Ideals and Filters of UP-Algebras in the Frame of Pentapartitioned Neutrosophic Structures

Anas Al-Masarwah* **and Haifa Al-Zghoul**
Department of Mathematics, Faculty of Science, Ajloun National University, P. O. Box 43, Ajloun 26810, Jordan.
E-mail: haifaazghoul@gmail.com
* Correspondence: anas.almasarwah@anu.edu.jo or almasarwah85@gmail.com

Abstract. The aim of this article is to apply the idea of pentapartitioned neutrosophic structures to UP-algebras. Fuzziness algebraic substructures, namely UP-subalgebras, near UP-filters, UP-filters, UP-ideals and strong UP-ideals of UP-algebras are modified and extended to introduce the notions of pentapartitioned neutrosophic UP-subalgebras, pentapartitioned neutrosophic near UP-filters, pentapartitioned neutrosophic UP-filters, pentapartitioned neutrosophic UP-ideals and pentapartitioned neutrosophic strong UP-ideals in UP-algebras and prove their generalizations. Furthermore, the relationship between pentapartitioned neutrosophic UP-subalgebras (resp., pentapartitioned neutrosophic near UP-filters, pentapartitioned neutrosophic UP-filters, pentapartitioned neutrosophic UP-ideals and pentapartitioned neutrosophic strong UP-ideals) in UP-algebras is discussed. After that, the conditions under which pentapartitioned neutrosophic UP-subalgebra can be pentapartitioned neutrosophic near UP-filter, and the condition under which pentapartitioned neutrosophic UP-filter can be pentapartitioned neutrosophic UP-ideal in UP-algebra are discovered. At last, some characterizations theorems of pentapartitioned neutrosophic structures in connection with UP-subalgebraic structures are presented and proved.

Keywords: UP-algebras, pentapartitioned neutrosophic structure, pentapartitioned neutrosophic UP-subalgebra, pentapartitioned neutrosophic near UP-filter, pentapartitioned neutrosophic UP-filter, pentapartitioned neutrosophic UP-ideal, pentapartitioned neutrosophic strong UP-ideal.

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1. Introduction

In several domains of mathematics, there are several types of algebraic structures, such as BCK-algebras [1], BCI-algebras [1], KU-algebras [2], SU-algebras [3] and others. A UP-algebra is one of algebraic structures discovered by Iampan [4] and it is known that the class of KU-algebras is a proper subclass of the class of UP-algebras. He presented the concepts of UP-subalgebra and UP-ideals. In [5], Iampan proved that the concept of UP-subalgebras is an extension of near UP-filters, near UP-filters is an extension of UP-filters, UP-filters is an extension of UP-ideals, and UP-ideals is an extension of strong UP-ideals. The study of UP-algebras offers a rich and fascinating area of inquiry for mathematicians and scientists alike, providing a powerful framework for understanding the underlying structures and behaviour of a wide range of mathematical systems.

In the field of fuzziness mathematics, Zadeh [6] implemented the notion of fuzziness structures as a generalization of classical (crisp) sets. This concept was merged with a crisp UP-algebra by Songsaeng et al. [7] to deal with uncertainty and fuzziness more accurately. In other words, the theory of fuzziness sets can be used within the framework of UP-algebras to explore fuzziness and imprecision in algebraic structures in UP-algebras. Atanassov [8] presented intuitionistic fuzzy sets which include the uncertainty degree called uncertainty margin. The uncertainty margin is defined as one minus the sum of membership and non-membership. Therefore, the intuitionistic fuzzy set is characterized by a membership function and non-membership function with a range $[0, 1]$. An intuitionistic fuzzy set is the extension of both classical (crisp) and fuzzy sets. This concept applied in several fields, such as medical diagnosis [9], modeling theories [10], pattern recognition [11] and decision making [12].

Smarandache [13] presented neutrosophic set theory that studies the origin, nature, and scope of neutralities and engagements with distinct ideational spectra. A neutrosophic set involves truth, indeterminacy and falsity based on three valued logics. Neutrosophic set is a powerful mathematical framework which extensions the perception of classical sets and (intuitionistic) fuzzy sets. Neutrosophic sets deal with the unspecified and inconsistent information that exists commonly in our daily life and these sets are important in algebras, see [14–16]. As a modification of neutrosophic sets, Wang et al. [17] defined single valued neutrosophic set as an instance of neutrosophic set which can be used in real scientific and engineering applications. Chatterjee et al. [18] defined the concept of quadripartitioned single valued neutrosophic structure as an extension of single valued neutrosophic structure and this concept involves truth, falsity, unknown and contradiction based on four valued logics. Mallick and Pramanik [19] implemented the investigation of pentapartioned neutrosophic structures as an extension of a single valued neutrosophic set [17] and quadripartitioned single valued neutrosophic set [18]. Here, indeterminacy is split into three parts as contradiction, ignorance and unknown membership function. Also, they introduced the concept of pentapartitioned neutrosophic Pythagorean set and establish a number of its properties. The concepts of pentapartitioned neutrosophic semi-open sets, pentapartitioned neutrosophic semi-closed sets, pentapartitioned neutrosophic semi-interior and pentapartitioned neutrosophic semi-closure in pentapartitioned neutrosophic topological spaces were introduced by Radha and Stanis Arul Mary [20].

Later, fuzzy UP-ideals, fuzzy UP-subalgebras and fuzzy UP-filters of UP-algebras considered by Somjanta et al. [7] and fuzzy translations of a fuzzy set in UP-algebras studied by Guntasow et al. [21]. Also, Kaijae et al. [22] studied and investigated anti-fuzzy UP-subalgebras and anti-fuzzy UP-ideals in UP-algebras. Kesorn et al. [23] connected intuitionistic fuzzy set theory with UP-algebras. In the context of neutrosophic UP-algebras, Songsaeng and Iampan [24] presented the concepts of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP ideals, and neutrosophic strongly UP-ideals of UP-algebras, and investigated many properties.

In this paper, we introduce the notions of pentapartitioned neutrosophic UP-subalgebras, pentapartitioned neutrosophic near UP-filters, pentapartitioned neutrosophic UP-filters, pentapartitioned neutrosophic UP-ideals and pentapartitioned neutrosophic strong UP-ideals in UP-algebras and prove their generalizations. Furthermore, the relationship between pentapartitioned neutrosophic UP-subalgebras (resp., pentapartitioned neutrosophic near UP-filters, pentapartitioned neutrosophic UP-filters, pentapartitioned neutrosophic UP-ideals and pentapartitioned neutrosophic strong UP-ideals) in UP-algebras is discussed. After that, the conditions under which pentapartitioned neutrosophic UP-subalgebra can be pentapartitioned neutrosophic near UP-filter, and the condition under which pentapartitioned neutrosophic UP-filter can be pentapartitioned neutrosophic UP-ideal in UP-algebra are established. At last, some characterizations results of pentapartitioned neutrosophic structures in connection with UP-subalgebraic structures are proposed and proved.

2. **Preliminaries**

In this section, a brief summary of some basic definitions and preliminary results related to this research, such as UP-algebras, subalgebras, ideals and filters in UP-algebras are presented. Thereafter, the main notions related to fuzzy sets, fuzzy subalgebras in UP-algebras, intuitionistic fuzzy sets and neutrosophic fuzzy sets, with some results and properties that will be of value for our later pursuits are mentioned. Throughout this article, Π (universe set) denotes a UP-algebra, unless otherwise specified.

2.1. *UP-Algebras and Some UP-Algebraic Substructures*

Definition 2.1. [4] An algebra $\Pi = (\Pi, \diamond, 0)$ is said to be a UP-algebra, where "Π" is a nonempty set, "$\diamond$" is a binary operation on Π, and "0" is a fixed element of Π if it satisfies the following postulates ($\forall h, j, k \in \Pi$):

(1) $(j \diamond k) \diamond ((h \diamond j) \diamond (h \diamond k)) = 0$,

(2) $0 \diamond h = h$,

(3) $h \diamond 0 = 0$,

(4) $h \diamond j = j \diamond h = 0 \Rightarrow h = j$.

Proposition 2.2. [4, 25] *Let* Π *be a UP-algebra. Then, the following assertions are valid* $(\forall h, j, k, c \in \Pi)$:

(1) $h \diamond h = 0$,

(2) $h \diamond j = 0$ *and* $j \diamond k = 0 \Rightarrow h \diamond k = 0$,

(3) $h \diamond j = 0 \Rightarrow (k \diamond h) \diamond (k \diamond j) = 0$,

(4) $h \diamond j = 0 \Rightarrow (j \diamond k) \diamond (h \diamond k) = 0$,

(5) $h \diamond (j \diamond h) = 0$,

(6) $(j \diamond h) \diamond h = 0 \Leftrightarrow h = j \diamond h$,

(7) $h \diamond (j \diamond j) = 0$,

(8) $(h \diamond (j \diamond k)) \diamond (h \diamond ((c \diamond j) \diamond (c \diamond k))) = 0$,

(9) $(((c \diamond h) \diamond (c \diamond j)) \diamond k) \diamond ((h \diamond j) \diamond k) = 0$,

(10) $((h \diamond j) \diamond k) \diamond (j \diamond k) = 0$,

(11) $h \diamond j = 0 \Rightarrow h \diamond (k \diamond j) = 0$,

(12) $((h \diamond j) \diamond k) \diamond (h \diamond (j \diamond k)) = 0$,

(13) $((h \diamond j) \diamond k) \diamond (j \diamond (c \diamond k)) = 0$.

Example 2.3. [4] Let $\Pi = \{0, v, b, n\}$ be a set with a binary operation "$\diamond$" defined by the following Cayley table:

TABLE 1. A UP-algebra $\Pi = \{0, v, b, n\}$ of Example 2.3

$\diamond$	0	v	b	n
0	0	v	b	n
v	0	0	0	0
b	0	v	0	n
n	0	v	b	0

Then, $(\Pi, \diamond, 0)$ is a UP-algebra.

In a UP-algebra Π, five types of special subsets are defined as follows.

Definition 2.4. [4, 7] A nonempty subset S of a UP-algebra Π $(\forall h, j, k \in \Pi)$ is called:

(1) a UP-subalgebra of Π if $h \diamond j \in S$ $\forall h, j \in S$;

(2) a near UP-filter of Π if

 (i) $0 \in S$,

 (ii) $j \in S \Rightarrow h \diamond j \in S$;

(3) a UP-filter of $\diamond$ if

 (i) $0 \in S$,

(ii) $h \diamond j \in S, h \in S \Rightarrow j \in S$;

(4) a UP-ideal of Π if

(i) $0 \in S$,

(ii) $h \diamond (j \diamond k) \in S, j \in S \Rightarrow h \diamond k \in S$;

(5) a strong UP-ideal of Π if

(i) $0 \in S$,

(ii) $(k \diamond j) \diamond (k \diamond h) \in S, j \in S \Rightarrow h \in S$.

In UP-algebras, Guntasow et al. [21] and Iampan [5] proved that the concept of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals and UP-ideals is a generalization of strong UP-ideals.

2.2. *Fuzziness and Intuitionistic Fuzziness Structures*

In 1965, the concept of a fuzzy structure was first considered by Zadeh [6] as the following definition.

Definition 2.5. A fuzzy structure Q in $\Pi \neq \phi$ (universe set) is a structure of the form:

$$Q = \{\langle h, \mu_Q(h)\rangle : h \in \Pi\},$$

where $\mu_Q : \Pi \to [0,1]$ is the degree of membership function of the element $h \in \Pi$.

Definition 2.6. [8] An intuitionistic fuzzy structure B in $\Pi \neq \phi$ (universe set) is a structure of the form:

$$\mathrm{B} = \{\langle h, \mu_B(h), \xi_B(h)\rangle \mid h \in \Pi\},$$

where the functions

$$\mu_B : \Pi \to [0,1] \text{ and } \xi_B : \Pi \to [0,1]$$

are the degree of membership and the degree of non-membership of the element $h \in \Pi$, respectively, and $(\forall h \in \Pi)$:

$$0 \leq \mu_B(h) + \xi_B(h) \leq 1.$$

2.3. *Neutrosophic Algebraic Substructures in UP-Algebras*

The notion of a neutrosophic structure introduced by Smarandache [13] as the following definition.

Definition 2.7. A neutrosophic structure Λ in $\Pi \neq \phi$ (universe set) is a structure of the form:

$$\Lambda = \{\langle h, \lambda_T(h), \lambda_I(h), \lambda_F(h)\rangle \mid h \in \Pi\},$$

where $\lambda_T : \Pi \to [0,1]$ is a truth, $\lambda_I : \Pi \to [0,1]$ is an indeterminate and $\lambda_F(h) : \Pi \to [0,1]$ is a false membership functions, and $(\forall h \in \Pi)$:

$$0 \leq \lambda_T(h) + \lambda_I(h) + \lambda_F(h) \leq 3.$$

All next definitions and examples, in this section, related to the connection between neutrosophic structure and some UP-algebraic substructures are mentioned in [24].

Definition 2.8. Let Λ be a neutrosophic structure of Π. Then, Λ is called a neutrosophic UP-subalgebra of Π if the following postulates are satisfied $(\forall h, j \in \Pi)$:

(1) $\lambda_T(h \diamond j) \geq \min\{\lambda_T(h), \lambda_T(j)\}$,
(2) $\lambda_I(h \diamond j) \leq \max\{\lambda_I(h), \lambda_I(j)\}$,
(3) $\lambda_F(h \diamond j) \geq \min\{\lambda_F(h), \lambda_F(j)\}$.

Example 2.9. Let $\Pi = \{0, v, b, n, l\}$ be a set with a binary operation "$\diamond$" defined by the following Cayley table:

TABLE 2. A UP-algebra $\Pi = \{0, v, b, n, l\}$ of Example 2.9

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	b	b	l
b	0	0	0	b	l
n	0	0	0	0	l
l	0	v	b	n	0

Define a neutrosophic structure Λ in Π as follows:

$$\Lambda = \begin{pmatrix} \Pi & 0 & v & b & n & l \\ \lambda_T & 0.9 & 0.7 & 0.5 & 0.3 & 0.3 \\ \lambda_I & 0 & 0.8 & 0.4 & 0.2 & 0.4 \\ \lambda_F & 1 & 0.6 & 0.8 & 0.3 & 0.2 \end{pmatrix}.$$

Then, for $b, l \in \Pi$ we have

$$\begin{aligned} \lambda_T(b \diamond l) = \lambda_T(l) &= 0.3 \\ &\geq \min\{\lambda_T(b), \lambda_T(l)\} \\ &= \min\{0.5, 0.3\} \\ &= 0.3, \end{aligned}$$

$$\begin{aligned}\lambda_I(b \diamond l) = \lambda_I(l) &= 0.4 \\ &\leq \max\{\lambda_I(b), \lambda_T(l)\} \\ &= \max\{0.4, 0.4\} \\ &= 0.4,\end{aligned}$$

$$\begin{aligned}\lambda_F(b \diamond l) = \lambda_F(l) &= 0.2 \\ &\geq \min\{\lambda_F(b), \lambda_T(l)\} \\ &= \min\{0.8, 0.2\} \\ &= 0.2.\end{aligned}$$

The remaining elements of Π can be verified similarly. Hence, Λ is a neutrosophic UP-subalgebra of Π.

Definition 2.10. Let Λ be a neutrosophic set of Π. Then, Λ is called a neutrosophic near UP-filter of Π if the condition (**K**), where

$$(\mathbf{K}) \quad (\forall h \in \Pi) \begin{pmatrix} \lambda_T(0) \geq \lambda_T(h), \\ \lambda_I(0) \leq \lambda_I(h), \\ \lambda_F(0) \geq \lambda_F(h) \end{pmatrix},$$

and following postulates are valid $(\forall h, j \in \Pi)$:

(1) $\lambda_T(h \diamond j) \geq \lambda_T(j)$,
(2) $\lambda_I(h \diamond j) \leq \lambda_I(j)$,
(3) $\lambda_F(h \diamond j) \geq \lambda_F(j)$.

Example 2.11. Let $\Pi = \{0, v, b, n, l\}$ be a set with a binary operation "$\diamond$" defined by the following Cayley table:

TABLE 3. A UP-algebra $\Pi = \{0, v, b, n, l\}$ of Example 2.11

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	v	b	l
b	0	0	0	v	l
n	0	0	0	0	l
l	0	v	b	n	0

Define a neutrosophic structure Λ in Π as follows:

$$\Lambda = \begin{pmatrix} \lambda & 0 & v & b & n & l \\ \lambda_T & 1 & 0.7 & 0.5 & 0.4 & 0.8 \\ \lambda_I & 0.1 & 0.2 & 0.3 & 0.7 & 0.6 \\ \lambda_F & 0.9 & 0.8 & 0.4 & 0.3 & 0.5 \end{pmatrix}.$$

Then,

$$\lambda_T(0) \geq \lambda_T(h), \lambda_I(0) \leq \lambda_I(h), \lambda_F(0) \geq \lambda_F(h) \forall h \in \Pi.$$

Also, for $v, n \in \Pi$ we have,

$$\lambda_T(v \diamond n) = \lambda_T(b) = 0.5 \geq \lambda_T(n) = 0.4,$$

$$\lambda_I(v \diamond n) = \lambda_I(b) = 0.3 \leq \lambda_I(n) = 0.7,$$

$$\lambda_F(v \diamond n) = \lambda_F(b) = 0.4 \geq \lambda_F(n) = 0.3.$$

The remaining elements of Π can be verified similarly. Hence, Λ is a neutrosophic near UP-filter of Π.

Definition 2.12. Let Λ be a neutrosophic structure in Π. Then, Λ is called a neutrosophic UP-filter of Π if the condition (**K**) of Definition 2.10 and the following postulates are valid ($\forall h, j \in \Pi$):

(1) $\lambda_T(j) \geq \min\{\lambda_T(h \diamond j), \lambda_T(h)\}$,
(2) $\lambda_I(j) \leq \max\{\lambda_I(h \diamond j), \lambda_I(h)\}$,
(3) $\lambda_F(j) \geq \min\{\lambda_F(h \diamond j), \lambda_F(h)\}$.

Example 2.13. Let $\Pi = \{0, v, b, n, l\}$ be a set with a binary operation " $\diamond$ " defined by the following Cayley table:

TABLE 4. A UP-algebra $\Pi = \{0, v, b, n, l\}$ of Example 2.13

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	b	n	l
b	0	0	0	n	n
n	0	v	b	0	n
l	0	v	b	0	0

Define a neutrosophic structure Λ in Π as follows:

$$\Lambda = \begin{pmatrix} \lambda & 0 & v & b & n & l \\ \lambda_T & 0.9 & 0.4 & 0.3 & 0.1 & 0.1 \\ \lambda_I & 0.2 & 0.3 & 0.7 & 0.8 & 0.8 \\ \lambda_F & 0.8 & 0.7 & 0.4 & 0.3 & 0.3 \end{pmatrix}.$$

Then,

$$\lambda_T(0) \geq \lambda_T(h), \lambda_I(0) \leq \lambda_I(h), \lambda_F(0) \geq \lambda_F(h) \forall h \in \Pi.$$

Also, for $b, v \in \Pi$ we have

$$\begin{aligned}\lambda_T(v) = 0.4 &\geq \min\{\lambda_T(b \diamond v), \lambda_T(b)\} \\ &= \min\{\lambda_T(0), \lambda_T(b)\} \\ &= \min\{0.9, 0.3\} = 0.3,\end{aligned}$$

$$\begin{aligned}\lambda_I(v) = 0.3 &\leq \max\{\lambda_I(b \diamond v), \lambda_I(b)\} \\ &= \max\{\lambda_I(0), \lambda_I(b)\} \\ &= \max\{0.2, 0.7\} = 0.7,\end{aligned}$$

$$\begin{aligned}\lambda_F(v) = 0.7 &\geq \min\{\lambda_F(b \diamond v), \lambda_F(b)\} \\ &= \min\{\lambda_F(0), \lambda_F(b)\} \\ &= \min\{0.8, 0.4\} = 0.4.\end{aligned}$$

The remaining elements of Π can be verified similarly. Hence, Λ is a neutrosophic UP-filter of Π.

Definition 2.14. Let Λ be a neutrosophic structure in Π. Then, Λ is called a neutrosophic UP-ideal of Π if the condition (**K**) of Definition 2.10 and the following postulates are valid ($\forall h, j, k \in \Pi$):

(1) $\lambda_T(h \diamond k) \geq \min\{\lambda_T(h \diamond (j \diamond k)), \lambda_T(j)\}$,
(2) $\lambda_I(h \diamond k) \leq \max\{\lambda_I(h \diamond (j \diamond k)), \lambda_I(j)\}$,
(3) $\lambda_F(h \diamond k) \geq \min\{\lambda_F(h \diamond (j \diamond k)), \lambda_F(j)\}$.

Example 2.15. Let $\Pi = \{0, v, b, n, l\}$ be a UP-algebra with a binary operation "$\diamond$" defined by the following Cayley table:

TABLE 5. A UP-algebra $\Pi = \{0, v, b, n, l\}$ of Example 2.15

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	b	n	l
b	0	0	0	b	l
n	0	0	0	0	l
l	0	v	b	n	0

Define a neutrosophic structure Λ in Π as follows:

$$\Lambda = \begin{pmatrix} \lambda & 0 & v & b & n & l \\ \lambda_T & 1 & 0.7 & 0.6 & 0.6 & 0.4 \\ \lambda_I & 0 & 0.3 & 0.5 & 0.5 & 0.7 \\ \lambda_F & 1 & 0.8 & 0.7 & 0.7 & 0.5 \end{pmatrix}.$$

Then,

$$\lambda_T(0) \geq \lambda_T(h), \lambda_I(0) \leq \lambda_I(h), \lambda_F(0) \geq \lambda_F(h) \forall h \in \Pi.$$

Also, for $b, n, v \in \Pi$ we have

$$\begin{aligned} \lambda_T(b \diamond v) = \lambda_T(0) = 1 &\geq \min\{\lambda_T(b \diamond (n \diamond v)), \lambda_T(n)\} \\ &= \min\{\lambda_T(b \diamond 0), \lambda_T(n)\} \\ &= \min\{1, 0.6\} \\ &= 0.6, \end{aligned}$$

$$\begin{aligned} \lambda_I(b \diamond v) = \lambda_I(0) = 0 &\leq \max\{\lambda_I(b \diamond (n \diamond v)), \lambda_I(n)\} \\ &= \max\{\lambda_I(b \diamond 0), \lambda_I(n)\} \\ &= \max\{0, 0.5\} \\ &= 0.5, \end{aligned}$$

$$\begin{aligned} \lambda_F(b \diamond v) = \lambda_F(0) = 1 &\geq \min\{\lambda_F(b \diamond (n \diamond v)), \lambda_F(n)\} \\ &= \min\{\lambda_F(b \diamond 0), \lambda_F(n)\} \\ &= \min\{1, 0.7\} \\ &= 0.7. \end{aligned}$$

The remaining elements of Π can be verified similarly. Hence, Λ is a neutrosophic UP-ideal of Π.

Definition 2.16. Let Λ be a neutrosophic structure in Π. Then, Λ is called a neutrosophic strong UP-ideal of Π if the condition (**K**) of Definition 2.10 and the following postulates are valid ($\forall h, j, k \in \Pi$):

(1) $\lambda_T(h) \geq \min\{\lambda_T((k \diamond j) \diamond (k \diamond h)), \lambda_T(j)\}$,
(2) $\lambda_I(h) \leq \max\{\lambda_I((k \diamond j) \diamond (k \diamond h)), \lambda_I(j)\}$,
(3) $\lambda_F(h) \geq \min\{\lambda_F((k \diamond j) \diamond (k \diamond h)), \lambda_F(j)\}$.

Example 2.17. Let $\Pi = \{0, v, b, n, l\}$ be a UP-algebra with a binary operation "$\diamond$" defined by the following Cayley table:

TABLE 6. A UP-algebra $\Pi = \{0, v, b, n, l\}$ of Example 2.17

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	b	n	l
b	0	v	0	n	l
n	0	v	0	0	l
l	0	v	0	n	0

Define a neutrosophic structure P in Π as follows:

$$P = \begin{pmatrix} \lambda & 0 & v & b & n & l \\ \lambda_T & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ \lambda_I & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ \lambda_F & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}.$$

Then,

$$\lambda_T(0) \geq \lambda_T(h), \lambda_I(0) \leq \lambda_I(h), \lambda_F(0) \geq \lambda_F(h) \forall h \in \Pi.$$

Also, for $n, b, l \in \Pi$ we have

$$\begin{aligned} \lambda_T(n) = 0.4 &\geq \min\{\lambda_T((l \diamond b) \diamond (l \diamond n)), \lambda_T(b)\} \\ &= \min\{\lambda_T((0 \diamond n), \lambda_T(b)\} \\ &= \min\{0.4, 0.4\} \\ &= 0.4, \\ \lambda_I(n) = 0.3 &\leq \max\{\lambda_I((l \diamond b) \diamond (l \diamond n)), \lambda_I(b)\} \\ &= \max\{\lambda_I((0 \diamond n), \lambda_I(b)\} \\ &= \max\{0.3, 0.3\} \\ &= 0.3 \\ \lambda_F(n) = 0.5 &\geq \min\{\lambda_F((l \diamond b) \diamond (l \diamond n)), \lambda_F(b)\} \\ &= \min\{\lambda_F((0 \diamond n), \lambda_F(b)\} \\ &= \min\{0.5, 0.5\} \\ &= 0.5. \end{aligned}$$

The remaining elements of Π can be verified similarly. Hence, P is a neutrosophic strong UP-ideal of Π.

2.4. *Pentapartitioned Neutrosophic Structures*

Definition 2.18. [19] A pentapartitioned neutrosophic structure P in Π (universe set) is a structure of the form:

$$P = \{\langle h, T_P(h), Q_P(h), E_P(h), D_P(h), F_P(h)\rangle \mid h \in \Pi\},$$

where, $T_P : \Pi \to [0,1]$ is a truth, $Q_P : \Pi \to [0,1]$ is an contradiction, $E_P : \Pi \to [0,1]$ is an ignorance, $D_P : \Pi \to [0,1]$ is an unknown and $F_P : \Pi \to [0,1]$ is a falsity membership functions, respectively such that $(\forall h \in \Pi)$:

$$0 \leq T_P(h) + Q_P(h) + E_P(h) + D_P(h) + F_P(h) \leq 5.$$

In this article, we use the symbol $P = \langle T_P, Q_P, E_P, D_P, F_P\rangle$ for the pentapartitioned neutrosophic structure

$$P = \{\langle h, T_P(h), Q_P(h), E_P(h), D_P(h), F_P(h)\rangle \mid h \in \Pi\}.$$

Example 2.19. Consider a UP-algebra Π which is given in Example 2.3. Then,

$$\Lambda = \begin{pmatrix} \Pi & 0 & v & b & n \\ T_P & 0.2 & 0.7 & 0.4 & 0.2 \\ Q_P & 0.3 & 0.1 & 0.2 & 0.3 \\ E_P & 0.5 & 0.2 & 0.1 & 0.4 \\ D_P & 0.2 & 0.3 & 0.6 & 0.5 \\ F_P & 0.4 & 0.3 & 0.3 & 0.6 \end{pmatrix}$$

is a pentapartitioned neutrosophic structure of Π.

3. Certain Types of Pentapartitioned Neutrosophic UP-Algebraic Substructures

In this section, we introduce the notions of pentapartitioned neutrosophic UP-subalgebras, pentapartitioned neutrosophic near UP-filters, pentapartitioned neutrosophic UP-filters, pentapartitioned neutrosophic UP-ideals, and pentapartitioned neutrosophic strong UP-ideals of UP-algebras. Based on these notions, certain necessary examples and properties with their generalizations are provided and discussed.

Definition 3.1. A pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P\rangle$$

is called a pentapartitioned neutrosophic UP-subalgebra of Π if the following postulates are valid $(\forall h, j, \in \Pi)$:

(1) $T_P(h \diamond j) \geq \min\{T_P(h), T_P(j)\}$,
(2) $Q_P(h \diamond j) \geq \min\{Q_P(h), Q_P(j)\}$,
(3) $E_P(h \diamond j) \leq \max\{E_P(h), E_P(j)\}$,
(4) $D_P(h \diamond j) \leq \max\{D_P(h), D_P(j)\}$,

(5) $F_P(h \diamond j) \leq \max\{F_P(h), F_P(j)\}$.

Proposition 3.2. *If* $P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$ *is a pentapartitioned neutrosophic UP-subalgebra of* Π, *then the condition* ($\boldsymbol{P}$) *is valid, where*

$$(\boldsymbol{P}) \quad \Big(\forall h \in \Pi\Big) \begin{pmatrix} T_P(0) \geq T_P(h), \\ Q_P(0) \geq Q_P(h), \\ E_P(0) \leq E_P(h), \\ D_P(0) \leq D_P(h), \\ F_P(0) \leq F_P(h) \end{pmatrix}.$$

Proof. Let $P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$ be a pentapartitioned neutrosophic UP-subalgebra of Π. Using (1) of Proposition 2.2, we have

$$\begin{aligned} T_P(0) = T_P(h \diamond h) \geq \min\{T_P(h), T_P(h)\} = T_P(h), \\ Q_P(0) = Q_P(h \diamond h) \geq \min\{Q_P(h), Q_P(h)\} = Q_P(h), \\ E_P(0) = E_P(h \diamond h) \leq \max\{E_P(h), E_P(h)\} = E_P(h), \\ D_P(0) = D_P(h \diamond h) \geq \min\{D_P(h), D_P(h)\} = D_P(h), \\ F_P(0) = F_P(h \diamond h) \leq \max\{F_P(h), F_P(h)\} = F_P(h). \end{aligned}$$

for all $h \in \Pi$. □

Example 3.3. Let $\Pi = \{0, v, b, n, l\}$ be a UP-algebra with a binary operation "$\diamond$" defined by the following Cayley table:

TABLE 7. A UP-algebra $\Pi = \{0, v, b, n, l\}$ of Example 3.3

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	v	n	l
b	0	0	0	n	l
n	0	0	0	0	l
l	0	0	0	0	0

Define a pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

in Π as follows:

$$P = \begin{pmatrix} \Pi & 0 & v & b & n & l \\ T_P & 0.7 & 0.4 & 0.5 & 0.2 & 0.6 \\ Q_P & 1 & 0.5 & 0.7 & 0.3 & 0.7 \\ E_P & 0 & 0.4 & 0.1 & 0.8 & 0.2 \\ D_P & 0.3 & 0.5 & 0.4 & 0.9 & 0.4 \\ F_P & 0 & 0.4 & 0.2 & 0.7 & 0.8 \end{pmatrix}.$$

Then, for $n, l \in \Pi$ we have

$$\begin{aligned} T_P(n \diamond l) = T_P(l) = 0.6 &\geq \min\{T_P(n), T_P(l)\} \\ &= \min\{0.2, 0.6\} \\ &= 0.2, \end{aligned}$$

$$\begin{aligned} Q_P(n \diamond l) = Q_P(l) = 0.7 &\geq \min\{Q_P(n), Q_P(l)\} \\ &= \min\{0.3, 0.7\} \\ &= 0.3, \end{aligned}$$

$$\begin{aligned} E_P(n \diamond l) = E_P(l) = 0.2 &\leq \max\{E_P(n), E_P(l)\} \\ &= \max\{0.8, 0.2\} \\ &= 0.8, \end{aligned}$$

$$\begin{aligned} D_P(n \diamond l) = D_P(l) = 0.4 &\leq \max\{D_P(n), D_P(l)\} \\ &= \max\{0.9, 0.4\} \\ &= 0.9, \end{aligned}$$

$$\begin{aligned} F_P(n \diamond l) = F_P(l) = 0.8 &\leq \max\{F_P(n), F_P(l)\} \\ &= \max\{0.7, 0.8\} \\ &= 0.8 \end{aligned}$$

The remaining elements of Π can be verified similarly. Hence, P is a pentapartitioned neutrosophic UP-subalgebra of Π.

Definition 3.4. A pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is called a pentapartitioned neutrosophic near UP-filter of Π if the condition $(\mathbf{P})$ of Proposition 3.2 and the following postulates are valid $(\forall h, j, \in \Pi)$:

$$\left(\begin{array}{c} T_P(h \diamond j) \geq T_P(j), \\ Q_P(h \diamond j) \geq Q_P(j), \\ E_P(h \diamond j) \leq E_P(j), \\ D_P(h \diamond j) \leq D_P(j), \\ F_P(h \diamond j) \leq F_P(j) \end{array} \right).$$

Example 3.5. Consider a UP-algebras Π which is given in Example 2.11. Define a pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

in Π as follows:

$$P = \left(\begin{array}{cccccc} \Pi & 0 & v & b & n & l \\ T_P & 1 & 0.6 & 0.5 & 0.4 & 0.1 \\ Q_P & 1 & 0.8 & 0.7 & 0.6 & 0.3 \\ E_P & 0 & 0.1 & 0.3 & 0.5 & 0.8 \\ D_P & 0.1 & 0.3 & 0.4 & 0.6 & 0.9 \\ F_P & 0.1 & 0.2 & 0.6 & 0.7 & 0.5 \end{array} \right).$$

Then, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$ $\forall h \in \Pi$. Also, for $l, v \in \Pi$ we have

$$\begin{array}{c} T_P(l \diamond v) = T_P(v) = 0.6 \geq T_P(v) = 0.6, \\ Q_P(l \diamond v) = Q_P(v) = 0.8 \geq Q_P(v) = 0.8, \\ E_P(l \diamond v) = E_P(v) = 0.1 \leq E_P(v) = 0.1, \\ D_P(l \diamond v) = D_P(v) = 0.3 \leq D_P(v) = 0.3, \\ F_P(l \diamond v) = F_P(v) = 0.2 \leq F_P(v) = 0.2. \end{array}$$

The remaining elements of Π can be verified similarly. Hence, P is a pentapartitioned neutrosophic near UP-filter of Π.

Definition 3.6. A pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is called a pentapartitioned neutrosophic UP-filter of Π if the condition ($\mathbf{P}$) of Proposition 3.2 and the following postulates are valid $(\forall h, j \in \Pi)$:

$$\left(\begin{array}{c} T_P(j) \geq \min \{T_P(h \diamond j), T_P(h)\}, \\ Q_P(j) \geq \min \{Q_P(h \diamond j), Q_P(h)\}, \\ E_P(j) \leq \max \{E_P(h \diamond j), E_P(h)\}, \\ D_P(j) \leq \max \{D_P(h \diamond j), D_P(h)\}, \\ F_P(j) \leq \max\{F_P(h \diamond j), F_P(h)\} \end{array} \right).$$

Example 3.7. Consider a UP-algebras Π which is given in Example 2.13. Define a pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

in Π as follows:

$$P = \begin{pmatrix} \Pi & 0 & v & b & n & l \\ T_P & 0.8 & 0.6 & 0.5 & 0.2 & 0.2 \\ Q_P & 0.9 & 0.7 & 0.6 & 0.4 & 0.4 \\ E_P & 0 & 0.2 & 0.4 & 0.7 & 0.7 \\ D_P & 0.1 & 0.3 & 0.5 & 0.9 & 0.9 \\ F_P & 0.1 & 0.2 & 0.9 & 0.3 & 0.3 \end{pmatrix},$$

Then, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$ $\forall h \in \Pi$. Also, for $b, l \in \Pi$ we have

$$\begin{aligned} T_P(l) = 0.2 &\geq \min\{T_P(b \diamond l), T_P(b)\} \\ &= \min\{T_P(n), T_P(b)\} \\ &= \min\{0.2, 0.5\} \\ &= 0.2, \end{aligned}$$

$$\begin{aligned} Q_P(l) = 0.4 &\geq \min\{Q_P(b \diamond l), Q_P(b)\} \\ &= \min\{Q_P(n), Q_P(b)\} \\ &= \min\{0.4, 0.6\} \\ &= 0.4, \end{aligned}$$

$$\begin{aligned} E_P(l) = 0.7 &\leq \max\{E_P(b \diamond l), E_P(b)\} \\ &= \max\{E_P(n), E_P(b)\} \\ &= \max\{0.7, 0.4\} \\ &= 0.7, \end{aligned}$$

$$\begin{aligned} D_P(l) = 0.9 &\leq \max\{D_P(b \diamond l), D_P(b)\} \\ &= \max\{D_P(n), D_P(b)\} \\ &= \max\{0.9, 0.5\} \\ &= 0.9, \end{aligned}$$

$$\begin{aligned} F_P(l) = 0.3 &\leq \max\{F_P(b \diamond l), F_P(b)\} \\ &= \max\{F_P(n), F_P(b)\} \\ &= \max\{0.3, 0.9\} \\ &= 0.9. \end{aligned}$$

The remaining elements of Π can be verified similarly. Hence, P is a pentapartitioned neutrosophic UP-filter of Π.

Definition 3.8. A pentapartitioned neutrosophic set is called a pentapartitioned neutrosophic UP-ideal of Π if the condition (**P**) of Proposition 3.2 and the following postulates are valid $(\forall h, j, k \in \Pi)$:

$$\begin{pmatrix} T_P(h \diamond k) \geq \min\{T_P(h \diamond (j \diamond k)), T_P(j)\}, \\ Q_P(h \diamond k) \geq \min\{Q_P(h \diamond (j \diamond k)), Q_P(j)\}, \\ E_P(h \diamond k) \leq \max\{E_P(h \diamond (j \diamond k)), E_P(j)\}, \\ D_P(h \diamond k) \leq \max\{D_P(h \diamond (j \diamond k)), D_P(j)\}, \\ F_P(h \diamond k) \leq \max\{F_P(h \diamond (j \diamond k)), F_P(j)\} \end{pmatrix}.$$

Example 3.9. Let $\Pi = \{0, v, b, n, l\}$ be a UP-algebra with a binary operation "$\diamond$" defined by the following Cayley table:

Table 8. A UP-algebra $\Pi = \{0, v, b, n, l\}$ of Example 3.9

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	b	n	l
b	0	0	0	0	l
n	0	0	b	0	l
l	0	0	0	0	0

Define a pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

in Π as follows:

$$P = \begin{pmatrix} \Pi & 0 & v & b & n & l \\ T_P & 0.8 & 0.5 & 0.4 & 0.5 & 0.4 \\ Q_P & 0.9 & 0.9 & 0.6 & 0.8 & 0.5 \\ E_P & 0.1 & 0.3 & 0.4 & 0.3 & 0.5 \\ D_P & 0.3 & 0.5 & 0.7 & 0.6 & 0.9 \\ F_P & 0 & 0.2 & 0.7 & 0.4 & 0.9 \end{pmatrix},$$

Then, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$ $\forall h \in \Pi$. Also, for $v, b, n \in \Pi$ we have

$$\begin{aligned} T_P(v \diamond n) = T_P(n) = 0.5 &\geq \min\{T_P(v \diamond (b \diamond n)), T_P(b)\} \\ &= \min\{T_P(0), T_P(b)\} \\ &= \min\{0.8, 0.4\} \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} Q_P(v \diamond n) = Q_P(n) = 0.8 &\geq \min\{Q_P(v \diamond (b \diamond n)), Q_P(b)\} \\ &= \min\{Q_P(0), Q_P(b)\} \\ &= \min\{0.9, 0.6\} \\ &= 0.6 \end{aligned}$$

$$\begin{aligned} E_P(v \diamond n) = E_P(n) = 0.3 &\leq \max\{E_P(v \diamond (b \diamond n)), E_P(b)\} \\ &= \max\{E_P(0), E_P(b)\} \\ &= \max\{0.1, 0.4\} \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} D_P(v \diamond n) = D_P(n) = 0.6 &\leq \max\{D_P(v \diamond (b \diamond n)), D_P(b)\} \\ &\leq \max\{D_P(0), D_P(b)\} \\ &\leq \max\{0.3, 0.7\} \\ &\leq 0.7 \end{aligned}$$

$$\begin{aligned} F_P(v \diamond n) = F_P(n) = 0.4 &\leq \max\{F_P(v \diamond (b \diamond n)), F_P(b)\} \\ &\leq \max\{F_P(0), F_P(b)\} \\ &\leq \max\{0, 0.7\} \\ &\leq 0.7 \end{aligned}$$

The remaining elements of Π can be verified similarly. Hence, P is a pentapartitioned neutrosophic UP-ideal of Π.

Definition 3.10. A pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is called a pentapartitioned neutrosophic strong UP-ideal of Π if the condition (**P**) of Proposition 3.2 and the following postulates are valid $(\forall h, j, k \in \Pi)$:

$$\begin{pmatrix} T_P(h) \geq \min\{T_P((k \diamond j) \diamond (k \diamond h)), T_P(j)\}, \\ Q_P(h) \geq \min\{Q_P((k \diamond j) \diamond (k \diamond h)), Q_P(j)\}, \\ E_P(h) \leq \max\{E_P((k \diamond j) \diamond (k \diamond h)), E_P(j)\}, \\ D_P(h) \leq \max\{D_P((k \diamond j) \diamond (k \diamond h)), D_P(j)\}, \\ F_P(h) \leq \max\{F_P((k \diamond j) \diamond (k \diamond h)), F_P(j)\} \end{pmatrix}.$$

Example 3.11. Consider a UP-algebras Π which is given in Example 2.17. Define a pentapartitioned neutrosophic structure

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

in Π as follows:

$$P = \begin{pmatrix} \Pi & 0 & v & b & n & l \\ T_P & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ Q_P & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ E_P & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ D_P & 0.9 & 0.9 & 0.9 & 0.9 & 0.9 \\ F_P & 0.7 & 0.7 & 0.7 & 0.7 & 0.7 \end{pmatrix}.$$

Then, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$ $\forall h \in \Pi$. Also, for $n, b, l \in \Pi$ we have

$$\begin{aligned} T_P(n) = 0.4 &\geq \min\{T_P((l \diamond b) \diamond (l \diamond n)), T_P(b)\} \\ &= \min\{T_P(0 \diamond n), T_P(b)\} \\ &= \min\{0.4, 0.4\} \\ &= 0.4, \end{aligned}$$

$$\begin{aligned} Q_P(n) = 0.5 &\geq \min\{Q_P((l \diamond b) \diamond (l \diamond n)), Q_P(b)\} \\ &= \min\{Q_P(0 \diamond n), Q_P(b)\} \\ &= \min\{0.5, 0.5\} \\ &= 0.5, \end{aligned}$$

$$\begin{aligned} E_P(n) = 0.3 &\leq \max\{E_P((l \diamond b) \diamond (l \diamond n)), E_P(b)\} \\ &= \max\{E_P(0 \diamond n), E_P(b)\} \\ &= \max\{0.3, 0.3\} \\ &= 0.3, \\ D_P(n) = 0.9 &\leq \max\{D_P((l \diamond b) \diamond (l \diamond n)), D_P(b)\} \\ &= \max\{D_P(0 \diamond n), D_P(b)\} \\ &= \max\{0.9, 0.9\} \\ &= 0.9, \\ F_P(n) = 0.7 &\leq \max\{F_P((l \diamond b) \diamond (l \diamond n)), F_P(b)\} \\ &= \max\{F_P(0 \diamond n), F_P(b)\} \\ &= \max\{0.7, 0.7\} \\ &= 0.7 \end{aligned}$$

The remaining elements of Π can be verified similarly. Hence, P is a pentapartitioned neutrosophic strong UP-ideal of Π.

Definition 3.12. Let $P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$ be a pentapartitioned neutrosophic structure of Π. Then, P is said to be a constant pentapartitioned neutrosophic structure in Π if $T_P(h) = T_P(0)$, $Q_P(h) = Q_P(0)$, $E_P(h) = E_P(0)$, $D_P(h) = D_P(0)$ and $F_P(h) = F_P(0)$ $\forall h \in \Pi$.

Theorem 3.13. *A pentapartitioned neutrosophic structure*

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

in Π *is constant if and only if it is a pentapartitioned neutrosophic strong UP-ideal of* Π.

Proof. Assume that P is a constant pentapartitioned neutrosophic structure in Π. Then, $T_P(h) = T_P(0)$, $Q_P(h) = Q_P(0)$, $E_P(h) = E_P(0)$, $D_P(h) = D_P(0)$ and $F_P(h) = F_P(0)$ for all $h \in \Pi$. This implies that, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$. Also, for all $h, j, k \in \Pi$, we get

$$\begin{aligned} \min\{T_P((k \diamond j) \diamond (k \diamond h)), T_P(j)\} &= \min\{T_P(0), T_P(0)\} \\ &= T_P(0) \\ &= T_P(h), \end{aligned}$$

$$\begin{aligned} \min\{Q_P((k \diamond j) \diamond (k \diamond h)), Q_P(j)\} &= \min\{Q_P(0), Q_P(0)\} \\ &= Q_P(0) \\ &= Q_P(h), \end{aligned}$$

$$\begin{aligned} \max\{E_P((k \diamond j) \diamond (k \diamond h)), E_P(j)\} &= \max\{E_P(0), E_P(0)\} \\ &= E_P(0) \\ &= E_P(h), \end{aligned}$$

$$\begin{aligned} \max\{D_P((k \diamond j) \diamond (k \diamond h)), D_P(j)\} &= \max\{D_P(0), D_P(0)\} \\ &= D_P(0) \\ &= D_P(h), \end{aligned}$$

$$\begin{aligned} \max\{F_P((k \diamond j) \diamond (k \diamond h)), F_P(j)\} &= \max\{F_P(0), F_P(0)\} \\ &= F_P(0) \\ &= F_P(h). \end{aligned}$$

Hence, P is a pentapartitioned neutrosophic strong UP-ideal of Π.

Conversely, assume that P is a pentapartitioned neutrosophic strong UP-ideal of Π. Then, for all $h, j, k \in \Pi$, we have

$$\begin{aligned} T_P(h) &\geq \min\{T_P((h \diamond 0) \diamond (h \diamond h)), T_P(0)\} \\ &= \min\{T_P(h \diamond h), T_P(0)\} \\ &= \min\{T_P(0), T_P(0)\} \\ &= T_P(0) \\ &\geq T_P(h), \end{aligned}$$

$$\begin{aligned} Q_P(h) &\geq \min\{Q_P((h \diamond 0) \diamond (h \diamond h)), Q_P(0)\} \\ &= \min\{Q_P(0 \diamond (h \diamond h)), Q_P(0)\} \\ &= \min\{Q_P(h \diamond h), Q_P(0)\} \\ &= \min\{Q_P(0), Q_P(0)\} \\ &= Q_P(0) \\ &\geq Q_P(h), \end{aligned}$$

$$\begin{aligned} E_P(h) &\leq \max\{E_P((h \diamond 0) \diamond (h \diamond h)), E_P(0)\} \\ &= \max\{E_P(0 \diamond (h \diamond h)), E_P(0)\} \\ &= \max\{E_P(h \diamond h), E_P(0)\} \\ &= \max\{E_P(0), E_P(0)\} \\ &= E_P(0) \\ &\leq E_P(h), \end{aligned}$$

$$\begin{aligned} D_P(h) &\leq \max\{D_P((h \diamond 0) \diamond (h \diamond h)), D_P(0)\} \\ &= \max\{D_P(0 \diamond (h \diamond h)), D_P(0)\} \\ &= \max\{D_P(h \diamond h), D_P(0)\} \\ &= \max\{D_P(0), D_P(0)\} \\ &= D_P(0) \\ &\leq D_P(h), \end{aligned}$$

$$
\begin{aligned}
F_P(h) &\leq \max\{F_P((h \diamond 0) \diamond (h \diamond h)), F_P(0)\} \\
&= \max\{F_P(0 \diamond (h \diamond h)), F_P(0)\} \\
&= \max\{F_P(h \diamond h), F_P(0)\} \\
&= \max\{F_P(0), F_P(0)\} \\
&= F_P(0) \\
&\leq F_P(h).
\end{aligned}
$$

Thus, $T_P(0) = T_P(h), Q_P(0) = Q_P(h), E_P(0) = E_P(h), D_P(0) = D_P(h)$, and $F_P(0) = F_P(h)$ $(\forall h \in \Pi)$. Hence, P is a constant. □

4. Some Relations of Pentapartitioned Neutrosophic UP-Algebraic Substructures

This section discusses the relations between pentapartitioned neutrosophic UP-subalgebras (resp., pentapartitioned neutrosophic near UP-filters, pentapartitioned neutrosophic UP-filters, pentapartitioned neutrosophic UP-ideals and pentapartitioned neutrosophic strong UP-ideals) in UP-algebras P.

Theorem 4.1. Every pentapartitioned neutrosophic near UP-filter of Π is a pentapartitioned neutrosophic UP-subalgebra.

Proof. Assume that

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic near UP-filter of Π. Then, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$. $\forall h \in \Pi$. Now, let $h, j \in \Pi$. Then,

$$
\begin{aligned}
T_P(h \diamond j) \geq T_P(j) \geq \min\{T_P(h), T_P(j)\}, \\
Q_P(h \diamond j) \geq Q_P(j) \geq \min\{Q_P(h), Q_P(j)\}, \\
E_P(h \diamond j) \leq E_P(j) \leq \max\{E_P(h), E_P(j)\}, \\
D_P(h \diamond j) \leq D_P(j) \leq \max\{D_P(h), D_P(j)\}, \\
F_P(h \diamond j) \leq F_P(j) \leq \max\{F_P(h), F_P(j)\}.
\end{aligned}
$$

Hence, P is a pentapartitioned neutrosophic UP-subalgebra of Π. □

The following example shows that the converse of Theorem 4.1 is not true.

Example 4.2. From Example 3.3, P is a pentapartitioned neutrosophic UP-subalgbra of Π. Since

$$E_P(v \diamond b) = 0.4 \nleq E_P(b) = 0.1,$$

P is not a pentapartitioned neutrosophic near UP-filter of Π.

Theorem 4.3. *Every pentapartitioned neutrosophic UP-filter of* Π *is a pentapartitioned neutrosophic near UP-filter.*

Proof. Assume that

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic UP-filter of Π. Then, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$ $\forall h \in \Pi$. Now, let $h, j \in \Pi$. Then,

$$\begin{aligned}
T_P(h \diamond j) &\geq \min\{T_P(j \diamond (h \diamond j)), T_P(j)\} \\
&= \min\{T_P(0), T_P(j)\} \\
&= T_P(j), \\
Q_P(h \diamond j) &\geq \min\{Q_P(j \diamond (h \diamond j)), Q_P(j)\} \\
&= \min\{Q_P(0), Q_P(j)\} \\
&= Q_P(j), \\
E_P(h \diamond j) &\leq \max\{E_P(j \diamond (h \diamond j)), E_P(j)\} \\
&= \max\{E_P(0), E_P(j)\} \\
&= E_P(j), \\
D_P(h \diamond j) &\leq \max\{D_P(j \diamond (h \diamond j)), D_P(j)\} \\
&= \max\{D_P(0), D_P(j)\} \\
&= D_P(j),
\end{aligned}$$

$$\begin{aligned}
F_P(h \diamond j) &\leq \max\{F_P(j \diamond (h \diamond j)), F_P(j)\} \\
&= \max\{F_P(0), F_P(j)\} \\
&= F_P(j)
\end{aligned}$$

Hence, P is a pentapartitioned neutrosophic near UP-filter of Π. □

The following example shows that the converse of Theorem 4.3 is not true.

Example 4.4. From Example 3.5, P is a pentapartitioned neutrosophic near UP-filter of Π. Since

$$T_P(b) = 0.5 \ngeq 0.6 = \min\{T_P(v \diamond b), T_P(v)\},$$

P is not a pentapartitioned neutrosophic UP-filter of Π.

Theorem 4.5. Every pentapartitioned neutrosophic UP-ideal of Π is a pentapartitioned neutrosophic UP-filter.

Proof. Assume that $P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$ is a pentapartitioned neutrosophic UP-ideal of Π. Then, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$. $\forall h \in \Pi$. Now, let $h, j \in \Pi$. Then,

$$\begin{aligned}
T_P(j) &= T_P(0 \diamond j) \\
&\geq \min\{T_P(0 \diamond (h \diamond j)), T_P(h)\} \\
&= \min\{T_P(h \diamond j), T_P(h)\}, \\
Q_P(j) &= Q_P(0 \diamond j) \\
&\geq \min\{Q_P(0 \diamond (h \diamond j)), Q_P(h)\} \\
&= \min\{Q_P(h \diamond j), Q_P(h)\}, \\
E_P(j) &= E_P(0 \diamond j) \\
&\leq \max\{E_P(0 \diamond (h \diamond j)), E_P(h)\} \\
&= \max\{E_P(h \diamond j), E_P(h)\}, \\
D_P(j) &= D_P(0 \diamond j) \\
&\leq \max\{D_P(0 \diamond (h \diamond j)), D_P(h)\} \\
&= \max\{D_P(h \diamond j), D_P(h)\}, \\
F_P(j) &= F_P(0 \diamond j) \\
&\leq \max\{F_P(0 \diamond (h \diamond j)), F_P(h)\} \\
&= \max\{F_P(h \diamond j), F_P(h)\}.
\end{aligned}$$

Hence, P is a pentapartitioned neutrosophic UP-filter of Π. □

The following example shows that the converse of Theorem 4.5 is not true.

Example 4.6. From Example 3.7, we have P is a pentapartitioned neutrosophic UP-filter of Π. Since

$$T_P(n \diamond l) = 0.2 \ngeq 0.5 = \min\{T_P(n \diamond (b \diamond l)), T_P(b)\},$$

P is not a pentapartitioned neutrosophic UP-ideal of Π.

Theorem 4.7. *Every pentapartitioned neutrosophic strong UP-ideal of* Π *is a pentapartitioned neutrosophic UP-ideal.*

Proof. Assume that

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic strong UP-ideal of Π. Then, $T_P(0) \geq T_P(h)$, $Q_P(0) \geq Q_P(h)$, $E_P(0) \leq E_P(h)$, $D_P(0) \leq D_P(h)$ and $F_P(0) \leq F_P(h)$ $\forall h \in \Pi$. Now, let $h, j, k \in \Pi$. Then,

$$\begin{aligned}
T_P(h \diamond k) = T_P(j) &\geq \min \{T_P(h \diamond (j \diamond k)), T_P(j)\}, \\
Q_P(h \diamond k) = Q_P(j) &\geq \min \{Q_P(h \diamond (j \diamond k)), Q_P(j)\}, \\
E_P(h \diamond k) = E_P(j) &\leq \max \{E_P(h \diamond (j \diamond k)), E_P(j)\}, \\
D_P(h \diamond k) = D_P(j) &\leq \max \{D_P(h \diamond (j \diamond k)), D_P(j)\}, \\
F_P(h \diamond k) = F_P(j) &\leq \max \{F_P(h \diamond (j \diamond k)), F_P(j)\}.
\end{aligned}$$

Hence, P is a pentapartitioned neutrosophic UP-ideal of Π. □

The following example shows that the converse of Theorem 4.7 is not true.

Example 4.8. From Example 3.9, P is a pentapartitioned neutrosophic UP-ideal of Π. Since

$$F_P(n) = 0.4 > 0 = \max \{F_P((b \diamond 0) \diamond (b \diamond n)), F_P(0)\},$$

P is not a pentapartitioned neutrosophic strong UP-ideal of Π.

Remark 4.9. Using Theorems 4.1, 4.3, 4.5 and 4.7; and Examples 4.2, 4.4, 4.6 and 4.8 we show the following:

- A pentapartitioned neutrosophic UP-subalgebra is an extension of a pentapartitioned neutrosophic near UP-filter.
- A pentapartitioned neutrosophic near UP-filter is an extension of a pentapartitioned neutrosophic UP-filter.
- A pentapartitioned neutrosophic UP-filter is an extension of a pentapartitioned neutrosophic UP-ideal.
- A pentapartitioned neutrosophic UP-ideal is an extension of pentapartitioned neutrosophic strong UP-ideals.
- Theorem 3.13 obtains that a pentapartitioned neutrosophic strong UP-ideal and a constant pentapartitioned neutrosophic strucure are coincided.

Theorem 4.10. *If*

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic UP-subalgebra of Π *satisfying the following condition* $(\forall h, j \in \Pi)$:

$$(h \diamond j \neq 0) \Rightarrow \begin{pmatrix} T_P(h) \geq T_P(j), \\ Q_P(h) \geq Q_P(j), \\ E_P(h) \leq E_P(j), \\ D_P(h) \leq D_P(j), \\ F_P(h) \leq F_P(j) \end{pmatrix},$$

then P *is a pentapartitioned neutrosophic near UP-filter of* Π.

Proof. Assume that

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic UP-subalgebra of Π satisfying the assumption. This implies that P satisfies the conditions in Proposition 3.2. Now, let $h, j \in \Pi$. Then, we have the following two cases:

Case (1). If $(h \diamond j = 0)$, then

$$\begin{aligned} T_P(h \diamond j) = T_P(0) \geq T_P(j), \\ Q_P(h \diamond j) = Q_P(0) \geq Q_P(j), \\ E_P(h \diamond j) = E_P(0) \leq E_P(j), \\ D_P(h \diamond j) = D_P(0) \leq D_P(j), \\ F_P(h \diamond j) = D_P(0) \leq F_P(j). \end{aligned}$$

Case (2). If $(h \diamond j \neq 0)$, then

$$\begin{aligned} T_P(h \diamond j) \geq \min\{T_P(h), T_P(j)\} = T_P(j), \\ Q_P(h \diamond j) \geq \min\{Q_P(h), Q_P(j)\} = Q_P(j), \\ E_P(h \diamond j) \leq \max\{E_P(h), E_P(j)\} = E_P(j), \\ D_P(h \diamond j) \leq \max\{D_P(h), D_P(j)\} = D_P(j), \\ F_P(h \diamond j) \leq \max\{F_P(h), F_P(j)\} = F_P(j). \end{aligned}$$

Thus, P is a pentapartitioned neutrosophic near UP-filter of Π. □

Theorem 4.11. *If*

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic UP-filter of Π *satisfying the following condition* $(\forall h, j, k \in \Pi)$:

$$\begin{pmatrix} T_P(j \diamond (h \diamond k)) = T_P(h \diamond (j \diamond k)) \\ Q_P(j \diamond (h \diamond k)) = Q_P(h \diamond (j \diamond k)) \\ E_P(j \diamond (h \diamond k)) = E_P(h \diamond (j \diamond k)) \\ D_P(j \diamond (h \diamond k)) = D_P(h \diamond (j \diamond k)) \\ F_P(j \diamond (h \diamond k)) = F_P(h \diamond (j \diamond k)) \end{pmatrix},$$

then P *is a pentapartitioned neutrosophic UP-ideal of* Π.

Proof. Assume that

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic UP-filter of Π satisfying the assumption. Then, P satisfies the conditions in Proposition 3.2. Now, let $h, j, k \in \Pi$. Then,

$$\begin{aligned} T_P(h \diamond k) &\geq \min \{T_P(j \diamond (h \diamond k)), T_P(j)\} \\ &= \min \{T_P(h \diamond (j \diamond k)), T_P(j)\}, \\ Q_P(h \diamond k) &\geq \min \{Q_P(j \diamond (h \diamond k)), Q_P(j)\} \\ &= \min \{Q_P(h \diamond (j \diamond k)), Q_P(j)\}, \\ E_P(h \diamond k) &\leq \max \{E_P(j \diamond (h \diamond k)), E_P(j)\} \\ &= \max \{E_P(h \diamond (j \diamond k)), E_P(j)\}, \\ D_P(h \diamond k) &\leq \max \{D_P(j \diamond (h \diamond k)), D_P(j)\} \\ &= \max \{D_P(h \diamond (j \diamond k)), D_P(j)\}, \\ F_P(h \diamond k) &\leq \max \{F_P(j \diamond (h \diamond k)), F_P(j)\} \\ &= \max \{F_P(h \diamond (j \diamond k)), F_P(j)\}, \end{aligned}$$

Therefore, P is a pentapartitioned neutrosophic UP-ideal of Π. □

5. UP-Algebraic Substructures and Pentapartitioned Neutrosophic Structures

This section investigates some results on certain types of UP-algebraic substructures in view of pentapartitioned neutrosophic structures.

Theorem 5.1. *If*

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic structure of Π *satisfying the condition* (**S1**), *where*

$$(\mathbf{S1})\quad (\forall h,j,k\in\Pi)(k\leq h\diamond j)\Rightarrow\left(\begin{array}{l} T_P(k)\geq\min\{T_P(h),T_P(j)\},\\ Q_P(k)\geq\min\{Q_P(h),Q_P(j)\},\\ E_P(k)\leq\max\{E_P(h),E_P(j)\},\\ D_P(k)\leq\max\{D_P(h),D_P(j)\},\\ F_P(k)\leq\max\{F_P(h),F_P(j)\}\end{array}\right),$$

then P is a pentapartitioned neutrosophic UP-subalgebra of Π.

Proof. Assume that

$$P=\langle T_P,Q_P,E_P,D_P,F_P\rangle$$

is a pentapartitioned neutrosophic structure of Π satisfying the condition $(\mathbf{S1})$. Let $h,j\in\Pi$. Then, by (1) of Proposition 2.2, $(h\diamond j)\diamond(h\diamond j)=0$, that is $h\diamond j\geq h\diamond j$. It follows from $(\mathbf{S1})$ that

$$\begin{array}{c} T_P(h\diamond j)\geq\min\{T_P(h),T_P(j)\},\\ Q_P(h\diamond j)\geq\min\{Q_P(h),Q_P(j)\},\\ E_P(h\diamond j)\leq\max\{E_P(h),E_P(j)\},\\ D_P(h\diamond j)\leq\max\{D_P(h),D_P(j)\},\\ F_P(h\diamond j)\leq\max\{F_P(h),F_P(j)\}.\end{array}$$

Hence, P is a pentapartitioned neutrosophic UP-subalgebra of Π. □

Theorem 5.2. *If*

$$P=\langle T_P,Q_P,E_P,D_P,F_P\rangle$$

is a pentapartitioned neutrosophic structure of Π *satisfying the condition* $(\mathbf{S2})$, *where*

$$(\mathbf{S2})\quad (\forall h,j,k\in\Pi)(k\leq h\diamond j)\Rightarrow\left(\begin{array}{l} T_P(j)\geq\min\{T_P(k),T_P(h)\},\\ Q_P(j)\geq\min\{Q_P(k),Q_P(h)\},\\ E_P(j)\leq\max\{E_P(k),E_P(h)\},\\ D_P(j)\leq\max\{D_P(k),D_P(h)\},\\ F_P(j)\leq\max\{F_P(k),F_P(h)\}\end{array}\right),$$

then P is a pentapartitioned neutrosophic UP-filter of Π.

Proof. Assume that

$$P=\langle T_P,Q_P,E_P,D_P,F_P\rangle$$

is a pentapartitioned neutrosophic structure of Π satisfying the condition $(\mathbf{S2})$. Let $h\in\Pi$. Then, by (3) of Definition 2.1, $h\diamond(h\diamond 0)=0$, that is $(h\leq h\diamond 0)$. It follows from $(\mathbf{S2})$ that

$$T_P(0) \geq \min\{T_P(h), T_P(h)\} = T_P(h),$$
$$Q_P(0) \geq \min\{Q_P(h), Q_P(h)\} = Q_P(h),$$
$$E_P(0) \leq \max\{E_P(h), E_P(h)\} = E_P(h),$$
$$D_P(0) \leq \max\{D_P(h), D_P(h)\} = D_P(h),$$
$$F_P(0) \leq \max\{F_P(h), F_P(h)\} = F_P(h).$$

Next, let $h, j \in \Pi$. Then, by (1) of Proposition 2.2, we have $(h \diamond j) \diamond (h \diamond j) = 0$, that is $h \diamond j \geq h \diamond j$. This implies that

$$T_P(j) \geq \min\{T_P(h \diamond j), T_P(h)\},$$
$$Q_P(j) \geq \min\{Q_P(h \diamond j), Q_P(h)\},$$
$$E_P(j) \leq \max\{E_P(h \diamond j), E_P(h)\},$$
$$D_P(j) \leq \max\{D_P(h \diamond j), D_P(h)\},$$
$$F_P(j) \leq \max\{F_P(h \diamond j), F_P(h)\}.$$

Thus, P is a pentapartitioned neutrosophic UP-filter of Π. □

Theorem 5.3. *If*

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic structure of Π *satisfying the condition* (**S3**), *where* $(\forall a, h, j, k \in \Pi)$

$$(\mathbf{S3}) \quad (a \leq h \diamond (j \diamond k)) \Rightarrow \begin{pmatrix} T_P(h \diamond k) \geq \min\{T_P(a), T_P(j)\}, \\ Q_P(h \diamond k) \geq \min\{Q_P(a), Q_P(j)\}, \\ E_P(h \diamond k) \leq \max\{E_P(a), E_P(j)\}, \\ D_P(h \diamond k) \leq \max\{D_P(a), D_P(j)\}, \\ F_P(h \diamond k) \leq \max\{F_P(a), F_P(j)\} \end{pmatrix},$$

then P *is a pentapartitioned neutrosophic UP-ideal of* Π.

Proof. Assume that

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic structure of Π satisfying the condition (**S3**). Let $h \in \Pi$. Then, by (3) of Definition 2.1, $h \diamond (0 \diamond (h \diamond 0)) = 0$, that is $h \leq 0 \diamond (h \diamond 0)$. It follows that

$$T_p(0) = T_P(0 \diamond 0) \geq \min\{T_P(h), T_P(h)\} = T_P(h),$$
$$Q_p(0) = Q_P(0 \diamond 0) \geq \min\{Q_P(h), Q_P(h)\} = Q_P(h),$$
$$E_p(0) = E_P(0 \diamond 0) \leq \max\{E_P(h), E_P(h)\} = E_P(h),$$
$$D_p(0) = D_P(0 \diamond 0) \leq \max\{D_P(h), D_P(h)\} = D_P(h),$$
$$F_p(0) = F_P(0 \diamond 0) \leq \max\{F_P(h), F_P(h)\} = F_P(h).$$

Next, let $h, j, k \in \Pi$. Then, by (1) of Definition 2.2, we have $(h \diamond (j \diamond k)) \diamond (h \diamond (j \diamond k)) = 0$, that is $h \diamond (j \diamond k) \geq h \diamond (j \diamond k)$. It follows that

$$\begin{aligned} T_P(h \diamond k) &\geq \min\{T_P(h \diamond (j \diamond k)), T_P(j)\}, \\ Q_P(h \diamond k) &\geq \min\{Q_P(h \diamond (j \diamond k)), Q_P(j)\}, \\ E_P(h \diamond k) &\leq \max\{E_P(h \diamond (j \diamond k)), E_P(j)\}, \\ D_P(h \diamond k) &\leq \max\{D_P(h \diamond (j \diamond k)), D_P(j)\}, \\ F_P(h \diamond k) &\leq \max\{F_P(h \diamond (j \diamond k)), F_P(j)\}. \end{aligned}$$

Hence, P is a pentapartitioned neutrosophic UP-ideal of Π. □

Theorem 5.4. *A pentapartitioned neutrosophic structure*

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

satisfies the condition (**S4**), *where*

$$(\mathbf{S4}) \quad (\forall h, j, k \in \Pi)(k \leq h \diamond j) \Rightarrow \begin{pmatrix} T_P(k) \geq T_P(j), \\ Q_P(k) \geq Q_P(j), \\ E_P(k) \leq E_P(j), \\ D_P(k) \leq D_P(j), \\ F_P(k) \leq F_P(j) \end{pmatrix}$$

if and only if P is a pentapartitioned neutrosophic strong UP-ideal of Π.

Proof. Assume that

$$P = \langle T_P, Q_P, E_P, D_P, F_P \rangle$$

is a pentapartitioned neutrosophic structure of Π satisfying the condition (**S4**). Let $h, j \in \Pi$. Then, By (3) of Definition 2.1 and (1) of Definition 2.2, $(h \diamond 0 = 0)$, that is $(h \leq 0 = j \diamond j)$. It follows from (**S4**) that

$$\begin{aligned} T_P(h) &\geq T_P(j), \\ Q_P(h) &\geq Q_P(j), \\ E_P(h) &\leq E_P(j), \\ D_P(h) &\leq D_P(j) \\ F_P(h) &\leq F_P(j). \end{aligned}$$

Similarly,

$$\begin{aligned} T_P(j) &\geq T_P(h), \\ Q_P(j) &\geq Q_P(h), \\ E_P(j) &\leq E_P(h), \\ D_P(j) &\leq D_P(h) \\ F_P(j) &\leq F_P(h). \end{aligned}$$

Then,

$$T_P(h) = T_P(j),$$
$$Q_P(h) = Q_P(j),$$
$$E_P(h) = E_P(j),$$
$$D_P(h) = D_P(j),$$
$$F_P(h) = F_P(j).$$

Thus, P is constant. Hence, by Theorem 3.13, P is a pentapartitioned neutrosophic strong UP-ideal of Π. □

6. Conclusions

In this paper, we introduced the notions of pentapartitioned neutrosophic UP-subalgebras, pentapartitioned neutrosophic near UP-filters, pentapartitioned neutrosophic UP-filters, pentapartitioned neutrosophic UP-ideals and pentapartitioned neutrosophic strong UP-ideals in UP-algebras and proved their generalizations. Furthermore, we discussed the relationship between pentapartitioned neutrosophic UP-subalgebras (resp., pentapartitioned neutrosophic near UP-filters, pentapartitioned neutrosophic UP-filters, pentapartitioned neutrosophic UP-ideals and pentapartitioned neutrosophic strong UP-ideals) in UP-algebras. After that, the conditions under which pentapartitioned neutrosophic UP-subalgebra can be pentapartitioned neutrosophic near UP-filter, and the condition under which pentapartitioned neutrosophic UP-filter can be pentapartitioned neutrosophic UP-ideal in UP-algebra were discovered. At last, we presented and proved some characterizations theorems of pentapartitioned neutrosophic structures in connection with UP-subalgebraic structures. In the future work, we will use the idea and results in this paper to study other algebraic structures, for example, KU-algebras, hoop algebras, MV-algebra and equality algebra.

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Received: Dec. 2, 2024. Accepted: June 4, 2025