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Neutrosophication of Neutrosophic B-Spline Curve Approximation Model

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Abstract: The neutrosophication process involves transforming crisp input value into neutrosophic value. It is a primary and crucial step for the operations within a neutrosophic set and system. Neutrosophic set theories are a generalization of intuitionistic fuzzy and fuzzy set theories, focusing on truth, indeterminacy, and falsity memberships independently. However, it isn't easy to generate a geometrical model such as a B-spline curve by using neutrosophic set theory through the neutrosophication process. Therefore, this paper used $(\alpha,\beta,\gamma)-\text{cut}$ operations in the neutrosophication process to construct the neutrosophic B-spline curves (NB-sC) models for approximation methods. Before generating the model, the neutrosophic control point (NCP) must first be introduced using the $(\alpha,\beta,\gamma)-\text{cut}$ operations in triangular form. After that, the NCP will be blended with the B-spline basis function to generate the NB-sCs using approximation methods. Next, some numerical examples of NB-sCs will be provided. Finally, the NB-sCs approximation models will be visualized, and their algorithm will be shown.

Keywords: Neutrosophication process; B-spline Curve; Approximation Method; $(\alpha,\beta,\gamma)-\text{cut}$ operations.

1. Introduction

Several theories have emerged to tackle uncertainties including probabilities theoretical, fuzzy set theory, intuitionistic fuzzy set theory, rough set theory, and much more. In 1999, Smarandache [1,2] introduced the concept of the neutrosophic set (NS), which involves an independent indeterminacy membership. This function serves as a generalization of various other set theories, including classic sets, fuzzy sets [3], and intuitionistic fuzzy sets [4]. In a NS, the level of uncertainty is precisely measured, and three distinct functions are used to quantify truth-membership (*T*), indeterminacy-membership (*I*), and false-membership (*F*). These functions are fully independent.

The neutrosophication process is important in converting provided values to neutrosophic values [5-9]. A few researchers have proposed some neutrosophication methods. Broumi and colleagues [10] described a neutrosophic trapezoidal function and its implications in MATLAB software. Karaaslan [11] investigated the neutrosophic Gaussian function and its use in decision-making. Chakraborty and colleagues [12] conducted research on neutrosophication and de-neutrosophication using the neutrosophic triangular function, as well as its applications.

Neutrosophic sets can be utilized to deal with challenges in curve and surface construction for computer-aided graphic designs (CAGD). This concept is required for building a smoother curve or surface model. NS and CAGD are techniques that can solve the ambiguity of control or data points. Wahab [13-15] developed a method for fuzzifying control points to create Bezier and B-spline curves. The study of fuzzy and intuitionistic fuzzy set theory with geometric modeling has expanded [16-21,40]. The notion was then expanded by Zakaria, who used the $(\alpha,\beta,\gamma)-\text{cut}$ function to create the fuzzified Bezier curve by using an approximation method [22]. Besides that, some studies used the neutrosophic set theory with geometric modeling [23-27,31-39].

This paper is summarized as follows. The initial section provides background details on the subject matter. In Section 2, some neutrosophic set theories and their basic properties are discussed. In Section 3, the generation of NCP through the neutrosophication process will be introduced. Next, Section 4 is the visualization of NB-sCs approximation by using a numerical example of NCP are discussed. Later, Section 5 is the neutrosophication of NB-sCs approximation models that will be presented for the selection of (α,β,γ) values. Finally, Section 6 is the conclusions and discussion parts.

2. Basic Properties

The fuzzy set only deals with membership, whereas the intuitionistic set deals with membership, non-membership, and uncertainty degrees and are dependent on each other. However, the NS treats the truth, indeterminacy, and falsity degrees independently [1]. A NS has three membership functions with the addition of the parameter "indeterminacy" to the NS specification [2]: a membership function, represented by the letter *T*; an indeterminacy membership function, denoted by the letter *I*; and a non-membership function, appointed by the letter *F*. This section will define neutrosophic set theory, neutrosophic relation notion, neutrosophic point, the $(\alpha,\beta,\gamma)-\text{cut}$ operations as the fundamentals and the triangular neutrosophic number. Therefore, the definitions are as follows.

Definition 1. [1] Let X be the universal set, with elements in X represented as x. The NS is an element in the structure as follows:

$$\hat{A}=\{\left\langle x:T_{\hat{A}(x)},I_{\hat{A}(x)},F_{\hat{A}(x)}\right\rangle \mid x\in X\} \tag{1}$$

where $T,I,F:X\rightarrow]^{-}0,1^{+}[$ represented as the degree of truth, indeterminacy, and falsity memberships respectively of the element $x\in X$ to the set $\hat{A}$ with the condition;

$$0^{-}\leq T_{\hat{A}}(x)+I_{\hat{A}}(x)+F_{\hat{A}}(x)\leq 3^{+} \tag{2}$$

There is no limit to the amount of $T_{\hat{A}}(x), I_{\hat{A}}(x)$ and $F_{\hat{A}}(x)$.

NS takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$. But in technical applications the real value of the interval $[0,1]$ will be used since $]^{-}0,1^{+}[$ are difficult to apply in real data such as scientific and engineering problem. Thus, the membership values used as follows:

$$\hat{A}=\{\left\langle x:T_{\hat{A}(x)},I_{\hat{A}(x)},F_{\hat{A}(x)}\right\rangle \mid x\in X\} \text{ and } T_{\hat{A}}(x),I_{\hat{A}}(x),F_{\hat{A}}(x)\in[0,1]$$

There is no limitation on the summation of $T_{\hat{A}}(x), I_{\hat{A}}(x)$ and $F_{\hat{A}}(x)$. But comply with the following conditions:

$$0\leq T_{\hat{A}}(x)+I_{\hat{A}}(x)+F_{\hat{A}}(x)\leq 3 \tag{3}$$

Definition 2. [23,24] Suppose $\hat{A}=\left\{\left\langle x:T_{\hat{A}(x)},I_{\hat{A}(x)},F_{\hat{A}(x)}\right\rangle \mid x\in\hat{A}\right\}$ and $\hat{B}=\left\{\left\langle y:T_{\hat{B}(y)},I_{\hat{B}(y)},F_{\hat{B}(y)}\right\rangle \mid y\in\hat{B}\right\}$ will be the neutrosophic elements. Thus, $NR=\left\{\left\langle (x,y):T_{(x,y)},I_{(x,y)},F_{(x,y)}\right\rangle \mid x\in\hat{A},y\in\hat{B}\right\}$ is a neutrosophic relation (NR) on $\hat{A}$ and $\hat{B}$.

Definition 3 [23,24] Neutrosophic set of $\hat{A}$ in space X is neutrosophic point (NP) and $\hat{A}=\{\hat{A}_i\}$ where $i=0,...,n$ is a set of NPs where $T_{\hat{A}}:X\to[0,1]$ as truth degree, $I_{\hat{A}}:X\to[0,1]$ as indeterminacy membership and $F_{\hat{A}}:X\to[0,1]$ as false membership with

$$T_{\hat{A}}(\hat{A})=\begin{cases}0 & \text{if } \hat{A}_i\notin\hat{A}\\ a\in(0,1) & \text{if } \hat{A}_i\,\hat{\in}\,\hat{A}\\ 1 & \text{if } \hat{A}_i\in\hat{A}\end{cases}$$
$$I_{\hat{A}}(\hat{A})=\begin{cases}0 & \text{if } \hat{A}_i\notin\hat{A}\\ b\in(0,1) & \text{if } \hat{A}_i\,\hat{\in}\,\hat{A}\\ 1 & \text{if } \hat{A}_i\in\hat{A}\end{cases} \tag{4}$$
$$F_{\hat{A}}(\hat{A})=\begin{cases}0 & \text{if } \hat{A}_i\notin\hat{A}\\ c\in(0,1) & \text{if } \hat{A}_i\,\hat{\in}\,\hat{A}\\ 1 & \text{if } \hat{A}_i\in\hat{A}\end{cases}$$

Definition 4 [29] The $(\alpha,\beta,\gamma)-\text{cut}$ of NS is denoted as $C_{(\alpha,\beta,\gamma)}$ where $\alpha,\beta,\gamma\in[0,1]$ and are fixed numbers, such that $0\le\alpha+\beta+\gamma\le 3$ is defined as $C_{(\alpha,\beta,\gamma)}=\left\{\begin{matrix}\left\langle T_{\hat{A}}(x),I_{\hat{A}}(x),F_{\hat{A}}(x)\right\rangle : x\in X,\\ T_{\hat{A}}(x)\ge\alpha, I_{\hat{A}}(x)\le\beta, F_{\hat{A}}(x)\le\gamma\end{matrix}\right\}$

Definition 5 [29] A NS $\hat{A}$ is mentioned in the whole conversation of real numbers $\mathbb{R}$ and it will be neutrosophic numbers if it follows the following properties:

i. $\hat{A}$ is normal if exist $x_0\in\mathbb{R}$ such that $T_{\hat{A}}(x_0)=1$ while $I_{\hat{A}}(x_0)=F_{\hat{A}}(x_0)=0$.

ii. $\hat{A}$ is convex set for truth function $T_{\hat{A}}(x)$ where $T_A(\mu x_1+(1-\mu)x_2)\ge\min\left(T_{\hat{A}}(x_1),T_{\hat{A}}(x_2)\right)$, $\forall x_1,x_2\in\mathbb{R}$, $\mu\in[0,1]$.

iii. $\hat{A}$ is concave set for indeterminacy $I_{\hat{A}}(x)$ and false function $F_{\hat{A}}(x)$ where $I_{\hat{A}}(\mu x_1+(1-\mu)x_2)\ge\max\left(I_{\hat{A}}(x_1),I_{\hat{A}}(x_2)\right)$, $\forall x_1,x_2\in\mathbb{R}$, $\mu\in[0,1]$ and $F_{\hat{A}}(\mu x_1+(1-\mu)x_2)\ge\max\left(F_{\hat{A}}(x_1),F_{\hat{A}}(x_2)\right)$, $\forall x_1,x_2\in\mathbb{R}$, $\mu\in[0,1]$.

Definition 6 [12] Suppose $w_{\hat{A}},u_{\hat{A}},y_{\hat{A}}\in[0,1]$. A triangular neutrosophic number for $\hat{A}=\left\langle(a_1,b_1,c_1);w_{\hat{A}},u_{\hat{A}},y_{\hat{A}}\right\rangle$ is a special NS on the real number set, since there are three memberships which are truth, indeterminacy, and falsity membership functions, the following functions are defined:

$$T_{\hat{A}}(x)=\begin{cases}\dfrac{(x-a)w_{\hat{A}}}{(b-a)w_{\hat{A}}} & (a\le x\le b)\\ w_{\hat{A}} & (x=b_1)\\ \dfrac{(c-x)w_{\hat{A}}}{(c-b)} & (b\le x\le c)\\ 0 & otherwise\end{cases}$$

$$I_{\hat{A}}(x)=\begin{cases}\dfrac{(b-x+u_{\hat{A}}(x-a))}{(b-a)} & (a\le x\le b)\\ \mathrm{u}_{\hat{A}} & (x=b)\\ \dfrac{(x-b+u_{\hat{A}}(c-x))}{(c-b)} & (b\le x\le c)\\ 1 & otherwise\end{cases} \tag{5}$$

$$F_{\hat{A}}(x)=\begin{cases}\dfrac{(b-x+y_{\hat{A}}(x-a))}{(b-a)} & (a\le x\le b)\\ \mathrm{y}_{\hat{A}} & (x=b)\\ \dfrac{(x-b+y_{\hat{A}}(c-x))}{(c-b)} & (b\le x\le c)\\ 1 & otherwise\end{cases}$$

Definition 6 can be visualized in Figure 1, Figure 1 shows the triangular neutrosophic number for set. $\hat{A}=\langle(a_1,b_1,c_1);w_{\hat{A}},u_{\hat{A}},y_{\hat{A}}\rangle$ where $T_{\hat{A}},I_{\hat{A}},F_{\hat{A}}\in[0,1]$ represented as truth, indeterminacy, and falsity membership.

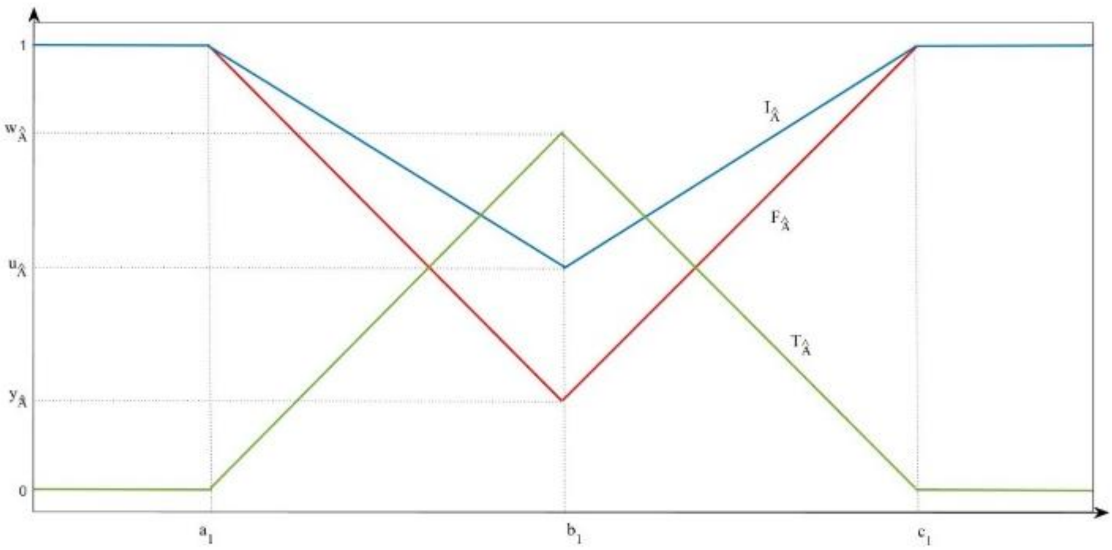

Figure 1 Triangular neutrosophic numbers for set $\hat{A}=\langle(a,b,c);w_{\hat{A}},u_{\hat{A}},y_{\hat{A}}\rangle$

3. Neutrosophication of Neutrosophic Control Point (NCP)

This section discussed the neutrosophic point relation (NPR), as well as the neutrosophication of neutrosophic control point (NCP) using $(\alpha,\beta,\gamma)-\text{cut}$ operations are introduced. NPR is built on the concept of NS, which was addressed in the preceding section. If P,Q is a collection of Euclidean universal space points and $P,Q\in\mathbf{R}^2$ then NPR is defined as follows:

Definition 7 [25] Let X,Y collection of universal space points with a non-empty set and $P,Q,I\in\mathbf{R}\times\mathbf{R}\times\mathbf{R}$, then NPR is defined as

$$\hat{R}=\left\{\left\langle\left(p_i,q_j\right),T_R(p_i,q_j),I_R(p_i,q_j),F_R(p_i,q_j)\right\rangle\Big|T_R(p_i,q_j),I_R(p_i,q_j),F_R(p_i,q_j)\in I\right\} \tag{6}$$

where $\left(p_i,q_j\right)$ is an ordered pair of coordinates and $\left(p_i,q_j\right)\in P\times Q$ while $T_R(p_i,q_j),I_R(p_i,q_j),F_R(p_i,q_j)$ are the truth membership, indeterminacy membership, and false membership that follow the condition of the neutrosophic set which is $0\le T_{\hat{A}}(x)+I_{\hat{A}}(x)+F_{\hat{A}}(x)\le 3$.

A spline's geometry is determined only by the data required to generate the curve [25]. The control point is crucial in the design, control, and manufacture of smooth curves [25]. The NCP is defined in this part by first applying the notion of fuzzy control point from Wahab et al. [14] in the following:

Definition 8 [25] Let $\hat{R}$ be a NPR, then $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ denoted as NCPs for membership truth, indeterminacy where $i=1,...,n+1$ and the position vector of $n+1$ as control polygon vertices.

$$\begin{aligned}\hat{P}_i^T &=\left\{\hat{p}_1^T,\hat{p}_2^T,...,\hat{p}_{n+1}^T\right\}\\ \hat{P}_i^I &=\left\{\hat{p}_1^I,\hat{p}_2^I,...,\hat{p}_{n+1}^I\right\}\\ \hat{P}_i^F &=\left\{\hat{p}_1^F,\hat{p}_2^F,...,\hat{p}_{n+1}^F\right\}\end{aligned} \tag{7}$$

Definition 9 is about a neutrosophication process as the operation of $(\alpha,\beta,\gamma)-\text{cut}$ in triangular forms. The neutrosophication of NCPs is based on the (α,β,γ) values when $\alpha,\beta,\gamma\in(0,1]$. Figure 2 demonstrates the neutrosophication of NCPs.

Definition 9 Let $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ be the set of NCPs where $i=1,...,n+1$. Then, ${}^{\alpha}\hat{P}_i^T$ is $\alpha-\text{cut}$ operation of NCPs for truth membership, ${}^{\beta}\hat{P}_i^I$ is the $\beta-\text{cut}$ operation of NCPs for indeterminacy membership, and ${}^{\gamma}\hat{P}_i^F$ is the $\gamma-\text{cut}$ operation of NCPs for falsity membership, where $\alpha,\beta,\gamma\in(0,1]$ and the neutrosophication of NCPs as follows:

$$\begin{aligned}{}^{\alpha}\hat{P}_i^T &=\left\langle{}^{\alpha}\hat{P}_i^{T(L)},\hat{P}_i^{T}\,{}^{\alpha}\hat{P}_i^{T(R)}\right\rangle=\left\langle\left[\frac{\left(\hat{P}_i^T-\hat{P}_i^{T(L)}\right)\alpha}{w}+\hat{P}_i^{T(L)}\right],\hat{P}_i^T,\left[\frac{-\left(\hat{P}_i^{T(R)}-\hat{P}_i^T\right)\alpha}{w}+\hat{P}_i^{T(R)}\right]\right\rangle\\ {}^{\beta}\hat{P}_i^I &=\left\langle{}^{\beta}\hat{P}_i^{I(L)},\hat{P}_i^{I}\,{}^{\beta}\hat{P}_i^{I(R)}\right\rangle=\left\langle\left[\frac{(1-\beta)\hat{P}_i^I+(\beta-u)\hat{P}_i^{I(L)}}{1-u}\right],\hat{P}_i^I,\left[\frac{(1-\beta)\hat{P}_i^I+(\beta-u)\hat{P}_i^{I(R)}}{1-u}\right]\right\rangle\\ {}^{\gamma}\hat{P}_i^F &=\left\langle{}^{\gamma}\hat{P}_i^{F(L)},\hat{P}_i^{F}\,{}^{\gamma}\hat{P}_i^{F(R)}\right\rangle=\left\langle\left[\frac{(1-\gamma)\hat{P}_i^F+(\gamma-y)\hat{P}_i^{F(L)}}{1-y}\right],\hat{P}_i^F,\left[\frac{(1-\gamma)\hat{P}_i^F+(\gamma-y)\hat{P}_i^{F(R)}}{1-y}\right]\right\rangle\end{aligned} \tag{8}$$

where $\left\langle {}^{\alpha}\hat{P}_i^{T(L)}, \hat{P}_i^{T}\ {}^{\alpha}\hat{P}_i^{T(R)} \right\rangle$ denoted as left NCP, mean, and right NCP for truth membership at any α values, $\left\langle {}^{\beta}\hat{P}_i^{I(L)}, \hat{P}_i^{I}\ {}^{\beta}\hat{P}_i^{I(R)} \right\rangle$ denoted as left NCP, mean, and right NCP for indeterminacy membership at any β values and $\left\langle {}^{\gamma}\hat{P}_i^{F(L)}, \hat{P}_i^{F}\ {}^{\gamma}\hat{P}_i^{F(R)} \right\rangle$ denoted as left NCP, mean, and right NCP for falsity membership at any γ values.

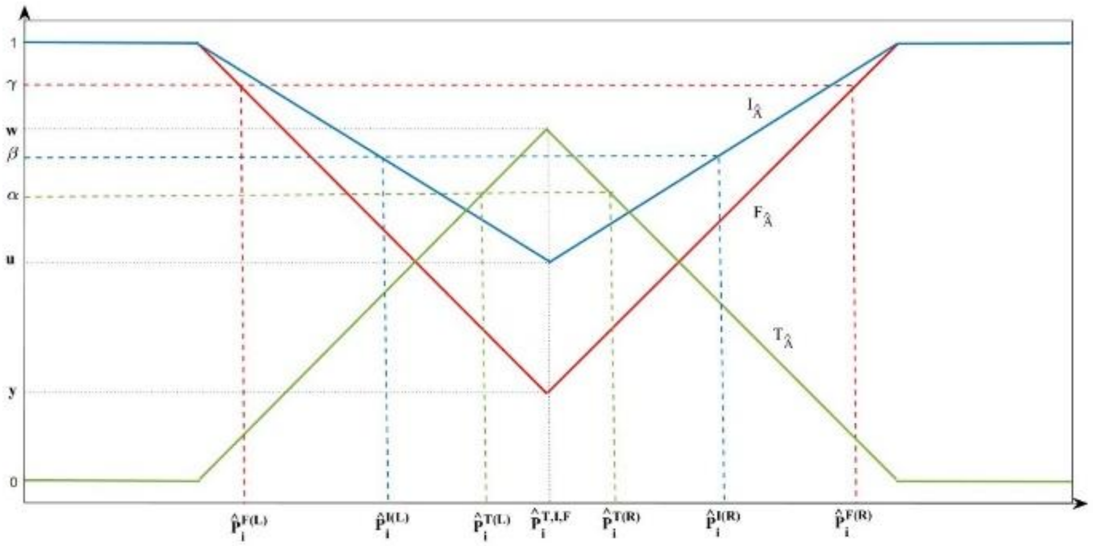

Figure 2. Neutrosophication of neutrosophic control points (NCPs)

4. Neutrosophic B-Spline Curve (NB-sC) Approximation Model

Based on the definition of NCPs in Definition 9, this section discusses the NB-sC approximation model. Before constructing the models, the definitions of crisp B-spline curve approximation are introduced in [30]. The next definition is NB-sC as mentioned in Definition 10 for truth, indeterminacy, and falsity memberships.

Definition 10 Let $\hat{P}_i^T = \left\{\hat{p}_1^T, \hat{p}_2^T, ..., \hat{p}_{n+1}^T\right\}, \hat{P}_i^I = \left\{\hat{p}_1^I, \hat{p}_2^I, ..., \hat{p}_{n+1}^I\right\}$ $\hat{P}_i^F = \left\{\hat{p}_1^F, \hat{p}_2^F, ..., \hat{p}_{n+1}^F\right\}$ where $i = 1, 2, ..., n+1$ is NCPs and NB-sC denoted by BsC with the vector along curve as parameter t. As a result of combining, it with the blending function, NB-sC is described as

$$\begin{aligned} BsC(t)^T &= \sum_{i=1}^{n+1} \hat{P}_i^T N_i^k(t) \\ BsC(t)^F &= \sum_{i=1}^{n+1} \hat{P}_i^F N_i^k(t) \\ BsC(t)^I &= \sum_{i=1}^{n+1} \hat{P}_i^I N_i^k(t) \end{aligned} \tag{10}$$

where $t_{\min} \le t \le t_{\max}$, $2 \le k \le n+1$ and k is the order of the basis function. N_i^k is the B-spline basis function for $i = 1, 2, ..., n+1$. The $N_i^k(t)$ is describe as

$$N_i^1(t) = \begin{cases} 1 & if \quad t_i \le t < t_{i+1} \\ 0 & otherwise \end{cases} \quad \text{and} \quad N_i^k(t) = \frac{(t - t_i)}{t_{i+k-1} - t_i} N_i^{k-1}(t) + \frac{(t_{i+k} - t)}{t_{i+k} - t_{i+1}} N_{i+1}^{k-1}(t).$$

To construct a B-spline model, the knot vector plays a significant influence on the B-spline basis function. $N_i^k(t)$ and there are a few types of knot vectors [30]. This study uses an open uniform knot vector for order $k = 4$, $[0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 2 \quad 2 \quad 2 \quad 2]$.

Table 1 shows the NCPs for truth, false, and indeterminacy memberships. Based on Table 1, Definition 8, and Definition 10, the visualization of the models is presented in Figure 3(a) for truth membership, Figure 3(b) for indeterminacy membership, and Figure 3(c) for falsity memberships by using the approximation method.

Table 1. Neutrosophic control points (NCPs) with their respective memberships

NCP $\hat{P}_i$	Truth $\hat{P}_i^T$	Indeterminacy $\hat{P}_i^I$	False $\hat{P}_i^F$
$\hat{P}_1 = 4$	0.8	0.2	0.2
$\hat{P}_2 = 8$	0.1	0.4	0.9
$\hat{P}_3 = 13$	0.4	0.1	0.4
$\hat{P}_4 = 19$	0.9	0.6	0.1
$\hat{P}_5 = 27$	0.2	0.9	0.8

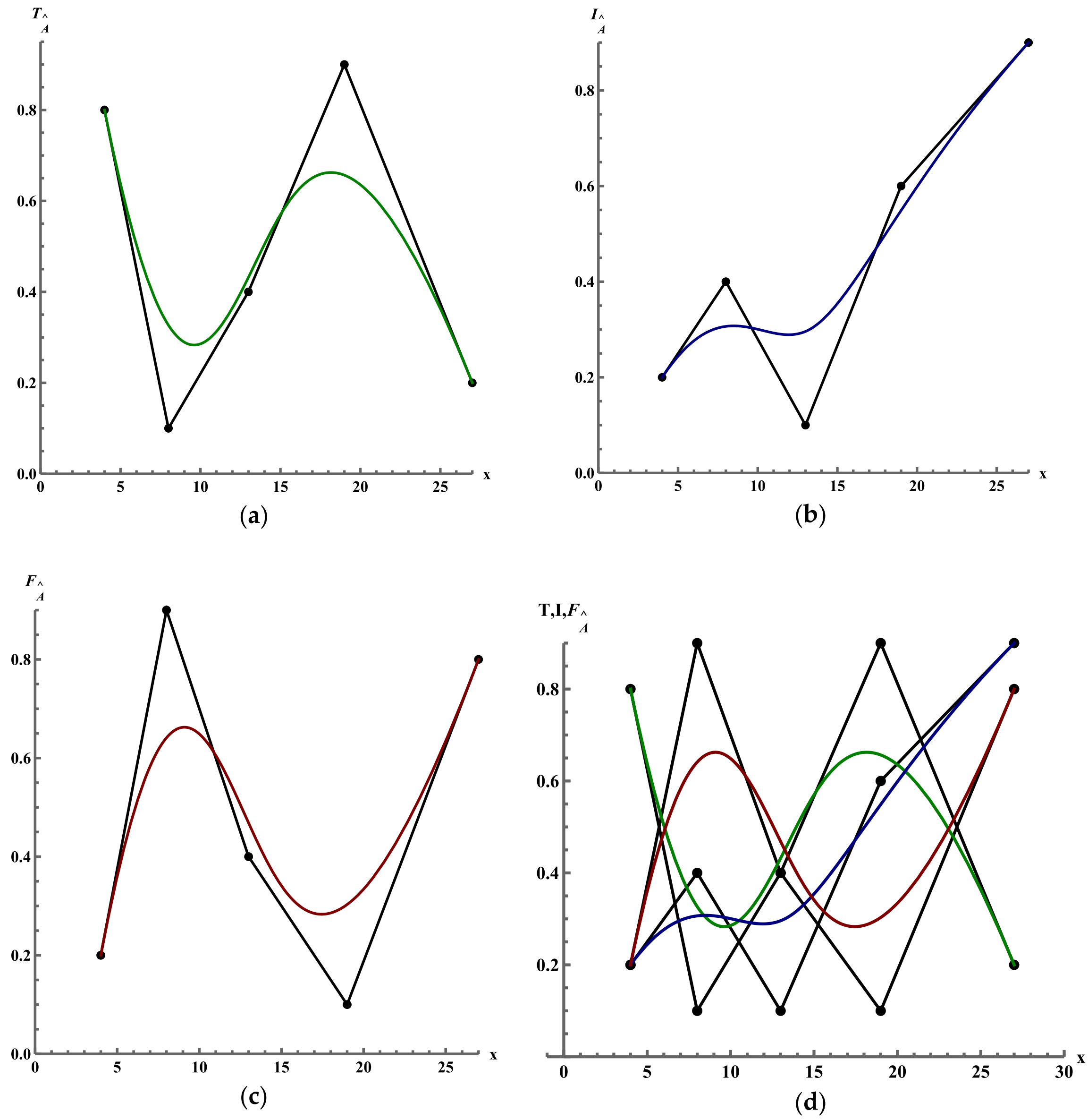

Figure 3. NB-sC approximation model for truth membership with their respective control points and polygon: (**a**) For truth membership; (**b**) For indeterminacy membership; (**c**) For falsity membership; (**d**) For all memberships (Truth, Indeterminacy and Falsity)

Figure 3(d) shows the NB-sC approximation models consist of truth, indeterminacy, and falsity memberships. The line connecting each control point is called a neutrosophic control polygon.

5. Neutrosophication of Neutrosophic B-Spline Curve (NB-sC) Approximation Model

In this section, the neutrosophication process of NB-sC by using the $(\alpha,\beta,\gamma)-\text{cut}$ is discussed. By using the NCPs that undergo $(\alpha,\beta,\gamma)-\text{cut}$ operation in Definition 9, the neutrosophication of NB-sC approximation is as follows:

Definition 11 Let $\left\langle {}^{\alpha}\hat{P}_i^{T(L)}, \hat{P}_i^{T}\ {}^{\alpha}\hat{P}_i^{T(R)} \right\rangle$, $\left\langle {}^{\beta}\hat{P}_i^{I(L)}, \hat{P}_i^{I}\ {}^{\beta}\hat{P}_i^{I(R)} \right\rangle$ and $\left\langle {}^{\gamma}\hat{P}_i^{F(L)}, \hat{P}_i^{F}\ {}^{\gamma}\hat{P}_i^{F(R)} \right\rangle$ be the NCPs of $(\alpha,\beta,\gamma)-\text{cut}$ where $i=1,2,\ldots,n+1$. NB-sC for truth, indeterminacy, and falsity denoted by ${}^{\alpha,\beta,\gamma}BsC^{(T,I,F)}$ while NB-sC for left and right for each membership represent as ${}^{\alpha,\beta,\gamma}BsC^{(T,I,F)(L,R)}$ with the vector along curve as parameter t. As a result of combining, it with the blending function, NB-sC is described as

$$
\begin{aligned}
&{}^{\alpha}BsC(t)^{T(L)} = \sum_{i=1}^{n+1} {}^{\alpha}\hat{P}_i^{T(L)} N_i^k(t) \quad {}^{\alpha}BsC(t)^{T} = \sum_{i=1}^{n+1} {}^{\alpha}\hat{P}_i^{T} N_i^k(t) \quad {}^{\alpha}BsC(t)^{T(R)} = \sum_{i=1}^{n+1} {}^{\alpha}\hat{P}_i^{T(R)} N_i^k(t) \\
&{}^{\beta}BsC(t)^{I(L)} = \sum_{i=1}^{n+1} {}^{\beta}\hat{P}_i^{I(L)} N_i^k(t) \quad {}^{\beta}BsC(t)^{I} = \sum_{i=1}^{n+1} {}^{\beta}\hat{P}_i^{I} N_i^k(t) \quad {}^{\beta}BsC(t)^{I(R)} = \sum_{i=1}^{n+1} {}^{\beta}\hat{P}_i^{I(R)} N_i^k(t) \\
&{}^{\gamma}BsC(t)^{F(L)} = \sum_{i=1}^{n+1} {}^{\gamma}\hat{P}_i^{F(L)} N_i^k(t) \quad {}^{\gamma}BsC(t)^{F} = \sum_{i=1}^{n+1} {}^{\gamma}\hat{P}_i^{F} N_i^k(t) \quad {}^{\gamma}BsC(t)^{F(R)} = \sum_{i=1}^{n+1} {}^{\gamma}\hat{P}_i^{F(R)} N_i^k(t)
\end{aligned}
\tag{11}
$$

where $t_{\min} \le t \le t_{\max}$ and $2 \le k \le n+1$ when k as the order of the basis function.

The neutrosophication process of NB-sCs was constructed by $(\alpha,\beta,\gamma)-\text{cut}$ operations for left and right NCPs which the (α,β,γ) values are $\alpha = 0.8, 0.1, 0.4, 0.9, 0.2$, $\beta = 0.2, 0.4, 0.1, 0.6, 0.9$ and $\gamma = 0.2, 0.9, 0.4, 0.1, 0.8$ where $\alpha_i, \beta_i, \gamma_i \in (0,1]$. The illustration for the selected (α,β,γ) values are presented in Figure 4-6 with their respective memberships. According to Definition 9, Table 2 displays the values of $\left\langle {}^{\alpha}\hat{P}_i^{T(L)}, \hat{P}_i^{T}\ {}^{\alpha}\hat{P}_i^{T(R)} \right\rangle$, $\left\langle {}^{\beta}\hat{P}_i^{I(L)}, \hat{P}_i^{I}\ {}^{\beta}\hat{P}_i^{I(R)} \right\rangle$ and $\left\langle {}^{\gamma}\hat{P}_i^{F(L)}, \hat{P}_i^{F}\ {}^{\gamma}\hat{P}_i^{F(R)} \right\rangle$ after the neutrosophication process. In this study, only neutrosophication for x-data is considered.

Table 2. Left and right neutrosophic control points (NCPs) with their respective memberships

Crisp values, $\hat{P}_i$	Triangular Neutrosophic Number, $\left\langle \hat{P}^{(L)}, \hat{P}\ \hat{P}^{(R)} \right\rangle$	Neutrosophication Process					
		$\alpha-\text{cut}$	$\left\langle {}^{\alpha}\hat{P}_i^{T(L)}, \hat{P}_i^{T}\ {}^{\alpha}\hat{P}_i^{T(R)} \right\rangle$	$\beta-\text{cut}$	$\left\langle {}^{\beta}\hat{P}_i^{I(L)}, \hat{P}_i^{I}\ {}^{\beta}\hat{P}_i^{I(R)} \right\rangle$	$\gamma-\text{cut}$	$\left\langle {}^{\gamma}\hat{P}_i^{F(L)}, \hat{P}_i^{F}\ {}^{\gamma}\hat{P}_i^{F(R)} \right\rangle$
$P_1 = 4$	$\langle 2,4,6 \rangle$	$\alpha_1 = 0.8$	$\langle 3.6,4,4.4 \rangle$	$\beta_1 = 0.2$	$\langle 3.6,4,4.4 \rangle$	$\gamma_1 = 0.2$	$\langle 3.6,4,4.4 \rangle$
$P_2 = 8$	$\langle 6,8,10 \rangle$	$\alpha_2 = 0.1$	$\langle 6.2,8,9.8 \rangle$	$\beta_2 = 0.4$	$\langle 7.2,8,8.8 \rangle$	$\gamma_2 = 0.9$	$\langle 6.2,8,9.8 \rangle$
$P_3 = 13$	$\langle 11,13,15 \rangle$	$\alpha_3 = 0.4$	$\langle 11.8,13,14.2 \rangle$	$\beta_3 = 0.1$	$\langle 12.8,13,13.2 \rangle$	$\gamma_3 = 0.4$	$\langle 11.8,13,14.2 \rangle$
$P_4 = 19$	$\langle 17,19,21 \rangle$	$\alpha_4 = 0.9$	$\langle 18.8,19,19.2 \rangle$	$\beta_4 = 0.6$	$\langle 17.8,19,20.2 \rangle$	$\gamma_4 = 0.1$	$\langle 18.8,19,19.2 \rangle$
$P_5 = 27$	$\langle 25,27,29 \rangle$	$\alpha_5 = 0.2$	$\langle 25.4,27,28.6 \rangle$	$\beta_5 = 0.9$	$\langle 25.2,27,28.8 \rangle$	$\gamma_5 = 0.8$	$\langle 25.4,27,28.6 \rangle$

According to Definition 5, the numerical examples in Table 1 meet the neutrosophic number requirements. Next, using Definition 6, the triangular neutrosophic number, labeled as $\left\langle \hat{P}_i^{(L)}, \hat{P}_i\ \hat{P}_i^{(R)} \right\rangle$, was shown in Table 2. For example, $\hat{P}_1^T = 4$ as a NCP after going through the triangular neutrosophic number will gets $\langle 2,4,6 \rangle$. Then, by using Definition 9 and Equation 8 which is the neutrosophication process by using $(\alpha,\beta,\gamma)-\mathrm{cut}$ operations for left and right NCPs, its will be $\langle 3.6,4,4.4 \rangle$ when $\alpha = 0.8$.

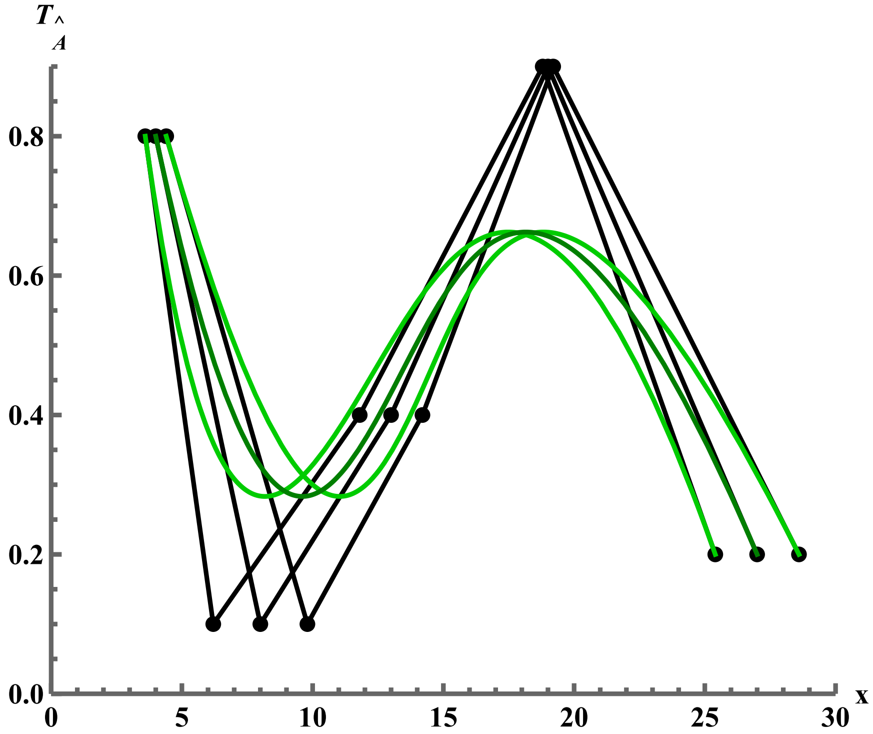

Figure 4. Neutrosophication of NB-sCs approximation for truth membership with their respective alpha values

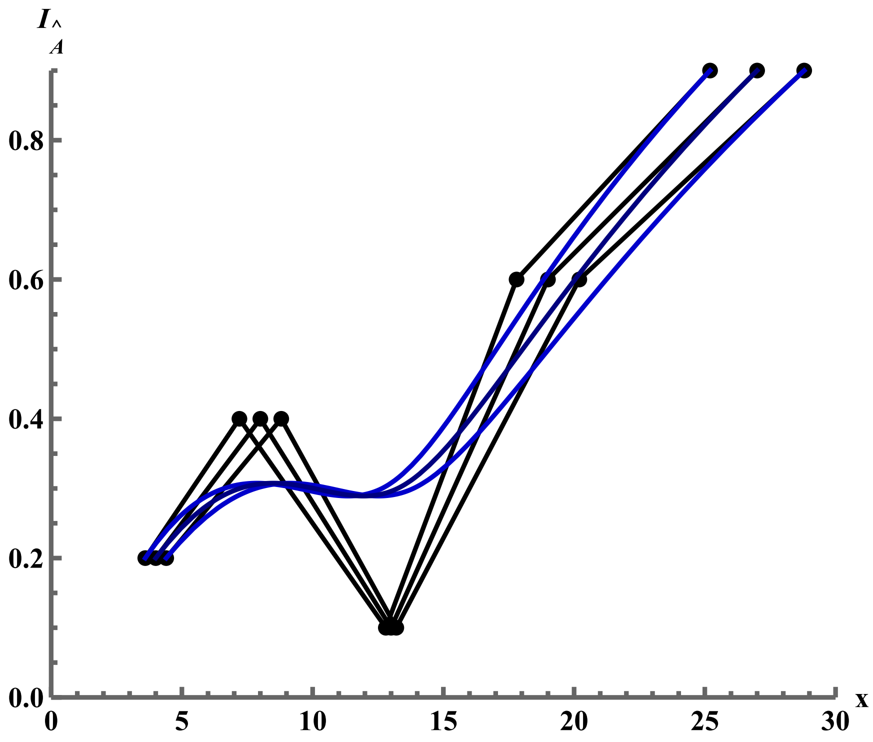

Figure 5. Neutrosophication of NB-sCs approximation for indeterminacy membership with their respective betta values

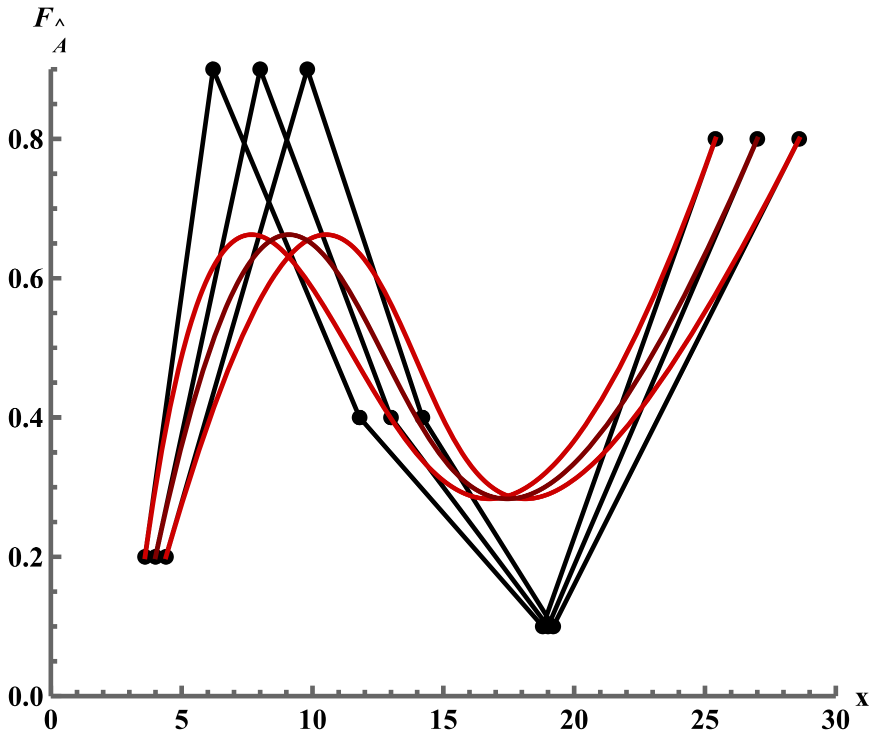

Figure 6. Neutrosophication of NB-sCs approximation for falsity membership with their respective gamma values

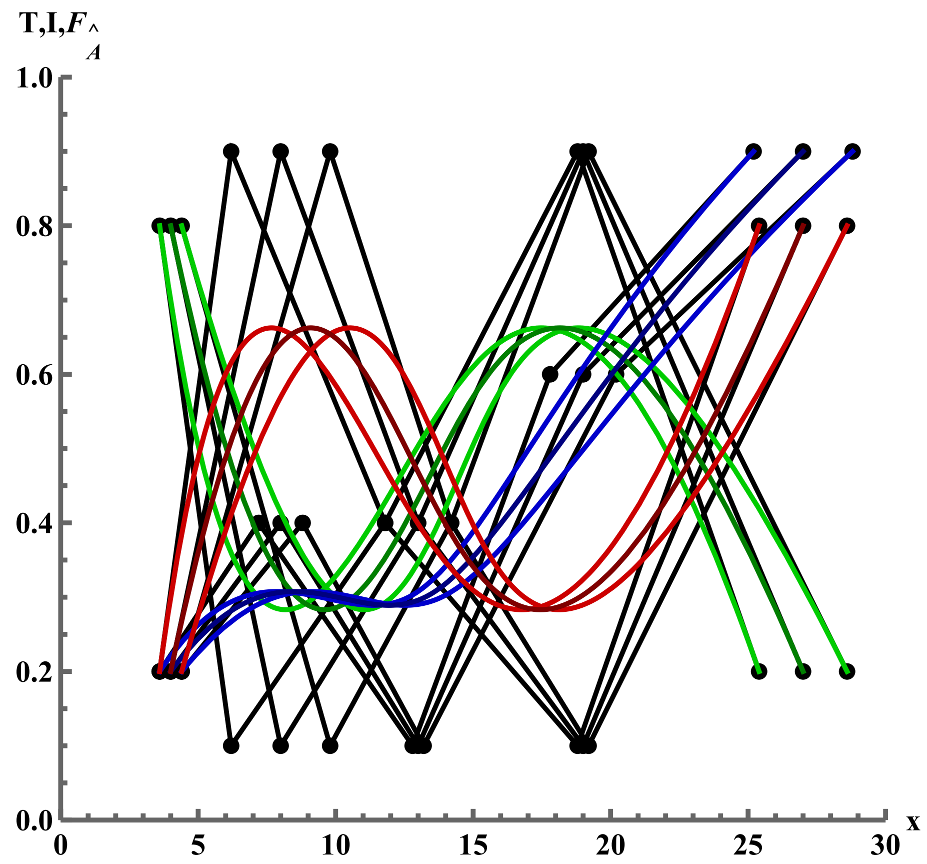

Figure 7. Neutrosophication of NB-sCs approximation with their respective memberships and (α,β,γ) values

Finally, Figure 7 shows the distinct curves between truth, indeterminacy, and falsity memberships for visualization of neutrosophication of NB-sCs approximation for selected. (α,β,γ) values. In the figure, the green curve denoted as NB-sCs for truth membership, the blue curve denoted as NB-sCs for indeterminacy membership and the red curve is denoted as NB-sCs for falsity membership. At the end of this section, an algorithm to generate the neutrosophication of NB-sCs, the algorithm as shown in Figure 8 is demonstrated.

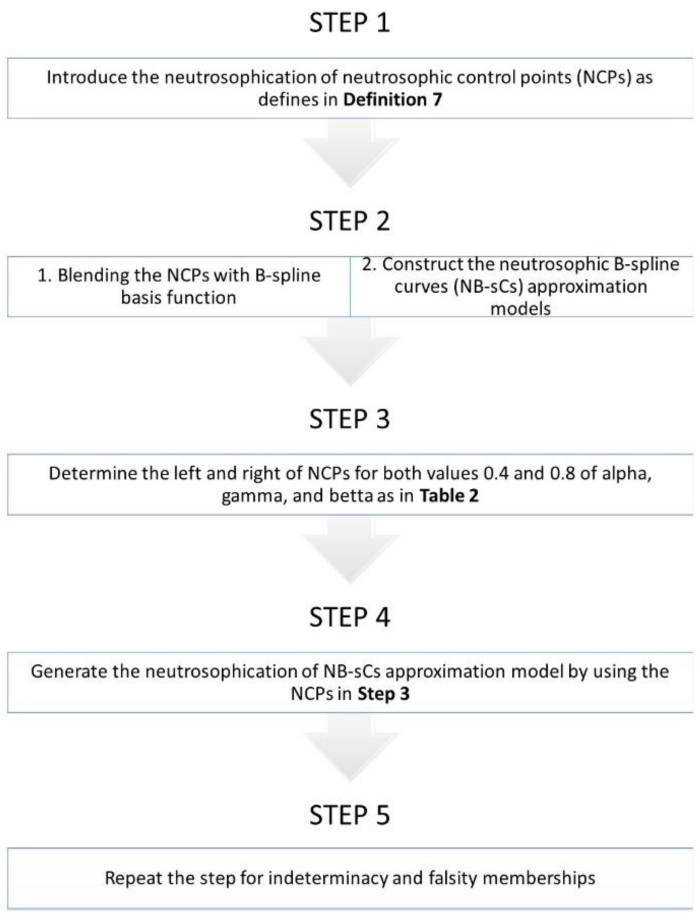

Figure 8. An algorithm to generate the neutrosophication of neutrosophic B-spline curves approximation models.

6. Discussion and Conclusion

This work has introduced the neutrosophication of the neutrosophic B-spline curve (NB-sC) approximation model for truth, indeterminacy, and falsity memberships by using $(\alpha,\beta,\gamma)-$cut operation. The operation of $(\alpha,\beta,\gamma)-$cut can be utilized to describe the degree of truth membership, indeterminacy, and non-membership based on the neutrosophication of neutrosophic control points (NCPs). The advantages of utilizing this method include the ability to define the (α,β,γ) values on the left and right NCPs of the NB-sCs, whether the values are the same or not depending on the nature of the neutrosophic data.

Future researchers can use other curve basis functions, such as the non-uniform rational B-splines (NURBS) geometric model, which could be used to simulate neutrosophic curves in the future. Furthermore, this research may be expanded to represent surfaces.

The neutrosophication of NB-sC can be utilized in 3D curve modeling to build road surface boundaries, closed boundaries of map perimeter modeling, and calculate the area region based on i-level values. This approach can also be used in engineering applications with uncertain collective data to remodel an entity based on its original uncertainty.

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