A Neutrosophic Topological Model for Analyzing Higher Education Internationalization Competence through Cross-Cultural Communication and Collaboration

Zesi Fan*
Taiyuan University of Technology,Taiyuan, 030024,China
*Corresponding author, E-mail: Jessevan852@163.com

Abstract-This paper presents a new way to measure internationalization competence using a neutrosophic topological model, focusing on how people communicate and work together across different cultures. We use neutrosophic logic, which considers truth, uncertainty, and falsehood, to represent each person's skills as a mathematical set. These sets help us analyze how cultural abilities combine or overlap during collaboration. By applying N-norm and N-conorm operations, we calculate shared skills and overall team strengths. The model also identifies gaps in competence through a complement operation. We provide clear equations and examples to show how the model works in practice. This approach offers a precise tool for assessing cross-cultural skills and lays the groundwork for future studies on cultural collaboration.

Keywords: Neutrosophic Logic, N-norm, N-conorm, Neutrosophic Topology, Internationalization Competence, Cross-Cultural Collaboration

1. Introduction

In today's globalized society, internationalization competence is a vital asset for individuals engaging in cross-cultural communication and collaboration. This competence involves the ability to navigate diverse cultural contexts, adapt to varying perspectives, and foster effective cooperation in multicultural settings [1]. Educational and professional environments increasingly prioritize these skills to promote global understanding and enhance collaborative outcomes [2]. However, while qualitative analyses of internationalization competence are prevalent, quantitative approaches to assess and model this multifaceted phenomenon remain underdeveloped [3]. The inherent complexity of cross-cultural interactions characterized by uncertainty, conflicting perceptions, and varying degrees of success necessitates a robust mathematical framework to capture these dynamics systematically.

Between 2019-2024 Smarandache introduced sixteen new types of topologies: NonStandard Topology, Largest Extended NonStandard Real Topology, Neutrosophic Triplet Weak/Strong Topologies, Neutrosophic Extended Triplet Weak/Strong Topologies, Neutrosophic Duplet Topology, Neutrosophic Extended Duplet Topology, Neutrosophic MultiSet Topology, NonStandard Neutrosophic Topology, NeutroTopology, AntiTopology, Refined Neutrosophic Topology, Refined Neutrosophic Crisp Topology, SuperHyperTopology, and Neutrosophic SuperHyperTopology [15].

To address this gap, we introduce a neutrosophic topological model designed to quantitatively evaluate internationalization competence, with a focus on cross-cultural communication and collaboration. Neutrosophic logic, pioneered by Smarandache [4], extends classical fuzzy logic by incorporating three components: truth (T), indeterminacy (I), and falsehood (F). This triadic structure is particularly adept at modeling social phenomena where ambiguity and contradiction are common, such as in cross-cultural interactions [5]. By representing individuals' competence profiles as neutrosophic sets, our framework enables a precise analysis of how cultural skills intersect and combine in collaborative contexts.

Our model leverages the mathematical constructs of N-norm and N-conorm, which generalize the T-norm and T-conorm from fuzzy logic to neutrosophic settings [6]. These operators define neutrosophic intersections and unions, allowing us to quantify collaborative potential and skill diversity among individuals. Additionally, we establish a neutrosophic topological space to rigorously study the interactions of competence attributes, such as language adaptability and cultural awareness. This approach provides a structured and scalable framework for assessing internationalization competence, bridging the divide between qualitative insights and mathematical rigor.

This study aims to lay the groundwork for future research on quantitative modeling of cultural competence. The following sections will elaborate on the theoretical foundations, methodology, and practical applications of the proposed model, demonstrating its clarity and utility in real-world settings.

2. Literature Review

Internationalization competence has become a focal point in research on globalization and multicultural collaboration. Scholars have underscored the importance of developing skills to communicate and collaborate effectively across cultural boundaries. Deardorff [1] defines intercultural competence as the ability to engage appropriately and effectively in diverse cultural contexts, emphasizing its significance in educational and professional domains. Spitzberg and Changnon [2] propose a comprehensive framework that integrates cognitive, affective, and behavioral dimensions of intercultural competence, highlighting its multidimensional nature. While these qualitative models provide valuable insights, they often lack the precision needed for systematic evaluation and comparison.

To overcome these limitations, researchers have turned to mathematical frameworks to model complex social phenomena. Fuzzy logic, introduced by Zadeh [7], offers a method to handle uncertainty and vagueness in human interactions. Atanassov [8] advanced this approach with intuitionistic fuzzy sets, which incorporate both membership and non-membership degrees, providing a more refined representation of uncertainty. However, these frameworks fall short in addressing the contradictory and indeterminate aspects of cross-cultural competence, such as conflicting cultural interpretations or incomplete assessments [9].

Neutrosophic logic, developed by Smarandache [4], addresses these challenges by introducing a triadic structure of truth, indeterminacy, and falsehood. This framework is particularly suited to modeling phenomena characterized by ambiguity and contradiction, as in cross-cultural collaboration [5]. Smarandache and Dezert [10] extended neutrosophic sets to applications in information fusion and decision-making under uncertainty, demonstrating their versatility. The introduction of N-norm and N-conorm operators by Smarandache [6] further enhanced neutrosophic logic by generalizing fuzzy T-norm and T-conorm, enabling the mathematical representation of intersections and unions in neutrosophic sets.

Neutrosophic topology, as explored by Smarandache [11], provides a mathematical structure to analyze the properties of neutrosophic sets within a topological space. This concept has been applied to domains such as decision-making and information analysis [12]. Wang et al. [13] investigated interval neutrosophic sets for computational applications, but their work does not specifically address cross-cultural competence. Coker [14] introduced intuitionistic fuzzy topological spaces, which share some similarities with neutrosophic topology but lack the capacity to model indeterminacy explicitly.

The literature highlights a critical gap in quantitative models for internationalization competence that account for uncertainty, contradiction, and dynamic interactions. Our study builds on the foundational contributions of Smarandache [4, 6, 11] and extends neutrosophic logic to develop a topological framework tailored to cross-cultural competence. By integrating N-norm, N-conorm, and neutrosophic topology, we propose a rigorous and adaptable model that advances both theoretical and practical approaches to assessing internationalization competence.

3. Preliminaries and definitions

In this section, we present the mathematical foundations and definitions of the neutrosophic topological framework we develop for analyzing internationalization competence.

3.1 Neutrosophic Set Structure

Let X be a non-empty set representing the universe of discourse (e.g., a set of individuals). A neutrosophic set A on X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$$

where:

$$T_A(x), I_A(x), F_A(x) \in [0,1]$$

and:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

Here:

— $T_A(x)$: degree of membership (truth) of x in A.

— $I_A(x)$: degree of indeterminacy of x in A.

— $F_A(x)$: degree of non-membership (falsehood) of x in A.

3.2 N-norm and N-conorm Operators

Following the generalization of triangular norms in fuzzy logic, we define:
Definition 1 (N-norm):
A mapping:

$$N: [0,1] \times [0,1] \rightarrow [0,1]$$

is called an N -norm if for all $a, b, c \in [0,1]$:

1. Boundary condition:

$$N(a, 1) = a \text{ and } N(1, a) = a.$$

2. Commutativity:

$$N(a, b) = N(b, a).$$

3. Associativity:

$$N(a, N(b, c)) = N(N(a, b), c).$$

4. Monotonicity:

If $a \leq c$ and $b \leq d$, then:

$$N(a, b) \leq N(c, d)$$

Definition 2 (N -conorm):
A mapping:

$$M: [0,1] \times [0,1] \rightarrow [0,1]$$

is called an N -conorm if for all $a, b, c \in [0,1]$:

1. Boundary condition:

$$M(a, 0) = a \text{ and } M(0, a) = a.$$

2. Commutativity:

$$M(a, b) = M(b, a)$$

3. Associativity:

$$M(a, M(b, c)) = M(M(a, b), c)$$

4. Monotonicity:

If $a \leq c$ and $b \leq d$, then:

$$M(a, b) \leq M(c, d)$$

3.3 Neutrosophic Intersection and Union

Using N-norm and N-conorm, we define:
Definition 3 (Neutrosophic Intersection):
For two neutrosophic sets A, B on X :

$$(A \cap_N B)(x) = \left(N(T_A(x), T_B(x)), M(I_A(x), I_B(x)), M(F_A(x), F_B(x))\right)$$

Definition 4 (Neutrosophic Union):

$$(A \cup_N B)(x) = \left(M(T_A(x), T_B(x)), N(I_A(x), I_B(x)), N(F_A(x), F_B(x))\right)$$

These definitions preserve the neutrosophic structure and ensure that intersections/unions of sets remain neutrosophic sets.

3.4 Complement of a Neutrosophic Set

Definition 5 (Neutrosophic Complement):
For a neutrosophic set A :

$$A^c(x) = (F_A(x), I_A(x), T_A(x)).$$

This operation models the reversal of membership and non-membership while preserving indeterminacy.

3.5 Neutrosophic Topological Space for Competence Analysis

We now introduce a mathematical space to rigorously analyze internationalization competence.
Definition 6 (Neutrosophic Topological Space):
A triple:

$$(X, \tau_N, \cap_N, \cup_N)$$

where:

— X is a non-empty set (individuals).

— $\tau_N \subseteq 2^X$ is a family of neutrosophic sets (open sets).

— $\cap_N, \cup_N$ are neutrosophic intersection and union (using N -norm and N -conorm). is called a neutrosophic topological space if:

— $\emptyset, X \in \tau_N$.

— $\forall A, B \in \tau_N, A \cap_N B \in \tau_N$.

1. $\forall \{A_\lambda\} \subseteq \tau_N, \cup_\lambda A_\lambda \in \tau_N$.

This formalizes how we can treat internationalization competence profiles as open sets in a mathematical topology, enabling us to study their interactions rigorously.

This mathematical framework lays the foundation for our proposed model and ensures that all subsequent numerical examples and applications in internationalization competence are grounded in robust mathematical structures.

4. Methedology

We now apply the neutrosophic topological structures developed in Section 3 to quantitatively represent and analyze internationalization competence.

4.1 Model Assumptions

Let:

$$X = \{S_1, S_2, \ldots, S_n\}$$

denote a set of n individuals (e.g., students or employees) whose internationalization competence we aim to evaluate.

We define a set of m core competence attributes:

$$A = \{A_1, A_2, \dots, A_m\}$$

such as:

— A_1 : language adaptability

— A_2 : cultural awareness

— A_3 : collaborative behavior

— and so on.

Each individual S_j has a neutrosophic competence profile for each attribute:

$$N_i(S_j) = (T_{ij}, I_{ij}, F_{ij}) \in [0,1]^3$$

where:

— T_{ij} : degree of demonstrated competence in attribute A_i.

— I_{ij} : uncertainty or lack of clarity in competence assessment.

— F_{ij} : degree of non-demonstration or failure in the attribute.

4.2 Aggregating Competence through N-norm and N-conorm

The neutrosophic intersection and union allow us to mathematically model how different individuals or attributes overlap or unite in collaborative, multicultural contexts.

Competence Intersection (Collaboration Potential)
To analyze the degree of successful collaboration between two individuals S_j and S_k for a specific attribute A_i, we compute:

$$N_i(S_j) \cap_N N_i(S_k) = \left(N(T_{ij}, T_{ik}), M(I_{ij}, I_{ik}), M(F_{ij}, F_{ik})\right)$$

This neutrosophic intersection quantifies how much their respective competence levels can jointly support effective international collaboration in that attribute.

Competence Union (Diversity of Skills)
Conversely, to measure the overall diversity of skills available in a pair or group (e.g., how broad their collective international competence is), we compute:

$$N_i(S_j) \cup_N N_i(S_k) = \left(M(T_{ij}, T_{ik}), N(I_{ij}, I_{ik}), N(F_{ij}, F_{ik})\right)$$

This union captures the maximum potential for international competence, regardless of overlap.

4.3 Modeling Collaborative Interaction as Neutrosophic Topological Spaces

To fully capture the collaborative environment of an international team, we define a neutrosophic topological space:

$$(X, \tau_N, \cap_N, \cup_N)$$

where:

τ_N consists of neutrosophic sets of individuals with shared competence levels.

$\cap_N$ models actual collaborative overlap (joint demonstration of competence).

$\cup_N$ models aggregate skill diversity (total capacity of the team).

This framework allows rigorous mathematical study of:

1) How strongly individuals' competencies overlap.
2) Where uncertainties (indeterminacy) are greatest in cross-cultural teams.
3) How gaps (falsehood components) limit effective internationalization.

4.4 Modeling Gaps and Resistance through Complements

To identify gaps or resistance in international collaboration, we define the neutrosophic complement:

$$N_i(S_j)^c = (F_{ij}, I_{ij}, T_{ij})$$

This mathematically represents:
Areas where an individual lacks international competence (F_{ij}).

How these gaps might reverse effective collaboration potential in practice.

4.5 Summary of Key Operators

For any attribute A_i and individuals S_j, S_k :

Neutrosophic Intersection (Collaboration Potential):

$$\left(N(T_{ij}, T_{ik}), M(I_{ij}, I_{ik}), M(F_{ij}, F_{ik})\right)$$

Neutrosophic Union (Aggregate Capacity):

$$\left(M(T_{ij}, T_{ik}), N(I_{ij}, I_{ik}), N(F_{ij}, F_{ik})\right)$$

Neutrosophic Complement (Resistance to Collaboration):

$$\left(F_{ij}, I_{ij}, T_{ij}\right)$$

These operators form the core of our mathematical model for internationalization competence.

5. Numerical Examples

To illustrate the mathematical model, let us consider a simplified scenario with three individuals:

$$X = \{S_1, S_2, S_3\}$$

and two competence attributes:

$$A_1 = \text{ Language Adaptability }, A_2 = \text{ Cultural Awareness}$$

5.1 Initial Neutrosophic Competence Profiles

For attribute A_1 :

Student	$\boldsymbol{T_{1j}}$	$\boldsymbol{I_{1j}}$	$\boldsymbol{F_{1j}}$
$\boldsymbol{S_1}$	0.7	0.2	0.1
$\boldsymbol{S_2}$	0.5	0.3	0.2
$\boldsymbol{S_3}$	0.6	0.25	0.15

For attribute A_2 :

Student	$\boldsymbol{T_{2j}}$	$\boldsymbol{I_{2j}}$	$\boldsymbol{F_{2j}}$
$\boldsymbol{S_1}$	0.8	0.15	0.05
$\boldsymbol{S_2}$	0.6	0.25	0.15
$\boldsymbol{S_3}$	0.65	0.2	0.15

5.2 Example 1: Collaboration Potential between S_1 and S_2 in A_1

We compute:

$$N\left(T_{1,1}, T_{1,2}\right) = \min(0.7,0.5) = 0.5$$
$$M\left(I_{1,1}, I_{1,2}\right) = \max(0.2,0.3) = 0.3$$
$$M\left(F_{1,1}, F_{1,2}\right) = \max(0.1,0.2) = 0.2$$

So:

$$N_1(S_1) \cap_N N_1(S_2) = (0.5,0.3,0.2)$$

5.3 Example 2: Aggregate Capacity (Skill Diversity) of S_1 and S_2 in A_1

$$M(T_{1,1}, T_{1,2}) = \max(0.7, 0.5) = 0.7$$
$$N(I_{1,1}, I_{1,2}) = \min(0.2, 0.3) = 0.2$$
$$N(F_{1,1}, F_{1,2}) = \min(0.1, 0.2) = 0.1$$

Thus:

$$N_1(S_1) \cup_N N_1(S_2) = (0.7, 0.2, 0.1)$$

5.4 Example 3: Collaboration Potential between S_1 and S_3 in A_2

$$N(T_{2,1}, T_{2,3}) = \min(0.8, 0.65) = 0.65$$
$$M(I_{2,1}, I_{2,3}) = \max(0.15, 0.2) = 0.2$$
$$M(F_{2,1}, F_{2,3}) = \max(0.05, 0.15) = 0.15$$

Thus:

$$N_2(S_1) \cap_N N_2(S_3) = (0.65, 0.2, 0.15)$$

5.5 Example 4: Neutrosophic Complement of S_3 in A_2

$$N_2(S_3)^c = (F_{2,3}, I_{2,3}, T_{2,3}) = (0.15, 0.2, 0.65)$$

5.6 Mathematical Observations

The mathematical analysis reveals distinct patterns in how the neutrosophic model evaluates cross-cultural skills. When assessing collaboration potential through intersection, the model selects the lowest truth value between two individuals, highlighting only their shared abilities in teamwork. In contrast, the union operation captures the highest truth value, showcasing the combined strengths and diversity of skills within a group. Uncertainty, or indeterminacy, is managed by choosing the maximum value for intersections and the minimum for unions, showing how ambiguity behaves in different collaborative scenarios. Additionally, the complement operation helps identify weaknesses by reversing competence values, pinpointing areas where an individual may struggle in cultural interactions.

6. Results Analysis

This section interprets the explicit numerical examples presented in Section 5, highlighting the mathematical behavior of the neutrosophic operators and their implications for analyzing internationalization competence.

6.1 Behavior of Neutrosophic Intersection

The intersection operator:

$$N_i(S_j) \cap_N N_i(S_k) = \left(N(T_{ij}, T_{ik}), M(I_{ij}, I_{ik}), M(F_{ij}, F_{ik})\right)$$

mathematically captures the minimal truth overlap and the maximal uncertainty and falsehood in collaborative performance.

For example, in Section 5.2:

$$(0.5, 0.3, 0.2)$$

This result highlights that collaboration between S_1 and S_2 in language adaptability is limited by the lower of their competence truth degrees. In mathematical terms:

$$N(a, b) = \min(a, b)$$

ensures that only shared capability is counted in the intersection.

6.2 Behavior of Neutrosophic Union

The union operator:

$$N_i(S_j) \cup_N N_i(S_k) = \left(M(T_{ij}, T_{ik}), N(I_{ij}, I_{ik}), N(F_{ij}, F_{ik})\right)$$

mathematically represents the maximal competence potential and the minimal uncertainty and falsehood.
For example, in Section 5.3:

$$(0.7, 0.2, 0.1)$$

Here, the maximum truth degree (0.7) reflects the best available skill from either student for that attribute. The neutrosophic structure ensures that the total potential of the pair is fully captured.

6.3 Complement Operator's Role

The neutrosophic complement:

$$N_i(S_j)^c = (F_{ij}, I_{ij}, T_{ij})$$

formally reverses the membership and non-membership components. For example, for S_3 in cultural awareness (Section 5.5):

$$(0.15, 0.2, 0.65)$$

This operation reveals the residual weakness in collaboration-useful for identifying areas requiring further training or support.

6.4 Indeterminacy as a Measure of Evaluation Uncertainty

The indeterminacy component:

$$I_{ij}^{*} = \max(I_{ij}, I_{ik}) \text{ (intersection)}$$
$$I_{ij}^{*} = \min(I_{ij}, I_{ik}) \text{ (union)}$$

reflects how uncertainty evolves mathematically in collaborative environments:

Intersection: uncertainty expands to the highest level in the pair.

Union: uncertainty shrinks to the smallest level in the pair.

This duality provides mathematical insight into how collaborative competence is affected by ambiguity in cross-cultural contexts.

6.5 Mathematical Consistency

All final neutrosophic evaluations:

$$(T^{*}, I^{*}, F^{*}) \in [0,1]^{3}$$

and:

$$0 \leq T^{*} + I^{*} + F^{*} \leq 3$$

This confirms that the neutrosophic structure is closed under the defined operators, ensuring consistent and reliable mathematical modeling.

The rigorous neutrosophic algebra captures the shared, combined, and residual gaps in cross-cultural skills. By explicitly modeling uncertainty and falsehood, the framework reveals not only performance levels but also areas of doubt and misalignment. These quantitative insights are critical for designing targeted interventions to strengthen international collaboration.

7. Conclusion

Our research developed a simple yet effective neutrosophic topological model to evaluate how well individuals collaborate across cultures. By using neutrosophic logic, we created a system that captures the strengths, uncertainties, and weaknesses in people's cultural skills. The N-norm and N-conorm operations allowed us to measure how skills work together or add up in a team, while the complement operation highlighted areas needing improvement. Our examples showed how this model can be applied in real-life scenarios, making it useful for schools and workplaces. This work provides a clear starting point for better understanding and improving cross-cultural teamwork, and we hope it inspires more research into practical tools for global collaboration.

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