 + 0.6 + 0.5}{3} = 0.6333$$

Average degree of indeterminacy=

$$\bar{I} = \frac{0.1 + 0.3 + 0.4}{3} = 0.2667$$

Average degree of falsehood=

$$\bar{F} = \frac{0.1 + 0.1 + 0.1}{3} = 0.1$$

Sustainability Index with $\lambda = 0.5$ =

$$\text{SustainabilityIndex} = 0.6333 \cdot (1 - 0.1) + 0.5 \cdot 0.2667 = 0.7033$$

These explicit calculations confirm the theoretical framework, making the model ready for application in real music education performance evaluations.

3. Case Study

Let's consider a scenario: a university music ensemble performs a piece that has three core dimensions:

1 Harmonic accuracy
2 Rhythmic stability
3 Expressiveness

Each dimension is evaluated using the neutrosophic triplet $\langle T, I, F \rangle$.
We collect assessments from 4 instructors for each dimension as :

Harmonic accuracy= $\langle 0.85,0.1,0.05 \rangle$, $\langle 0.9,0.05,0.05 \rangle$,$\langle 0.7,0.2,0.1 \rangle$ $\langle 0.8,0.1,0.1 \rangle$,

Rhythmic stability =$\langle 0.7,0.2,0.1 \rangle$, $\langle 0.6,0.3,0.1 \rangle$, $\langle 0.55,0.35,0.1 \rangle$ $\langle 0.65,0.25,0.1 \rangle$,

Expressiveness = $\langle 0.6,0.3,0.1 \rangle$, $\langle 0.5,0.4,0.1 \rangle$, $\langle 0.45,0.45,0.1 \rangle$ $\langle 0.55,0.35,0.1 \rangle$,

3.1 Numeric Matrix Representation

For easier calculation, we represent these as matrices:

Harmonic Accuracy=

$$H = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.8 & 0.1 & 0.1 \\ 0.85 & 0.1 & 0.05 \\ 0.7 & 0.2 & 0.1 \end{bmatrix}$$

Rhythmic Stability=

$$R = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.65 & 0.25 & 0.1 \\ 0.7 & 0.2 & 0.1 \\ 0.55 & 0.35 & 0.1 \end{bmatrix}$$

Expressiveness=

$$E = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.55 & 0.35 & 0.1 \\ 0.6 & 0.3 & 0.1 \\ 0.45 & 0.45 & 0.1 \end{bmatrix}$$

3.2 Averages

We compute the average neutrosophic triplet for each dimension:

Harmonic Accuracy =

$$\bar{T}_H = \frac{0.9 + 0.8 + 0.85 + 0.7}{4} = 0.8125$$

$$\bar{I}_H = \frac{0.05 + 0.1 + 0.1 + 0.2}{4} = 0.1125$$

$$\bar{F}_H = \frac{0.05 + 0.1 + 0.05 + 0.1}{4} = 0.075$$

Rhythmic Stability =

$$\bar{T}_R = \frac{0.6 + 0.65 + 0.7 + 0.55}{4} = 0.625$$

$$\bar{I}_R = \frac{0.3 + 0.25 + 0.2 + 0.35}{4} = 0.275$$

$$\bar{F}_R = \frac{0.1 + 0.1 + 0.1 + 0.1}{4} = 0.1$$

Expressiveness =

$$\bar{T}_E = \frac{0.5 + 0.55 + 0.6 + 0.45}{4} = 0.525$$

$$\bar{I}_E = \frac{0.4 + 0.35 + 0.3 + 0.45}{4} = 0.375$$

$$\bar{F}_E = \frac{0.1 + 0.1 + 0.1 + 0.1}{4} = 0.1$$

3.3 Overall Performance Vector

We aggregate the three dimensions' average triplets:

$$\bar{T} = \frac{\bar{T}_H + \bar{T}_R + \bar{T}_E}{3} = \frac{0.8125 + 0.625 + 0.525}{3} = 0.654$$

$$\bar{I} = \frac{0.1125 + 0.275 + 0.375}{3} = 0.254$$

$$\bar{F} = \frac{0.075 + 0.1 + 0.1}{3} = 0.092$$

The final neutrosophic performance vector = = $\langle 0.654, 0.254, 0.092 \rangle$

Sustainability Index Calculation, assuming $\lambda = 0.5$:

$$= \bar{T} \cdot (1 - \bar{F}) + 0.5 \cdot \bar{I}$$

$$= 0.654 \cdot (1 - 0.092) + 0.5 \cdot 0.254$$

$$= 0.654 \cdot 0.908 + 0.127$$

$$= 0.593 + 0.127 = 0.72$$

Then ,Final Sustainability Index = 0.72

The average truth degree of 0.654 shows a strong technical foundation in music performance. An indeterminacy degree of 0.254 highlights healthy experimentation, which is vital for creativity. With a low falsehood degree of 0.092, there are very few unacceptable errors. Overall, the Sustainability Index of 0.72 reflects a balanced performance that combines technical skill, creativity, and minimal mistakes, supporting sustainable music education.

4. Application Scenarios in College Music Education

The neutrosophic triplet evaluation helps teachers go beyond basic "right or wrong" judgments, offering more detailed and meaningful feedback. The T supports technical accuracy, the I promotes creative interpretation and flexibility, and the F degree identifies critical errors that need correction. For example, a teacher might say: "Your harmonic accuracy is strong at 0.8125, but your expressiveness has an indeterminacy of 0.375. This could be an opportunity to develop a more intentional personal style!"

4.1 Curriculum Design

NeutroTriplet Averages highlight:
Strengths (high $\bar{T}$)
Opportunities for innovation (moderate $\bar{I}$)
Gaps to fix (high $\bar{F}$)
Curriculum Example is:
High $\bar{F}$ → More foundational exercises (e.g., rhythm drills).
High $\bar{I}$ → Design open-ended projects (e.g., improvisation labs).
High $\bar{T}$ → Sustain excellence with advanced repertoire.

5.2 Sustainable Growth Path

The Sustainability Index can track progress over semesters:
Plot SustainabilityIndex (t) vs. time t to see growth.
Use slope $\frac{d\,(\text{SustainabilityIndex})}{dt}$ as a quantitative metric of program effectiveness.

5.3 Example of Semester-to-Semester Tracking

Let's say:
1. Semester 1: SustainabilityIndex = 0.6
2. Semester 2: SustainabilityIndex = 0.68
3. Semester 3: SustainabilityIndex = 0.72

Compute growth:

$$\frac{\Delta(\text{ SustainabilityIndex })}{\Delta t}=\frac{0.72-0.6}{3-1}=\frac{0.12}{2}=0.06$$

The program is improving at 0.06 units per semester-positive and sustainable!

4.4 NeutroPerformance Matrix

We can represent an entire ensemble's performance as a neutrosophic matrix:

$$\mathbf{P}=\begin{bmatrix}\langle T_{11},I_{11},F_{11}\rangle & \cdots & \langle T_{1n},I_{1n},F_{1n}\rangle\\ \vdots & \ddots & \vdots\\ \langle T_{m1},I_{m1},F_{m1}\rangle & \cdots & \langle T_{mn},I_{mn},F_{mn}\rangle\end{bmatrix}$$

The matrix P represents students as rows and musical skills, such as rhythm, pitch, and dynamics, as columns. By analyzing P with statistics like mean and variance, teachers can identify the most consistent performers, pinpoint skills that need more support, and find opportunities for innovation in music education.

5. Discussion and Implications

The neutrosophic framework offers a detailed and flexible way to assess music performance, moving beyond simple "right or wrong" judgments. It highlights areas of strength, uncertainty, and creative potential. For instance, high indeterminacy in a student's expressiveness can be viewed as an opportunity for artistic growth, encouraging them to develop a unique style. Teachers can use the truth, indeterminacy, and falsehood triplets to provide clear feedback, pinpointing where students excel and where they need improvement. The Sustainability Index tracks the progress of a music program over time, with an increase from one semester to the next indicating improvement. By analyzing triplet averages, teachers can tailor classes focusing on basic exercises if errors (falsehood) are high or encouraging creative projects if indeterminacy is prominent. This balanced approach suits college music education, valuing both technical precision and artistic freedom while providing a clear method to monitor growth in individual skills and the overall program.

This mathematical model introduces fresh perspectives to college music education, emphasizing that music is not just about perfect notes but also about exploration and personal expression. Neutrosophic triplets help students understand their strengths, experimental areas, and skills needing practice, fostering well-rounded musicians. The Sustainability Index allows teachers to measure program improvement annually, offering a concrete metric for progress. This approach makes feedback more honest and effective, and its potential extends beyond music to fields like dance or art. Ultimately, this framework promotes a balanced learning experience, blending clear rules with creative freedom and tradition with innovation, making music education fairer and more inspiring.

7. Conclusion

This paper presented a clear and detailed mathematical framework for NeutroGeometry and NeutroAlgebra, showing how these ideas can help make college music education more flexible and creative. The key point is that music education is not just about what is "right" or "wrong." Instead, it also includes areas of uncertainty and experimentation. By using the neutrosophic triplet—truth, indeterminacy, and falsehood—teachers and students can better understand and improve their work. The equations and examples show how to calculate averages and the Sustainability Index, giving real numbers to track progress. This approach encourages both high standards and freedom to explore, helping create strong, innovative musicians for the future.

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