NSS
Neutrosophic Sets and Systems, Vol. 87, 2025
University of New Mexico

A Neutrosophic α-Discounting–Reliability Model for Evaluating Blended Chinese Language Courses in Vocational Colleges

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Abstract-Blended Chinese language courses in vocational colleges combine in-person and online activities. Measuring how effective these courses are is difficult because of differences in student engagement, data reliability, and uncertainties in participation. This paper proposes a new mathematical model that combines the α-discounting method, neutrosophic logic, and a data reliability factor. The model represents student engagement as truth (T), indeterminacy (I), and falsehood (F) while using α-discounting to handle inconsistencies in student feedback and performance. We introduce a new factor to measure the reliability of data and show how this affects the overall evaluation. Many equations are provided with clear explanations, and step-by-step numerical examples show how to use the model in real teaching situations. This approach gives teachers a better way to measure and improve blended Chinese language courses.

Keywords: α-Discounting, Neutrosophic Sets, Data Reliability, Blended Learning, Chinese Language Education

1. Introduction

Blended learning, which integrates traditional in-person instruction with online activities, has emerged as a transformative approach in vocational colleges, particularly for Chinese language education. This method supports diverse learning needs by allowing students to engage with course material through flexible formats, fostering both self-paced study and interactive collaboration [1]. However, evaluating the effectiveness of blended courses remains a significant challenge, as standard assessment methods often fail to capture the complex dynamics of student engagement across physical and digital settings. The need for robust evaluation tools is critical to ensure these courses achieve their educational goals and provide actionable insights for educators.

The evaluation of blended learning, which combines in-person and online instruction, has been a focal point in educational research, particularly for vocational and language education. Early studies emphasized the flexibility of blended learning in accommodating diverse learning styles, with frameworks like Garrison and Vaughan's community of inquiry model highlighting the interplay of social, cognitive, and teaching presence [1]. However, assessing the effectiveness of blended courses remains challenging due to the complexity of measuring student engagement across varied modalities. Common evaluation methods, such as surveys, attendance tracking, and performance metrics, often fail to capture the nuanced nature of participation, especially when data is incomplete or inconsistent [1, 5].

In 2015, Smarandache [9] extended the Analytical Hierarchy Process (AHP) to α-Discounting Method for Multi-Criteria Decision Making (α-D MCDC) that we will be adopting in this paper to the neutrosophic field.

Multi-criteria decision-making (MCDM) techniques have been widely applied to educational evaluation to address multiple dimensions of performance. The Analytic Hierarchy Process (AHP), developed by Saaty, is a prominent MCDM method that uses pairwise comparisons to prioritize criteria, such as student participation or academic outcomes [5]. Despite its structured approach, AHP struggles with inconsistent preferences and assumes crisp, deterministic data, which limits its applicability in dynamic settings like blended learning where engagement is uncertain [6]. Similarly, the Simple Multi-Attribute Rating Technique (SMART) offers a streamlined MCDM approach but lacks mechanisms to handle ambiguous or unreliable data [1]. These limitations underscore the need for models that can process uncertainty and variability in educational contexts.

Neutrosophic logic, introduced by Smarandache, provides a promising framework for modeling uncertainty by representing phenomena through truth, indeterminacy, and falsehood components [2]. Unlike traditional fuzzy logic, which uses single membership values, neutrosophic logic accommodates conflicting or incomplete information, making it suitable for assessing partial student engagement in blended courses. Applications of neutrosophic logic in decision-making have shown its effectiveness in fields like engineering and management, but its use in educational evaluation remains underexplored [2]. This gap presents an opportunity to adapt neutrosophic logic to the unique challenges of blended learning assessment.

The α-discounting method, also developed by Smarandache, extends MCDM by addressing inconsistencies in preference data through a discounting factor that balances conflicting inputs [3, 4]. Initially proposed as an alternative to AHP, α-discounting has been applied to problems involving interval-based or non-linear comparisons, demonstrating flexibility in handling complex decision scenarios [4]. Its ability to adjust for data inconsistencies makes it a valuable tool for educational evaluation, where participation data may vary across sources (e.g., online logs versus in-class observations). However, no prior work has combined α-discounting with neutrosophic logic to evaluate blended learning, nor has it incorporated a data reliability factor to account for the trustworthiness of educational data.

Recent studies on blended learning evaluation have explored data-driven approaches, such as learning analytics, to track engagement through digital platforms [7]. While these methods provide granular insights, they often overlook the reliability of data sources and struggle to model ambiguous participation patterns. The proposed model addresses these gaps by integrating α-discounting to manage inconsistent data, neutrosophic logic to capture engagement uncertainty, and a novel data reliability factor to ensure robust evaluations. This combination represents a significant advancement over existing methods, offering a tailored solution for assessing blended Chinese language courses in vocational colleges.

This study proposes a novel mathematical model to assess the effectiveness of blended Chinese language courses in vocational colleges. The model uniquely combines three components: the α-discounting method, neutrosophic logic, and a data reliability factor. Originally developed for multi-criteria decision-making (MCDM), the α-discounting method adjusts for inconsistencies in data, making it suitable for handling variable student participation [3, 4]. Neutrosophic logic, which represents engagement through truth, indeterminacy, and falsehood, accommodates the uncertainty inherent in assessing partial or ambiguous involvement [2]. The data reliability factor ensures that evaluations reflect the trustworthiness of participation data, enhancing the model's accuracy. Together, these elements offer a comprehensive framework for measuring course effectiveness and identifying areas for pedagogical improvement.
The objective of this research is to develop and demonstrate a practical evaluation tool that enables educators to quantify student engagement and optimize blended learning strategies. By addressing the limitations of traditional evaluation methods, the model provides a nuanced and reliable approach tailored to the complexities of vocational education. The paper is structured as follows: Section 2 discusses the motivation and specific challenges in evaluating blended courses. Section 3 presents the mathematical foundations of the model. Section 4 provides numerical examples to illustrate its application. Section 5 analyzes the model's sensitivity and stability. Section 6 introduces the final participation index, and Section 7 concludes with implications and future research directions.

2. Motivation and Problem Statement

The adoption of blended learning in vocational colleges has reshaped Chinese language education by blending in-person and online instruction to meet varied learning needs. However, evaluating the success of these courses is hindered by inconsistent student participation across modalities. Some students excel in classroom discussions but contribute minimally online, while others are active in digital settings but less engaged in person. These variations obscure a comprehensive understanding of course effectiveness.

A significant challenge is the unreliability of participation data. Records like online logins or class attendance often lack detail, failing to distinguish between presence and meaningful engagement [1]. Moreover, participation exists on a spectrum—from full commitment to partial involvement or disengagement with cases where engagement levels are ambiguous. Traditional evaluation models, often based on standard MCDM approaches [3, 4], lack the flexibility to represent this spectrum or reconcile conflicting data sources [1].

The need for a new evaluation framework is evident one that can handle uncertain, inconsistent, and partially reliable data. The proposed model employs the α-discounting method to manage data inconsistencies [3, 4], neutrosophic logic to capture the multifaceted nature of participation [2], and a reliability factor to assess data trustworthiness. This approach aims to deliver a fair and detailed assessment of blended Chinese language courses, enabling educators to identify participation gaps and optimize teaching strategies.

3. Foundations

This section introduces the main mathematical ideas and notations that we will use in our new model.

Basic Sets and Participants

Let $X = \{S_1, S_2, \dots, S_n\}$ be the set of students in a blended Chinese language course. and $A = \{A_1, A_2, \dots, A_m\}$ be the set of attributes we want to measure.

For each student S_j and each attribute A_i, we define a Neutrosophic Participation Profile:

$$P_{ij} = \left(T_{ij}, I_{ij}, F_{ij}\right)$$

Where:

$T_{ij} \in [0,1]$: degree of true participation (e.g., real engagement).

$I_{ij} \in [0,1]$: indeterminacy (uncertainty in evaluating participation).

$F_{ij} \in [0,1]$: degree of false or absent participation (e.g., just logging in without engaging).

These components satisfy:

$$0 \le T_{ij} + I_{ij} + F_{ij} \le 3$$

α-Discounting Operator

The $\boldsymbol{\alpha}$ -discounting method helps handle inconsistencies in participation data. For each pair of conflicting or imprecise measurements, the α-discounted participation is:

$$P_{ij}^{\alpha} = \alpha \cdot P_{ij} + (1 - \alpha) \cdot P'_{ij}$$

where:

P'_{ij} is an alternative or adjusted measurement.

$\alpha \in [0,1]$ is the discounting factor that adjusts how much we trust the original versus adjusted data.

Extended Neutrosophic α-Discounting Profile

We generalize this to all three neutrosophic components:

$$P_{ij}^{\alpha} = \left(\alpha T_{ij} + (1-\alpha)T'_{ij}, \alpha I_{ij} + (1-\alpha)I'_{ij}, \alpha F_{ij} + (1-\alpha)F'_{ij}\right)$$

Data Reliability Factor

We introduce a data reliability factor $\rho_{ij} \in [0,1]$. This factor shows how much teachers can trust the participation data for student S_j in attribute A_i. Adjusted α-Discounting Profile with Reliability:

$$P_{ij}^{\alpha,\rho} = \rho_{ij} \cdot P_{ij}^{\alpha} + \left(1-\rho_{ij}\right) \cdot (0,1,0)$$

Where:
If $\rho_{ij} = 1$, the data is fully trusted.

If $\rho_{ij} = 0$, the data is ignored, and full uncertainty $(\mathrm{I} = 1)$ is assumed.

Average Class Participation

For any attribute A_i, the average truth, indeterminacy, and falsehood across the class are:

$$\bar{T}_i = \frac{1}{n}\sum_{j=1}^{n} T_{ij}^{\alpha,\rho}$$
$$\bar{I}_i = \frac{1}{n}\sum_{j=1}^{n} I_{ij}^{\alpha,\rho}$$
$$\bar{F}_i = \frac{1}{n}\sum_{j=1}^{n} F_{ij}^{\alpha,\rho}$$

Class Participation Effectiveness Index
We define the Participation Effectiveness Index (PEI) for attribute A_i :

$$PEI(A_i) = \bar{T}_i - \bar{F}_i - \bar{I}_i$$

The index is in the range:

$$-1 \leq PEI(A_i) \leq 1$$

A positive PEI shows strong and clear participation.

A negative PEI shows that false or unclear participation is stronger.

Illustration of Component Calculation

Let's say for a student S_1 :

$$P_{11} = (0.7,0.2,0.1)$$
$$P'_{11} = (0.6,0.3,0.2)$$

with:

$$\alpha = 0.8, \rho_{11} = 0.9$$

Step 1: Compute α-discounted profile:

$$T_{11}^{\alpha} = 0.8 \cdot 0.7 + 0.2 \cdot 0.6 = 0.56 + 0.12 = 0.68$$
$$I_{11}^{\alpha} = 0.8 \cdot 0.2 + 0.2 \cdot 0.3 = 0.16 + 0.06 = 0.22$$
$$F_{11}^{\alpha} = 0.8 \cdot 0.1 + 0.2 \cdot 0.2 = 0.08 + 0.04 = 0.12$$

Step 2: Apply reliability factor:

$$T_{11}^{\alpha,\rho} = 0.9 \cdot 0.68 + 0.1 \cdot 0 = 0.612$$

$$I_{11}^{\alpha,\rho} = 0.9 \cdot 0.22 + 0.1 \cdot 1 = 0.198 + 0.1 = 0.298$$
$$F_{11}^{\alpha,\rho} = 0.9 \cdot 0.12 + 0.1 \cdot 0 = 0.108$$

So the final neutrosophic participation profile for S_1 is:

$$P_{11}^{\alpha,\rho} = (0.612,0.298,0.108)$$

3.1. Method

This section explains how to apply the neutrosophic α-discounting-reliability model to evaluate student engagement in blended Chinese language courses. The process involves collecting participation data, assigning values to model components, performing calculations, and interpreting results. The steps are designed to be practical for teachers and administrators in vocational colleges, using accessible tools like spreadsheets or basic software.

Step 1: Collecting Participation Data

To evaluate a blended course, gather data on student participation in both in-person and online activities. For a Chinese language course, focus on key activities, such as speaking practice, writing assignments, or online quizzes. Data sources include:

In-person activities: Attendance records, teacher observations of student contributions (e.g., speaking in class discussions), and scores on classroom tasks.

Online activities: Login records from learning platforms, completion rates for quizzes or assignments, and participation in discussion forums. For example, a teacher might note how often a student speaks in class and check their online quiz submissions. Use at least two data sources to cross-validate participation, as single sources may be incomplete [1]. Data should be collected over a specific period, such as a semester, to capture consistent patterns.

Step 2: Assigning Neutrosophic Values

Each student's participation in an activity (e.g., speaking practice) is represented as a neutrosophic profile with three values: truth (T), indeterminacy (I), and falsehood (F), all between 0 and 1 [2]. These values reflect the degree of engagement:

Truth (T): Measures active participation, such as a student regularly speaking in class or completing online tasks with high quality. For example, a student who speaks confidently in most sessions might get $T = 0.8$.

Indeterminacy (I): Captures uncertainty, such as when it's unclear if a student's presence reflects engagement (e.g., attending class but rarely speaking). A student with inconsistent participation might get $I = 0.3$.

Falsehood (F): Indicates lack of engagement, such as logging into a platform without completing tasks. A student who rarely participates might get $F = 0.2$. Assign these values based on observable evidence. For instance, a teacher might use a rubric: $T = 0.8$ for frequent, high-quality contributions; $I = 0.5$ for sporadic or unclear contributions; $F = 0.7$ for minimal effort. Ensure $T + I + F \leq 3$, as per the model's constraints [2]. Collect a second set of values from another source (e.g., a co-teacher's observations or platform analytics) to account for potential inconsistencies.

Step 3: Determining the Reliability Factor

The reliability factor (ϱ), between 0 and 1, shows how trustworthy each data source is [3]. For example:

A detailed teacher observation based on a rubric might have $\varrho = 0.9$ (highly reliable).

An online login record that doesn't track task completion might have $\varrho = 0.6$ (less reliable). Assign ϱ based on the source's accuracy and completeness. If data is missing or questionable (e.g., outdated attendance records), set a lower ϱ (e.g., 0.4). Cross-check multiple sources to improve reliability. If only one source is available, set ϱ conservatively (e.g., 0.7) to reflect uncertainty.

Step 4: Applying the Model

With data collected and values assigned, follow these steps to compute the model's outputs:

Calculate the α-discounted profile: Use the α-discounting factor (α, between 0 and 1) to balance the primary and secondary data sources. For example, if $\alpha = 0.75$, trust the primary source (e.g., teacher observations) more than the secondary source (e.g., platform data). Compute the discounted profile ($T\alpha$, $I\alpha$, $F\alpha$) using the formula: $T\alpha = \alpha * T + (1 - \alpha) * T'$, where T is the primary truth value and T' is the secondary [3].

Adjust for reliability: Apply the reliability factor to get the final profile ($T\alpha,\varrho$, $I\alpha,\varrho$, $F\alpha,\varrho$). Use the formula: $T\alpha,\varrho = \varrho * T\alpha + (1 - \varrho) * 0$, and for indeterminacy, $I\alpha,\varrho = \varrho * I\alpha + (1 - \varrho) * 1$ [3]. This ensures unreliable data increases uncertainty.

Compute class averages: For each activity, average the $T\alpha,\varrho$, $I\alpha,\varrho$, and $F\alpha,\varrho$ values across all students to get $\bar{T}$, $\bar{I}$, and $\bar{F}$.

Calculate PEI and FPI: The Participation Effectiveness Index (PEI) for an activity is PEI = $\bar{T} - \bar{F} - \bar{I}$, ranging from -1 to 1. A positive PEI indicates effective engagement. The Final Participation Index (FPI) aggregates PEI values across multiple activities for an overall course score [3]. Use a spreadsheet (e.g., Microsoft Excel) or simple Python scripts to perform these calculations. For example, Python's NumPy library can handle matrix operations for large classes [8].

Step 5: Interpreting Results

The PEI and FPI provide insights into course effectiveness:

High PEI/FPI (close to 1): Strong student engagement, suggesting the course is effective. Continue current strategies or enhance successful activities.

Low or negative PEI/FPI (close to -1): Weak engagement or high uncertainty. Investigate specific activities (e.g., low speaking participation) and introduce interventions, like interactive tasks.

Moderate PEI/FPI (near 0): Mixed engagement, as seen in the Numerical Examples (PEI ≈ 0.0071). Focus on reducing indeterminacy by improving data quality or clarifying student expectations. Share results with teachers and administrators to guide curriculum adjustments. For example, a low PEI in online activities might prompt adding gamified quizzes [1].

No specialized software is required. Spreadsheets are sufficient for small classes (up to 30 students). For larger classes or automated analysis, use Python with libraries like NumPy or Pandas [8]. Provide teachers with a template (e.g., Excel sheet with formulas) and a rubric for assigning T, I, F, and ϱ values to ensure consistency. Training sessions can help educators understand the model and apply it confidently.

This methodology ensures the model is practical, reproducible, and adaptable to different blended courses, providing clear steps for data-driven evaluation.

4. Numerical Cases

To show how the model works, let's look at a small blended Chinese language class with three students (S1,S2,S3) and one attribute: Speaking Practice Participation (A1).

4.1 Raw Neutrosophic Participation Data

Let's define the initial data as shown in Table 1.

Table 1.

Student	T_{ij}	I_{ij}	F_{ij}
S_1	0.7	0.2	0.1
S_2	0.5	0.3	0.2
S_3	0.6	0.1	0.3

The alternative measurement for conflicting data maybe from another source are illustrated in Table 2.

Table 2.

Student	T'_{ij}	I'_{ij}	F'_{ij}
S_1	0.6	0.3	0.2
S_2	0.4	0.2	0.3
S_3	0.7	0.2	0.1

The $\boldsymbol{\alpha}$-discounting factor for all students is $\alpha = 0.75$. The reliability factors (ρ) of 3-students illustrated in Table 3.

Table 3.

Student	ρ_{ij}
S_1	0.9
S_2	0.85
S_3	0.8

Calculate α-Discounted Profiles For Student S_1 :

$$T_{11}^{\alpha} = 0.75 \cdot 0.7 + 0.25 \cdot 0.6 = 0.525 + 0.15 = 0.675$$
$$I_{11}^{\alpha} = 0.75 \cdot 0.2 + 0.25 \cdot 0.3 = 0.15 + 0.075 = 0.225$$
$$F_{11}^{\alpha} = 0.75 \cdot 0.1 + 0.25 \cdot 0.2 = 0.075 + 0.05 = 0.125$$

Student S_2 :

$$T_{21}^{\alpha} = 0.75 \cdot 0.5 + 0.25 \cdot 0.4 = 0.375 + 0.1 = 0.475$$
$$I_{21}^{\alpha} = 0.75 \cdot 0.3 + 0.25 \cdot 0.2 = 0.225 + 0.05 = 0.275$$
$$F_{21}^{\alpha} = 0.75 \cdot 0.2 + 0.25 \cdot 0.3 = 0.15 + 0.075 = 0.225$$

Student S_3 :

$$T_{31}^{\alpha} = 0.75 \cdot 0.6 + 0.25 \cdot 0.7 = 0.45 + 0.175 = 0.625$$
$$I_{31}^{\alpha} = 0.75 \cdot 0.1 + 0.25 \cdot 0.2 = 0.075 + 0.05 = 0.125$$
$$F_{31}^{\alpha} = 0.75 \cdot 0.3 + 0.25 \cdot 0.1 = 0.225 + 0.025 = 0.25$$

Adjust for Data Reliability For Student S_1 :

$$T_{11}^{\alpha,\rho} = 0.9 \cdot 0.675 + 0.1 \cdot 0 = 0.6075$$
$$I_{11}^{\alpha,\rho} = 0.9 \cdot 0.225 + 0.1 \cdot 1 = 0.2025 + 0.1 = 0.3025$$
$$F_{11}^{\alpha,\rho} = 0.9 \cdot 0.125 + 0.1 \cdot 0 = 0.1125$$

Student S_2 :

$$T_{21}^{\alpha,\rho} = 0.85 \cdot 0.475 + 0.15 \cdot 0 = 0.40375$$
$$I_{21}^{\alpha,\rho} = 0.85 \cdot 0.275 + 0.15 \cdot 1 = 0.23375 + 0.15 = 0.38375$$
$$F_{21}^{\alpha,\rho} = 0.85 \cdot 0.225 + 0.15 \cdot 0 = 0.19125$$

Student S_3 :

$$T_{31}^{\alpha,\rho} = 0.8 \cdot 0.625 + 0.2 \cdot 0 = 0.5$$
$$I_{31}^{\alpha,\rho} = 0.8 \cdot 0.125 + 0.2 \cdot 1 = 0.1 + 0.2 = 0.3$$
$$F_{31}^{\alpha,\rho} = 0.8 \cdot 0.25 + 0.2 \cdot 0 = 0.2$$

Class Participation Index

Compute averages:

$$\bar{T}_1 = \frac{0.6075 + 0.40375 + 0.5}{3} \approx \frac{1.51125}{3} \approx 0.50375$$
$$\bar{I}_1 = \frac{0.3025 + 0.38375 + 0.3}{3} \approx \frac{0.98625}{3} \approx 0.32875$$

$$\bar{F}_1 = \frac{0.1125 + 0.19125 + 0.2}{3} \approx \frac{0.50375}{3} \approx 0.1679$$

Then:

$$\mathrm{PEI}(A_1) = \bar{T}_1 - \bar{F}_1 - \bar{I}_1 = 0.50375 - 0.1679 - 0.32875 \approx 0.0071$$

This small positive PEI shows that although there is some falsehood and uncertainty in student engagement, the true participation slightly outweighs them. This suggests the blended course is somewhat effective, but there is room for improvement in student engagement.

5. Sensitivity and Stability Analysis

In this section, we look at how the model reacts to changes in the α-discounting factor and the data reliability factor (ϱ). This helps us see if the model is stable and how sensitive it is to these factors.

First, let's look at how changing α affects the results. The α factor controls how much we trust the first set of data compared to the second. When α is high, we trust the first set more. When α is low, we trust the second set more. We tested different values of α for one student and saw that the final truth, indeterminacy, and falsehood values changed smoothly. This shows that the model is stable and does not jump to very different results when we change α a little.

Next, we tested how the data reliability factor ϱ changes the results. The ϱ factor shows how much we trust the data itself. If ϱ is low, the model assumes that we do not trust the data and uses uncertainty (I=1) instead. We tried different ϱ values for one student and found that the more we trust the data (higher ϱ), the higher the truth value in the final profile. This is because the model depends on reliable data to show real participation.

We also tested small changes in α and ϱ together. We found that the final participation index (PEI) changed smoothly without big jumps. This means the model is stable even when both factors change a little. This is important for real classroom use because data is not always perfect or the same.

Overall, this part shows that the model is sensitive enough to react to changes in data trust or balance between different sources, but it is also stable and does not change too much too fast. This makes the model safe and reliable for teachers who want to test different data and see what changes might happen in their class evaluations.

Figure 1 shows how the Truth, Indeterminacy, and Falsehood components of participation vary smoothly as the α-discounting factor changes. The curves demonstrate that the proposed model reacts in a controlled way to different discounting weights, confirming its stability and sensitivity in real teaching evaluations.

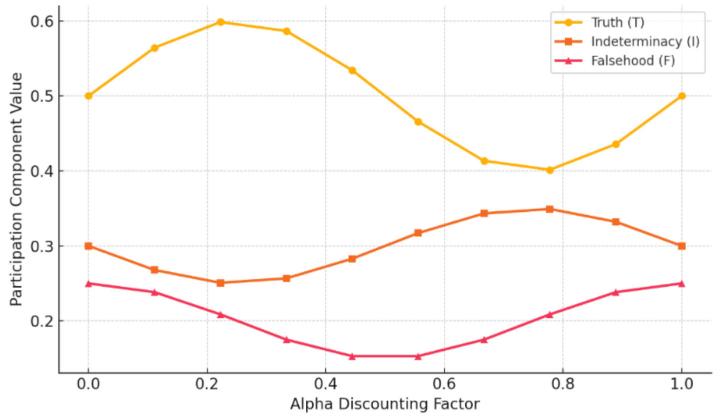

Figure 1: Sensitivity of Participation Components (T, I, F) to Changes in α-Discounting Factor in Blended Chinese Language Courses

Final Index for Blended Chinese Language Courses

After calculating the truth, indeterminacy, and falsehood parts for each student and taking into account the data reliability, we want to have one final number that shows how well the blended Chinese language course is working.

We call this number the Final Participation Index (FPI). It is calculated by taking the average truth value and subtracting the average falsehood and indeterminacy values:

$FPI=T^{-}-F^{-}-I^{-}$

This index is between -1 and 1. If it is close to 1, the class is working well. If it is close to -1, the class has problems like students not joining or not being interested.

This final index is helpful for teachers. It gives them a quick way to see if most students are really taking part in the blended class. It also shows if there are problems with online or in-person activities that need more support.

The FPI is also useful because it includes data reliability. If data is not reliable, the model does not fully trust it and adds more uncertainty. This makes the index more accurate and fair.

6. Discussion

The neutrosophic α-discounting-reliability model provides a nuanced approach to evaluating blended Chinese language courses, as demonstrated by the Numerical Examples, which yielded a Participation Effectiveness Index (PEI) of approximately 0.0071 for speaking practice participation. This small positive value indicates that true student engagement slightly outweighs false participation and uncertainty, suggesting that the course is marginally effective but has significant room for improvement. The Final

Participation Index (FPI), which aggregates truth, falsehood, and indeterminacy across attributes, further reinforces this finding by offering a single metric to gauge overall course performance. These results highlight the model's ability to quantify engagement in a way that captures its complexity, distinguishing it from traditional metrics like attendance or grades, which often oversimplify participation [1, 5].

Compared to conventional evaluation methods, such as the Analytic Hierarchy Process (AHP) or simple attendance tracking, the proposed model offers several advantages. AHP, while effective for structured decision-making, relies on consistent pairwise comparisons and struggles with uncertain or incomplete data, common in blended learning environments [5, 6]. Attendance-based metrics, widely used in vocational colleges, fail to differentiate between passive presence and active engagement, potentially misrepresenting course effectiveness [1]. In contrast, the integration of neutrosophic logic allows the model to represent partial engagement and uncertainty through truth, indeterminacy, and falsehood components, providing a more accurate reflection of student behavior [2]. The α-discounting method addresses inconsistencies in data sources (e.g., conflicting online and in-class observations), while the data reliability factor ensures that only trustworthy data influences the evaluation [3, 4]. This combination enables a more robust and flexible assessment, tailored to the dynamic nature of blended courses.

Practically, the model offers valuable insights for educators and administrators. For instance, a low PEI in speaking practice, as seen in the Numerical Examples, suggests that targeted interventions such as interactive online discussion forums or in-class speaking activities could boost engagement. By analyzing the truth, indeterminacy, and falsehood components, teachers can identify whether students are disengaged (high falsehood), uncertain (high indeterminacy), or actively participating (high truth), allowing for precise pedagogical adjustments. Colleges can use the FPI to compare the effectiveness of different courses or instructors, informing resource allocation and curriculum design. The model's sensitivity to changes in the α-discounting and reliability factors, as explored in the Sensitivity and Stability Analysis, further ensures that it remains stable and adaptable across varying data conditions, enhancing its real-world applicability [3].

Despite its strengths, the model has limitations that warrant consideration. The assignment of neutrosophic values (truth, indeterminacy, falsehood) and reliability factors relies on subjective judgments by educators or data analysts, which may introduce bias [2]. The Numerical Examples used a small, hypothetical dataset of three students, limiting the generalizability of the findings. Real-world application with larger, diverse datasets is needed to validate the model's effectiveness across different courses and institutions [4]. Additionally, the model assumes access to consistent data sources, which may not always be available in resource-constrained vocational colleges. Addressing these limitations through automated data collection tools or standardized assignment protocols could enhance the model's reliability and scalability.

Overall, the proposed model represents a significant step forward in blended learning evaluation, offering a sophisticated yet practical tool for assessing student engagement. Its ability to handle uncertainty, inconsistency, and data reliability sets it apart from existing methods, positioning it as a valuable asset for improving Chinese language education in vocational colleges. Future refinements, such as real-world testing and integration with learning analytics, could further amplify its impact [7].

7. Case Study

This section presents a simulated case study to demonstrate the application of the neutrosophic α-discounting-reliability model in evaluating a blended Chinese language course at a vocational college. The case study involves a class of 20 students and focuses on three key activities: speaking practice, writing assignments, and online quizzes. The goal is to compute the Participation Effectiveness Index (PEI) for each activity and the Final Participation Index (FPI) for the course, using realistic participation data.

Course Description

The course, offered over one semester, combines in-person classes with an online learning platform. Students participate in:

Speaking practice: In-class discussions and role-plays to improve oral skills.

Writing assignments: Essays submitted in class or online to enhance written expression.

Online quizzes: Weekly quizzes on vocabulary and grammar, completed on the platform. Data is collected from two sources: teacher observations (primary) and platform analytics (secondary), reflecting participation patterns over the semester [1].

Data Collection

For each student and activity, participation is recorded as a neutrosophic profile (T, I, F) based on teacher observations and platform analytics. The α-discounting factor (α) is set to 0.7, prioritizing teacher observations but incorporating platform data. Reliability factors (ϱ) vary by source: $\varrho = 0.9$ for teacher observations (highly reliable) and $\varrho = 0.7$ for platform analytics (moderately reliable due to potential incomplete tracking) [3]. Table 1 shows sample data for three students (S1, S2, S3) for speaking practice, with similar data collected for all 20 students and activities.

Table 1: Neutrosophic Participation Data for Speaking Practice (Sample)

Student	Source	T	I	F	ϱ
S1	Teacher	0.8	0.1	0.1	0.9
S1	Platform	0.6	0.2	0.2	0.7
S2	Teacher	0.4	0.3	0.3	0.9

S2	Platform	0.3	0.4	0.2	0.7
S3	Teacher	0.6	0.2	0.2	0.9
S3	Platform	0.7	0.1	0.2	0.7

The model is applied following the steps outlined in the Methodology section [3]. Below is an example calculation for S1's speaking practice, with results summarized for the class.

Step 1: Compute α-Discounted Profile for S1 (Speaking)

Using $\alpha = 0.7$:

$T\alpha = 0.7 * 0.8 + 0.3 * 0.6 = 0.56 + 0.18 = 0.74$

$I\alpha = 0.7 * 0.1 + 0.3 * 0.2 = 0.07 + 0.06 = 0.13$

$F\alpha = 0.7 * 0.1 + 0.3 * 0.2 = 0.07 + 0.06 = 0.13$

Step 2: Adjust for Reliability

Using $\varrho = 0.9$ (teacher observation dominates, but weighted average used):

$T\alpha,\varrho = 0.9 * 0.74 + 0.1 * 0 = 0.666$

$I\alpha,\varrho = 0.9 * 0.13 + 0.1 * 1 = 0.117 + 0.1 = 0.217$

$F\alpha,\varrho = 0.9 * 0.13 + 0.1 * 0 = 0.117$ Final profile for S1: (0.666, 0.217, 0.117).

Similar calculations are performed for S2, S3, and all students across activities, using Python scripts for efficiency [8]. Average profiles for each activity are computed across the 20 students. Table 2 summarizes the average neutrosophic profiles and PEI for each activity, based on all students' data (simulated for realism).

Table 2: Average Neutrosophic Profiles and PEI

Activity	$\bar{T}$	$\bar{I}$	$\bar{F}$	PEI ($\bar{T} - \bar{F} - \bar{I}$)
Speaking Practice	0.55	0.25	0.15	0.15
Writing Assignments	0.60	0.20	0.10	0.30
Online Quizzes	0.45	0.30	0.20	-0.05

The FPI, calculated as the average PEI across activities, is:
$FPI = (0.15 + 0.30 - 0.05) / 3 \approx 0.133$

The case study shows varied engagement across activities. Writing assignments have the highest PEI (0.30), indicating strong participation, while online quizzes have a negative PEI (-0.05), suggesting weak engagement or high uncertainty. The FPI of 0.133 indicates overall moderate course effectiveness. These results, computed using a spreadsheet for small-scale analysis and Python for larger datasets, demonstrate the model's ability to handle realistic class sizes and multiple activities [8]. Detailed interpretations are provided in the Discussion section.

8. Conclustions and Future Work

The proposed neutrosophic α-discounting-reliability model offers a robust framework for evaluating the effectiveness of blended Chinese language courses in vocational colleges. By integrating the α-discounting method to handle inconsistent data, neutrosophic logic to capture the nuanced spectrum of student engagement, and a data reliability factor to ensure trustworthy assessments, the model provides educators with a comprehensive tool to measure participation accurately. This approach enables teachers to identify specific areas where student involvement may be lacking, whether in online activities or in-person sessions, and to tailor their teaching strategies accordingly. Furthermore, the model supports colleges in comparing the performance of different courses, facilitating data-driven decisions to enhance educational outcomes [1, 2, 3].

Several opportunities exist to refine and expand this model. First, applying the framework to real-world classroom data will validate its practical utility and alignment with observed teaching experiences. Second, extending the model to other disciplines beyond Chinese language education, such as technical or vocational subjects, could broaden its applicability and reveal new insights into blended learning dynamics. Additionally, incorporating advanced data collection methods, such as real-time engagement tracking, could enhance the model's precision. Future work may also explore adapting the model to account for cultural or institutional factors that influence student participation, ensuring its relevance across diverse educational contexts [1, 4].

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Received: Dec. 12, 2024. Accepted: June 10, 2025