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A Neutrosophic α-Discounting IndetermHyperSoft Framework for Evaluating Agricultural Product Export Trade Quality under Uncertainty

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Abstract: This research presents a novel mathematical framework for evaluating the quality of agricultural export products under conditions of uncertainty and inconsistency. The proposed model integrates Neutrosophic logic, α-Discounting multi-criteria decision-making, and IndetermHyperSoft set theory to systematically analyze expert evaluations that contain vague, indeterminate, or conflicting information. In many real-world trade situations, quality assessment data is collected from various authorities, laboratories, or stakeholders, resulting in incomplete or contradictory judgments. Classical decision-making approaches fail to address such indeterminacy in a structured and quantitative manner. In this study, we propose an integrated decision matrix that accommodates multi-attribute Neutrosophic evaluations. To strengthen the analysis, three original mathematical indicators are introduced: the α-Neutrosophic Dominance Score, the Neutrosophic Contrast Ratio, and the Neutrosophic Rejection Index. These indicators provide a quantitative basis for ranking, filtering, and validating decision alternatives under uncertain information. The α-Discounting method is used to resolve inconsistencies between preference structures by introducing adaptive parameters that transform unsolvable systems into solvable ones. The model is demonstrated using a comprehensive case study involving the evaluation of mango export quality across multiple criteria and exporting countries. All mathematical formulations, definitions, and calculations are presented in full detail to ensure clarity, reproducibility, and academic rigor.

Keywords: Neutrosophic decision-making, α-discounting method, IndetermHyperSoft set, agricultural export quality, uncertainty modeling, inconsistent preferences, multi-criteria analysis

1. Introduction

Agricultural exports play a major role in the economies of many countries. As global markets grow more connected, the quality of exported goods, like fruits and vegetables, is critical for success in international trade [1]. These products must meet strict standards for appearance, chemical safety, packaging, and regulatory compliance. Assessing quality involves multiple criteria and relies on judgments from inspectors, exporters, and certification bodies. However, these evaluations are often inconsistent. Experts may disagree on product traits, data may be incomplete, and subjective factors, such as visual appeal, are hard to quantify [2]. Traditional decision-making tools, designed for consistent data, struggle with these real-world uncertainties.

This study introduces a new mathematical framework to address these challenges. It combines Neutrosophic logic, α-Discounting, and IndetermHyperSoft sets. Neutrosophic logic, developed by Smarandache, captures truth, falsity, and indeterminacy, making it ideal for handling uncertain or hesitant assessments [3]. The α-Discounting method adjusts inconsistent judgments to create solvable systems [4]. IndetermHyperSoft sets manage multi-attribute data with indeterminate values [5]. Together, these tools form a robust system for dealing with uncertainty and contradictions in quality evaluations. The framework introduces three new indicators: the α-Neutrosophic Dominance Score, the Neutrosophic Contrast Ratio, and the Neutrosophic Rejection Index. These enable objective ranking, better differentiation, and exclusion of unreliable decisions. The model is validated through a case study on mango export quality, with clear calculations for transparency and reproducibility.

2. Literature Review

Evaluating the quality of exported products has been studied using various decision-making models. The Analytic Hierarchy Process (AHP) is a common method that compares alternatives in pairs [6]. However, AHP assumes consistent judgments, which are often unrealistic when experts provide conflicting or incomplete data. Fuzzy set theory addresses vague criteria by allowing partial truth values [7]. Yet, it cannot handle cases where data is both vague and indeterminate.

The Analytical Hierarchy Process (AHP) was extended by Smarandache in 2015 to the α-D MCDM method, adjusting the conflicting preferences to find consistent solutions [11]

Neutrosophic logic, introduced by Smarandache, extends fuzzy sets by including truth, indeterminacy, and falsity, making it suitable for conflicting or uncertain data [3, 8]. Smarandache's work on Neutrosophic sets has been applied to multi-criteria decision-making, offering a flexible approach for complex problems [9]. Soft Set theory and its extension, HyperSoft Sets, manage uncertainty across multiple attributes [10]. Smarandache's IndetermHyperSoft Sets further allow for indeterminate data, better reflecting real-world scenarios [5]. The α-Discounting method, also explored by Smarandache, adjusts inconsistent judgments by applying discount factors, enabling solvable systems [4, 11].

While these approaches are valuable, few studies integrate them into a single framework. Most models either overlook indeterminacy or handle inconsistencies manually. This study combines Neutrosophic logic, α-Discounting, and IndetermHyperSoft Sets, building on Smarandache's contributions [3, 4, 5]. It introduces tools like the α-Neutrosophic Dominance Score for ranking and the Neutrosophic Rejection Index for excluding low-quality options, providing a structured solution for evaluating export quality under uncertainty.

3. Mathematical Framework and Model Construction

This section presents the complete mathematical structure of the proposed model, which combines Neutrosophic logic, α-Discounting, and IndetermHyperSoft Sets to evaluate agricultural export quality. The model is built step-by-step, beginning with key definitions and followed by the construction of the decision matrix, transformation into equations, and final calculation indicators.

Neutrosophic Set Basics

A Neutrosophic set is a mathematical structure that allows the representation of uncertainty, inconsistency, and contradiction. It assigns to each element three independent values: the degree of truth (denoted T), the degree of indeterminacy (denoted I), and the degree of falsity (denoted F), where each value lies in the interval [0,1], and their sum is not necessarily 1.

Formally, for an element x in universe U:

$$x \in A \Rightarrow A(x) = (T(x), I(x), F(x)), \text{ where } T, I, F \in [0,1]$$

This format allows flexibility in modeling real opinions or reports, especially when there is missing or uncertain information.

IndetermHyperSoft Set (IHSS)

Let U be the universe of discourse (for example, exported products), and let there be multiple parameters or attributes $A_1, A_2, \dots, A_n$ (such as packaging, chemical level, color, etc.). Each attribute A_i has its own set of possible values. The IndetermHyperSoft Set is a function defined as:

$$F: A_1 \times A_2 \times \cdots \times A_n \to \mathcal{P}(U)$$

This function maps each combination of attribute values to a subset of U. However, unlike classical sets, this structure allows indeterminacy, so the image of a function may include undefined, conflicting, or incomplete data.

Each evaluation in our decision matrix will now take the form of a Neutrosophic triplet:

$$D_{ij} = \left(T_{ij}, I_{ij}, F_{ij}\right)$$

where D_{ij} is the Neutrosophic evaluation of alternative i under criterion j.

Construction of the $\boldsymbol{\alpha}$-Discounted Neutrosophic System

Suppose we have m alternatives and n evaluation criteria. The decision matrix is structured as:

$$D = \begin{bmatrix} (T_{11}, I_{11}, F_{11}) & \cdots & (T_{1n}, I_{1n}, F_{1n}) \\ \vdots & \ddots & \vdots \\ (T_{m1}, I_{m1}, F_{m1}) & \cdots & (T_{mn}, I_{mn}, F_{mn}) \end{bmatrix}$$

Because expert judgments may be inconsistent, we apply $\boldsymbol{\alpha}$-discounting parameters to adjust the influence of each evaluation. Let $\alpha_j \in (0,1]$ be the discount factor for criterion j. The discounted Neutrosophic value becomes:

$$D_{ij}^{(\alpha)} = \left(\alpha_j T_{ij}, \alpha_j I_{ij}, \alpha_j F_{ij}\right)$$

These parameters help in reducing the weight of evaluations that are considered uncertain or conflicting and are essential to solving contradictory systems.

α-Neutrosophic Dominance Score (α-NDS)

To rank the alternatives, we define a new indicator called the α-Neutrosophic Dominance Score, which measures the net strength of each alternative by combining discounted truth and falsity values:

$$\alpha - \mathrm{NDS}_i = \sum_{j=1}^{n} \alpha_j \cdot \left(T_{ij} - F_{ij}\right)$$

This score increases as the truth value becomes higher, and the falsity becomes lower. The alternative with the highest α-NDS is considered the most preferred.

Neutrosophic Contrast Ratio (NCR)

This index measures how clearly an evaluation stands out from being ambiguous. It compares the difference between truth and falsity, normalized by the level of indeterminacy:

$$\mathrm{NCR}_i = \frac{|T_i - F_i|}{1 + I_i}$$

This helps identify evaluations that are precise and reliable. A high NCR means low confusion and high clarity.

Neutrosophic Rejection Index (NRI)

To remove unreliable alternatives, we define the Neutrosophic Rejection Index. It detects entries where uncertainty and falsity dominate over truth:

$$\mathrm{NRI}_i = \frac{I_i + F_i}{T_i + \varepsilon}, \text{ with } \varepsilon > 0 \text{ small}$$

If the NRI of an alternative exceeds a predefined threshold, it is rejected from the decision process.

Normalization

After calculating the α-discounted scores or the composite indicators, normalization may be used to express the values on a common scale:

$$\hat{x}_i = \frac{x_i}{\sum_{k=1}^{m} x_k}$$

This ensures the final scores form a comparative ranking across all alternatives.

4. Case Study: Evaluating Mango Export Quality

In this case study, we want to evaluate the export quality of mangoes from three countries. Each country is an alternative. We will evaluate them based on three criteria:

1. Packaging Quality
2. Chemical Residue Level
3. Visual Appearance

The evaluation is given by three experts. Their opinions are converted into Neutrosophic values:
T: how much the expert believes the product meets the standard
I: how much the expert is unsure
F: how much the expert believes the product fails the standard

Step 1: Raw Decision Matrix
Each value below is written as a triplet (T, I, F) as shown in Table 1.

Table 1. Decision Matrix

Country	Packaging	Residue	Appearance
A	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.9, 0.05, 0.05)
B	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.2, 0.2)
C	(0.4, 0.3, 0.3)	(0.6, 0.2, 0.2)	(0.5, 0.4, 0.1)

Step 2: α-Discounting Parameters
Suppose the decision-makers choose these **α values** based on trust in the data for each criterion:
Packaging: α_1=1.0
Residue: α_2=0.9
Appearance: α_3=0.8

Step 3: Apply α-Discounting
Now we multiply each component of each triplet by its corresponding α.
Country A:
Packaging: $(1.0)(0.8,0.1,0.1) = (0.8,0.1,0.1)$
Residue: $(0.9)(0.7,0.2,0.1) = (0.63,0.18,0.09)$
Appearance: $(0.8)(0.9,0.05,0.05) = (0.72,0.04,0.04)$
Country B:
Packaging: $(1.0)(0.6,0.3,0.1) = (0.6,0.3,0.1)$
Residue: $(0.9)(0.5,0.4,0.1) = (0.45,0.36,0.09)$
Appearance: $(0.8)(0.6,0.2,0.2) = (0.48,0.16,0.16)$
Country C:
Packaging: $(1.0)(0.4,0.3,0.3) = (0.4,0.3,0.3)$
Residue: $(0.9)(0.6,0.2,0.2) = (0.54,0.18,0.18)$
Appearance: $(0.8)(0.5,0.4,0.1) = (0.4,0.32,0.08)$

Step 4: Calculate α-Neutrosophic Dominance Score (α-NDS)

$$\alpha - \text{NDS} = \sum \left(\alpha_j \cdot (T - F)\right)$$

Country A:
Packaging: $1.0(0.8 - 0.1) = 0.7$
Residue: $0.9(0.7 - 0.1) = 0.54$
Appearance: $0.8(0.9 - 0.05) = 0.68$

Total $\alpha -$ NDS $(A) = 0.7 + 0.54 + 0.68 = 1.92$
Country B:
Packaging: $1.0(0.6 - 0.1) = 0.5$
Residue: $0.9(0.5 - 0.1) = 0.36$
Appearance: $0.8(0.6 - 0.2) = 0.32$
Total α-NDS $(B) = 0.5 + 0.36 + 0.32 = 1.18$
Country C:
Packaging: $1.0(0.4 - 0.3) = 0.1$
Residue: $0.9(0.6 - 0.2) = 0.36$
Appearance: $0.8(0.5 - 0.1) = 0.32$
Total α-NDS (C) $= 0.1 + 0.36 + 0.32 = 0.78$

Step 5: Calculate NCR for each Country

$$\text{NCR} = \frac{|T - F|}{1 + I}$$

We use the average $T, I,$ and F values across all criteria for each country:
Country A:
Average $\text{T} = \frac{0.8+0.63+0.72}{3} = 0.716$
Average $I = \frac{0.1+0.18+0.04}{3} = 0.106$
Average $F = \frac{0.1+0.09+0.04}{3} = 0.076$

$$\text{NCR}_A = \frac{|0.716 - 0.076|}{1 + 0.106} = \frac{0.64}{1.106} \approx 0.578$$

Country B:
Average $\text{T} = (0.6 + 0.45 + 0.48)/3 = 0.51$
Average $\text{I} = (0.3 + 0.36 + 0.16)/3 = 0.273$
Average $F = (0.1 + 0.09 + 0.16)/3 = 0.116$

$$\text{NCR}_B = \frac{|0.51 - 0.116|}{1 + 0.273} = \frac{0.394}{1.273} \approx 0.309$$

Country C:
Average $\text{T} = (0.4 + 0.54 + 0.4)/3 = 0.447$
Average $\text{I} = (0.3 + 0.18 + 0.32)/3 = 0.267$
Average $F = (0.3 + 0.18 + 0.08)/3 = 0.187$

$$\text{NCR}_C = \frac{|0.447 - 0.187|}{1 + 0.267} = \frac{0.26}{1.267} \approx 0.205$$

Step 6: Calculate NRI for each Country (Table 2)

$$\text{NRI} = \frac{I + F}{T + \varepsilon}, \varepsilon = 0.001$$

Country A:

$$\text{NRI}_A = \frac{0.106 + 0.076}{0.716 + 0.001} = \frac{0.182}{0.717} \approx 0.254$$

Country B:

$$\text{NRI}_B = \frac{0.273 + 0.116}{0.51 + 0.001} = \frac{0.389}{0.511} \approx 0.761$$

Country C:

$$\mathrm{NRI}_C = \frac{0.267 + 0.187}{0.447 + 0.001} = \frac{0.454}{0.448} \approx 1.013$$

Table 2. Final Ranking Summary

Country	α-NDS	NCR	NRI	Decision
A	1.92	0.578	0.254	Selected
B	1.18	0.309	0.761	Acceptable
C	0.78	0.205	1.013	Rejected

Country A is the best option. It has the highest dominance score, good clarity (NCR), and low uncertainty (NRI). Country C is rejected because the uncertainty and falsity outweigh the trust.

5. Results and Analysis

The results of the case study show that the proposed Neutrosophic α-Discounting IndetermHyperSoft model is effective in handling uncertain, incomplete, and conflicting information. Country A was selected as the best option because it had the highest dominance score, low rejection index, and a strong contrast between good and bad evaluations. Country C was rejected due to its high level of indeterminacy and falsity compared to its truth score.

To understand the strength of our method, we now compare it with two well-known decision-making methods: the Analytic Hierarchy Process (AHP) and the Fuzzy Multi-Criteria Decision-Making (Fuzzy MCDM).

Comparison with AHP

The AHP method works well when all comparisons are consistent and complete. It builds a preference matrix by comparing each option with every other one. However, if there are contradictions in the expert opinions, the AHP result becomes unreliable. In our case study, Country B and Country C received mixed evaluations with high uncertainty. AHP cannot directly process indeterminacy. It also cannot use values like "I am not sure" or "possibly acceptable" unless they are forced into a numeric value, which may reduce accuracy.

If we tried to apply AHP to this case, we would first have to remove or replace all indeterminate values. This would lead to data loss or artificial results. Our proposed method, on the other hand, keeps all the original information, including truth, falsity, and indeterminacy, and uses it mathematically.

5.1 Comparison with Fuzzy MCDM

Fuzzy MCDM improves on AHP by allowing partial membership values between 0 and 1. This helps model vague data, such as "high quality" or "medium risk." However, fuzzy sets only use one value to represent each evaluation, which is usually interpreted as a degree of truth. Fuzzy models do not separate indeterminacy and falsity from the truth. In real situations, an evaluation might be partly true, partly false, and partly unknown at the same time.

In the case study, Country C had high falsity and indeterminacy. Fuzzy methods would mix these values into a single score, which could give a false impression of quality. Our Neutrosophic model keeps these three parts separate and handles them directly. This gives a clearer and more complete picture of each alternative.

5.2 Benefits of the Proposed Model

The Neutrosophic α-Discounting model has several advantages. It keeps uncertain and conflicting information without needing to simplify it. It allows the decision-maker to reduce the weight of unreliable data using α-discounting. It also gives three useful indicators: the Dominance Score to rank options, the Contrast Ratio to measure clarity, and the Rejection Index to filter out weak choices. These tools work together to give a full and fair analysis.

When information is clear and consistent, traditional methods may work. But when information is uncertain or inconsistent, the proposed model gives better, safer, and more realistic decisions.

6. Discussion

The main purpose of this study was to build a reliable mathematical model for evaluating agricultural export quality in situations where the data is uncertain or inconsistent. The results of the case study confirmed that the proposed model works well in handling complex evaluations involving multiple criteria and vague expert judgments.

One important feature of the model is its ability to keep and use indeterminacy in the analysis. In real trade situations, experts often give opinions that are not fully clear. For example, a product may be said to be "possibly acceptable" or "generally okay but with some issues." These types of answers cannot be used directly in classic models like AHP or Fuzzy MCDM. However, our model accepts such inputs as part of the Neutrosophic structure, using the indeterminacy value to reflect hesitation or incomplete knowledge.

The use of α-discounting gives the decision-maker control over how much to trust each part of the data. If a particular criterion is based on low-confidence data, it can be discounted with a lower α value. This flexible approach improves the fairness of the decision-making process. It also reduces the effect of extreme or inconsistent judgments.

The three mathematical indicators used in this study α-Neutrosophic Dominance Score, Neutrosophic Contrast Ratio, and Neutrosophic Rejection Index—work together to make the decision process more complete. The dominance score ranks all alternatives by comparing their truth and falsity values. The contrast ratio shows how clear or sharp an evaluation is. The rejection index helps to remove options that have too much uncertainty

or too much falsity. These indicators give more than just one final answer; they offer deeper understanding of each alternative's strength and weakness.
Another benefit of the proposed model is that it does not require a perfect decision matrix. Even if the expert inputs are not consistent or are missing some information, the model still works. The α-discounting technique transforms inconsistent systems into solvable ones. This allows decision-makers to continue working with real-world data without forcing it into artificial formats.

In the case study, Country A had the highest dominance score and was clearly the best choice. This result matched what we would expect based on the expert evaluations. Country C had high falsity and indeterminacy and was correctly rejected. Country B had middle values and was ranked between the two. The model successfully handled the differences in the evaluations and gave fair and logical results.
Whole, the model shows great promise for other real-world problems that involve uncertain data, not only in agriculture but also in areas such as healthcare, environmental planning, and supply chain selection. The combination of Neutrosophic logic and α-discounting gives a new, structured way to solve decision problems where uncertainty is high and consistency is not guaranteed.

7. Conclusion

This study introduced a new mathematical framework for evaluating agricultural export quality when information is uncertain, incomplete, or inconsistent. The model combines Neutrosophic logic, α-discounting, and IndetermHyperSoft sets to build a complete decision-making structure that can handle real-world data more effectively than traditional methods. The framework allows experts to express their evaluations in the form of truth, indeterminacy, and falsity values. These values are not simplified or forced into fixed categories, which helps preserve the real meaning of expert judgments. By applying α-discounting, the model adjusts the influence of each evaluation to solve contradictions and reduce the effect of unreliable data.
Three original indicators were developed in this paper: the α-Neutrosophic Dominance Score, the Neutrosophic Contrast Ratio, and the Neutrosophic Rejection Index. These indicators work together to support ranking, clarity measurement, and filtering weak options. The case study on mango exports showed that the proposed model could successfully select the best option, clearly distinguish between alternatives, and reject low-quality choices. Unlike other methods such as AHP or Fuzzy MCDM, this model did not require the data to be fully consistent or clearly defined. It worked directly with uncertain and conflicting evaluations and produced fair, useful results.
This framework is not limited to agriculture. It can also be applied in many other fields where decisions must be made under uncertain or unclear conditions. In future work, this model can be extended by including more criteria, more alternatives, or using software tools for automatic computation.

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