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On Symbolic 2-Plithogenic Hyperbolic Functions

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Abstract. In this article, we have introduced and studied the concept of symbolic 2-plithogenic hyperbolic functions and the symbolic 2-plithogenic identities. Also, we have discussed the derivatives and integrals of the symbolic 2-plithogenic hyperbolic functions and presented the logarithmic forms of the inverse symbolic 2-plithogenic hyperbolic functions.

Keywords: Symbolic 2-plithogenic function; hyperbolic fraction; symbolic 2-plithogenic derivatives and symbolic 2-plithogenic integrals.

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1. **Introduction**

The concept of refined neutrosophic structure was studied by many authors in [1–4]. Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation [11].

In [10], the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings are studied. In [12, 13] was studied the concepts of symbolic 2-plithogenic vector spaces and modules.

Recent works have significantly advanced the theoretical and applied aspects of fuzzy and neutrosophic structures, including the development of various neutrosophic ideals in ternary semigroups [14], the exploration of intuitionistic and fuzzy algebraic systems [15, 19, 22], novel weighted and trigonometric operators in complex fuzzy environments [16, 17], and the integration of neutrosophic models in graph theory and numerical analysis [20, 21, 26, 27]; these efforts are further extended by applications in intelligent systems, health technologies, and topological frameworks [18, 23–25].

In [6], Alhasan Yaser et al., presented the concept of the symbolic plithogenic differentials calculus, where the symbolic plithogenic differentiable is defined. Also in [7], they discussed integrations in symbolic plithogenic field, where they presented direct methods for solving integrations of symbolic plithogenic functions. Further the indefinite plithogenic trigonometric integrals was discussed in [8].

Recently, in [5], Alhasan Yaser et al., introduced the idea of the neutrosophic hyperbolic functions and studied the derivatives and integrals of the neutrosophic hyperbolic functions and inverse neutrosophic hyperbolic functions. As an extension, in [9], the concept of 2-refined neutrosophic hyperbolic functions with its differential and integrals was introduced and studied.

Motivated by these works, we introduced the concept of symbolic 2-plithogenic hyperbolic functions, the symbolic 2-plithogenic hyperbolic identities in this article. Also, we have studied derivatives and integrals of the symbolic 2-plithogenic hyperbolic functions and presented the logarithmic forms of the inverse symbolic 2-plithogenic hyperbolic functions.

2. Preliminaries

Definition 2.1. [10] Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2-SP_R = \left\{a_0 + a_1P_1 + a_2P_2; a_i \in R, P_j^2 = P_j, P_1 \times P_2 = P_{max(1,2)} = P_2\right\}$$

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2$

Multiplication:

$[a_0 + a_1P_1 + a_2P_2].[b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_1P_1P_1 = (a_0b_0) + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2$.

It is clear that $2-SP_R$ is a ring. If R is a field, then $2-SP_R$ is called a symbolic 2-plithogenic field. Also, if R is commutative, then $2-SP_R$ is commutative, and if R has a unity (1), than $2-SP_R$ has the same unity (1).

Definition 2.2. [6] The rules of the symbolic plithogenic derivatives

Let $PN_r = a_0 + a_1P_1 + a_2P_2 + \cdots + a_rP_r$ and $PN_s = b_0 + b_1P_1 + b_2P_2 + \cdots + b_sP_s \in SPS$ then we can prove each of the following:

(1) $\frac{d}{dx}[PC] = 0$; where $PN = c_0+c_1P_1+c_2P_2+\cdots+c_rP_r$ is symbolic plithogenic constant.

(2) $\frac{d}{dx}[PN_sx + PN_r] = PN_s$

(3) $\frac{d}{dx}[PN_sx^n] = nPN_sx^{n-1}$; n is real number.

(4) $\frac{d}{dx}[e^{PN_sx+PN_r}] = PN_se^{PN_sx+PN_r}$

Definition 2.3. [7] Let $f : SPS \to SPS$ to evaluate $\int f(x, PN)dx$, where $PN = d_0+d_1P_1+d_2P_2 + \cdots + d_nP_n$, put : $x = g(u) \Rightarrow dx = g'(u)du$, by substitution we get

$$\int f(x, PN)dx = \int f(u)g'(u)du$$

then we can directly integral it.

Theorem 2.4. *[7] If $\int f(x, PN)dx = \varphi(x, PN)$, then:*

$$\int PN_rf(PN_sx + PN_n)dx = \frac{PN_r}{PN_s}\varphi(PN_sx + PN_n) + PC$$

provided that $\frac{PN_r}{PN_s}$ is divisible, where $PN_r = a_0 + a_1P_1 + a_2P_2 + \cdots + a_rP_r$ and $PN_n = c_0 + c_1P_1 + c_2P_2 + \cdots + c_rP_r \in SPS$ is symbolic plithogenic constant.

3. Symbolic 2-Plithogenic Hyperbolic Functions

Let $f(x, P_1, P_2) = e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}$, then we can express the symbolic 2-plithogenic function as follows:

$$\begin{aligned} f(x, P_1, P_2) &= e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} \\ &= \frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} + e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2} \\ &\quad + \frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} - e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2} \end{aligned}$$

The odd function is called the symbolic 2-plithogenic hyperbolic sine function of x and the even function is called the symbolic 2-plithogenic hyperbolic cosine function of x.

Hence for all $x \in (-\infty, +\infty)$

$$sinh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) = \frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} - e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2}$$

$$cosh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) = \frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} + e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2}$$

where $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers.

Now, let us write the definition of symbolic 2-plithogenic hyperbolic functions as follows:

Definition 3.1. The symbolic 2-plithogenic hyperbolic sine function of x is

$sinh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$= \frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} - e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2}$$

Definition 3.2. The symbolic 2-plithogenic hyperbolic cosine function of x is

$cosh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$= \frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} + e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2}$$

Definition 3.3. The symbolic 2-plithogenic hyperbolic tangent function of x is

$tanh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$= \frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} - e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} + e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}$$

Definition 3.4. The symbolic 2-plithogenic hyperbolic cotangent function of x is

$coth((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$= \frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} + e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} - e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}$$

Definition 3.5. The symbolic 2-plithogenic hyperbolic secant function of x is

$sech((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$= \frac{1}{cosh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}$$

$$= \frac{2}{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} + e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}$$

Definition 3.6. The symbolic 2-plithogenic hyperbolic cosecant function of x is

$cosech((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$= \frac{1}{sinh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}$$

$$= \frac{2}{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} - e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}$$

4. Symbolic 2-Plithogenic Hyperbolic Identities

(1) $cosh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) + sinh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$= e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)}$$

(2) $cosh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) - sinh((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$= e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}$$

(3) $cosh^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) - sinh^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) = 1$

(4) $sech^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) + tanh^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) = 1$

(5) $coth^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) - cosech^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) = 1$

(6) $sinh(2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)))$

$$= 2sinh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))cosh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

(7) $cosh(2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)))$

$$= 2sinh^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) + 1$$

$$= 2cosh^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) - 1$$

$$= cosh^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) + sinh^2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$$

(8) $sinh^2(2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))) = \frac{cosh(2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)))-1}{2}$

(9) $cosh^2(2((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))) = \frac{cosh(2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)))+1}{2}$

Proof. It is easy to prove the symbolic 2-plithogenic hyperbolic identities. For example let us see the proof of identity (7) and (5).

Proof of (7):

$$cosh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))+sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$
$$= \left[\frac{e^{(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)}+e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2}\right]^2$$
$$+ \left[\frac{e^{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}-e^{-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2}\right]^2$$

$$= \frac{2e^{2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}+2e^{-2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{4}$$

$$= \frac{e^{2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}+e^{-2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}}{2}$$

$$= cosh(2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)))$$

Moreover,

$$cosh(2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)))$$

$$= cosh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) + sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

$$= 1 + sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) + sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

$$= 1 + 2sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

And,

$$cosh(2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)))$$

$$= cosh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) + sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

$$= cosh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) + cosh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) - 1$$

$$= 2cosh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) - 1$$

Proof of (5):

$$cosh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) - sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) = 1$$

Divide both sides by $sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$

$$\frac{cosh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}{sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))} - 1 = \frac{1}{sinh^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))}$$

$coth^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) = 1+cosech^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$

$coth^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) - cosech^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) = 1$□

5. Derivatives of Symbolic 2-Plithogenic hyperbolic functions

(1) $\frac{d}{dx}[sinh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]$

$$= (a_1+b_1P_1+c_1P_2)cosh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

(2) $\frac{d}{dx}[cosh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]$

$$= (a_1+b_1P_1+c_1P_2)sinh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

(3) $\frac{d}{dx}[tanh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]$

$$= (a_1+b_1P_1+c_1P_2)sech^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

(4) $\frac{d}{dx}[coth((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]$

$$= -(a_1+b_1P_1+c_1P_2)cosech^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

(5) $\frac{d}{dx}[sech((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]$

$$= -(a_1+b_1P_1+c_1P_2)sech((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) \\ tanh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

(6) $\frac{d}{dx}[cosech((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]$

$$= -(a_1+b_1P_1+c_1P_2)cosech((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) \\ coth((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))$$

Example 5.1.

(1) $\frac{d}{dx}[sinh((3+6P_1+P_2)x^3+(5+7P_1+6P_2))]$
$$= 3(3+6P_1+P_2)x^2cosh((3+6P_1+P_2)x^3+(5+7P_1+6P_2))$$

(2) $\frac{d}{dx}[cosh(2((-1-P_1-5P_2)x+(3P_1+P_2)))]$

$$= \frac{d}{dx}[2cosh^2((-1-P_1-5P_2)x+(3P_1+P_2))-1]$$

$$= -4(1+P_1+5P_2)cosh((-1-P_1-5P_2)x+(3P_1+P_2))$$

$$sinh((-1-P_1-5P_2)x+(3P_1+P_2))$$

(3) $\frac{d}{dx}[cosh^2((5-3P_1-2P_2)x+(-2-P_1+P_2)))]$

$$=\frac{d}{dx}[\frac{cosh(2((5-3P_1-2P_2)x+(-2-P_1+P_2)))+1}{2}]$$

$$=(5-3P_1-2P_2)sinh(2((5-3P_1-2P_2)x+(-2-P_1+P_2)))$$

(4) $\frac{d}{dx}[coth((1-4P_1)x+(-4+7P_1+9P_2)))]$

$$=(-1+4P_1)cosech^2((1-4P_1)x+(-4+7P_1+9P_2))$$

(5) $\frac{d}{dx}[tanh\sqrt{((1+P_1+2P_2)^2x^2+P_1-P_2)}]$

$$=\frac{2(1+P_1+2P_2)^2xsech^2\sqrt{((1+P_1+2P_2)^2x^2+P_1-P_2)}}{2\sqrt{((1+P_1+2P_2)^2x^2+P_1-P_2)}}$$

$$=\frac{(1+P_1^2+4P_2^2+2P_1+4P_1P_2+4P_2)xsech^2\sqrt{((1+P_1+2P_2)^2x^2+P_1-P_2)}}{\sqrt{((1+P_1+2P_2)^2x^2+P_1-P_2)}}$$

$$=\frac{(1+3P_1+12P_2)xsech^2\sqrt{((1+P_1+2P_2)^2x^2+P_1-P_2)}}{\sqrt{((1+P_1+2P_2)^2x^2+P_1-P_2)}}$$

6. Derivatives of Inverse Symbolic 2-Plithogenic Hyperbolic Functions

(1) $\frac{d}{dx}[sinh^{-1}((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]=\frac{(a_1+b_1P_1+c_1P_2)}{\sqrt{1+((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))^2}}$.

(2) $\frac{d}{dx}[cosh^{-1}((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]=\frac{(a_1+b_1P_1+c_1P_2)}{\sqrt{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))^2-1}}$;
$x>\frac{1-a_2-b_2P_1-c_2}{a_1+b_1P_1+c_1P_2}$.

(3) $\frac{d}{dx}[tanh^{-1}((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]=\frac{(a_1+b_1P_1+c_1P_2)}{1-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))^2}$;
$|(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)|<1$.

(4) $\frac{d}{dx}[coth^{-1}((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]=\frac{(a_1+b_1P_1+c_1P_2)}{1-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))^2}$;
$|(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)|>1$.

(5) $\frac{d}{dx}[sech^{-1}((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]$

$$=\frac{(-a_1-b_1P_1-c_1P_2)}{((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))\sqrt{1-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))^2}};$$

$$\frac{-a_2-b_2P_1-c_2P_2}{a_1+b_1P_1+c_1P_2}<x<\frac{1-a_2-b_2P_1-c_2P_2}{a_1+b_1P_1+c_1P_2}$$

(6) $\frac{d}{dx}[cosech^{-1}((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))]))$

$= \frac{(-a_1-b_1P_1-c_1P_2)}{|((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))|\sqrt{1+((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))^2}}; x \neq \frac{-a_2-b_2P_1-c_2P_2}{a_1+b_1P_1+c_1P_2}$

Example 6.1.

(1) $\frac{d}{dx}[sinh^{-1}((3+P_1-2P_2)x+(P_1+5P_2))] = \frac{3+P_1-2P_2}{\sqrt{1+((3+P_1-2P_2)x+(P_1+5P_2))^2}}$

(2) $\frac{d}{dx}[cosh^{-1}((2+3P_1+4P_2)x+(-1-P_1-P_2))] = \frac{2+3P_1+4P_2}{\sqrt{((2+3P_1+4P_2)x+(-1-P_1-P_2))^2-1}}$

(3) $\frac{d}{dx}[coth^{-1}(cos((5+6P_1-2P_2)x+(3P_1-4P_2)))]$

$$= \frac{-(5+6P_1-2P_2)sin((5+6P_1-2P_2)x+(3P_1-4P_2))}{1-cos^2((5+6P_1-2P_2)x+(3P_1-4P_2))}$$

$$= \frac{-(5+6P_1-2P_2)sin((5+6P_1-2P_2)x+(3P_1-4P_2))}{sin^2((5+6P_1-2P_2)x+(3P_1-4P_2))}$$

$$= \frac{-5-6P_1+2P_2}{sin((5+6P_1-2P_2)x+(3P_1-4P_2))}$$

$$= (-5-6P_1+2P_2)cosec((5+6P_1-2P_2)x+(3P_1-4P_2))$$

(4) $\frac{d}{dx}[(4+P_1+P_2)xsinh^{-1}((P_1+P_2)x+(2+P_1-7P_2))]$

$= (4+P_1+P_2)sinh^{-1}((P_1+P_2)x+(2+P_1-7P_2)) + (4+P_1+P_2)x\frac{P_1+P_2}{\sqrt{1+((P_1+P_2)x+(2+P_1-7P_2))^2}}$

$= (4+P_1+P_2)[sinh^{-1}((P_1+P_2)x+(2+P_1-7P_2)) + \frac{(P_1+P_2)x}{\sqrt{1+((P_1+P_2)x+(2+P_1-7P_2))^2}}$

(5) $\frac{d}{dx}[cosech^{-1}((8+9P_1+7P_2)x+(1-3P_1+11P_2))]$

$$= \frac{-(8+9P_1+7P_2)}{|(8+9P_1+7P_2)x+(1-3P_1+11P_2)|\sqrt{1+((8+9P_1+7P_2)x+(1-3P_1+11P_2))^2}}$$

7. Integrals of Symbolic 2-Plithogenic Hyperbolic Functions

Let $a_1 \neq 0, a_1 \neq -b_1, a_1+b_1+c_1 \neq 0$ and $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers, while P_1, P_2 are indeterminacy. Then

(1) $\int sinh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))dx$

$$= (\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)cosh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))+C$$

(2) $\int cosh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))dx$

$$= (\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)sinh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))+C$$

(3) $\int sech^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))dx$

$$= (\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)tanh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))+C$$

(4) $\int cosech^2((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))dx$

$=$

$-(\frac{1}{a_1}-\frac{b_1}{a_1(a_1+b_1)}P_1-\frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)coth((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))+C$

(5)

$\int sech((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))tanh((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))dx$

$=$

$-(\frac{1}{a_1}-\frac{b_1}{a_1(a_1+b_1)}P_1-\frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)sech((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))+C$

(6)

$\int cosech((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))coth((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))dx$

$=-(\frac{1}{a_1}-\frac{b_1}{a_1(a_1+b_1)}P_1-\frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)cosech((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))+C$

where $C=a_0+b_0P_1+c_0P_2$ is integration constant and a_0,b_0,c_0 are real numbers.

Example 7.1.

(1) $\int cosh((4+P_1+P_2)x+(2+P_1-7P_2))dx$

$=(\frac{1}{4}-\frac{1}{20}P_1-\frac{1}{30}P_2)sinh((4+P_1+P_2)x+(2+P_1-7P_2))+C$

(2) $\int sinh^2((3+P_1-2P_2)x+(1+P_1+P_2))dx$

$=\int \frac{cosh(2((3+P_1-2P_2)x+(1+P_1+P_2)))-1}{2}dx$

$=\int \frac{cosh((6+2P_1-4P_2)x+(2+2P_1+2P_2)))-1}{2}dx$

$=(\frac{1}{12}-\frac{1}{48}P_1+\frac{1}{16}P_2)sinh((4+P_1+P_2)x+(2+P_1-7P_2))-\frac{x}{2}+C$

(3) $\int tanh((4-2P_1+P_2)x+(2-3P_1-5P_2))dx$

$=\int \frac{sinh((4-2P_1+P_2)x+(2-3P_1-5P_2))}{cosh((4-2P_1+P_2)x+(2-3P_1-5P_2))}dx$

Put

$u=cosh((4-2P_1+P_2)x+(2-3P_1-5P_2))$

$\Rightarrow du=(4-2P_1+P_2)sinh((4-2P_1+P_2)x+(2-3P_1-5P_2))dx$

$\Rightarrow \frac{du}{4-2P_1+P_2}=sinh((4-2P_1+P_2)x+(2-3P_1-5P_2))dx$

$\int tanh((4-2P_1+P_2)x+(2-3P_1-5P_2))dx$

$=\frac{1}{4-2P_1+P_2}\int\frac{du}{u}$

$=\frac{1}{4-2P_1+P_2}ln|u|+C$

$=\frac{1}{4-2P_1+P_2}ln|cosh((4-2P_1+P_2)x+(2-3P_1-5P_2))|+C$

$=(\frac{1}{4}+\frac{1}{4}P_1-\frac{1}{6}P_2)ln|cosh((4-2P_1+P_2)x+(2-3P_1-5P_2))|+C$

(4) $\int sinh^3((3+5P_1+7P_2)x-11P_1+13P_2))cosh((3+5P_1+7P_2)x-11P_1+13P_2))dx$

Put $u = sinh((3+5P_1+7P_2)x - 11P_1 + 13P_2))$

$\Rightarrow du = (3+5P_1+7P_2)cosh((3+5P_1+7P_2)x - 11P_1 + 13P_2))dx$

$\Rightarrow \frac{du}{3+5P_1+7P_2} = cosh((3+5P_1+7P_2)x - 11P_1 + 13P_2))dx$

Now

$$\int sinh^3((3+5P_1+7P_2)x - 11P_1 + 13P_2))cosh((3+5P_1+7P_2)x - 11P_1 + 13P_2))dx$$
$$= \frac{1}{3+5P_1+7P_2}\int u^3 du$$
$$= \frac{1}{3+5P_1+7P_2}\frac{u^4}{4} + C$$
$$= (\frac{1}{3} - \frac{5}{24)}P_1 - \frac{7}{120}P_2)\frac{sinh^4((3+5P_1+7P_2)x-11P_1+13P_2))}{4} + C$$
$$= (\frac{1}{12} - \frac{5}{96}P_1 - \frac{7}{480}P_2)sinh^4((3+5P_1+7P_2)x - 11P_1 + 13P_2)) + C$$

(5) $\int [(4+5P_1+6P_2)x + (5-P_1-P_2)]cosh[((4+5P_1+6P_2)x + (5-P_1-P_2))^2]dx$

Put $u = ((4+5P_1+6P_2)x + (5-P_1-P_2))^2$

$\Rightarrow du = 2(4+5P_1+6P_2)((4+5P_1+6P_2)x + (5-P_1-P_2))dx$

$\Rightarrow \frac{du}{2(4+5P_1+6P_2)} = [(4+5P_1+6P_2)x + (5-P_1-P_2)]dx$

Now

$$\int [(4+5P_1+6P_2)x + (5-P_1-P_2)]cosh[((4+5P_1+6P_2)x + (5-P_1-P_2))^2]dx$$

$$= \frac{1}{2(4+5P_1+6P_2)}\int coshudu$$

$$= \frac{1}{2(4+5P_1+6P_2)}sinhu + C$$

$$= (\frac{1}{8} - \frac{5}{72}P_1 - \frac{1}{45}P_2)sinh[((4+5P_1+6P_2)x + (5-P_1-P_2))^2] + C$$

8. Logarithmic forms of inverse Symbolic 2-Plithogenic hyperbolic functions

(1) $sinh^{-1}((a_1+b_1P_1+c_1P_2)x + (a_2+b_2P_1+c_2P_2))$

$$= ln[(a_1+b_1P_1+c_1P_2)x + (a_2+b_2P_1+c_2P_2) + \sqrt{((a_1+b_1P_1+c_1P_2)x + (a_2+b_2P_1+c_2P_2))^2 + 1}]$$

$$|(a_1+b_1P_1+c_1P_2)x + (a_2+b_2P_1+c_2P_2)| < \infty$$

(2) $cosh^{-1}((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$= ln[(a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2) + \sqrt{((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))^2 - 1}]$

$x \geq \frac{1-(a_2+b_2P_1+c_2P_2)}{a_1+b_1P_1+c_1P_2}$

(3) $tanh^{-1}((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$= \frac{1}{2}ln(\frac{1+(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)}{1-(a_1+b_1P_1+c_1P_2)x-(a_2+b_2P_1+c_2P_2)}; |(a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)| < 1$

(4) $coth^{-1}((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$= \frac{1}{2}ln(\frac{(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)+1}{(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)-1}; |(a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)| > 1$

(5) $sech^{-1}((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) = ln(\frac{1+\sqrt{1-((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))^2}}{(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)};$

$0 < ((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)) < 1$

(6) $cosech^{-1}((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$= ln(\frac{1}{(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)} + \frac{1+\sqrt{1+((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2))^2}}{|(a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)|}); x \neq -\frac{a_2+b_2P_1+c_2P_2}{a_1+b_1P_1+c_1P_2}$

Proof. For example let us see the proof of the result (3).

Let $u = tanh^{-1}((a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2))$

$$(a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2) = tanhu = \frac{e^u - e^{-u}}{e^u + e^{-u}}$$

$$\frac{1 + \frac{e^u-e^{-u}}{e^u+e^{-u}}}{1 - \frac{e^u-e^{-u}}{e^u+e^{-u}}} = \frac{1 + (a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)}{1 - (a_1 + b_1P_1 + c_1P_2)x - (a_2 + b_2P_1 + c_2P_2)}$$

$$e^{2u} = \frac{1 + (a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)}{1 - (a_1 + b_1P_1 + c_1P_2)x - (a_2 + b_2P_1 + c_2P_2)}$$

Taking log on both sides we get

$$u = \frac{1}{2}ln\left(\frac{1 + (a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)}{1 - (a_1 + b_1P_1 + c_1P_2)x - (a_2 + b_2P_1 + c_2P_2)}\right)$$

$$tanh^{-1}((a_1+b_1P_1+c_1P_2)x+(a_2+b_2P_1+c_2P_2)) = \frac{1}{2}ln\left(\frac{1 + (a_1 + b_1P_1 + c_1P_2)x + (a_2 + b_2P_1 + c_2P_2)}{1 - (a_1 + b_1P_1 + c_1P_2)x - (a_2 + b_2P_1 + c_2P_2)}\right)$$

□

9. Integrals of inverse Symbolic 2-Plithogenic hyperbolic functions

(1) $\int \frac{1}{\sqrt{(a_1+b_1P_1+c_1P_2)^2+x^2}}dx = sinh^{-1}(\frac{x}{a_1+b_1P_1+c_1P_2}) + C =$
$ln(x + \sqrt{(a_1 + b_1P_1 + c_1P_2)^2 + x^2}) + C$

(2) $\int \frac{1}{\sqrt{x^2-(a_1+b_1P_1+c_1P_2)^2}}dx = cosh^{-1}(\frac{x}{a_1+b_1P_1+c_1P_2}) + C$

$= ln(x + \sqrt{x^2 - (a_1 + b_1P_1 + c_1P_2)^2}) + C; x > a_1 + b_1P_1 + c_1P_2$

(3) $\int \frac{1}{(a_1+b_1P_1+c_1P_2)^2-x^2}dx$

$$= (\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)tanh^{-1}(\frac{x}{a_1+b_1P_1+c_1P_2}) + C$$

$$= (\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)coth^{-1}(\frac{x}{a_1+b_1P_1+c_1P_2}) + C$$

$$= \frac{1}{2}(\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)ln|\frac{a_1+b_1P_1+c_1P_2+x}{a_1+b_1P_1+c_1P_2-x}| + C$$

(4) $\int \frac{1}{x\sqrt{(a_1+b_1P_1+c_1P_2)^2-x^2}}dx$

$$= -(\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)sech^{-1}|\frac{x}{a_1+b_1P_1+c_1P_2}| + C$$

$$= -(\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)ln(\frac{a_1+b_1P_1+c_1P_2+\sqrt{(a_1+b_1P_1+c_1P_2)^2-x^2}}{|x|} + C)$$

(5) $\int \frac{1}{x\sqrt{(a_1+b_1P_1+c_1P_2)^2+x^2}}dx$

$$= -(\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)cosech^{-1}|\frac{x}{a_1+b_1P_1+c_1P_2}| + C$$

$$= -(\frac{1}{a_1} - \frac{b_1}{a_1(a_1+b_1)}P_1 - \frac{c_1}{(a_1+b_1)(a_1+b_1+c_1)}P_2)ln(\frac{a_1+b_1P_1+c_1P_2+\sqrt{(a_1+b_1P_1+c_1P_2)^2+x^2}}{|x|} + C; x \neq 0$$

Example 9.1.

(1) $\int \frac{1}{\sqrt{(3+5P_1-2P_2)^2+x^2}}dx = sinh^{-1}[\frac{x}{3+5P_1-2P_2}] + C = sinh^{-1}(\frac{1}{3} - \frac{5}{24}P_1 + \frac{1}{24}P_2)x + C$

(or)

$\int \frac{1}{\sqrt{(3+5P_1-2P_2)^2+x^2}}dx$

$$= ln(x + \sqrt{(3 + 5P_1 - 2P_2)^2 + x^2}) + C$$

$$= ln(x + \sqrt{9 + 25P_1^2 + 4P_2^2 + 30P_1 - 20P_1P_2 - 12P_2 + x^2}) + C$$

$$= ln(x + \sqrt{9 + 25P_1 + 4P_2 + 30P_1 - 20P_2 - 12P_2 + x^2}) + C$$

$$= ln(x + \sqrt{9 + 55P_1 - 28P_2 + x^2}) + C$$

(2) $\int \frac{e^{(5+P_2)x}}{(-2+P_1-3P_2)^2 - e^{2(5+P_2)x}} dx$

Let $u = e^{(5+P_2)x} \Rightarrow du = (5 + P_2)e^{(5+P_2)x} dx \Rightarrow \frac{du}{5+P_2} = e^{(5+P_2)x} dx$

$\int \frac{e^{(5+P_2)x}}{(-2+P_1-3P_2)^2 - e^{2(5+P_2)x}} dx$

$$= \frac{1}{5+P_2} \int \frac{1}{(-2+P_1-3P_2)^2 - u^2} du$$

$$= \frac{1}{5+P_2} [\frac{1}{-2+P_1-3P_2} tanh^{-1}(\frac{u}{-2+P_1-3P_2}] + C$$

$$= \frac{1}{(5+P_2)(-2+P_1-3P_2)} tanh^{-1}(\frac{u}{-2+P_1-3P_2}] + C$$

$$= \frac{1}{-10+5P_1-19P_2)} tanh^{-1}(\frac{u}{-2+P_1-3P_2}] + C$$

$$= (-\frac{1}{10} - \frac{1}{10}P_1 + \frac{19}{120}P_2) tanh^{-1}(-\frac{1}{2} + \frac{1}{2}P_1 + \frac{3}{4}P_2)u + C$$

$$= (-\frac{1}{10} - \frac{1}{10}P_1 + \frac{19}{120}P_2) tanh^{-1}((-\frac{1}{2} + \frac{1}{2}P_1 + \frac{3}{4}P_2)e^{(5+P_2)x}) + C$$

10. Conclusions

This article is an extensio of the studies on symbolic 2-plithogenic derivatives and integrals. In this article we have introduced the concept of symbolic 2-plithogenic hyperbolic functions and symbolic 2-plithogenic inverse hyperbolic functions with its differential and integrals.
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References

[1] Abobala, M., "On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations", Journal Of Mathematics, Hindawi, 2021.

[2] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021.

[3] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., "The Algebraic Creativity In The Neutrosophic Square Matrices", Neutrosophic Sets and Systems, Vol. 40, pp.1-11, 2021.

[4] Adeleke, E. O., Agboola, A. A. A., and Smarandache, F., "Refined neutrosophic rings II", International Journal of Neutrosophic Science, vol. 2, pp. 89-94, 2020.

[5] Alhasan, Y. A., Musa, I. A., Abdelgawad, E. A., and Sheen, S., "The Neutrosophic Hyperbolic Functions", International Journal of Neutrosophic Science", Vol. 20, 2023.

[6] Alhasan, Y. A., Smarandache, F., and Abdulfatah, R. A., "The symbolic plithogenic differentials calculus", Neutrosophic Sets and Systems, Vol. 60, 2023.

[7] Alhasan, Y. A., Smarandache, F., and Abdulfatah, R. A., "The indefinite symbolic plithogenic integrals", Neutrosophic Sets and Systems, Vol. 60, 2023.

[8] Alhasan, Y. A., Sheen, S., Abdulfatah, R. A., and Ahmed, M. M., "The indefinite symbolic plithogenic trigonometric integrals", Neutrosophic Sets and Systems, Vol. 67, 2024.

[9] Alhasan, Y. A., Alfahal, A., and Abdulfatah, R. A., "The 2-refined neutrosophic hyperbolic functions with its differential and integrals", Neutrosophic Sets and Systems, Vol. 75, pp.1-14, 2025.

[10] Merkepci, H., and Abobala, M., "On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[11] Smarandache, F., "Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.

[12] Taffach, N., "An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces", Neutrosophic Sets and Systems, Vol 54, 2023.

[13] Taffach, N., and Ben Othman, K., "An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.

[14] Rajalakshmi, A., Hatamleh, R., Al-Husban, A., Muthu Kumaran, K. L., & Malchijah Raj, M. S. (2025). Various (ζ_1, ζ_2) neutrosophic ideals of an ordered ternary semigroups. *Communications on Applied Nonlinear Analysis*, **32**(3), 400–417.

[15] Hatamleh, R., Al-Husban, A., Sundarakannan, N., & Malchijah Raj, M. S. (2025). Complex cubic intuitionistic fuzzy set applied to subbisemirings of bisemirings using homomorphism. *Communications on Applied Nonlinear Analysis*, **32**(3), 418–435.

[16] Hatamleh, R., Al-Husban, A., Palanikumar, M., & Sundareswari, K. (2025). Different Weighted Operators such as Generalized Averaging and Generalized Geometric based on Trigonometric $\wp$-rung Interval-Valued Approach. *Communications on Applied Nonlinear Analysis*, **32**(5), 91–101.

[17] Hatamleh, R., Al-Husban, A., Sundareswari, K., Balaj, G., & Palanikumar, M. (2025). Complex Tangent Trigonometric Approach Applied to (γ, τ)-Rung Fuzzy Set using Weighted Averaging, Geometric Operators and its Extension. *Communications on Applied Nonlinear Analysis*, **32**(5), 133–144.

[18] Shihadeh, A., Matarneh, K. A. M., Hatamleh, R., Al-Qadri, M. O., & Al-Husban, A. (2024). On The Two-Fold Fuzzy n-Refined Neutrosophic Rings For $2 \leq 3$. *Neutrosophic Sets and Systems*, **68**, 8–25.

[19] Shihadeh, A., Matarneh, K. A. M., Hatamleh, R., Hijazeen, R. B. Y., Al-Qadri, M. O., & Al-Husban, A. (2024). An Example of Two-Fold Fuzzy Algebras Based On Neutrosophic Real Numbers. *Neutrosophic Sets and Systems*, **67**, 169–178.

[20] Abubaker, A. A., Hatamleh, R., Matarneh, K., & Al-Husban, A. (2024). On the Numerical Solutions for Some Neutrosophic Singular Boundary Value Problems by Using (LPM) Polynomials. *International Journal of Neutrosophic Science*, **25**(2), 197–205.

[21] Ahmad, A., Hatamleh, R., Matarneh, K., & Al-Husban, A. (2025). On the Irreversible k-Threshold Conversion Number for Some Graph Products and Neutrosophic Graphs. *International Journal of Neutrosophic Science*, **25**(2), 183–196.

[22] Shihadeh, A., Hatamleh, R., Palanikumar, M., & Al-Husban, A. (2025). New algebraic structures towards different (α, β) intuitionistic fuzzy ideals and its characterization of an ordered ternary semigroup. *Communications on Applied Nonlinear Analysis*, **32**(6), 568–578.

[23] Hatamleh, R., Heilat, A. S., Palanikumar, M., & Al-Husban, A. (2025). Characterization of interaction aggregating operators setting interval-valued Pythagorean neutrosophic set. *Neutrosophic Sets and Systems*, **81**, 285–305.

[24] Hatamleh, R., Al-Husban, A., Zubair, S. A. M., Elamin, M., Saeed, M. M., Abdolmaleki, E., ... & Khattak, A. M. (2025). AI-Assisted Wearable Devices for Promoting Human Health and Strength Using Complex Interval-Valued Picture Fuzzy Soft Relations. *European Journal of Pure and Applied Mathematics*, **18**(1), 5523–5523.

[25] Al-Husban, A., Saeed, M. M., Nordo, G., Fujita, T., Khattak, A. M., Hatamleh, R., ... & Armada, C. L. (2025). A Comprehensive Study of Bipolar Vague Soft Expert P-Open Sets in Bipolar Vague Soft Expert Topological Spaces with Applications to Cancer Diagnosis. *European Journal of Pure and Applied Mathematics*, **18**(2), 5900–5900.

[26] Hatamleh, R., *On the Form of Correlation Function for a Class of Nonstationary Field with a Zero Spectrum*, Rocky Mountain Journal of Mathematics, Vol. 33(1), 2003.

[27] Hatamleh, R., Zolotarev, V. A., *On Two-Dimensional Model Representations of One Class of Commuting Operators*, Ukrainian Mathematical Journal, Vol. 66(1), pp. 122–144, 2014.

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