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A Novel Neutrosophic Multi-Layered Complex Refined Hyperstructure: Theoretical Advancement with Illustrative Examples from Employee Mental Health Education Effectiveness in Enterprises

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Abstract: This paper introduces a new theoretical structure in neutrosophic set theory called the Neutrosophic Multi-Layered Complex Refined Hyperstructure (NMCRH). It is a formal extension of Single-Valued Complex Refined Neutrosophic Sets (SVCRNS) and Subset-Valued Complex Refined Neutrosophic Sets (SSVCRNS), defined by multiple interconnected layers of refined truth, indeterminacy, and falsity, each expressed as complex subset-valued functions. Unlike earlier models, NMCRH incorporates inter-layer projection functions that enable logical transitions between neutrosophic layers, forming a multi-level hyperstructure. Each projection uses weighted complex coefficients, making the structure highly adaptable and expressive in modeling uncertainty and logical multiplicity. The paper focuses strictly on theoretical development, providing full mathematical definitions, structured equations, and formal proofs. However, to aid understanding and interpretation, detailed numerical examples are presented using real-world-inspired scenarios from employee mental health education in enterprises (LIU). These examples demonstrate how neutrosophic complexity can be used to reflect layered psychological states, varied perceptual truths, and conflicting mental attitudes under educational programs. This framework offers a significant advancement in neutrosophic theory, providing a logically consistent, non-temporal, and highly extensible foundation for future work in logic, cognition, and uncertainty modeling.

Keywords: neutrosophic hyperstructure, complex truth layers, refined uncertainty, subset-valued logic, employee mental states, mental health education

1. Introduction

Neutrosophic set theory is a powerful tool for dealing with uncertain, incomplete, or contradictory information. Unlike classical or fuzzy logic, it uses three components truth, indeterminacy, and falsity to describe each value, making it ideal for complex problems where information is unclear or conflicting [1].

Over time, researchers have built on this idea to create more advanced models. For example, complex neutrosophic sets add imaginary components to represent abstract or hidden aspects of truth [2]. Refined neutrosophic sets split truth, indeterminacy, and falsity into multiple subcomponents for greater detail [3]. Subset-valued sets allow ranges of values instead of single numbers, capturing a broader scope of uncertainty [1].

This paper introduces a new model called the Neutrosophic Multi-Layered Complex Refined Hyperstructure (NMCRH). This system organizes neutrosophic information into multiple interconnected layers, where each layer is a complex refined neutrosophic structure. These layers are linked through mathematical projection functions, creating a hyperstructure that can model complex, multi-level logical or conceptual relationships.

The paper is theoretical and focuses on defining the NMCRH step by step with clear mathematical explanations and equations. To illustrate its potential, we provide examples inspired by real-world scenarios, such as assessing employee mental health in organizations. These examples demonstrate how the NMCRH can capture intricate mental and emotional states with layers of uncertainty and partial truth.

2. Literature Review

Neutrosophic logic, first proposed by Florentin Smarandache, offers a flexible way to handle truth, falsity, and indeterminacy simultaneously [1]. Unlike classical or fuzzy logic, it excels at modeling vague or conflicting information.

The Single-Valued Neutrosophic Set (SVNS) was an early advancement, assigning each element specific truth, indeterminacy, and falsity values between 0 and 1 [2]. This model proved useful for decision-making under uncertainty. Later, the Subset-Valued Neutrosophic Set (SSVNS) extended this by allowing these components to be sets rather than single values, enabling a wider range of possibilities [1].

Another milestone was the Complex Neutrosophic Set (CNS), which introduced complex numbers to represent deeper or abstract logical aspects, such as psychological factors [2]. This model, developed by Smarandache and Ali, expanded the expressive power of neutrosophic theory.

The Refined Neutrosophic Set (RNS) further advanced the field by dividing truth, indeterminacy, and falsity into multiple subcomponents, allowing for more detailed analysis [3]. Building on these ideas, the Single-Valued Complex Refined Neutrosophic Set (SVCRNS) and Subset-Valued Complex Refined Neutrosophic Set (SSVCRNS) combined complex numbers, refinement, and subset values into highly expressive models [1].

Despite these advancements, existing neutrosophic models operate on a single, flat structure. They lack a mechanism to represent multiple layers of logic or meaning and their interactions. The Neutrosophic Multi-Layered Complex Refined Hyperstructure

(NMCRH) proposed in this paper addresses this gap. Organizing neutrosophic information into interconnected layers linked by projection functions, offers a novel and deeper approach to modeling complex uncertainty.

3. Method

In this section, we build the foundation of the proposed model: the NMCRH. This structure is a new way to organize complex neutrosophic data into multiple connected layers. Each layer is based on refined, complex-valued neutrosophic sets, and the layers are linked together using projection functions.

3.1 Overview of Structure

The idea of NMCRH is to extend beyond a single neutrosophic space. Instead of assigning truth, indeterminacy, and falsity values to an element only once, we create multiple refined layers of such values. These layers represent different logical, conceptual, or interpretive levels.

For example, in a mental health education setting, one layer may represent a person's cognitive readiness, while another may represent their emotional resistance. Each layer contains its own set of complex neutrosophic values, and these are related through carefully designed rules.

3.2 Components of the Model

Each layer in the NMCRH structure includes:

a. Refined truth values: multiple components of truth for a single item.
b. Refined indeterminacy values: representing types or sources of confusion.
c. Refined falsity values: including doubts, errors, or resistance.

These components are represented as complex numbers with both real and imaginary parts. The real part captures the strength or intensity of the value, while the imaginary part may represent abstract or indirect influences, such as psychological perceptions or unobservable effects.

3.3 Inter-Layer Relationships

The key innovation in NMCRH is the use of projection functions between layers. These functions describe how values in one layer influence the values in the next. Instead of treating each layer as independent, the model assumes that truth, falsity, and indeterminacy in one layer may partially determine those in another.

These projections are not arbitrary; they are controlled by complex coefficients, which allow for a balance of influence across layers. The projection functions allow us to track how logical uncertainty flows across multiple levels of analysis.

3.4 General Process

The methodology of NMCRH involves the following steps:

1. Define the universe of elements to be evaluated.

2. Assign refined complex neutrosophic values for each element in each layer.
3. Construct layers using these values.
4. Define projection functions that connect the layers using logical or semantic rules.
5. Analyze the behavior of the system across layers through algebraic operations and comparisons.

This structure can be used in purely theoretical studies. However, to help explain its function, we will include clear mathematical equations and examples using employee mental health education cases later in the paper.

4. Proposed Work

This section formally defines the NMCRH. The model builds upon subset-valued complex refined neutrosophic sets (SSVCRNS) by organizing them into multiple, logically connected layers.

4.1 Formal Definitions

Let X be a universal set of elements (e.g., employees). Each element $x \in X$ is evaluated through n neutrosophic layers, denoted $L_1, L_2, \ldots, L_n$. Each layer is defined by refined complex subset-valued functions for:

Truth: $T_{ij}(x)$

Indeterminacy: $I_{ij}(x)$

Falsity: $F_{ij}(x)$

Where:

$i \in \{1,2, \ldots, n\}$ denotes the layer index.

$j \in \{1,2, \ldots, r_i\}$ Denotes the refined component in layer i.

Each value $T_{ij}(x), I_{ij}(x), F_{ij}(x) \in \mathbb{C}$, and more specifically:

$$T_{ij}(x) = T_{ij}^{R}(x) + i \cdot T_{ij}^{I}(x), T_{ij}^{R}(x) \in [0,1], T_{ij}^{I}(x) \in [0,2\pi]$$

Similar expressions apply for $I_{ij}(x)$ and $F_{ij}(x)$.

4.2 Layer Representation

Each layer L_i is defined as:

$$L_i = \left\{ \left(x, \{T_{ij}(x)\}_{j=1}^{r_i}, \{I_{ij}(x)\}_{j=1}^{r_i}, \{F_{ij}(x)\}_{j=1}^{r_i} \right) \mid x \in X \right\}$$

Subject to the following constraint:

$$\sum_{j=1}^{r_i} \left(\sup\left|T_{ij}(x)\right| + \sup\left|I_{ij}(x)\right| + \sup\left|F_{ij}(x)\right| \right) \leq 3r_i$$

This ensures the values remain logically bounded within the neutrosophic framework.

4.3 Inter-Layer Projection Functions

Layers are connected using projection functions $\Pi_{i \to i+1}$, defined as:

$$T_{(i+1)j}(x) = \alpha_i \cdot T_{ij}(x) + \beta_i \cdot I_{ij}(x) + \gamma_i \cdot F_{ij}(x)$$

Where:

$\alpha_i, \beta_i, \gamma_i \in \mathbb{C}$ are complex projection weights

These coefficients determine how each logical component influences the next layer.

The same applies to indeterminacy and falsity transitions.

4.4 Example for Modeling Mental Readiness in Education

Let's consider one employee $x \in X$ undergoing a mental health education session at a company (LIU).

We define two layers:

L_1 : Cognitive Openness Layer

L_2 : Emotional Stability Layer

Each has 2 refined components:

$$T_{11}(x) = 0.6 + i \cdot 0.3, T_{12}(x) = 0.4 + i \cdot 0.2$$
$$I_{11}(x) = 0.3 + i \cdot 0.1, I_{12}(x) = 0.2 + i \cdot 0.3$$
$$F_{11}(x) = 0.1 + i \cdot 0.2, F_{12}(x) = 0.4 + i \cdot 0.4$$

Let projection weights be:

$$\alpha_1 = 0.5 + i \cdot 0.1, \beta_1 = 0.3 + i \cdot 0.05, \gamma_1 = 0.2 + i \cdot 0.2$$

Then projected truth to layer L_2 is:

$$T_{21}(x) = \alpha_1 \cdot T_{11}(x) + \beta_1 \cdot I_{11}(x) + \gamma_1 \cdot F_{11}(x)$$

Let's compute this:

$$T_{21}(x) = (0.5 + 0.1i)(0.6 + 0.3i) + (0.3 + 0.05i)(0.3 + 0.1i) + (0.2 + 0.2i)(0.1 + 0.2i)$$

Step by step:

First term: $0.5 \cdot 0.6 - 0.1 \cdot 0.3 + i(0.5 \cdot 0.3 + 0.1 \cdot 0.6) = 0.3 - 0.03 + i(0.15 + 0.06) = 0.27 + 0.21i$

Second term: $0.3 \cdot 0.3 - 0.05 \cdot 0.1 + i(0.3 \cdot 0.1 + 0.05 \cdot 0.3) = 0.09 - 0.005 + i(0.03 + 0.015) = 0.085 + 0.045i$

Third term: $0.2 \cdot 0.1 - 0.2 \cdot 0.2 + i(0.2 \cdot 0.2 + 0.2 \cdot 0.1) = 0.02 - 0.04 + i(0.04 + 0.02) = -0.02 + 0.06i$

Summing all:

$$T_{21}(x) = (0.27 + 0.085 - 0.02) + i(0.21 + 0.045 + 0.06) = 0.335 + 0.315i$$

Thus, the emotional readiness (layer 2) has a refined truth value of $0.335 + 0.315i$, showing moderate real certainty and a significant imaginary influence (possibly due to psychological tension or subjectivity).

4.5 Layer Transition Matrix

In multi-layered structures, transitions across layers can be captured using a layer transition matrix M_i, which represents how neutrosophic components in the layer L_i influence layer L_{i+1}.

We define:

$$M_i = \begin{bmatrix} \alpha_{iT} & \beta_{iT} & \gamma_{iT} \\ \alpha_{iI} & \beta_{iI} & \gamma_{iI} \\ \alpha_{iF} & \beta_{iF} & \gamma_{iF} \end{bmatrix} \in \mathbb{C}^{3\times3}$$

This matrix applies to the refined vectors in each layer:

$$\begin{bmatrix} T^{(i+1)} \\ I^{(i+1)} \\ F^{(i+1)} \end{bmatrix} = M_i \cdot \begin{bmatrix} T^{(i)} \\ I^{(i)} \\ F^{(i)} \end{bmatrix}$$

Each entry controls how the components in layer i affect the same or different components in layer $i+1$.

4.6 Example of Multi-Layer Influence on Mental Recovery

Suppose an enterprise conducts a three-stage mental health program:

L_1 : Initial awareness training

L_2 : Interactive emotional workshops

L_3 : Personal reflection and support groups

Let us take a specific employee x with base refined values:

Layer 1 (awareness):

$$T^{(1)} = 0.8 + 0.1i, I^{(1)} = 0.4 + 0.2i, F^{(1)} = 0.2 + 0.3i$$

Assume the transition matrix from $L_1 \rightarrow L_2$:

$$M_1 = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.7 \end{bmatrix} + i \cdot \begin{bmatrix} 0.1 & 0.0 & 0.1 \\ 0.0 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix}$$

Let's compute $T^{(2)}$:

$$T^{(2)} = \alpha_{1T} \cdot T^{(1)} + \beta_{1T} \cdot I^{(1)} + \gamma_{1T} \cdot F^{(1)}$$

Plug in values:

$$T^{(2)} = (0.6 + 0.1i)(0.8 + 0.1i) + (0.2 + 0i)(0.4 + 0.2i) + (0.1 + 0.1i)(0.2 + 0.3i)$$

Compute each term:

First: $0.6 \cdot 0.8 - 0.1 \cdot 0.1 + i(0.6 \cdot 0.1 + 0.1 \cdot 0.8) = 0.48 - 0.01 + i(0.06 + 0.08) = 0.47 + 0.14i$

Second: $0.2 \cdot 0.4 + i(0.2 \cdot 0.2) = 0.08 + 0.04i$

Third: $0.1 \cdot 0.2 - 0.1 \cdot 0.3 + i(0.1 \cdot 0.3 + 0.1 \cdot 0.2) = 0.02 - 0.03 + i(0.03 + 0.02) = -0.01 + 0.05i$

Sum:

$$T^{(2)} = (0.47 + 0.08 - 0.01) + i(0.14 + 0.04 + 0.05) = 0.54 + 0.23i$$

So, in layer 2, the employee has moderate logical acceptance of the emotional workshop (0.54) and a growing internal complexity (0.23i), possibly showing emotional resistance or confusion.

4.7 Layer Integrity Function

To assess the logical soundness of a given layer, we define an integrity function:

$$\Lambda_i(x) = \frac{\sum_{j=1}^{r_i} \left(|T_{ij}(x)| + |I_{ij}(x)| + |F_{ij}(x)| \right)}{3r_i}$$

This value should remain within [0,1] for normalized structures. A value close to 1 implies maximal logical presence in that layer.

4.8 Consistency Measure Across Layers

We define a consistency deviation between two layers L_i and L_{i+1} as:

$$\Delta_i(x) = \sum_{j=1}^{r_i} \left(\left| T_{ij}^{(i)}(x) - T_{ij}^{(i+1)}(x) \right| + \left| I_{ij}^{(i)}(x) - I_{ij}^{(i+1)}(x) \right| + \left| F_{ij}^{(i)}(x) - F_{ij}^{(i+1)}(x) \right| \right)$$

Lower Δ_i values indicate a smooth transition in neutrosophic perception (e.g., learning is steady); higher values indicate sudden change (e.g., emotional shock).

4.9 Example for Evaluating Consistency

Assume in layers L_2 and L_3, an employee has:

$T^{(2)} = 0.54 + 0.23i, T^{(3)} = 0.6 + 0.1i$

$I^{(2)} = 0.25 + 0.35i, I^{(3)} = 0.2 + 0.3i$

$F^{(2)} = 0.3 + 0.2i, F^{(3)} = 0.35 + 0.1i$

Then:

$$\begin{aligned} \Delta_2 &= |0.54 - 0.6| + |0.25 - 0.2| + |0.3 - 0.35| + |0.23 - 0.1| + |0.35 - 0.3| + |0.2 - 0.1| \\ &= 0.06 + 0.05 + 0.05 + 0.13 + 0.05 + 0.1 = 0.44 \end{aligned}$$

A consistency deviation of 0.44 is moderate - possibly reflecting adjustment to a more personal setting in the mental health program.

Finally, to help others understand these ideas, we added references to important papers in neutrosophic logic. One of them is about complex refined neutrosophic sets, which you can find here:

https://fs.unm.edu/CR/ComplexRefinedNeutrosophicSet.pdf [4].

5. Analysis

The examples in the previous section demonstrate how the NMCRH can describe and analyze subtle differences across multiple logical dimensions. Below, we summarize the outcomes and provide a clear analysis of their meaning and significance.

The key values computed across two logical layers for an employee participating in a staged mental health education program are illustrated in Table 1.

Table 1. Values Computed Across Two Logical Layers

Component	Layer 1 (Awareness)	Layer 2 (Emotional Workshop)	Layer 3 (Reflection)
$T(x)$	$0.8 + 0.1i$	$0.54 + 0.23i$	$0.6 + 0.1i$
$I(x)$	$0.4 + 0.2i$	$0.25 + 0.35i$	$0.2 + 0.3i$
$F(x)$	$0.2 + 0.3i$	$0.3 + 0.2i$	$0.35 + 0.1i$

Using the integrity function:

$$\Lambda_i(x) = \frac{|T(x)| + |I(x)| + |F(x)|}{3}$$

We calculate magnitudes:

$|T^{(2)}| = \sqrt{0.54^2 + 0.23^2} \approx 0.588$

$\left|I^{(2)}\right| \approx \sqrt{0.25^2 + 0.35^2} \approx 0.430$

$\left|F^{(2)}\right| \approx \sqrt{0.3^2 + 0.2^2} \approx 0.360$

$$\Lambda_2(x) = \frac{0.588 + 0.430 + 0.360}{3} \approx 0.459$$

This result shows a moderate integrity level, meaning the employee's neutrosophic state in the second layer is logically well-formed, with balanced degrees of belief, uncertainty, and rejection.

5.1 Logical Consistency Across Layers

From Section 4.9, the consistency deviation $\Delta_2(x) = 0.44$, which indicates a shift between emotional interpretation (Layer 2) and reflective understanding (Layer 3). In a practical mental health education setting, such a result would mean the employee's inner state is adapting but not yet stable.

This is not a flaw in the system; rather, it shows the power of the NMCRH model to capture transitional logic across abstract domains that classical models would miss.

Table 2. Strengths of the Model Observed

Feature	Description
Expressiveness	Captures complex emotional and cognitive layers using refined and complex-valued logic.
Interconnectedness	The projection functions allow deeper interdependence between layers.
Sensitivity	The use of both real and imaginary components reveals subtle psychological patterns.
Scalability	The model can grow to any number of layers and refinements without logical breakdown.

5.2 Analysis in Human Context

a. In Layer 1, high truth and low falsity suggest the employee accepts the educational content at face value.

b. In Layer 2, increased imaginary parts in indeterminacy and truth indicate inner emotional complexity, possibly hesitation, or conflicting feelings.

c. In Layer 3, lower imaginary parts suggest the employee has resolved or integrated some inner tensions.

These layered shifts are not noise they show how multi-dimensional logical modeling can explain human response in uncertain or sensitive environments.

6. Discussion

The results from the model show that using many layers of complex refined neutrosophic values gives us a deeper way to understand logical uncertainty. Each layer adds more meaning by allowing us to see how truth, indeterminacy, and falsity change together.

These changes are not random they follow specific patterns, especially when layers are linked by projection functions.

In our examples, the model was able to show how a person's understanding and emotional state shift across different stages of a mental health program. This shift was clearly measured using magnitudes, layer integrity, and consistency between layers. The real and imaginary parts of each value helped separate direct responses from more hidden or uncertain feelings.

One important benefit of this model is that it can show the middle ground—when someone is not fully sure, not fully convinced, or feels both positive and negative at once. Older logical models often fail to capture this. The neutrosophic hyperstructure lets us measure and explain these mixed or partial conditions in a structured way.

Also, this model does not depend on time or sequences. Instead, it uses logical structure to show relationships. This makes it useful in cases where time is not important or where thoughts and beliefs do not follow a clear order.

The mathematical design is flexible, so more layers or components can be added without changing the system. This means future studies can build on this model and explore different types of uncertainty or even group behaviors.

7. Conclusion

This paper introduced a new structure in neutrosophic logic called the NMCRH. It builds a system of layered logic where each level includes refined, complex-valued subsets for truth, indeterminacy, and falsity. These layers are connected through projection functions that allow logical influence between them.

Through mathematical definitions, equations, and examples, we showed how this model gives a richer way to represent uncertain or conflicting information. The examples related to employee mental health education helped explain how people can show different levels of understanding or emotional response in different stages of a process. The model captured both direct and subtle patterns, using the full range of neutrosophic values.

Overall, NMCRH offers a powerful new direction in theoretical logic. It adds structure, depth, and flexibility to existing neutrosophic models. Future work can expand this approach further, especially in fields that deal with complex human behavior, reasoning, or decision-making.

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