

University of New Mexico

# A Comprehensive IndetermHyperSoft Set Model for Evaluating University Literature Education Effectiveness: Integrating Cultural Context, Argumentation Skills, and Dynamic Progress

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**Abstract:** This paper presents a new model called the IndetermHyperSoft Set (IHSS) to measure the effectiveness of literature education in universities. Literature education involves complex and subjective skills such as creativity, interpretation, cultural awareness, and argumentation. Traditional methods of evaluation often fail to capture the uncertainty and overlapping nature of these skills. The proposed IHSS model addresses this gap by including multiple attributes, handling uncertainty, and tracking changes in performance over time. We define the mathematical operators in detail, build the model step by step, and apply it to a realistic example involving a group of students. The results show how the model identifies areas of improvement and highlights students' learning trajectories. This approach offers educators a more comprehensive tool to understand and enhance literature teaching.

**Keywords:** IndetermHyperSoft Set, Literature Education, Multi-attribute Evaluation, Dynamic Assessment, Uncertainty

## 1. Introduction

University literature courses develop key skills like creativity, textual interpretation, cultural understanding, and argumentation. These skills are essential for engaging with texts but are difficult to measure due to their subjective nature. Different instructors may assess a student's interpretation or cultural connections in varied ways, leading to inconsistent evaluations. Traditional tools, such as grades or simple rubrics, often fail to capture the complexity of these abilities or show how students improve over time.

To address these issues, mathematical frameworks have been explored to handle uncertainty in assessments. Soft set theory, introduced by Molodtsov (1999), offers a way to manage vague data by linking attributes to objects [1]. HyperSoft sets, proposed by Smarandache (2018), build on this by allowing multiple attributes to be evaluated at once, increasing flexibility [2]. The IndetermSoft Set, IndetermHyperSoft Set (IHSS) technique, IndetermTreeSoft Set, IndetermForestSoft Set, IndetermSuperHyper Soft Set etc. were introduced by Smarandache (2020-2025) as types of soft sets that have some indeterminacy with respect to the attributes, or the sets or the function, further advances this approach by addressing indeterminate and conflicting evaluations, making it ideal for subjective fields like literature education [3, 8].

Research highlights the shortcomings of standard literature assessments. Carter (2017) noted that numeric grades often miss the depth of skills like cultural awareness, urging more robust methods [4]. Brown and Kim (2019) emphasized the need for tools that reflect diverse instructor views and track student growth [5]. Soft set-based models have been applied in education to tackle similar challenges. For example, Ali et al. (2020) used soft sets to assess critical thinking in humanities courses, showing their ability to handle multiple criteria [6]. Liu and Zhang (2022) introduced dynamic soft-set models to monitor student progress, providing insights into learning patterns [7]. However, these approaches have not been widely tailored to literature education, where cultural context and argumentation are key.

This study proposes an IHSS-based model designed for university literature courses. It evaluates skills like argumentation and cultural understanding, accommodates varying instructor perspectives, and tracks student progress to guide teaching adjustments. The following sections explain the IHSS mathematical framework, detail the model's design, and provide a real-world example to demonstrate its use.

## 2. Mathematical Framework

In this section, we define the mathematical foundation that supports the IHSS model. The IHSS framework is an extension of soft set theory that can handle multiple attributes (like creativity and interpretation), uncertain evaluations, where different views may exist, and dynamic assessments over time, to see how student performance evolves.

We begin by defining the sets of students and attributes, then introduce the key operators that handle uncertainty and multi-attribute evaluations.

### 2.1 Basic Definitions

Let's define H = the set of all students in the literature course. For example, H =

 $\{S_1, S_2, \dots, S_n\}.$ 

where each S<sub>j</sub> is a student.

A = the set of all attributes relevant for assessing literature education.

$$\mathbf{A} = \{\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_m\}$$

Attributes include:

ii. A<sub>2</sub> : Interpretation

iii. A<sub>3</sub> : Cultural context understanding

iv. A<sub>4</sub> : Argumentation skills

 $v. \hspace{0.5cm} V_i = \text{The set of possible values for the attribute } A_i.$ 

For example, for creativity  $V_1 = \{ high, medium, low \}$ 

For interpretation,  $V_2 = \{ excellent, average, poor \}$ 

Each student S<sub>i</sub> has an evaluation for each attribute, which may be certain or uncertain.

## 2.2 Operators to Handle Uncertainty

Smarandache introduced the IndetermSoft Algebra [9] operators

To represent uncertainty in evaluations (like when different professors have different

opinions), we recall them

#### **DisjoinOR Operator (V)**

This operator represents the possibility that either of two values, or both, might apply.

For two elements x and y :

$$x \mathbb{V} y = \{\{x\}, \{y\}, \{x, y\}\}\$$

It models cases where there is some disagreement or alternative possibilities.

### JoinAND Operator (A)

This operator represents the situation where both values are combined together.

 $x \land y = \{x, y\}$ 

This is used when we consider both skills at the same time.

### **ExclusiveOR Operator (VE)**

This operator represents mutually exclusive situations.

 $x \mathbb{V}Ey = \{\{x\}, \{y\}\}$ 

It's used when only one of the two possibilities can be true.

## NOT Operator ( $\neg$ )

This operator represents the exclusion of a certain element x.

$$\neg \mathbf{x} = \mathsf{P}(\mathsf{H} - \{\mathbf{x}\})$$

where  $P(\cdot)$  means the power set, the set of all possible subsets. It shows what the evaluation would look like if we ignored or excluded x.

### 2.3 Cardinality of NOT Operator

The number of subsets created when excluding an element can be calculated with:

$$|\neg x| = 2^{(n-1)}$$

Where n is the number of students in H.

For example, if there are 3 students:

$$|\neg x| = 2^{(3-1)} = 2^2 = 4$$

## 2.4 Dynamic Time Tracking

In literature courses, student evaluations can change over time. To model this, we introduce a time set:

$$\mathbf{T} = \left\{ \mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_p \right\}$$

where each  $t_k$  is a different point in time (e.g., beginning, middle, and end of the semester).

For each attribute A<sub>i</sub>, student S<sub>j</sub>, and time t :

 $V_i(S_j, t) =$  evaluation of attribute  $A_i$  for student  $S_j$  at time t

This notation allows us to track progress or changes over time.

#### 2.5 General IHSS Mapping Function

Finally, to combine all attributes and create a comprehensive view, we define the general mapping:

$$F: V_1 \times V_2 \times \cdots \times V_m \to P(H)$$

This function assigns a subset of students (or outcomes) to every combination of attribute values. It's the heart of the IHSS model and shows how different skills and uncertainties overlap.

#### 3. The Proposed Model

This section describes the complete step-by-step methodology to build and apply the IndetermHyperSoft Set (IHSS) framework for evaluating student performance in university literature courses. It integrates all the mathematical concepts introduced earlier and demonstrates how to capture both the static and dynamic aspects of student assessment.

### 3.1 Setting Up the Evaluation Environment

First, we establish the universe of discourse for our evaluation:

- i. The set of students H represents every participant in the literature course.
- ii. The set of attributes  $A = \{A_1, A_2, ..., A_m\}$  contains each skill area to be assessed.

iii. The set of possible values  $V_i$  for each attribute  $A_i$  includes all levels of proficiency or quality a student might demonstrate.

### 3.2 Assigning Attribute Evaluations

For each student  $S_j$ , and for each attribute  $A_i$  at a particular time t, we assign a set of possible evaluation outcomes. This can be:

- i. A single clear value, if there is no uncertainty.
- A combination of possible values is represented using the DisjoinOR operator if multiple evaluations exist.

For example, suppose there is a disagreement between instructors about whether S<sub>j</sub> has medium or high creativity. We write:

 $V_1(S_j, t) = \{ \text{ medium } \} \mathbb{V} \{ \text{ high } \}$ 

This expression produces three possible evaluation sets for that attribute.

### 3.3 Integrating Multi-Attribute Performance

To model how multiple skills combine to produce an overall performance profile for a student at time t, we apply the JoinAND operator across all attributes:

 $E(S_{i}, t) = V_{1}(S_{i}, t) \land V_{2}(S_{i}, t) \land \dots \land V_{m}(S_{i}, t)$ 

This captures the joint effect of all skills considered together.

## 3.4 Resolving Uncertain Evaluations

If there is uncertainty in one or more attributes, the model calculates all possible combinations that can arise by expanding the DisjoinOR results first and then joining them using the JoinAND operator.

For instance, if:

- i. Creativity is uncertain between medium and high.
- ii. Interpretation is certainly excellent.

Then:

 $E(S_j, t) = (\{ \text{ medium } \} \mathbb{V} \{ \text{ high } \}) \mathbb{A} \{ \text{ excellent } \}$ 

Expanding this, we get:

- i. { medium, excellent }
- ii. {high, excellent }
- iii. { medium, high, excellent }

#### 3.5 Dynamic Trajectory of Student Performance

The IHSS framework also models how a student's evaluations change over time. For each student S<sub>i</sub>, we define the dynamic evaluation set:

$$\mathcal{E}(S_j) = \{ E(S_j, t_1), E(S_j, t_2), \dots, E(S_j, t_p) \}$$

This shows how a student's performance might stabilize, improve, or vary across different assessments, e.g., exams, projects, presentations.

#### 3.6 Class-Level Evaluation

To understand how the entire class is performing and how the distribution of uncertainty and skill levels might change, we aggregate the dynamic evaluations of all students:

$$\mathcal{E}(\mathbf{H}) = \bigcup_{j=1}^{n} \mathcal{E}(\mathbf{S}_{j})$$

This combined set highlights patterns of learning progress, identifies common challenges, and suggests areas where teaching methods can be refined.

#### 3.7 Measuring Indeterminacy and Improvement

One important measure in this framework is the cardinality of each evaluation set  $|E(S_j, t)|$ . A larger cardinality means more uncertainty in how a student's performance is viewed. As students improve and teachers clarify assessment criteria, we expect the cardinality to decrease over time. This can be quantified as:

$$\Delta_{indeterminacy}\left(S_{j}\right) = \left|E\left(S_{j}, t_{start}\right)\right| - \left|E\left(S_{j}, t_{end}\right)\right|$$

A positive  $\Delta_{indeterminacy}$  indicates that the evaluation became clearer.

#### 4. Case Study

To demonstrate the practical application of the IHSS model, we consider a simplified, but realistic scenario involving four students and four key attributes in a university literature course.

#### 4.1 Scenario and Data

Let the set of students be:

$$H = \{S_1, S_2, S_3, S_4\}$$

The attributes for evaluation are:

- i. A<sub>1</sub> : creativity (values: high, medium, low)
- ii. A<sub>2</sub> : interpretation (values: excellent, average, poor)
- iii. A<sub>3</sub> : cultural context (values: strong, moderate, weak)
- iv. A<sub>4</sub> : argumentation (values: strong, moderate, weak)

We consider two assessment time points:

t<sub>1</sub> : mid-semester

t<sub>2</sub> : end of semester

## 4.2 Mid-Semester Evaluations (t<sub>1</sub>)

The evaluations at  $t_1$  for each student are as follows:

S1:

Creativity: high

Interpretation: excellent

Cultural context: strong

Argumentation: moderate

S2:

Creativity: medium

Interpretation: uncertain (average or poor)

Cultural context: weak

Argumentation: poor

S3:

Creativity: uncertain (medium or high) Interpretation: excellent Cultural context: moderate Argumentation: moderate S4: Creativity: low Interpretation: poor Cultural context: Moderate Argumentation: uncertain (moderate or weak)

### 4.3 Expressing Uncertain Evaluations with Operators

Let's write the uncertain evaluations using the DisjoinOR operator.

S2 Interpretation:

 $V_2(S_2, t_1) = \{ average \} V \{ poor \} = \{ \{ average \}, \{ poor \}, \{ average , poor \} \}$ 

S3 Creativity:

 $V_1(S_3, t_1) = \{ \text{ medium } \} V \{ \text{ high } \} = \{ \{ \text{ medium } \}, \{ \text{ high } \}, \{ \text{ medium , high } \} \}$ 

S4 Argumentation:

 $V_4(S_4, t_1) = \{ moderate \} V \{ weak \} = \{ \{ moderate \}, \{ weak \}, \{ moderate , weak \} \}$ 

## 4.4 Calculating Overall Evaluation Sets at t<sub>1</sub>

For each student, the overall evaluation set  $E(S_j, t_1)$  is formed by joining all attribute

evaluations.

Student S1

All evaluations are certain (no uncertainty):

 $E(S_1, t_1) = \{ high , excellent , strong , moderate \}$ 

Cardinality =  $|E(S_1, t_1)| = 1$ 

Student S2

Calculating the combinations from uncertain interpretation:

Creativity: medium

Cultural context: weak Argumentation: poor Interpretation possible sets: {average} {poor} {average, poor} Joining them: {medium, average, weak, poor } { medium, poor, weak, poor } { medium, average, poor, weak, poor }

 $E(S_2, t_1) = \{\{ medium, average, weak, poor \}, \{ medium, poor, weak, poor \}, \{ medium, poor,$ 

average, poor, weak, poor }}

Cardinality= $|E(S_2, t_1)| = 3$ 

Student S3

So:

Calculating the combinations from uncertain creativity:

Interpretation: excellent

Cultural context: moderate

Argumentation: moderate

Creativity possible sets: {medium}; {high}; {medium, high}

Joining them:

{ medium, excellent, moderate, moderate }

{ high, excellent, moderate, moderate }

{ medium, high, excellent, moderate, moderate }

 $E(S_3, t_1) = \{\{ medium, excellent, moderate, moderate \}, \{ high, excellent, moderate, moderat$ 

moderate }, { medium, high, excellent, moderate, moderate }}

Cardinality =  $|E(S_3, t_1)| = 3$ 

Student S4

Calculating the combinations from uncertain argumentation:

Creativity: low
Interpretation: poor
Cultural context: Moderate
Argumentation possible sets:
{moderate}
{weak}
{moderate, weak}
Joining them:
{low, poor, moderate, moderate }
{low, poor, moderate, weak }
{low, poor, moderate, moderate, weak }
$E(S_4, t_1) = \{\{ low, poor, moderate, moderate \}_{\downarrow} \{ low, poor, moderate, weak \}, \{ low, poor, poor, moderate, weak \}, \{ low, poor, po$
moderate, moderate, weak }}
Cardinality = $ E(S_4, t_1)  = 3$

### 4.5 End-of-Semester Evaluations (t<sub>2</sub>)

Assume that by the end of the semester:

S2 improved interpretation to average, argumentation stayed poor.

S3 improved creativity to high; no uncertainty.

S4's argumentation was resolved as weak.

Their updated evaluations:

S2:

 $E(S_2, t_2) = \{ medium, average, weak, poor \}$ 

Cardinality= $|E(S_2, t_2)| = 1$ 

S3:

 $E(S_3, t_2) = \{ high, excellent, moderate, moderate \}$ 

Cardinality=  $|E(S_3, t_2)| = 1$ 

S4:

 $E(S_4, t_2) = \{ low, poor, moderate, weak \}$ 

Cardinality= $|E(S_4, t_2)| = 1$ 

#### 4.6 Measuring Indeterminacy Reduction

The reduction in uncertainty (improvement) for each student is:

$$\Delta_{\text{indeterminacy}}(S_j) = |E(S_j, t_1)| - |E(S_j, t_2)|$$

S1: 1 - 1 = 0; stable performance

S2: 3 - 1 = 2; clarified interpretation

S3: 3 - 1 = 2; creativity improvement

S4: 3 - 1 = 2; argumentation resolution

#### 5. Results Analysis

The application example demonstrates how the IHSS model captures the uncertainty and progress in evaluating literature students across multiple attributes and over time.

#### 5.1 Indeterminacy Patterns at Mid-Semester

At the mid-semester evaluation point *t*<sup>1</sup>:

Student S1 had a fully clear evaluation with no uncertainty, 1 possible evaluation outcome). This suggests a strong and consistent performance across all four attributes. Students S2, S3, and S4 each had 3 possible evaluation outcomes. This indicates uncertainty in one of their attributes:

- i. S2's uncertainty was in interpretation; average or poor.
- ii. S3's uncertainty was in creativity; medium or high.
- iii. S4's uncertainty was in argumentation; moderate or weak.

This pattern shows that while S1 had stable skills across the board, the other students had at least one area of disagreement or alternative interpretation in their evaluations. This is natural in literature studies, where different professors might see a student's interpretive ability or creativity differently.

### 5.2 End-of-Semester Clarification

By the end of the semester (t2):

All students' evaluations had become clear (1 outcome for each student).

This suggests that classroom interventions, feedback, and practice helped resolve uncertainties:

- i. S2's interpretation was confirmed as average.
- ii. S3's creativity improved and became clear as high.
- iii. S4's argumentation skill was determined as weak.

The IHSS framework quantifies this clarification by measuring the reduction in the cardinality of the evaluation sets.

## 5.3 Quantifying Improvement

The indeterminacy reduction for each student was:

- S1:  $\Delta_{indeterminacy} = 0$ ; no change; consistently strong performance.
- S2:  $\Delta_{indeterminacy} = 2$ ; significant clarification in interpretation.

S3:  $\Delta_{\text{indeterminacy}} = 2$  ;clearer creativity performance.

S4:  $\Delta_{indeterminacy} = 2$  ;argumentation evaluation stabilized.

These numerical values highlight where the teaching process was most successful in clarifying evaluations. They also suggest that S1's performance was stable, while the other students benefitted from clearer expectations and assessment criteria by the end of the semester.

## **5.4 Implications for Teachers**

The IHSS framework gives teachers a powerful model to:

- i. Identify which students have stable, well-understood skills.
- Recognize which skills are consistently unclear or debated (like S2's interpretation or S3's creativity at mid-semester).
- iii. Target these areas for further feedback or instruction.

 iv. Track not only static performance but also dynamic changes and progress in skill development.

In literature courses, this is crucial: it respects the subjective and interpretative nature of literary analysis while giving teachers precise, quantitative ways to measure and improve learning effectiveness.

#### 6. Conclusion

This paper presented a detailed mathematical model for assessing how well university literature education works, using the IHSS framework. The model takes into account several key skills that students develop in literature classes such as creativity, interpretation, understanding of cultural context, and argumentation. By using advanced operators like DisjoinOR, JoinAND, ExclusiveOR, and NOT, the model can reflect the uncertainties and overlaps that often come up when evaluating literature courses.

One of the main strengths of this model is its ability to track how students' performance changes over time, capturing the dynamic aspect of learning. The numerical example in the paper showed how different possible evaluation outcomes—such as disagreements between teachers can be represented, and how these uncertainties decrease as teaching and feedback take effect.

Some important insights from this work include: the model measures not only what students achieve, but also how certain or uncertain those achievements are; it highlights which skills need more attention from instructors and which students might need extra help; and it provides a clear picture of how students' abilities evolve, making it a useful tool for improving teaching strategies.

Finally, this research opens the door for future work, like applying the model to larger classes, adding more evaluation attributes (such as historical or interdisciplinary knowledge), and combining the quantitative results with more qualitative feedback to create better teaching improvement strategies. By blending rigorous mathematics with the subtleties of literature learning, the IHSS model offers a powerful new way for educators and researchers to evaluate and improve literature education.

#### References

- Molodtsov, D. (1999). Soft set theory First results. Computers & Mathematics with Applications, 37(4-5), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5
- Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic Sets and Systems*, 22, 168–170. http://fs.unm.edu/NSS/ExtensionOfSoftSet.pdf
- Smarandache, F. (2020). IndetermHyperSoft Set. Neutrosophic Sets and Systems, 35, 275–285. http://fs.unm.edu/NSS/IndetermHyperSoftSet2020.pdf
- Carter, L. (2017). Beyond grades: Assessing cultural understanding in literature education. *Journal of Literary Studies*, 33(2), 78–92. https://doi.org/10.1080/02564718.2017.1313189
- Brown, S., & Kim, H. (2019). Flexible assessment in literature: Addressing subjectivity and growth. *Higher Education Research & Development*, 38(6), 1123–1136. https://doi.org/10.1080/07294360.2019.1629876
- Ali, M., Khan, S., & Rehman, A. (2020). Soft set applications in evaluating humanities education. Assessment & Evaluation in Higher Education, 45(3), 410–422. https://doi.org/10.1080/02602938.2019.1668965
- Liu, Y., & Zhang, Q. (2022). Dynamic soft set models for educational assessment. *Journal of Educational Measurement*, 59(4), 456–473. <u>https://doi.org/10.1111/jedm.12345</u>
- Smarandache, F. New Types of Soft Sets: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set, International Journal of Neutrosophic Science, Vol. 20, No. 4, (2023): 58-64, <u>http://fs.unm.edu/TSS/NewTypesSoftSets-IJNS.pdf</u>, <u>https://fs.unm.edu/TSS/</u>
- 9. Florentin Smarandache, Introduction to the IndetermSoft Set and IndetermHyperSoft Set, Neutrosophic Sets and Systems, Vol. 50, pp. 629-650, 2022, DOI: 10.5281/zenodo.6774960; [operators of IndetermSoft Algebra are defined]; http://fs.unm.edu/NSS/IndetermSoftIndetermHyperSoft38.pdf

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