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A New Technique to Solve Game Matrix with Neutrosophic Payoffs

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Abstract. Matrix games are extensively applied to conflicting situations that frequently arise in real world since it gives the ability to the decision maker to make more informed decisions. However, modeling of such situations often cannot be done by conventional techniques as the payoffs may not be concretely determined due to uncertainty present in the system. This uncertainty can be handled in numerous ways but neutrosophic set theory plays an important role in examining intricacy, inadequacy, enigma and self-contradictory parameters in real life problems. This article develops a more structured technique to solve neutrosophic game matrix with payoffs as Single Valued Trapezoidal Neutrosophic (SVTrN) numbers. This method converts the considered game matrix to interval valued game matrix problem by using (α, β, γ)- cut on SVTrN numbers. This interval valued game matrix problem is further converted to a crisp game matrix (pessimistic, optimistic and moderate)problem by using a ranking function. Then, these problems are solved by maxmin theorem if saddle point exist. In case of no saddle point or many saddle points, the problem is solved by converting it to linear programming primal-dual problem. The proposed technique can be applied to a wider range of game theory problems existing in practical, as the data encountered in practice is often imprecise with some level of hesitation, inconsistent or incompleteness which can best be described using single valued neutrosophic number. Numerical illustrations are provided to demonstrate the methodology and to prove the vitality of the proposed method.

Keywords: Neutrosophic Matrix Game ; Single Valued Trapezoidal Neutrosophic numbers; Zero Sum Two Person Game;(α, β, γ)-cut sets.

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1. Introduction

Scholars started to study game theory in a systematic way after Neumann(1947)1's Theory of Games and Economics Behaviour and Nash(1951)[2]'s Non co-operative games. Generally, games are divided in two categories: (i) Zero sum games and (ii) Non-zero Sum games. In this article, we investigate zero sum game model which are also known as matrix game. Since, real world conflict situations are full of uncertainty, which can not be tackled by crisp set theory. So to deal with uncertainty Zadeh[3] introduced fuzzy set theory which has been widely used by many scholars. Butnariu et. al[4] introduced fuzzy set in game theory and fuzzy game. Compose, L.[5, 6] discussed the maximum-minimum equilibrium of fuzzy matrix game. Bector et al[7] elaborated concept of fuzzy set in mathematical programming and in matrix games. Dubious[8] presented matrix games with fuzzy payoffs. Vijay and Chandra[9] discussed matrix games with fuzzy goals and fuzzy payoffs by using fuzzy linear optimization problem. Vijay, A. Mehra and Chandra[10] presented a fuzzy relation approach to solve fuzzy matrix games. Fuzzy set theory deals with only membership degree of the elements to represent uncertainty. To over come this problem Antanassov[11, 12] introduced Intuitionistic Fuzzy Set(IFS) theory which is a generalization of fuzzy set theory. IFS helps to determine the uncertainty of non membership degree also. An IFS is characterized by membership and non-membership functions representing the degree of belongingness and the degree of non-belongingness such that sum of these two values is less than or equal to one. Concept of intuitionistic fuzzy set has been applied by several scholars[13, 14, 15, 16, 17, 18, 19, 20, 21] to study intuitionistic fuzzy matrix games. Interval Valued Intutionistics Fuzzy(IVIF) sets are extension of the IFS. Garg and Kumar[22] discussed the TOPSIS method under the IVIF environment. E.Fathy[23] suggested a method which takes all parameters or decision variables as IVIF numbers.

In real life, the parameters of matrix game problems may have some indeterminacy or inconsistency along with uncertainty. An IFS is unable to handle such indeterminacy in the information efficiently. To deal with such indeterminacy in the data, Smarandache[24, 25] introduced the concept of Neutrosophic set theory as a extension of the IFS, which deals with indeterminate, inconsistent and incomplete circumstances. In the NS(Neutrosophic Set), the degree of indeterminacy membership is independent of the degree of falsity-membership and the degree of truth membership. Kadali et al.[26] used the technique "Non-homogeneous Poisson process with neutrosophic logic" in evaluating and deterring crime based on crime data analysis. They also applied reinforcement learning in crime cluster data for customizing hyperparameters. Software reliability growth models are also being used for improving the uncertain cluster of crime. Dhanalakshmi P.[27] came up with the new concept of rough fermatean neutrosophic sets and examined cosine similarity of these sets. Also, discussed its applications in Medical Diagnosis. Salama A. A. et al.[28] proposed a Neutrosophic model for measuring and evaluating the Role of digital transformation in improving sustainable performance in Egyptian Universities. This model may be used to develop strategies for improving sustainable performance. They applied this model in 10 Egyptian universities and showed that Egyptian universites should focused on investing in two digital transformation indicators, the percentage of course offered online and the percentage of students using digital learning platforms to improve their sustainable performance.

Wang et al.[29] introduced single value neutrosophic set as an extension of the NS. Many scholars have applied the NS in decision making problems[30, 31, 32, 33, 34, 35, 39]. Khalid et al.[36] come up with a new idea for single valued neutrosophic fuzzy soft set. Aries et al.[37] presented a neutrosophic model for non-co-operative games to handle the contemporary conflicting political circumstances. In their study, they present a solution to neutrosophic matrix games with payoffs as single valued triangular neutrosophic(SVTN) numbers and nicely apply it by finding the solution A voter problem. Bhaumik et al.[38] developed a ranking approach to solve bi-matrix games based on (α, β, γ)-cuts set of a SVTN. Vinod et al.[40] gave a technique to solve matrix games with (SVTN) as payoffs. They used a ranking function for de-neutrosophication, based on ambiguity index and value index. Seikh et al.[41] came up a nonlinear programming model to solve matrix games with payoffs of single valued neutrosophic numbers. They first obtained two multi-objective programming(MOP) problems from the matrix game problem, then these MOP problems are converted to bi-objective programming(BOP) problems. Finally, they used Lexicographic method to find optimal strategies for prayers. Sharma et al.[42] presented a method to solve Two-Person Zero-Sum matrix games With payoff as (SVTN). In their paper, they used Mellin's Transformation to get crisp game problem from neutrosophic game problem.

The proposed article aims to providing an efficient method for solving neutrosophic matrix game. The major novelties of the proposed method are pointed as:

- The projected technique generates the optimal solution for considered problem in pessimistic($\lambda = 0$), optimistic($\lambda = 1$) and moderate($\lambda = 0.5$) matrix game problems.
- The proposed method is based on (α, β, γ)-cuts set values and decision maker attitude $\lambda \in [0, 1]$. Hence, decision maker has flexibility to vary these parameter according to his requirements.
- The proposed technique is better than oriented or traditional method as it covers wider range of problems. Since SVTN numbers, Triangular or Trapezodial IFN and Triangular or Trapezodial fuzzy numbers are the special cases of SVTrN numbers.

The rest of this study is arranged as follows: the next section is subdivided into three subsections: Subsection 2.1 reviewed some basic definitions about NS and SVTrN numbers. In Subsection 2.2, arithmetic operations of SVTrN Numbers is proposed. Subsection 2.3, cut sets of SVTrN Numbers are defined. In section 3, the mathematical formation and methodology to find solution of game matrix whose entries are SVTrN number is discussed. Numerical illustration is address in Section 4. The result and discussion of this paper is given in Section 5. Finally, the conclusion is presented in Section 6.

2. Basic Definitions and Terminology

2.1. *Basic Definitions*

This section reviews some basic definitions pertaining to neutrosophic set theory.

Definition 2.1. [24] A neutrosophic set $\widetilde{N}$ is characterized by its three functions-the truth membership function $\mu_{\widetilde{N}}$, indeterminacy membership function $\pi_{\widetilde{N}}$ and falsity membership function $\nu_{\widetilde{N}}$ which associates with $u_i \in U = \{u_1, u_2, \ldots, u_n\}$(the universal set) a degree in the interval $[0, 1]$. A NS $\widetilde{N}$ consists of the following form

$$\widetilde{N} = \{\langle u_i, \mu_{\widetilde{N}}(u_i), \pi_{\widetilde{N}}(u_i), \nu_{\widetilde{N}}(u_i)\rangle : u_i \in U\},$$

where the functions $\mu_{\widetilde{N}} : U \to [0, 1]$; *i.e.*, $u_i \in U \to \mu_{\widetilde{N}}(u_i) \in [0, 1]$, $\pi_{\widetilde{N}} : U \to [0, 1]$; *i.e.*, $u_i \in U \to \pi_{\widetilde{N}}(u_i) \in [0, 1]$ and $\nu_{\widetilde{N}} : U \to [0, 1]$; *i.e.*, $u_i \in U \to \nu_{\widetilde{N}}(u_i) \in [0, 1]$ define the degree of truth, indeterminacy and falsity membership functions of an element$u_i \in U$ respectively.

Definition 2.2. [39] A single valued trapezoidal neutrosophic(SVTrN) number is defined by $\widetilde{p}^N = (p_1, p_2, p_3, p_4; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N})$, where $p_1, p_2, p_3, p_4 \in \mathbb{R}$ and $t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N} \in [0, 1]$ with condition that $p_1 \leq p_2 \leq p_3 \leq p_4$. The truth, indeterminacy and falsity membership functions of $\widetilde{p}^N$ are given as follows:

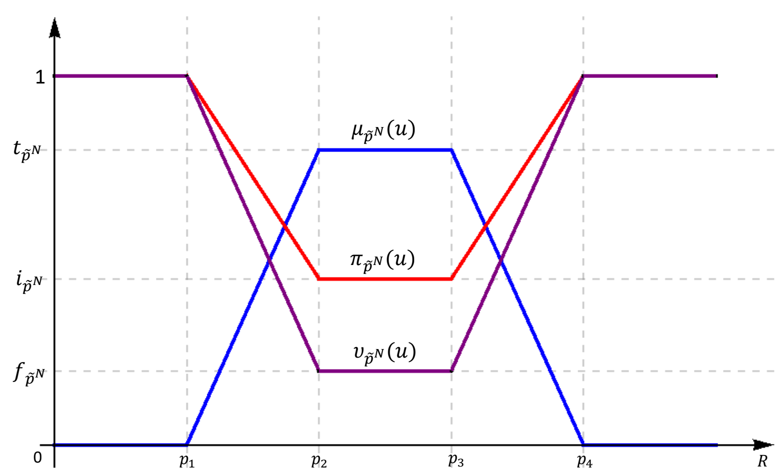

FIGURE 1. SVTrN-number

$$\mu_{\widetilde{p}^N}(u) = \begin{cases} t_{\widetilde{p}^N}\left(\dfrac{u-p_1}{p_2-p_1}\right); & \text{if } p_1 \le u \le p_2 \\ t_{\widetilde{p}^N}; & \text{if } p_2 \le u \le p_3 \\ t_{\widetilde{P}^N}\left(\dfrac{p_4-u}{p_4-p_3}\right); & \text{if } p_3 \le u \le p_4 \\ 0; & \text{if otherwise.} \end{cases}$$

$$\pi_{\widetilde{p}^N}(u) = \begin{cases} \dfrac{(p_2-u)+i_{\widetilde{p}^N}(u-p_1)}{p_2-p_1}; & \text{if } p_1 \le u \le p_2 \\ i_{\widetilde{p}^N}; & \text{if } p_2 \le u \le p_3 \\ \dfrac{(u-p_3)+i_{\widetilde{p}^N}(p_4-u)}{p_4-p_3}; & \text{if } p_3 \le u \le p_4 \\ 1; & \text{if otherwise.} \end{cases}$$

$$\nu_{\widetilde{p}^N}(u) = \begin{cases} \dfrac{(p_2-u)+f_{\widetilde{p}^N}(u-p_1)}{p_2-p_1}; & \text{if } p_1 \le u \le p_2 \\ f_{\widetilde{p}^N}; & \text{if } p_2 \le u \le p_3 \\ \dfrac{(u-p_3)+f_{\widetilde{p}^N}(p_4-u)}{p_4-p_3}; & \text{if } p_3 \le u \le p_4 \\ 1; & \text{if otherwise.} \end{cases}$$

where $t_{\widetilde{p}^N}$, $i_{\widetilde{p}^N}$ and $f_{\widetilde{p}^N}$ represent the maximum truth, minimum-indeterminacy and minimum falsity membership grade respectively. The geometric representation of SVTrN is shown in figure 1.

2.2. *Arithmetic Operations of SVTrN numbers*

Let $\widetilde{p}^N = (p_1, p_2, p_3, p_4; t_{\widetilde{P}^N}, i_{\widetilde{P}^N}, f_{\widetilde{P}^N})$ and $\widetilde{q}^N = (q_1, q_2, q_3, q_4; t_{\widetilde{q}^N}, i_{\widetilde{q}^N}, f_{\widetilde{q}^N})$ be two SVTrN numbers and the operators$(\wedge, \vee)$ be the operators(min, max) respectively and $k \neq 0$ be any real number. The following are the SVTrN numbers' arithmetic operations[39]:

Addition:

$$\widetilde{p}^N \oplus \widetilde{q}^N = (p_1+q_1, p_2+q_2, p_3+q_3; p_4+q_4; t_{\widetilde{p}^N} \wedge t_{\widetilde{q}^N}, i_{\widetilde{p}^N} \vee i_{\widetilde{q}^N}, f_{\widetilde{p}^N} \vee f_{\widetilde{q}^N}).$$

Subtraction:

$$\widetilde{p}^N \ominus \widetilde{q}^N = (p_1-q_4, p_2-q_3, p_3-q_2, p_4+q_1; t_{\widetilde{p}^N} \wedge t_{\widetilde{q}^N}, i_{\widetilde{p}^N} \vee i_{\widetilde{q}^N}, f_{\widetilde{p}^N} \vee f_{\widetilde{q}^N}).$$

Multiplication:

$$\widetilde{p}^N \otimes \widetilde{q}^N = \begin{cases} (p_1q_1, p_2q_2, p_3q_3; p_4q_4; t_{\widetilde{p}^N} \wedge t_{\widetilde{q}^N}, i_{\widetilde{p}^N} \vee i_{\widetilde{q}^N}, f_{\widetilde{p}^N} \vee f_{\widetilde{q}^N}) & \text{if } \widetilde{p}^N \succeq 0 \text{ and } \widetilde{q}^N \succeq 0. \\ (p_1q_4, p_2q_3, p_3q_2; p_4q_1; t_{\widetilde{p}^N} \wedge t_{\widetilde{q}^N}, i_{\widetilde{p}^N} \vee i_{\widetilde{q}^N}, f_{\widetilde{p}^N} \vee f_{\widetilde{q}^N}) & \text{if } \widetilde{p}^N \preccurlyeq 0 \text{ and } \widetilde{q}^N \succeq 0. \\ (p_4q_4, p_3q_3, p_2q_2; p_4q_4; t_{\widetilde{p}^N} \wedge t_{\widetilde{q}^N}, i_{\widetilde{p}^N} \vee i_{\widetilde{q}^N}, f_{\widetilde{p}^N} \vee f_{\widetilde{q}^N}) & \text{if } \widetilde{p}^N \preccurlyeq 0 \text{ and } \widetilde{q}^N \preccurlyeq 0. \end{cases}$$

Division:

$$\widetilde{p}^N \div \widetilde{q}^N = \begin{cases} (\frac{p_1}{q_4}, \frac{p_2}{q_3}, \frac{p_3}{q_2}; \frac{p_4}{q_1}; t_{\widetilde{p}^N} \wedge t_{\widetilde{q}^N}, i_{\widetilde{p}^N} \vee i_{\widetilde{q}^N}, f_{\widetilde{p}^N} \vee f_{\widetilde{q}^N}) & \text{if } \widetilde{p}^N \succeq 0 \text{ and } \widetilde{q}^N \succeq 0. \\ (\frac{p_4}{q_4}, \frac{p_3}{q_3}, \frac{p_2}{q_2}; \frac{p_1}{q_1}; t_{\widetilde{p}^N} \wedge t_{\widetilde{q}^N}, i_{\widetilde{p}^N} \vee i_{\widetilde{q}^N}, f_{\widetilde{p}^N} \vee f_{\widetilde{q}^N}) & \text{if } \widetilde{p}^N \preccurlyeq 0 \text{ and } \widetilde{q}^N \succeq 0. \\ (\frac{p_4}{q_1}, \frac{p_3}{q_2}, \frac{p_2}{q_3}; \frac{p_1}{q_4}; t_{\widetilde{p}^N} \wedge t_{\widetilde{q}^N}, i_{\widetilde{p}^N} \vee i_{\widetilde{q}^N}, f_{\widetilde{p}^N} \vee f_{\widetilde{q}^N}) & \text{if } \widetilde{p}^N \preccurlyeq 0 \text{ and } \widetilde{q}^N \preccurlyeq 0. \end{cases}$$

Scalar multiplication:

$$k\widetilde{p}^N = \begin{cases} (kp_1, kp_2, kp_3, kp_4; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N}) & \text{if } k > 0 \\ (kp_4, kp_3, kp_2, kp_1; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N}) & \text{if } k > 0 \end{cases}$$

Thus, the set of all SVTrN numbers is closed with respected to all the above stated arithmetic operations.

2.3. *Cut sets for SVTrN*

Definition 2.3. An α-cut set of truth function of a single valued trapezoidal neutrosophic(SVTrN) number $\widetilde{p}^N = (p_1, p_2, p_3, p_4; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N})$ is the following crisp subset of $\mathbb{R}$:

$$\widetilde{p}^N_\alpha = \{u | \mu_{\widetilde{p}^N}(u) \geq \alpha\},$$

where $0 \leq \alpha \leq t_{\widetilde{p}^N}$.

Clearly, any α cut set of $\widetilde{p}^N$ for truth membership function is a closed interval, given by

$$\widetilde{p}^N_\alpha = \left[a^L_{\widetilde{p}^N}(\alpha), a^R_{\widetilde{p}^N}(\alpha)\right] = \left[\frac{(t_{\widetilde{p}^N} - \alpha)p_1 + \alpha p_2}{t_{\widetilde{p}^N}}, \frac{(t_{\widetilde{p}^N} - \alpha)p_4 + \alpha p_3}{t_{\widetilde{p}^N}}\right]$$

Definition 2.4. A β-cut set of indeterminacy function of a single valued trapezoidal neutrosophic(SVTrN) number $\widetilde{p}^N = (p_1, p_2, p_3, p_4; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N})$ is the following crisp subset of $\mathbb{R}$:

$$\widetilde{p}^N_\beta = \{u | \pi_{\widetilde{p}^N}(u) \leq \beta\},$$

where $i_{\widetilde{p}^N} \leq \beta \leq 1$.

Clearly, any β cut set of $\widetilde{p}^N$ for indeterminacy membership function is a closed interval, given by

$$\widetilde{p}^N_\beta = \left[a^L_{\widetilde{p}^N}(\beta), a^R_{\widetilde{p}^N}(\beta)\right] = \left[\frac{(1-\beta)p_2 + (\beta - i_{\widetilde{p}^N})p_1}{1 - i_{\widetilde{p}^N}}, \frac{(1-\beta)p_3 + (\beta - i_{\widetilde{p}^N})p_4}{1 - i_{\widetilde{p}^N}}\right]$$

Definition 2.5. A γ-cut set of falsity function of a single valued trapezoidal neutrosophic(SVTrN) number $\widetilde{p}^N = (p_1, p_2, p_3, p_4; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N})$ is the following crisp subset of $\mathbb{R}$:

$$\widetilde{p}^N_\gamma = \{u | \nu_{\widetilde{p}^N}(u) \leq \gamma\}$$

where $f_{\widetilde{p}^N} \leq \gamma \leq 1$.

Clearly, any γ cut set of $\widetilde{p}^N$ for indeterminacy membership function is a closed interval, given by

$$\widetilde{p}^N_\gamma = \left[a^L_{\widetilde{p}^N}(\gamma), a^R_{\widetilde{p}^N}(\gamma)\right] = \left[\frac{(1-\gamma)p_2 + (\gamma - t_{\widetilde{p}^N})p_1}{1 - f_{\widetilde{p}^N}}, \frac{(1-\gamma)p_3 + (\gamma - i_{\widetilde{p}^N})p_4}{1 - f_{\widetilde{p}^N}}\right]$$

Definition 2.6. An (α, β, γ)-cut set of a single valued trapezoidal neutrosophic(SVTrN) number $\widetilde{p}^N = (p_1, p_2, p_3, p_4; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N})$ is the following crisp subset of $\mathbb{R}$:

$$\widetilde{p}^N_{<\alpha,\beta,\gamma>} = \{u | \mu_{\widetilde{p}^N}(u) \geq \alpha, \pi_{\widetilde{p}^N}(u) \leq \beta, \nu_{\widetilde{p}^N}(u) \leq \gamma\},$$

where $0 \leq \alpha \leq t_{\widetilde{p}^N}, i_{\widetilde{p}^N} \leq \beta \leq 1, f_{\widetilde{p}^N} \leq \gamma \leq 1$ and $0 \leq \alpha + \beta + \gamma \leq 3$.

It can be easily proven that for a a single valued trapezoidal neutrosophic(SVTrN) number $\widetilde{p}^N = (p_1, p_2, p_3, p_4; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N})$ and for any $\alpha \in [0, t_{\widetilde{p}^N}]$, $\beta \in [i_{\widetilde{p}^N}, 1]$ and $\gamma \in [f_{\widetilde{\mathcal{P}}}, 1]$,where $0 \leq \alpha + \beta + \gamma \leq 3$

$$I_{\widetilde{p}^N} = \widetilde{p}^N_{<\alpha,\beta,\gamma>} = \widetilde{p}^N_\alpha \wedge \widetilde{p}^N_\beta \wedge \widetilde{p}^N_\gamma \tag{1}$$

where the symbol " $\wedge$ " denote the minimum between $\widetilde{p}^N_\alpha, \widetilde{p}^N_\beta$ and $\widetilde{p}^N_\gamma$.

3. Mathematical Formulation of Game Matrix with SVTrN number

Two person zero sum($TPZS$) game is a two player matrix game where player 'X' and player 'Y' both have finite pure strategy sets. Let $S_1 = \{\delta_1, \delta_2, ..., ..., \delta_m\}$ and $S_2 = \{\gamma_1, \gamma_2, ..., ..., \gamma_n\}$ be strategy sets for player 'X' and player 'Y' respectively. The problem that we are trying to solve is a zero sum game(two players) where the payoffs are SVTrN numbers. Then the payoffs for strategy sets S_1 and S_2 may be represented by the following $m \times n$ payoff matrix:

$$\widetilde{\mathcal{P}}^N = \begin{matrix} & \gamma_1 & \gamma_2 & \gamma_3 & \cdots & \gamma_n \\ \delta_1 & \widetilde{p}^N_{11} & \widetilde{p}^N_{12} & \widetilde{p}^N_{13} & \cdots & \widetilde{p}^N_{1n} \\ \delta_2 & \widetilde{p}^N_{21} & \widetilde{p}^N_{22} & \widetilde{p}^N_{23} & \cdots & \widetilde{p}^N_{2n} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \delta_m & \widetilde{p}^N_{m1} & \widetilde{p}^N_{m2} & \widetilde{p}^N_{m3} & \cdots & \widetilde{p}^N_{mn} \end{matrix} \tag{2}$$

where $\widetilde{p}^N_{ij} = (p^1_{ij}, p^2_{1j}, p^3_{ij}, p^4_{ij}; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N})$ is the payoff when player 'X' plays the strategy i and player 'Y' plays strategy j.

4. Methodology to Solve Neutrosophic Game Matrix

To find solution of the above problem , the following steps are needed:

Step 1: Obtain (α, β, γ)-cut of each SVtrN number

Let $\widetilde{p}^N = (p_1, p_2, p_3, p_4; t_{\widetilde{p}^N}, i_{\widetilde{p}^N}, f_{\widetilde{p}^N})$ be a SVtrN number. Taking (α, β, γ)-cut on the SVTrN number $\widetilde{p}^N$, from definition(2.11) an interval $I_{\widetilde{p}^N} = \widetilde{p}^N_{<\alpha,\beta,\gamma>} = \widetilde{p}^N_\alpha \wedge \widetilde{p}^N_\beta \wedge \widetilde{p}^N_\gamma$ is obtained. Where the symbol "$\wedge$" denotes the minimum between $\widetilde{p}^N_\alpha, \widetilde{p}^N_\beta$ and $\widetilde{p}^N_\gamma$. Thus $\widetilde{p}^N_{<\alpha,\beta,\gamma>}$ is a closed crisp interval, and $(\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{ij} = \left[a^L_{\widetilde{p}^N_{ij}}(\alpha, \beta, \gamma), a^R_{\widetilde{p}^N_{ij}}(\alpha, \beta, \gamma)\right]$ $\forall\ i = 1, 2, ..., ..., m$ and $j = 1, 2, ..., ..., n$.

Step 2: The interval valued Matrix Game for the considered problem is obtained

By using Step 1, the considered matrix game(2) is transformed to the following interval valued(IV) game matrix:

$$\mathcal{P}^{IV} = \begin{array}{c} \\ \delta_1 \\ \delta_2 \\ \vdots \\ \delta_m \end{array} \begin{array}{c} \begin{array}{ccccc} \gamma_1 & \gamma_2 & \gamma_3 & \cdots & \gamma_n \end{array} \\ \begin{pmatrix} (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{11} & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{12} & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{13} & \cdots & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{1n} \\ (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{21} & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{22} & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{23} & \cdots & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{m1} & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{m2} & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{m3} & \cdots & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{mn} \end{pmatrix} \end{array}$$

where $(\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{ij} = \left[a^L_{\widetilde{p}^N_{ij}}(\alpha, \beta, \gamma), a^R_{\widetilde{p}^N_{ij}}(\alpha, \beta, \gamma)\right]$ is the payoff when player 'X' plays the strategy i and player 'Y' plays strategy j.

Step 3: Conversion of crisp interval valued game matrix to crisp game matrix

The crisp game matrix p_λ is obtained from the interval valued game matrix P^{IV} by taking following ranking function $\mathcal{R}$.

$$\mathcal{R}\left((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{ij}\right) = \mathcal{R}\left(\left[a^L_{\widetilde{p}^N_{ij}}(\alpha, \beta, \gamma), a^R_{\widetilde{p}^N_{ij}}(\alpha, \beta, \gamma)\right]\right) = (1-\lambda)a^L_{\widetilde{p}^N_{ij}}(\alpha, \beta, \gamma) + \lambda a^R_{\widetilde{p}^N_{ij}}(\alpha, \beta, \gamma).$$

Where$\lambda \in [0, 1]$ is index that defines decision makers attitude.$\lambda = 0$ pessimistic, $\lambda = 1$ optimistic and $\lambda = 0.5$ moderate. then crisp game matrix is obtained as follows:

$$p_\lambda = \mathcal{R}\left(p^{IV}\right) = \begin{array}{c} \\ \delta_1 \\ \delta_2 \\ \vdots \\ \delta_m \end{array} \begin{array}{c} \begin{array}{ccccc} \gamma_1 & \gamma_2 & \gamma_3 & \cdots & \gamma_n \end{array} \\ \begin{pmatrix} \mathcal{R}(\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{11}) & \mathcal{R}(\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{12}) & \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{13}) & \cdots & \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{1n}) \\ \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{21}) & \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{22}) & (\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{23}) & \cdots & \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{2n}) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{m1}) & \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{m2}) & \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{m3}) & \cdots & \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{mn}) \end{pmatrix} \end{array} \tag{3}$$

Step 4: Solving crisps game matrix whenp_λ has a saddle point

If crisp game matrix(3) has a saddle point. Then problem(3) can be solved by using maxmin method. Else go to step 5.

Step 5:Solving crisps game matrix whenp_λ has no or many saddle point

If crisp game matrix(3) has no saddle point or multiple saddle points. Then matrix game problem p_λ is transformed to the following linear programming primal-dual problems.

Let probabilities of selecting strategies $(1, 2, ..., m)$ and $(1, 2, ..., n)$ for player X and Player Y be $(p_1, p_2, ..., p_m)$ and $(q_1, q_2, ..., q_n)$ respectively and V and W be value of game for player X and Player Y respectively.

Therefore, linear programming primal-dual problems corresponding to player X and player Y are

$$\begin{cases} \text{Max } V \\ \text{subject to} \\ \sum\limits_{i=1}^{m} \mathcal{R}((\widetilde{\mathcal{P}}^N_{<\alpha,\beta,\gamma>})_{ij})p_i \geq V \quad (j = 1, 2, ..., n) \\ p_i \geq 0 \quad (i = 1, 2, ..., m) \end{cases} \tag{4}$$

and

$$\begin{cases} \text{Min } W \\ \text{subject to} \\ \sum\limits_{j=1}^{n} \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{ij})q_i \leq W \quad (i = 1, 2, ..., m) \\ q_j \geq 0 \quad (j = 1, 2, ..., n) \end{cases} \tag{5}$$

Above primal-dual(6) and (7) pair is equivalent to the following primal dual pair of LPPs.

$$\begin{cases} \text{Min } \frac{1}{V} = x_1 + x_2 + ... + x_m \\ \text{subject to} \\ \sum_{i=1}^{m} \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{ij})x_i \geq 1 \quad (j = 1, 2, ..., n) \\ x_i \geq 0 \quad (i = 1, 2, ..., m) \end{cases} \tag{6}$$

and

$$\begin{cases} \text{Max } \frac{1}{W} = y_1 + y_2 + ... + y_n \\ \text{subject to} \\ \sum_{j=1}^{n} \mathcal{R}((\widetilde{p}^N_{<\alpha,\beta,\gamma>})_{ij})q_i \leq 1 \quad (i = 1, 2, ..., m) \\ y_j \geq 0 \quad (j = 1, 2, ..., n) \end{cases} \tag{7}$$

where $x_1 = \frac{p_1}{V}, x_2 = \frac{p_2}{V}, ...$ and $x_m = \frac{p_m}{V}$ and $y_1 = \frac{q_1}{W}, y_2 = \frac{q_2}{W}, ...$ and $y_n = \frac{q_n}{W}$.These linear programming primal-dual problems are solved by using simplex method.

The method described above provides more flexibility as the decision maker (based upon his expert knowledge) can modify the result by controlling α, β, γ. Observe that the parameter λ denotes the DM's preferences and the DM can analyze the worst and best possible game values to make a more informed decision.

5. **Numerical Illustrations**

Example 5.1. Assume player 'X' and player 'Y' in a *TPZS* game with the following payoff matrix:

$$\widetilde{\mathcal{P}}^N = \begin{bmatrix} (175, 180, 185, 190; 0.6, 0.5, 0.6) & (150, 155, 157, 158; 0.8, 0.2, 0.6) \\ \\ (80, 87, 92, 100; 0.7, 0.3, .02) & (175, 180, 185, 190; 0.3, 0.7, 0.6) \end{bmatrix}$$

where all the elements of $\widetilde{\mathcal{P}}^N$ are in the form of SVTrN numbers.

Let $\alpha = 0.3$, $\beta = 0.8$ and $\gamma = 0.7$. By using (α, β, γ)-cut we obtain following payoff matrix(Interval-Valued game matrix):

$$\mathcal{P}^{IV} = \begin{bmatrix} [178.75, 186.25] & [153.75, 157.25] \\ \\ [83, 96.57] & [180, 185] \end{bmatrix}$$

By using the ranking function defined in Step 3, the pessimistic game problem(corresponding to $\lambda = 0$) is obtained as follows

$$\mathcal{P}^{p}_{\lambda=0} = \begin{bmatrix} 178.75 & 153.75 \\ \\ 83 & 180 \end{bmatrix}$$

Since the game matrix has no saddle point, so LPP corresponding to player X is

$$\begin{cases} \min\ Z_X = \frac{1}{V} = x_1 + x_2 \\ \text{subject to} \\ 178.75x_1 + 83x_2 \geq 1 \\ 153.75x_1 + 180x_2 \geq 1 \\ x_1, x_2 \geq 0. \end{cases} \tag{8}$$

where $x_1 = \frac{p_1}{V}$ and $x_2 = \frac{p_2}{V}$.
And LPP corresponding to player Y is

$$\begin{cases} \max\ Z_Y = \frac{1}{W} = y_1 + y_2 \\ \text{subject to} \\ 178.75y_1 + 153.75y_2 \leq 1 \\ 83y_1 + 180y_2 \leq 1 \\ y_1, y_2 \geq 0. \end{cases} \tag{9}$$

where $y_1 = \frac{q_1}{W}$ and $y_2 = \frac{q_2}{W}$.
Solving the above LPPs, we get optimum solution (O.S.) for player X and Y as $(0.796, 0.204)$ and $(0.216, 0.784)$, respectively. Therefore, the value of the game corresponding to these strategies is **159.124**.
By using the ranking function defined in Step 3, the optimistic game problem(corresponding to $\lambda = 1$) is obtained as follows

$$\mathcal{P}^{o}_{\lambda=1} = \begin{bmatrix} 186.25 & 157.25 \\ \\ 96.57 & 185 \end{bmatrix}$$

Therefore, LPP corresponding to player X is

$$\begin{cases} \min\ Z_X = \frac{1}{V} = x_1 + x_2 \\ \text{subject to} \\ 186.25x_1 + 96.57x_2 \geq 1 \\ 157.25x_1 + 185x_2 \geq 1 \\ x_1, x_2 \geq 0. \end{cases} \tag{10}$$

where $x_1 = \frac{p_1}{V}$ and $x_2 = \frac{p_2}{V}$.

And LPP corresponding to player Y is

$$\begin{cases} \max\ Z_Y = \frac{1}{W} = y_1 + y_2 \\ \text{subject to} \\ 186.25y_1 + 157.25y_2 \le 1 \\ 96.57y_1 + 185y_2 \le 1 \\ y_1, y_2 \ge 0. \end{cases} \tag{11}$$

where $y_1 = \frac{q_1}{W}$ and $y_2 = \frac{q_2}{W}$.

Solving the above LPPs, we get O.S. for player X and Y as $(0.753, 0.247)$ and $(0.236, 0.764)$, respectively. Therefore, the value of the game corresponding to these strategies is **164.103**.

By using the ranking function defined in Step 3, the moderate game problem(corresponding to $\lambda = 0.5$) is obtained as follows

$$\mathcal{P}^m_{\lambda=0.5} = \begin{bmatrix} 182.5 & 155.5 \\ \\ 89.79 & 182.5 \end{bmatrix}$$

Therefore, LPP corresponding to player X is

$$\begin{cases} \min\ Z_X = \frac{1}{V} = x_1 + x_2 \\ \text{subject to} \\ 182.5x_1 + 89.79x_2 \ge 1 \\ 155.5x_1 + 182.5x_2 \ge 1 \\ x_1, x_2 \ge 0. \end{cases} \tag{12}$$

where $x_1 = \frac{p_1}{V}$ and $x_2 = \frac{p_2}{V}$.

And LPP corresponding to player Y is

$$\begin{cases} \max\ Z_Y = \frac{1}{W} = y_1 + y_2 \\ \text{subject to} \\ 182.5y_1 + 155.5y_2 \le 1 \\ 89.79y_1 + 182.5y_2 \le 1 \\ y_1, y_2 \ge 0. \end{cases} \tag{13}$$

where $y_1 = \frac{q_1}{W}$ and $y_2 = \frac{q_2}{W}$.

Solving the above LPPs, we get O.S. for player X and Y as $(0.774, 0.226)$ and $(0.226, 0.774)$, respectively. Therefore, the value of the game corresponding to these strategies is **161.59**.

6. Result and Discussion

The numerical example discussed in section 4 is solved for $\lambda = 0, 0.5$ and 1, which gives the worst, the moderate and the best possible solutions respectively. The proposed technique thus gives flexibility to decision maker by observing different scenarios and making more informed decision to get the desired results. Table 2 represents the optimal solution of the considered problem for different values of λ.

Table 1. Solutions for different values of λ

DM's attitude	λ	Optimum strategies for player X	Optimum strategies for player Y	Value of Game
Pessimistic	0	(0.796,0.204)	(0.216,0.784)	159.124
Moderate	0.5	(0.774,0.226)	(0.226,0.774)	161.59
Moderate	1	(0.753,0.247)	(0.236,0.764)	164.103

7. Conclusion

In this work, a technique to solve a neutrosophic game matrix with payoffs as SVTrN numbers is established. In this study, the neutrosophic game matrix is converted to interval valued game matrix, which is transformed to crisp game matrix problems by taking a ranking function. This crisp game matrix problem is solved by using minimax method if saddle point exist otherwise it is converted to linear programming primal-dual pair and hence simplex method is applied to obtain the optimum value of the given game problem. The proposed technique is computationally efficient and can be applied to a wider range of game theory problems existing in practical, as the data encountered in practice is often imprecise with some level of hesitation, inconsistent or incompleteness which can best be described using single valued neutrosophic number. Numerical illustrations are provided to demonstrate the methodology and to prove the vitality of the proposed method.

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