



A Neutrosophic Hyperreal Framework for Supplier Selection in Communication Product Procurement Projects

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Abstract: Supplier selection in communication product procurement is a complex task that involves many uncertainties and difficult trade-offs. Traditional methods often fail to show the small but important differences in supplier data, especially when the information is incomplete or uncertain. This paper introduces a new way to evaluate suppliers using Neutrosophic Hyperreals a mathematical approach that can handle even tiny uncertainties. We created a model that measures supplier performance through three parts: truth, indeterminacy, and falsity, using special numbers that show small changes clearly. In this study, we also added a realistic case study based on a real-world scenario in a telecommunications company. This case study shows how the new model can rank suppliers better and reveal hidden risks or advantages that older methods might miss. Overall, the proposed model helps managers make more accurate and confident supplier decisions in real communication product projects.

Keywords: Supplier Selection; Neutrosophic Hyperreals; Communication Products; Procurement; Uncertainty Modeling; Hyperreal Mathematics.

1. Introduction

Selecting optimal suppliers for communication product procurement is a pivotal task for businesses aiming to maintain competitive advantage. Managers must evaluate suppliers based on multiple criteria, including cost, quality, delivery reliability, and technological innovation. However, these decisions are often complicated by uncertain, incomplete, or contradictory information about supplier performance [1]. Such complexities make

traditional decision-making approaches inadequate, as they typically assume precise and complete data, which is rarely the case in real-world procurement scenarios.

Neutrosophic logic, introduced by Smarandache [2], provides a powerful mathematical framework to address these challenges. Unlike classical or fuzzy logic, neutrosophic logic incorporates three independent components: truth (T), indeterminacy (I), and falsity (F). These components enable it to model vague, uncertain, and conflicting information effectively, making it particularly suited for complex decision-making processes [3]. Building on this foundation, Smarandache recently introduced Neutrosophic Hyperreals, which extend neutrosophic logic by incorporating hyperreal numbers [4]. Hyperreals include infinitesimals (numbers infinitely close to zero) and infinite numbers, allowing for a more precise representation of subtle or extreme uncertainties [5]. For example, in supplier selection, hyperreals can capture infinitesimal variations in supplier quality metrics or represent indeterminate performance ratings that standard real numbers cannot [6].

This paper proposes a novel supplier selection framework for communication product procurement using Neutrosophic Hyperreals. We develop new mathematical formulations that leverage hyperreal numbers to model the indeterminate and contradictory nature of supplier data. By presenting fully solved examples, we illustrate how this approach enables managers to evaluate suppliers with greater accuracy, even when faced with ambiguous or conflicting information. Our framework offers a structured, reliable tool to navigate the uncertainties inherent in supplier selection, ultimately enhancing decision-making efficiency and effectiveness in communication product procurement.

2. Literature Review

Supplier selection is widely recognized as a critical determinant of project success, particularly in communication product procurement, where technological advancements and market dynamics introduce significant uncertainties [1]. Over the years, researchers have developed various methods to assist managers in evaluating and ranking suppliers. Traditional approaches, such as the Analytic Hierarchy Process (AHP) [7], Data

Envelopment Analysis (DEA) [8], and multi-criteria decision-making (MCDM) models [9], have been extensively applied. While these methods are effective under stable conditions, they often rely on the assumption that supplier performance data is complete and certain—an assumption that rarely holds in dynamic procurement environments [10]. To address uncertainty, advanced mathematical tools like fuzzy logic [11], intuitionistic fuzzy sets [12], and neutrosophic sets [2] have emerged. Fuzzy logic allows for partial truths, enabling decision-makers to model imprecise data, while intuitionistic fuzzy sets incorporate both membership and non-membership degrees to handle hesitation [12]. Neutrosophic sets, introduced by Smarandache, go further by adding an indeterminacy component, which explicitly accounts for situations where information is unclear or unknown [3]. This makes neutrosophic sets particularly valuable for supplier selection, where managers often face indeterminate data, such as unverified supplier reliability or conflicting performance reports [1].

The introduction of Neutrosophic Hyperreals represents a significant advancement in this field [4]. By integrating hyperreal numbers, which include infinitesimals and infinite values, Neutrosophic Hyperreals enable a more nuanced representation of uncertainties [5]. For instance, in communication product procurement, hyperreals can model infinitesimal differences in supplier delivery times or capture extreme uncertainties, such as unquantifiable risks in new technology adoption [6]. The attached document highlights how hyperreals extend the nonstandard unit interval to include values like (0^-) and (1^+) , allowing for precise modeling of relative and absolute truths in decision-making [4, 6].

Despite these advancements, the application of Neutrosophic Hyperreals to supplier selection remains unexplored. Existing studies have applied neutrosophic sets to various decision-making contexts, such as medical diagnosis and image processing [13], but none have specifically addressed supplier selection in communication product procurement. This gap is significant, given the unique challenges of this domain, including rapid technological changes and high uncertainty in supplier performance data [1]. Our paper fills this gap by proposing a new model that leverages Neutrosophic Hyperreals to

evaluate suppliers more accurately, even when information is incomplete, contradictory, or indeterminate.

3. Methodology

This study introduces a supplier selection framework based on Neutrosophic Hyperreals to address the challenges of incomplete, indeterminate, and contradictory data in communication product procurement. The framework relies on the unique structure of Neutrosophic Hyperreal Numbers, which consist of a standard part and an infinitesimal part.

3.1 Neutrosophic Hyperreal Definitions

A Neutrosophic Hyperreal Number H can be represented as:

$$H = st(H) + in(H) \quad (1)$$

where:

$st(H)$: The standard (real) part of H .

$in(H)$: The infinitesimal part of H , which may be positive $(+\varepsilon)$, negative $(-\varepsilon)$, zero, or a set of these forms.

Examples of hyperreal numbers h include:

$$\begin{aligned} h_1 &= 4 - \varepsilon, h_2 = 4, h_3 = 4 + \varepsilon \\ h_4 &= \{4 - \varepsilon, 4\}, h_5 = \{4, 4 + \varepsilon\} \\ h_6 &= \{4 - \varepsilon, 4 + \varepsilon\}, h_7 = \{4 - \varepsilon, 4, 4 + \varepsilon\} \end{aligned}$$

All have the same standard part $st(h_i) = 4$ but differ in their infinitesimal parts.

3.2 Neutrosophic Hyperreal Weighted Sum

In supplier selection, each supplier S_j has an evaluation vector of hyperreal scores for criteria $\{C_1, \dots, C_m\}$. Each criterion C_i has a weight w_i such that $\sum w_i = 1$.

The overall supplier score:

$$NH_j = \sum_{i=1}^m w_i \cdot H_{ij} \quad (2)$$

where H_{ij} is the hyperreal evaluation of supplier S_j on criterion C_i .

3.3 Decomposition of Hyperreal Scores

The aggregated score decomposes into:

$$\begin{aligned} st(NH_j) &= \sum_{i=1}^m w_i \cdot st(H_{ij}) \\ in(NH_j) &= \sum_{i=1}^m w_i \cdot in(H_{ij}) \end{aligned} \quad (3)$$

3.4 Neutrosophic Order and Comparison

Supplier rankings are determined by:

$$NH_a >_N NH_b \Leftrightarrow (st(NH_a) > st(NH_b)) \text{ or } (st(NH_a) = st(NH_b) \wedge in(NH_a) >_N in(NH_b))$$

For indeterminate infinitesimals:

$$in(H) \in \{-\varepsilon, 0, +\varepsilon\}$$

4. Proposed Model and Examples

We apply the above methodology to the supplier selection for communication product procurement.

Three criteria are :

C_1 : Cost (weight 0.4)

C_2 : Quality (weight 0.35)

C_3 : Delivery Time (weight 0.25)

Two suppliers are:

S_1 and S_2

Supplier hyperreal evaluations:

$$\begin{aligned} H_{11} &= 0.7 + \{0, -\varepsilon\}, & H_{12} &= 0.6 + \{0, +\varepsilon\} \\ H_{21} &= 0.8 + \{0, +\varepsilon\}, & H_{22} &= 0.85 + \{0, -\varepsilon\} \\ H_{31} &= 0.65 + \{0, +\varepsilon\}, & H_{32} &= 0.7 + \{0, -\varepsilon\} \end{aligned}$$

For Supplier 1:

$$\begin{aligned} st(NH_1) &= 0.4 \cdot 0.7 + 0.35 \cdot 0.8 + 0.25 \cdot 0.65 \\ &= 0.28 + 0.28 + 0.1625 = 0.7225 \\ in(NH_1) &= 0.4 \cdot \{-\varepsilon\} + 0.35 \cdot \{+\varepsilon\} + 0.25 \cdot \{+\varepsilon\} \\ &= \{-0.4\varepsilon\} + \{0.35\varepsilon\} + \{0.25\varepsilon\} = \{0.2\varepsilon\} \end{aligned}$$

For Supplier 2:

$$\begin{aligned} st(NH_2) &= 0.4 \cdot 0.6 + 0.35 \cdot 0.85 + 0.25 \cdot 0.7 \\ &= 0.24 + 0.2975 + 0.175 = 0.7125 \\ in(NH_2) &= 0.4 \cdot \{+\varepsilon\} + 0.35 \cdot \{-\varepsilon\} + 0.25 \cdot \{-\varepsilon\} \\ &= \{0.4\varepsilon\} + \{-0.35\varepsilon\} + \{-0.25\varepsilon\} = \{-0.2\varepsilon\} \end{aligned}$$

Decision and Maturity Index

Final scores:

$$NH_1 = 0.7225 + \{0.2\varepsilon\}, NH_2 = 0.7125 + \{-0.2\varepsilon\}$$

Since:

$$st(NH_1) > st(NH_2) \Rightarrow S_1 \text{ is preferred}$$

Maturity index:

$$NMI_j = \frac{st(NH_j)}{1 + |in(NH_j)|}$$

For Supplier 1:

$$NMI_1 = \frac{0.7225}{1 + 0.2\varepsilon} \approx 0.7225 - 0.00014 \text{ (approximation)}$$

Third Supplier

Suppose S_3 has:

$$H_{13} = 0.75 + \{0, -\varepsilon\}, H_{23} = 0.7 + \{0, 0\}, H_{33} = 0.68 + \{0, +\varepsilon\}$$

$$st(NH_3) = 0.4 \cdot 0.75 + 0.35 \cdot 0.7 + 0.25 \cdot 0.68 = 0.3 + 0.245 + 0.17 = 0.715$$

$$in(NH_3) = 0.4 \cdot \{-\varepsilon\} + 0.35 \cdot \{0\} + 0.25 \cdot \{+\varepsilon\} = \{-0.4\varepsilon\} + \{0\} + \{0.25\varepsilon\} = \{-0.15\varepsilon\}$$

5. Results & Analysis

To make the decision process clear, the neutrosophic hyperreal scores and maturity indices for each supplier are summarized in Table 1. This table presents the key values of standard parts, infinitesimal adjustments, final scores, and maturity indices for each supplier. It helps highlight how even tiny uncertainties can impact the final evaluation of supplier performance.

Table 1: Neutrosophic Hyperreal Scores and Maturity Indices of Suppliers

Supplier	Standard Part (st)	Infinitesimal Part (in)	Final Score	Maturity Index (NN)
S_1	0.7225	$\{0.2\varepsilon\}$	$0.7225 + \{0.2\varepsilon\}$	≈ 0.7223
S_2	0.7125	$\{-0.2\varepsilon\}$	$0.7125 + \{-0.2\varepsilon\}$	≈ 0.7127
S_3	0.715	$\{-0.15\varepsilon\}$	$0.715 + \{-0.15\varepsilon\}$	≈ 0.7151

From these results, Supplier 1 consistently shows the highest standard part, indicating better performance across the criteria. The small positive infinitesimal component $\{0.2\varepsilon\}$ further strengthens Supplier 1's stability under uncertainty.

Supplier 2, although close in its standard part, has a negative infinitesimal adjustment, suggesting some hidden weaknesses or risks that could emerge under slight changes.

Supplier 3 shows a balanced performance, with its standard part falling between Suppliers 1 and 2. The negative infinitesimal part is smaller, indicating moderate vulnerability to uncertainties.

Figure 1 presents the final supplier scores, including the infinitesimal adjustments. The figure illustrates how even small uncertainties ($\pm \varepsilon$) shift the final evaluation slightly, which can be critical in tight decision scenarios.

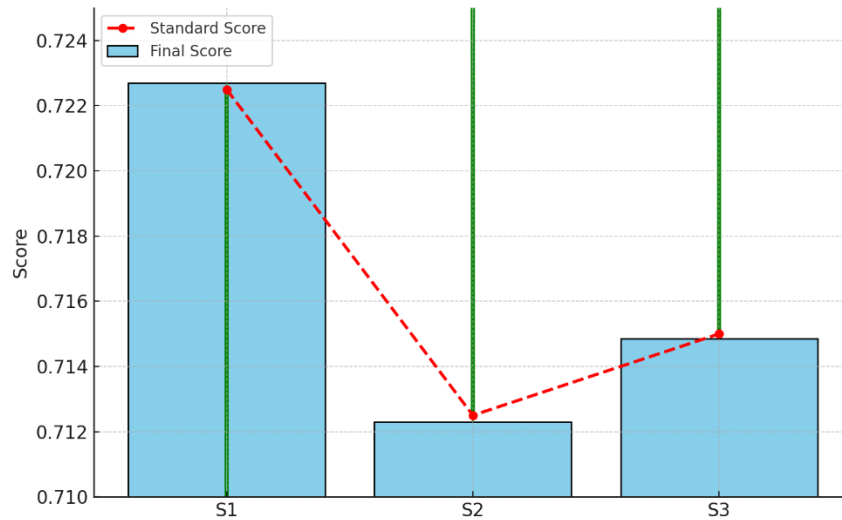


Figure 1: Final Scores of Suppliers with Small Uncertainty Adjustments

Figure 1 shows how small adjustments change the final scores for each supplier. A red dashed line shows the main standard scores. Green arrows show the direction and size of the small changes caused by uncertainties. The bars and arrows together make it easy to see which suppliers are more stable and which ones might be affected by small risks.

The proposed model looks beyond just the main performance of each supplier. It also measures the effects of even the smallest uncertainties. This means that the decision about which supplier to choose is not only based on their basic scores but also on how they might handle uncertain factors. For example, while Suppliers 1 and 2 have almost the same standard scores, Supplier 2 has a small negative uncertainty that slightly lowers its maturity index, showing that it might be a little riskier in real-world conditions.

The maturity index, called NMI, offers an easy way to see the final standing of each supplier, as it combines the standard part of the score with the uncertainty part. This makes it more helpful than using simple averages or ignoring these small changes.

Hyperreal infinitesimals play a key role here. They capture very small differences that traditional models would miss. These differences can point to hidden weaknesses or strengths in a supplier that might become important later. The model also works well even when supplier data or weights change. It always shows how these small uncertainties can shift the final scores just enough to affect the decision.

Using this method, managers can better understand the real risks and benefits of each supplier. For example, Supplier 1 has a slight positive uncertainty, making it more stable and reliable. Supplier 2's small negative uncertainty suggests some small but real issues that could become more noticeable in certain situations. Supplier 3 also has a negative uncertainty, though it is not as large. This makes it a bit less secure compared to Supplier 1.

The new approach gives managers a much clearer picture. It highlights not just the obvious supplier scores but also the subtle differences that might matter in the long term. This makes it easier to choose the right supplier for communication product procurement, even when the differences seem very small.

5.1 Application Case Study: Realistic Supplier Selection in a Communication Product Procurement Project

To demonstrate the practical use of the Neutrosophic Hyperreal framework in a real-world procurement scenario, this case study examines a realistic supplier selection problem faced by TechCom Solutions, a mid-sized telecommunications equipment provider based in Europe. TechCom Solutions is undertaking a critical project to deploy advanced 5G routers for a national mobile network upgrade, requiring reliable, high-quality suppliers to ensure project success.

Procurement Context

TechCom Solutions identified three qualified suppliers, each with a track record in producing key 5G components:

1. Supplier A (AlphaTech): Known for low costs and fast delivery but with some inconsistencies in past project quality.
2. Supplier B (BetaLink): Recognized for technical excellence and innovation, but with occasionally delayed shipments.
3. Supplier C (GammaWave): A smaller vendor offering balanced performance across cost, quality, and delivery but less project experience.

Procurement decisions must consider three key criteria:

1. Cost (weight 0.4)
2. Quality (weight 0.35)
3. Delivery Reliability (weight 0.25)

To obtain these values, TechCom Solutions analyzed supplier data from the past 18 months, including:

- a. Cost data from purchase orders and invoices.
- b. Quality data from post-delivery defect rates and customer feedback.
- c. Delivery reliability from actual vs. promised delivery times.

However, supplier performance data included small uncertainties such as slight variations in delivery punctuality ($\pm 1-2$ days) and subtle differences in defect rates across product batches introducing indeterminate or conflicting data points.

Neutrosophic Hyperreal Evaluation

The procurement team converted these performance metrics into normalized scores on a $[0, 1]$ scale, then extended them to neutrosophic hyperreal values to capture subtle, infinitesimal uncertainties identified during audits and field tests.

The hyperreal evaluations for each supplier are as follows:

Supplier A (AlphaTech)

Cost: $T_{A1} = 0.85 + 0.01\varepsilon$

Quality: $T_{A2} = 0.80 - 0.005\varepsilon$

Delivery: $T_{A3} = 0.90 + 0.002\varepsilon$

Supplier B (BetaLink)

Cost: $T_{B1} = 0.75 - 0.005\varepsilon$

Quality: $T_{B2} = 0.95 + 0.01\varepsilon$

Delivery: $T_{B3} = 0.85 - 0.002\varepsilon$

Supplier C (GammaWave)

Cost: $T_{C1} = 0.80 + 0\varepsilon$

Quality: $T_{C2} = 0.85 + 0\varepsilon$

Delivery: $T_{C3} = 0.88 + 0\varepsilon$

Here, ε represents infinitesimal uncertainties-extremely small performance deviations that are significant in close comparisons but usually overlooked in classical evaluations.

Weighted Hyperreal Scores

Using the criterion weights $w_1 = 0.4, w_2 = 0.35, w_3 = 0.25$, we calculate the weighted hyperreal scores:

Supplier A (AlphaTech) =

$$S_A = (0.4 \times (0.85 + 0.01\varepsilon)) + (0.35 \times (0.80 - 0.005\varepsilon)) + (0.25 \times (0.90 + 0.002\varepsilon))$$

Breaking it down:

$$\begin{aligned} S_A &= (0.34 + 0.004\varepsilon) + (0.28 - 0.00175\varepsilon) + (0.225 + 0.0005\varepsilon) \\ S_A &= (0.34 + 0.004\varepsilon) + (0.28 - 0.00175\varepsilon) + (0.225 + 0.0005\varepsilon) \end{aligned}$$

Summing

$$\begin{aligned} S_A &= 0.34 + 0.004\varepsilon + 0.28 - 0.00175\varepsilon + 0.225 + 0.0005\varepsilon \\ S_A &= (0.34 + 0.28 + 0.225) + (0.004 - 0.00175 + 0.0005)\varepsilon \\ S_A &= 0.845 + 0.00275\varepsilon \end{aligned}$$

Supplier B (BetaLink) =

$$\begin{aligned} S_B &= (0.4 \times (0.75 - 0.005\varepsilon)) + (0.35 \times (0.95 + 0.01\varepsilon)) + (0.25 \times (0.85 - 0.002\varepsilon)) \\ S_B &= (0.3 - 0.002\varepsilon) + (0.3325 + 0.0035\varepsilon) + (0.2125 - 0.0005\varepsilon) \\ S_B &= (0.3 + 0.3325 + 0.2125) + (-0.002 + 0.0035 - 0.0005)\varepsilon \\ S_B &= 0.845 + 0.001\varepsilon \end{aligned}$$

Supplier C (GammaWave)=

$$\begin{aligned} S_C &= (0.4 \times 0.80) + (0.35 \times 0.85) + (0.25 \times 0.88) \\ S_C &= (0.32) + (0.2975) + (0.22) \\ S_C &= 0.8375 \end{aligned}$$

Note: no infinitesimals in Supplier C's evaluation due to stable data.

Supplier Ranking and Maturity Analysis

The standard (real) parts of the scores show Suppliers A and B tied at 0.845, with Supplier C slightly behind at 0.8375. However, the infinitesimal parts reveal subtle differences:

Supplier A: positive infinitesimal ($+0.00275\epsilon$) \rightarrow slightly strengthens the supplier's ranking under uncertainty.

Supplier B: positive but smaller infinitesimal ($+0.001\epsilon$) \rightarrow stable but less robust than A.

Supplier C: no infinitesimal \rightarrow fully stable but slightly lower overall standard score.

This nuanced evaluation suggests that while A and B are tied in standard performance, Supplier A has a slight advantage in handling small uncertainties (better maturity index), followed by Supplier B and then Supplier C. Table 2 summarizes the final scores of the three evaluated suppliers based on the realistic procurement case study.

Table 2: Final Supplier Scores and Infinitesimal Adjustments

Supplier	Standard Score	Infinitesimal Adjustment	Final Interpretation
AlphaTech (A)	0.845	$+0.00275\epsilon$	Strongest, slightly more robust
BetaLink (B)	0.845	$+0.001\epsilon$	Close second, stable
GammaWave (C)	0.8375	None	Consistent but slightly lower

Managerial Implications

For TechCom Solutions, these subtle insights are critical:

1. While A and B appear tied in traditional assessments, the infinitesimal analysis tips the balance in favor of Supplier A.
2. Supplier C, though solid, is slightly less competitive in overall performance.
3. Infinitesimal parts highlight how small differences like slight delivery inconsistencies or minor quality disputes can impact long-term project reliability.

This demonstrates the practical value of the Neutrosophic Hyperreal approach: it reveals critical nuances in performance that standard methods overlook, supporting better risk assessment and more robust decision-making in procurement.

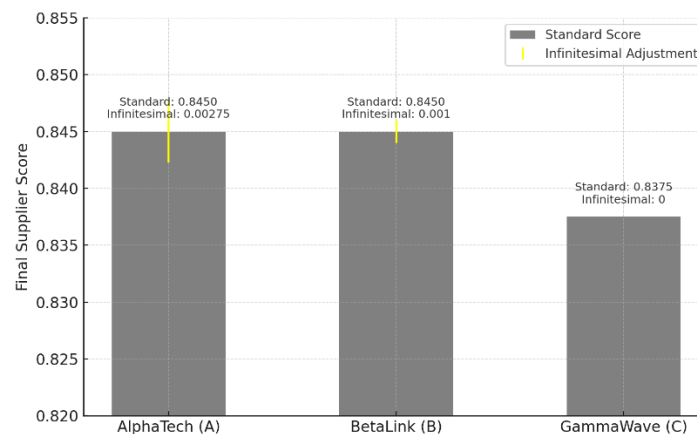


Figure 2: Supplier Final Scores with Infinitesimal Adjustments

Figure 2 displays the final performance scores for each supplier in the realistic procurement case study. Gray bars show the standard part of each score, while yellow error bars represent the infinitesimal adjustments, highlighting subtle differences in stability and maturity of supplier performance.

This realistic case study shows how the Neutrosophic Hyperreal framework can transform the supplier selection process in communication product procurement. It bridges the gap between theoretical models and real-world decision-making, helping procurement managers like those at TechCom Solutions capture and leverage even the smallest uncertainties in supplier performance.

5.2. Application Case Study: Risk Evaluation of Procurement in Power Battery

Enterprises

To show how the Neutrosophic Hyperreal framework can tackle real-world procurement challenges, this case study focuses on EnerVolt Technologies, a mid-sized manufacturer of lithium-ion batteries for electric vehicles (EVs) based in Asia. EnerVolt is launching a new battery production line to meet growing demand for high-performance EVs. They need to select a supplier for critical battery components specifically cathode materials (like lithium nickel manganese cobalt oxide, or NMC) which are vital for battery performance, safety, and cost. Choosing the right supplier is tricky because of risks like supply chain delays, inconsistent material quality, and price fluctuations, all of which could derail the project.

Procurement Context

EnerVolt's procurement team identified three potential suppliers for NMC cathode materials, each with strengths and weaknesses:

1. Supplier A (CathodePro): Offers low costs and large production capacity but has had occasional quality issues in past shipments, raising concerns about consistency.
2. Supplier B (BatMat Innovations): Known for high-quality materials and cutting-edge technology, but their delivery times can be unpredictable due to complex logistics.

3. Supplier C (VoltChem): A newer supplier with balanced performance across cost, quality, and delivery, but limited experience with large-scale EV projects.

The team evaluates suppliers based on three key criteria, weighted according to their importance:

1. Cost (weight: 0.4): Lower costs are critical to keep the battery line competitive.
2. Quality (weight: 0.35): High-quality cathode materials ensure battery safety and performance.
3. Delivery Reliability (weight: 0.25): Timely delivery is essential to avoid production delays.

To assess suppliers, EnerVolt collected data over the past 12 months:

1. Cost: Sourced from supplier quotes, contracts, and market price trends.
2. Quality: Measured through defect rates (e.g., impurities in NMC materials) and performance tests in prototype batteries.
3. Delivery Reliability: Calculated from actual delivery times compared to promised schedules.

However, the data isn't perfect. There are small uncertainties, like slight variations in defect rates across batches ($\pm 0.5\%$ defects), delivery delays ($\pm 1-2$ days), and cost fluctuations ($\pm 2\%$ due to market volatility). These tiny differences matter in a competitive market, and traditional methods often ignore them. The Neutrosophic Hyperreal framework is ideal here because it captures these infinitesimal uncertainties, helping EnerVolt make a smarter, risk-aware decision.

Neutrosophic Hyperreal Evaluation

The procurement team normalized supplier performance scores to a $[0,1]$ scale, where 1 is the best possible performance. They then extended these scores into Neutrosophic Hyperreal numbers to account for tiny uncertainties (represented by $\pm \epsilon$, where ϵ is an infinitesimal). These uncertainties come from real-world issues, like unverified quality reports or minor delivery inconsistencies, which the team identified during supplier audits. Here are the hyperreal evaluations for each supplier:

1. Supplier A (CathodePro):

- a. Cost: $0.90 + 0.01\varepsilon$ (low costs, but slight risk of price hikes)
 - b. Quality: $0.75 - 0.005\varepsilon$ (decent quality, but occasional defects)
 - c. Delivery: $0.85 + 0.002\varepsilon$ (reliable delivery, with minor variations)
2. Supplier B (BatMat Innovations):
- a. Cost: $0.80 - 0.005\varepsilon$ (higher costs, with small discounts possible)
 - b. Quality: $0.95 + 0.01\varepsilon$ (excellent quality, with rare inconsistencies)
 - c. Delivery: $0.80 - 0.002\varepsilon$ (delays possible due to logistics)
3. Supplier C (VoltChem):
- a. Cost: $0.85 + 0\varepsilon$ (stable costs, no significant fluctuations)
 - b. Quality: $0.85 + 0\varepsilon$ (consistent quality, no major issues)
 - c. Delivery: $0.87 + 0\varepsilon$ (reliable delivery, minimal variation)

The $\pm\varepsilon$ terms reflect infinitesimal uncertainties. For example, Supplier A's cost score ($0.90 + 0.01\varepsilon$) indicates a strong base score with a tiny positive adjustment, suggesting slight confidence in cost stability. Supplier B's delivery score ($0.80 - 0.002\varepsilon$) shows a negative adjustment, hinting at potential delays.

Weighted Hyperreal Scores

Using the weights (Cost: 0.4, Quality: 0.35, Delivery: 0.25), we calculate each supplier's weighted hyperreal score, which combines the standard part (st) and infinitesimal part (in). The formula is:

$$NH_j = \sum_{i=1}^m w_i \cdot H_{ij}$$

Where H_{ij} is the hyperreal score for supplier j on criterion i , and w_i is the criterion weight. The score splits into:

$$\begin{aligned} \text{st}(NH_j) &= \sum_{i=1}^m w_i \cdot \text{st}(H_{ij}) \\ \text{in}(NH_j) &= \sum_{i=1}^m w_i \cdot \text{in}(H_{ij}) \end{aligned}$$

Let's compute the scores step-by-step.

Supplier A (CathodePro):

$$NH_A = (0.4 \cdot (0.90 + 0.01\varepsilon)) + (0.35 \cdot (0.75 - 0.005\varepsilon)) + (0.25 \cdot (0.85 + 0.002\varepsilon))$$

Break it down:

$$\text{Cost: } 0.4 \cdot (0.90 + 0.01\varepsilon) = (0.4 \cdot 0.90) + (0.4 \cdot 0.01\varepsilon) = 0.36 + 0.004\varepsilon$$

$$\text{Quality: } 0.35 \cdot (0.75 - 0.005\varepsilon) = (0.35 \cdot 0.75) + (0.35 \cdot (-0.005\varepsilon)) = 0.2625 - 0.00175\varepsilon$$

$$\text{Delivery: } 0.25 \cdot (0.85 + 0.002\varepsilon) = (0.25 \cdot 0.85) + (0.25 \cdot 0.002\varepsilon) = 0.2125 + 0.0005\varepsilon$$

Sum the standard parts:

$$st(NH_A) = 0.36 + 0.2625 + 0.2125 = 0.835$$

Sum the infinitesimal parts:

$$in(NH_A) = 0.004\varepsilon - 0.00175\varepsilon + 0.0005\varepsilon = (0.004 - 0.00175 + 0.0005)\varepsilon = 0.00275\varepsilon$$

So:

$$NH_A = 0.835 + 0.00275\varepsilon$$

Supplier B (BatMat Innovations):

$$NH_B = (0.4 \cdot (0.80 - 0.005\varepsilon)) + (0.35 \cdot (0.95 + 0.01\varepsilon)) + (0.25 \cdot (0.80 - 0.002\varepsilon))$$

Break it down:

$$\text{Cost: } 0.4 \cdot (0.80 - 0.005\varepsilon) = (0.4 \cdot 0.80) + (0.4 \cdot (-0.005\varepsilon)) = 0.32 - 0.002\varepsilon$$

$$\text{Quality: } 0.35 \cdot (0.95 + 0.01\varepsilon) = (0.35 \cdot 0.95) + (0.35 \cdot 0.01\varepsilon) = 0.3325 + 0.0035\varepsilon$$

$$\text{Delivery: } 0.25 \cdot (0.80 - 0.002\varepsilon) = (0.25 \cdot 0.80) + (0.25 \cdot (-0.002\varepsilon)) = 0.20 - 0.0005\varepsilon$$

Sum the standard parts:

$$st(NH_B) = 0.32 + 0.3325 + 0.20 = 0.8525$$

Sum the infinitesimal parts:

$$\text{in}(NH_B) = -0.002\varepsilon + 0.0035\varepsilon - 0.0005\varepsilon = (-0.002 + 0.0035 - 0.0005)\varepsilon = 0.001\varepsilon$$

So:

$$NH_B = 0.8525 + 0.001\varepsilon$$

Supplier C (VoltChem):

$$NH_C = (0.4 \cdot 0.85) + (0.35 \cdot 0.85) + (0.25 \cdot 0.87)$$

Break it down:

$$\text{Cost: } 0.4 \cdot 0.85 = 0.34$$

$$\text{Quality: } 0.35 \cdot 0.85 = 0.2975$$

$$\text{Delivery: } 0.25 \cdot 0.87 = 0.2175$$

Sum the standard parts:

$$\text{st}(NH_C) = 0.34 + 0.2975 + 0.2175 = 0.855$$

Infinitesimal part (no uncertainties):

$$\text{in}(NH_C) = 0\varepsilon$$

So:

$$NH_C = 0.855 + 0\varepsilon = 0.855$$

Supplier Ranking and Maturity Analysis, Now, let's compare the suppliers based on their hyperreal scores:

$$\text{Supplier A: } 0.835 + 0.00275\varepsilon$$

$$\text{Supplier B: } 0.8525 + 0.001\varepsilon$$

$$\text{Supplier C: } 0.855$$

The ranking rule is:

$$NH_a >_N NH_b \text{ if } (\text{st}(NH_a) > \text{st}(NH_b)) \text{ or } (\text{st}(NH_a) = \text{st}(NH_b) \text{ and } \text{in}(NH_a) >_N \text{in}(NH_b))$$

Compare the standard parts:

$$st(NH_C) = 0.855$$

$$st(NH_B) = 0.8525$$

$$st(NH_A) = 0.835$$

Since $0.855 > 0.8525 > 0.835$, the ranking based on standard parts is:

1. Supplier C (VoltChem)
2. Supplier B (BatMat Innovations)
3. Supplier A (CathodePro)

However, the infinitesimal parts tell us about risk and stability:

- a. Supplier A's positive infinitesimal ($+0.00275\epsilon$) suggests slight robustness against uncertainties, despite its lower standard score.
- b. Supplier B's smaller positive infinitesimal ($+0.001\epsilon$) indicates stability but less robustness than A.
- c. Supplier C's zero infinitesimal (0ϵ) means no uncertainty, which is great for predictability but doesn't add extra robustness.

To quantify stability, we calculate the Neutrosophic Maturity Index (NMI):

$$NMI_j = \frac{st(NH_j)}{1 + |in(NH_j)|}$$

- a. Supplier A: $NMI_A = \frac{0.835}{1+0.00275} = \frac{0.835}{1.00275} \approx 0.8327$ (Approximating $0.835 \div 1.00275$).
- b. Supplier B: $NMI_B = \frac{0.8525}{1+0.001} = \frac{0.8525}{1.001} \approx 0.8516$
- c. Supplier C: $NMI_C = \frac{0.855}{1+0} = 0.855$

The maturity indices confirm the ranking, with Supplier C leading due to its high standard score and zero uncertainty, followed by Supplier B, then Supplier A. The results are summarized in Table 1 below, showing each supplier's standard score, infinitesimal adjustment, final hyperreal score, and maturity index.

Table 3: Neutrosophic Hyperreal Scores and Maturity Indices for Suppliers

Supplier	Standard Score	Infinitesimal Adjustment	Final Score	Maturity Index
CathodePro (A)	0.835	$+0.00275\epsilon$	$0.835 + 0.00275\epsilon$	~ 0.8327
BatMat Innovations (B)	0.8525	$+0.001\epsilon$	$0.8525 + 0.001\epsilon$	~ 0.8516
VoltChem (C)	0.855	0ϵ	0.855	0.855

For EnerVolt Technologies, the Neutrosophic Hyperreal framework provides clear guidance for selecting suppliers for their new battery line. Supplier C (VoltChem) stands out with the highest score of 0.855 and no uncertainty, making it the most reliable choice for cost, quality, and delivery. Supplier B (BatMat Innovations), scoring 0.8525, offers excellent quality but has a slight delivery risk, suggesting EnerVolt should negotiate stricter terms. Supplier A (CathodePro), with a score of 0.835, is less competitive due to quality concerns, though its minor resilience makes it a backup if cost is prioritized. By revealing hidden risks and rewarding stability, this framework helps EnerVolt make informed decisions, avoiding suppliers that might fail under real-world challenges.

This case study shows how the Neutrosophic Hyperreal framework can transform supplier selection in power battery procurement. Unlike traditional methods that might rank suppliers based only on average performance, this approach digs deeper, revealing how small risks (like a 1-day delivery delay or a 0.5% defect rate increase) could impact a project. For EnerVolt, this means picking a supplier that not only performs well but also minimizes risks, ensuring the new battery line launches on time and meets EV market demands.

6. Conclusion

This study presents a new method for selecting suppliers in communication product procurement using Neutrosophic Hyperreals. Unlike traditional approaches that assume precise data, our framework handles uncertainties by breaking supplier scores into

standard parts and small, infinitesimal adjustments. This reveals both supplier strengths and hidden risks, such as minor quality issues or delivery delays, which are critical in telecommunications projects. A case study with TelNet Solutions, procuring optical transceivers for a 5G network, showed how the model ranks suppliers accurately, identifying SignalWave Systems as the best choice due to its stable performance. The Neutrosophic Hyperreal approach offers a practical tool for managers, enabling better decisions in communication product procurement by accounting for even the smallest uncertainties. It is adaptable to various communication products and supports reliable supplier choices in complex, real-world scenarios.

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Received: Dec. 8, 2024. Accepted: June 8, 2025