University of New Mexico

COMPLEX NEUTROSOPHIC VAGUE SOFT GRAPHS

SURIYAKUMAR G.[1], V. J. SUDHAKAR[1,*], SYED TAHIR HUSSAINY[1] and MANA DONGANONT[2,*]
[1]Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi, Affiliated to Thiruvalluvar University, Vellore, Tamil Nadu, India;gansuriyakumar@gmail.com; vjsvec1@gmail.com;small1maths@gmail.com
[2]School of Science, University of Phayao, Phayao 56000, Thailand; mana.do@up.ac.th
*Correspondence: mana.do@up.ac.th;vjsvec1@gmail.com;

ABSTRACT. In this paper, we present and expand on the idea of complex neutrosophic vague soft graphs (CNVSGs), a sophisticated framework for simulating vagueness, uncertainty and indeterminacy in complex systems. We characterise a strong complex neutrosophic vague soft graph (SCNVSG) as one in which, under soft set parameters, both the edges and vertices have complex-valued memberships and high levels of neutrosophic vagueness. A revised metric that takes into consideration a cumulative strength and uncertainty of connections within the graph properties, we also present the complement of a complex neutrosophic vague soft graph and the complement of a strong complex neutrosophic vague soft graph. In uncertain networks, a complement operations provide information about inverse behaviors and dual linkage. We further explore the notion of isomorphism and provide criteria for structural equivalence under complex neutrosophic inaccurate mappings. To demonstrate how the proposed ideas can be used, examples are provided. The theoretical results in this paper provide the way for future studies of complex uncertain networks, particularly in areas such as information fusion, cognitive modelling and decision support systems.

Keywords: Soft Graph; Neutrosophic Soft Graph; Complex Neutrosophic Vague Soft Graph.

---

1. Introduction

Zadeh [22] developed the idea of fuzzy set theory to address the challenges associated with handling uncertainties. Numerous scholars have since looked into fuzzy logic and fuzzy set theory in an effort to address a variety of real-world issues involving vague and unsure environments. Smarandache's [15] proposal of neutrosophic sets (NSs) is a potent mathematical tool for handling inconsistent, ambiguous and incomplete data in the real world. Neutrosophic sets are characterized individually by a truth-membership function (T), an indeterminacy-membership function (I), and a falsity-membership function (F), each of which can take values within the standard or non-standard interval from 0 to 1.

Akram [4–7] distinguishes two separate soft computing methodologies for expressing ambiguity and uncertainty: fuzzy sets and soft sets. They employed soft computing models to study graph ambiguity and uncertainty. A few aspects of regular fuzzy soft graphs have been investigated. Then he presented intuitionistic fuzzy soft graphs and their applications, as well as neutrosophic soft graphs. Soft graphs include parameters for each vertex and edge. According to Satham Hussain [14], ambiguous sets and neutrosophic soft sets are an effective technique for describing data with vague information. This approach produced neutrosophic vague soft graphs. This allows decision-makers to adapt their opinions to their own areas of competence.

Satham Hussain [13] introduced the concepts of *neutrosophic vague graphs* and *strong neutrosophic vague graphs*, which served as a foundation for the subsequent in-depth research. This innovative theoretical framework integrates characteristics of both neutrosophic graphs and vague graphs. In this context, the sum of the membership values for truth, indeterminacy, and falsity lies between 0 and 2, as the truth and falsity memberships are considered dependent variables. Neutrosophic vague graphs are viewed as counterparts within this framework. The development of this theory is intended to support applications in operations research and in addressing challenges in social networks. A particularly pressing issue in the latter domain is the proliferation of fake or fraudulent profiles. The neutrosophic vague graph model offers a novel approach to tackling such challenges. Furthermore, several essential properties of *strong neutrosophic vague graphs*, *complete neutrosophic vague graphs*, and *self-complementary neutrosophic vague graphs* were investigated, and the proposed concepts were illustrated with appropriate examples.

Said Broumi [12] proposed the complicated neutrosophic soft set model, which mixes neutrosophic and soft sets. The fundamental operations of set theory, as well as other concepts related to this model structure are given and demonstrated. To demonstrate the use of this paradigm, an example of a decision-making dilemma with subjective and ambiguous information is offered.

In this paper, we provide complex neutrosophic vague soft graphs as well as strong complex neutrosophic vague soft graphs and constant complex neutrosophic vague soft graphs. Furthermore, with the proper inspection, we established some amazing properties of the complement and self-complement of the complicated neutrosophic vague soft graph.

To clearly highlight the value of the complex neutrosophic vague soft graph model, we give a method-specific comparison with existing neutrosophic-based techniques:

- **Standard Neutrosophic Graph:** By assigning three membership degrees—truth (T), indeterminacy (I) and falsehood (F)—to each vertex and edge, with each value falling between $[0, 1]$, standard neutrosophic graphs go beyond classical graphs. Their ability to handle sophisticated vagueness or numerous criteria is limited, yet they are fundamental for modelling simple uncertain relations.

- **Neutrosophic Soft Graphs:** Neutrosophic soft graphs enable parameter-based modelling by fusing the idea of a soft set with neutrosophic graphs. Each vertex or edge relies on a variety of soft qualities, which increases flexibility without specifically solving the vagueness issue.
- **Neutrosophic Vague Graphs:** The characteristics of vague set theory and neutrosophic logic are combined in neutrosophic vague graphs. Each relationship is defined by imprecise degrees (truth-membership and falsity-membership bounded by a degree of hesitation or vagueness) in addition to T, I and F components. This model provides a more accurate representation of many real-world uncertainties and is especially helpful when making decisions, including hesitation or incomplete knowledge.
- **Neutrosophic Bipolar Graphs:** Neutrosophic bipolar graphs, which are essential for conflict analysis, psychology and negotiation systems, model contradictory or bipolar information (such as support vs. resistance) by allowing positive and negative neutrosophic values to coexist inside the same framework.
- **Interval-Valued Neutrosophic Graphs:** By permitting T, I and F to be intervals inside $[0, 1]$, interval-valued neutrosophic graphs enhance uncertainty handling by recording a range of potential membership values. In situations where accurate assessments are challenging, this paradigm is more resilient.
- **Complex Neutrosophic Vague Soft Graph:** A thorough combination of complex neutrosophic numbers, vague sets, and soft sets is represented by the complex neutrosophic vague soft graph. It models ambiguity and many criteria while capturing truth, indeterminacy and falsity with complex values. Because of this, CNVSG is ideally suited for high-dimensional, dynamic decision-making systems, including intelligent transportation systems, large data analytic and medical diagnosis.

2. Preliminaries

This section provides a basic description and example to support the key findings.

Definition 1. *(Adapted from [4]) A **fuzzy soft graph** $G = (G^*, S, K, Q)$ is defined as an ordered quadruple that satisfies the following conditions:*

- $G^* = (V, E)$ *is a simple graph,*
- Q *is a non-empty set of parameters,*
- (S, Q) *is a fuzzy set defined over the vertex set* V*,*
- (K, Q) *is a fuzzy set defined over the edge set* E*,*

- *For each $a \in Q$, the pair $(S(a), K(a))$ forms a fuzzy graph associated with G^*, such that:*

$$K(a)(xy) \leq \min\{S(a)(x), S(a)(y)\}$$

holds for all $x, y \in V$.

For convenience, the fuzzy graph $(S(a), K(a))$ is denoted by $H(a)$. Thus, a fuzzy soft graph can be viewed as a family of fuzzy graphs parameterized by elements of Q. The class of all fuzzy soft graphs over G^* is represented by $\mathfrak{S}(G^*)$.

Definition 2. *[6] An* intuitionistic fuzzy soft graph $G = (G^*, S, K, Q)$ *is defined as an ordered quadruple that satisfies the following conditions:*

- *$G^* = (V, E)$ is a simple graph,*
- *Q is a non-empty set of parameters,*
- *(S, Q) constitutes an intuitionistic fuzzy set defined over the vertex set V,*
- *(K, Q) represents an intuitionistic fuzzy set defined over the edge set E,*
- *For each $a \in Q$, the pair $(S(a), K(a))$ forms an intuitionistic fuzzy graph based on G^*. Specifically,*

$$\eth_{K(a)}(xy) \leq \min\left\{\eth_{S(a)}(x), \eth_{S(a)}(y)\right\}$$
$$\wp_{K(a)}(xy) \geq \max\left\{\wp_{S(a)}(x), \wp_{S(a)}(y)\right\}$$

for all $a \in Q$ and $x, y \in V$. For convenience, the intuitionistic fuzzy graph $(S(a), K(a))$ is denoted by $H(a)$. Thus, an intuitionistic fuzzy soft graph can be viewed as a parameterized collection of intuitionistic fuzzy graphs.

Definition 3. *[7] A neutrosophic soft graph $G = (G^*, S, K, Q)$ is defined as a quadruple satisfying the following conditions:*

- *$G^* = (V, E)$ represents a simple (i.e., undirected and without loops or multiple edges) graph,*
- *Q is a non-empty collection of parameters,*
- *(S, Q) denotes a neutrosophic set defined over the vertex set V,*
- *(K, Q) denotes a neutrosophic set defined over the edge set E,*
- *For every $a \in Q$, the pair $(S(a), K(a))$ forms a neutrosophic subgraph of G^*. Specifically, the following conditions hold:*

$$\eth_{K(a)}(xy) \leq \min\{\eth_{S(a)}(x), \eth_{S(a)}(y)\},$$
$$\Im_{K(a)}(xy) \leq \min\{\Im_{S(a)}(x), \Im_{S(a)}(y)\},$$
$$\wp_{K(a)}(xy) \leq \max\{\wp_{S(a)}(x), \wp_{S(a)}(y)\},$$

for all $a \in Q$ and $x, y \in V$. The neutrosophic graph associated with a parameter a is denoted by $H(a) = (S(a), K(a))$. Thus, a neutrosophic soft graph can be interpreted as a parameterized collection of neutrosophic graphs.

Definition 4. *[13] A neutrosophic vague soft graph $G = (G^*, S, K, Q)$ is defined as an ordered quadruple that satisfies the following criteria:*

- $G^* = (V, E)$ *represents a simple graph,*
- Q *is a non-empty collection of parameters,*
- (S, Q) *denotes a neutrosophic vague set defined over the vertex set* V*,*
- (K, Q) *denotes a neutrosophic vague set defined over the edge set* E*,*
- *For every parameter* $a \in Q$*, the pair* $(S(a), K(a))$ *forms a neutrosophic vague graph corresponding to* G^**. Specifically, the following conditions hold:*

$$\eth^-_{K(a)}(xy) \leq \min\{\eth^-_{S(a)}(x), \eth^-_{S(a)}(y)\}, \quad \eth^+_{K(a)}(xy) \leq \min\{\eth^+_{S(a)}(x), \eth^+_{S(a)}(y)\},$$
$$\Im^-_{K(a)}(xy) \leq \min\{\Im^-_{S(a)}(x), \Im^-_{S(a)}(y)\}, \quad \Im^+_{K(a)}(xy) \leq \min\{\Im^+_{S(a)}(x), \Im^+_{S(a)}(y)\},$$
$$\wp^-_{K(a)}(xy) \leq \max\{\wp^-_{S(a)}(x), \wp^-_{S(a)}(y)\}, \quad \wp^+_{K(a)}(xy) \leq \max\{\wp^+_{S(a)}(x), \wp^+_{S(a)}(y)\},$$

for all $a \in Q$ *and* $x, y \in V$*. The neutrosophic vague graph* $(S(a), K(a))$ *is denoted by* $H(a)$*. Therefore, a neutrosophic vague soft graph can be regarded as a parameterized collection of neutrosophic vague graphs indexed by elements of* Q*.*

Definition 5. [13] *Let the graph be denoted as* $G^* = (R, S)$*. A neutrosophic vague graph on* G^**, or simply a neutrosophic vague graph, is defined as a pair* $G = (A, B)$*, where:*

- $A = (\hat{\eth}_A, \hat{\Im}_A, \hat{\wp}_A)$ *represents a neutrosophic vague set on the vertex set* R*, and*
- $B = (\hat{\eth}_B, \hat{\Im}_B, \hat{\wp}_B)$ *represents a neutrosophic vague set on the edge set* $S \subseteq R \times R$*.*

1. Vertex Set: *Let* $R = \{v_1, v_2, \dots, v_n\}$ *be the set of vertices. The mappings:*

$$\eth^-_A : R \to [0, 1], \quad \Im^-_A : R \to [0, 1], \quad \wp^-_A : R \to [0, 1]$$

correspond to the lower bounds of the truth-membership, indeterminacy-membership, and falsity-membership functions, respectively. These satisfy the constraint:

$$\wp^-_A = 1 - \eth^+_A.$$

Similarly, the upper bounds are defined as:

$$\eth^+_A : R \to [0, 1], \quad \Im^+_A : R \to [0, 1], \quad \wp^+_A : R \to [0, 1]$$

which obey the condition:

$$\wp^+_A = 1 - \eth^-_A.$$

These functions respectively represent the degrees of truth, indeterminacy, and falsity associated with each vertex $v_i \in R$*, satisfying the bounds:*

$$0 \leq \eth^-_A(v_i) + \Im^-_A(v_i) + \wp^-_A(v_i) \leq 2,$$

$$0 \leq \eth^+_A(v_i) + \Im^+_A(v_i) + \wp^+_A(v_i) \leq 2.$$

2. Edge Set: *Let $S \subseteq R \times R$ represent the set of edges. Define the mappings:*

$$\eth_B^- : R \times R \to [0,1], \quad \Im_B^- : R \times R \to [0,1], \quad \wp_B^- : R \times R \to [0,1],$$

$$\eth_B^+ : R \times R \to [0,1], \quad \Im_B^+ : R \times R \to [0,1], \quad \wp_B^+ : R \times R \to [0,1],$$

which specify the lower and upper bounds for the truth-membership, indeterminacy-membership, and falsity-membership values of the edge $(v_i, v_j) \in S$, respectively. These functions satisfy:

$$0 \leq \eth_B^-(v_iv_j) + \Im_B^-(v_iv_j) + \wp_B^-(v_iv_j) \leq 2,$$

$$0 \leq \eth_B^+(v_iv_j) + \Im_B^+(v_iv_j) + \wp_B^+(v_iv_j) \leq 2.$$

Additionally, the edge-related membership values are constrained in relation to the corresponding vertex values as follows:

$$\eth_B^-(v_iv_j) \leq \min\{\eth_A^-(v_i), \eth_A^-(v_j)\}, \quad \Im_B^-(v_iv_j) \leq \min\{\Im_A^-(v_i), \Im_A^-(v_j)\},$$

$$\wp_B^-(v_iv_j) \leq \max\{\wp_A^-(v_i), \wp_A^-(v_j)\},$$

$$\eth_B^+(v_iv_j) \leq \min\{\eth_A^+(v_i), \eth_A^+(v_j)\}, \quad \Im_B^+(v_iv_j) \leq \min\{\Im_A^+(v_i), \Im_A^+(v_j)\},$$

$$\wp_B^+(v_iv_j) \leq \max\{\wp_A^+(v_i), \wp_A^+(v_j)\}.$$

Definition 6. *[10] Assume $U \neq \phi$. A complex fuzzy set A is an entity with the following structure:*

$$A = \{(x, \rho_A(x)) : x \in U\} = \{x, \eth_A(x)e^{iw_A(x)}\}$$

where $i = \sqrt{-1}, \eth_A(x) \in [0,1]$ and $0 \leq w_A(x) \leq 2\pi$.

Definition 7. *[1] Assume $U \neq \phi$. An item of the type complex intuitionistic fuzzy set A*

$$A = \{(x, \eth_A(x), \wp_A(x)) : x \in U\}$$

$$A = \{(x, \eth_A(x)e^{i\alpha_A(x)}, \wp_A(x)e^{i\gamma_A(x)}) : x \in U\}$$

where $i = \sqrt{-1}, \eth_A(x), \wp_A(x) \in [0,1], \alpha_A(x), \gamma_A(x) \in [0, 2\pi]$ and $0^- \leq \eth_A(x) + \wp_A(x) \leq 1^+$.

Definition 8. *[1] Assume $U \neq \phi$. An object with the shape of a complex neutrosophic set A is*

$$A = \{(x, \eth_A(x), \Im_A(x), \wp_F(x)) : x \in U\}$$

$$A = \{(x, \eth_A(x)e^{i\alpha_A(x)}, \Im_A(x)e^{i\beta_A(x)}, \wp_A(x)e^{I\gamma_A(x)}) : x \in U\}$$

where $i = \sqrt{-1}, \eth_A(x), \Im_A(x), \wp_A(x) \in [0,1], \alpha_A(x), \beta_A(x), \gamma_A(x) \in [0, 2\pi]$ and $0^- \leq \eth_A(x) + \Im_A(x) + \wp_A(x) \leq 3^+$.

Definition 9. *[9] When A is a complex neutrosophic set on X and B is a complex neutrosophic relation in X, then $G = (A, B)$ is a complex neutrosophic graph on $X \neq \phi$.*

$$\eth_B(xy)e^{i\alpha_B(xy)} \leq \min\{\eth_A(x), \eth_A(y)\}e^{i\min\{\alpha_A(x), \alpha_A(y)\}}$$

$$\Im_B(xy)e^{i\beta_B(xy)} \leq \min\{\Im_A(x), \Im_A(y)\}e^{i\min\{\beta_A(x),\beta_A(y)\}}$$
$$\wp_B(xy)e^{i\gamma_B(xy)} \leq \max\{\wp_A(x), \wp_A(y)\}e^{i\max\{\gamma_A(x),\gamma_A(y)\}}$$

3. COMPLEX NEUTROSOPHIC VAGUE SOFT GRAPHS

Definition 10. *A **Complex Neutrosophic Vague Soft Graph (CNVSG)** is defined as an ordered quadruple $G = (G^*, S, K, P)$ that satisfies the following conditions:*

- *$G^* = (V, E)$ is a simple graph.*
- *P is a non-empty set of parameters.*
- *(S, P) constitutes a complex neutrosophic vague soft (CNVS) set over the vertex set V.*
- *(K, P) represents a CNVS set over the edge set E.*
- *For each parameter $p \in P$, the pair $(S(p), K(p))$ forms a complex neutrosophic vague graph (CNVG) on G^*.*

1. Let $V = \{l_1, l_2, \ldots, l_n\}$, and consider the functions:

$$\eth^-_{S(p)} : V \to [0,1], \quad \Im^-_{S(p)} : V \to [0,1], \quad \wp^-_{S(p)} : V \to [0,1]$$
$$\eth^+_{S(p)} : V \to [0,1], \quad \Im^+_{S(p)} : V \to [0,1], \quad \wp^+_{S(p)} : V \to [0,1]$$

These functions represent, respectively, the lower and upper bounds of the degrees of truth-membership, indeterminacy-membership, and falsity-membership for each vertex $l_i \in V$ under the parameter $p \in P$.

These functions satisfy:

$$\wp^-_{S(p)} = 1 - \eth^+_{S(p)}, \quad \wp^+_{S(p)} = 1 - \eth^-_{S(p)}$$

Additionally, let $A_{S(p)}, B_{S(p)}, C_{S(p)} \in [0, 2\pi]$ be the respective argument functions associated with each membership function. Then, for each vertex l_i:

$$0 \leq \eth^-_{S(p)}(l_i)e^{iA_{S(p)}(l_i)} + \Im^-_{S(p)}(l_i)e^{iB_{S(p)}(l_i)} + \wp^-_{S(p)}(l_i)e^{iC_{S(p)}(l_i)} \leq 2$$
$$0 \leq \eth^+_{S(p)}(l_i)e^{iA_{S(p)}(l_i)} + \Im^+_{S(p)}(l_i)e^{iB_{S(p)}(l_i)} + \wp^+_{S(p)}(l_i)e^{iC_{S(p)}(l_i)} \leq 2$$

2. For the edge set $E \subseteq V \times V$, the corresponding functions are defined as:

$$\eth^-_{K(p)} : E \to [0,1], \quad \Im^-_{K(p)} : E \to [0,1], \quad \wp^-_{K(p)} : E \to [0,1]$$
$$\eth^+_{K(p)} : E \to [0,1], \quad \Im^+_{K(p)} : E \to [0,1], \quad \wp^+_{K(p)} : E \to [0,1]$$

These functions denote the lower and upper degrees of truth-membership, indeterminacy-membership, and falsity-membership for each edge $l_il_j \in E$ with respect to parameter p. The argument functions $A_{K(p)}, B_{K(p)}, C_{K(p)} \in [0, 2\pi]$ are also associated with the respective membership functions.

For each edge $l_i l_j \in E$, the following conditions must be satisfied:

$$\eth^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} \leq \min\{\eth^{-}_{S(p)}(l_i), \eth^{-}_{S(p)}(l_j)\}e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}}$$
$$\eth^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} \leq \min\{\eth^{+}_{S(p)}(l_i), \eth^{+}_{S(p)}(l_j)\}e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}}$$
$$\Im^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} \leq \min\{\Im^{-}_{S(p)}(l_i), \Im^{-}_{S(p)}(l_j)\}e^{i\min\{B_{S(p)}(l_i),B_{S(p)}(l_j)\}}$$
$$\Im^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} \leq \min\{\Im^{+}_{S(p)}(l_i), \Im^{+}_{S(p)}(l_j)\}e^{i\min\{B_{S(p)}(l_i),B_{S(p)}(l_j)\}}$$
$$\wp^{-}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} \leq \max\{\wp^{-}_{S(p)}(l_i), \wp^{-}_{S(p)}(l_j)\}e^{i\max\{C_{S(p)}(l_i),C_{S(p)}(l_j)\}}$$
$$\wp^{+}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} \leq \max\{\wp^{+}_{S(p)}(l_i), \wp^{+}_{S(p)}(l_j)\}e^{i\max\{C_{S(p)}(l_i),C_{S(p)}(l_j)\}}$$

Furthermore, each edge satisfies the bounded magnitude condition:

$$0 \leq \eth^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} + \Im^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} + \wp^{-}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} \leq 2$$
$$0 \leq \eth^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} + \Im^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} + \wp^{+}_{K(p)}(l_il_j)e^{iC_{S(p)}(l_il_j)} \leq 2$$

The pair $(S(p), K(p))$ is collectively referred to as the complex neutrosophic vague graph $\mathbb{G}$ for simplicity. A CNVSG, therefore, represents a parameterized structure extending the concept of a complex neutrosophic vague graph.

Example 1. *Assume $V = \{l_1, l_2, l_3\}$ and $E = \{l_1l_2, l_2l_3, l_1l_3\}$ is a graph G^*. Let (S, P) be a complex neutrosophic vague soft set (CNVSS) over V and $P = (p_1, p_2)$ be a set of parameters. The complex neutrosophic vague approximation function $S : P \to P(V)$ is defined as*

$$S(p_1) = \begin{cases} \{l_1, (.30e^{i\pi(1.5)}, .70e^{i\pi(1.6)}, .60e^{i\pi(1.4)})^-, (.40e^{i\pi(1.6)}, .80e^{i\pi(1.7)}, .60e^{i\pi(1.7)})^+\} \\ \{l_2, (.20e^{i\pi(1.3)}, .60e^{i\pi(1.7)}, .70e^{i\pi(1.4)})^-, (.30e^{i\pi(1.4)}, .80e^{i\pi(1.6)}, .50e^{i\pi(1.5)})^+\} \\ \{l_3, (.10e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .80e^{i\pi(1.6)})^-, (.30e^{i\pi(1.7)}, .70e^{i\pi(1.6)}, .70e^{i\pi(1.4)})^+\} \end{cases}$$

$$S(p_2) = \begin{cases} \{l_1, (.10e^{i\pi(1.7)}, .50e^{i\pi(1.4)}, .80e^{i\pi(1.6)})^-, (.20e^{i\pi(1.6)}, .70e^{i\pi(1.5)}, .60e^{i\pi(1.5)})^+\} \\ \{l_2, (.20e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .80e^{i\pi(1.7)})^-, (.20e^{i\pi(1.6)}, .60e^{i\pi(1.7)}, .70e^{i\pi(1.8)})^+\} \\ \{l_3, (.20e^{i\pi(1.2)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.4)})^-, (.40e^{i\pi(1.8)}, .80e^{i\pi(1.6)}, .50e^{i\pi(1.5)})^+\} \end{cases}$$

Let (K, P) be a CNVSS over E and let $P = (p_1, p_2)$ be a set of parameters. The complex neutrosophic vague approximation function $K : P \to P(E)$ is defined by

$$K(p_1) = \begin{cases} \{l_1l_2, (.20e^{i\pi(1.2)}, .50e^{i\pi(1.6)}, .70e^{i\pi(1.3)})^-, (.20e^{i\pi(1.3)}, .70e^{i\pi(1.5)}, .60e^{i\pi(1.6)})^+\} \\ \{l_2l_3, (.10e^{i\pi(1.3)}, .50e^{i\pi(1.4)}, .80e^{i\pi(1.5)})^-, (.20e^{i\pi(1.4)}, .60e^{i\pi(1.6)}, .70e^{i\pi(1.3)})^+\} \\ \{l_1l_3, (.10e^{i\pi(1.4)}, .70e^{i\pi(1.4)}, .70e^{i\pi(1.5)})^-, (.30e^{i\pi(1.5)}, .60e^{i\pi(1.5)}, .60e^{i\pi(1.6)})^+\} \end{cases}$$

$$K(p_2) = \begin{cases} \{l_1l_2, (.10e^{i\pi(1.5)}, .50e^{i\pi(1.2)}, .70e^{i\pi(1.7)})^-, (.10e^{i\pi(1.6)}, .60e^{i\pi(1.4)}, .60e^{i\pi(1.8)})^+\} \\ \{l_2l_3, (.10e^{i\pi(1.2)}, .40e^{i\pi(1.3)}, .80e^{i\pi(1.6)})^-, (.20e^{i\pi(1.5)}, .50e^{i\pi(1.4)}, .60e^{i\pi(1.7)})^+\} \\ \{l_1l_3, (.10e^{i\pi(1.2)}, .40e^{i\pi(1.2)}, .60e^{i\pi(1.5)})^-, (.20e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .50e^{i\pi(1.4)})^+\} \end{cases}$$

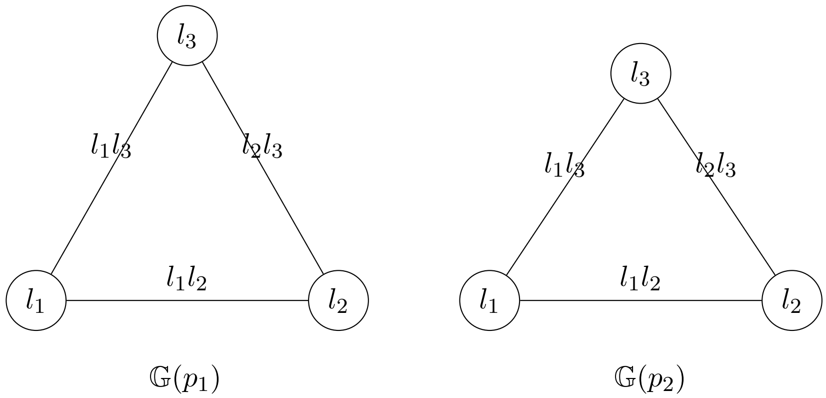


Figure 1: CNVSGs $G = \{\mathbb{G}(p_1), \mathbb{G}(p_2)\}$.

Definition 11. *A CNVSG* $G = (G^*, S, K, P)$ *is considered complete if*

$$\eth^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min\{\eth^-_{S(p)}(l_i), \eth^-_{S(p)}(l_j)\}e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\eth^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min\{\eth^+_{S(p)}(l_i), \eth^+_{S(p)}(l_j)\}e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$

$$\Im^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min\{\Im^-_{S(p)}(l_i), \Im^-_{S(p)}(l_j)\}e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}}$$
$$\Im^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min\{\Im^+_{S(p)}(l_i), \Im^+_{S(p)}(l_j)\}e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}}$$

$$\wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max\{\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)\}e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$
$$\wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max\{\wp^+_{S(p)}(l_i), \wp^+_{S(p)}(l_j)\}e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$

for all $l_i, l_j \in V$ *and* $l_il_j \in E$.

Example 2. *Let* $V = \{l_1, l_2, l_3, l_4\}$ *and* $E = \{l_1l_2, l_2l_3, l_3l_4, l_1l_4, l_2l_4\}$ *be a graph* G^* *by assumption. Let* S *and* K *be full complex neutrosophic vague soft subsets of* V *and* E*, respectively, as shown. Let* (S, P) *be a CNVSS over* V *and let* $P = p$ *be a set of parameters. The complete complex neutrosophic vague approximation function* $S : P \to P(V)$ *is defined by*

$$S(p) = \begin{cases} \{l_1, (.20e^{i\pi(1.7)}, .70e^{i\pi(1.6)}, .40e^{i\pi(1.3)})^-, (.40e^{i\pi(1.2)}, .60e^{i\pi(1.5)}, .40e^{i\pi(1.3)})^+\} \\ \{l_2, (.30e^{i\pi(1.5)}, .50e^{i\pi(1.3)}, .50e^{i\pi(1.5)})^-, (.30e^{i\pi(1.4)}, .40e^{i\pi(1.8)}, .60e^{i\pi(1.7)})^+\} \\ \{l_3, (.10e^{i\pi(1.4)}, .60e^{i\pi(1.4)}, .60e^{i\pi(1.7)})^-, (.30e^{i\pi(1.3)}, .50e^{i\pi(1.4)}, .70e^{i\pi(1.5)})^+\} \\ \{l_4, (.50e^{i\pi(1.3)}, .40e^{i\pi(1.5)}, .30e^{i\pi(1.8)})^-, (.60e^{i\pi(1.6)}, .50e^{i\pi(1.3)}, .50e^{i\pi(1.4)})^+\} \end{cases}$$

Let (K, P) be a complete CNVSS over E and let $P = p$ be a set of parameters. The complete complex neutrosophic vague approximation function $K : P \to P(E)$ is defined by

$$K(p) = \begin{cases} \{l_1l_2, (.20e^{i\pi(1.5)}, .50e^{i\pi(1.3)}, .50e^{i\pi(1.5)})^-, (.30e^{i\pi(1.2)}, .40e^{i\pi(1.5)}, .60e^{i\pi(1.3)})^+\} \\ \{l_2l_3, (.10e^{i\pi(1.4)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.7)})^-, (.30e^{i\pi(1.3)}, .40e^{i\pi(1.4)}, .70e^{i\pi(1.7)})^+\} \\ \{l_3l_4, (.10e^{i\pi(1.3)}, .40e^{i\pi(1.4)}, .60e^{i\pi(1.8)})^-, (.30e^{i\pi(1.3)}, .50e^{i\pi(1.3)}, .70e^{i\pi(1.5)})^+\} \\ \{l_1l_4, (.20e^{i\pi(1.3)}, .40e^{i\pi(1.5)}, .40e^{i\pi(1.8)})^-, (.40e^{i\pi(1.2)}, .50e^{i\pi(1.3)}, .50e^{i\pi(1.4)})^+\} \\ \{l_2l_4, (.30e^{i\pi(1.3)}, .40e^{i\pi(1.3)}, .50e^{i\pi(1.8)})^-, (.30e^{i\pi(1.4)}, .40e^{i\pi(1.3)}, .50e^{i\pi(1.7)})^+\} \end{cases}$$

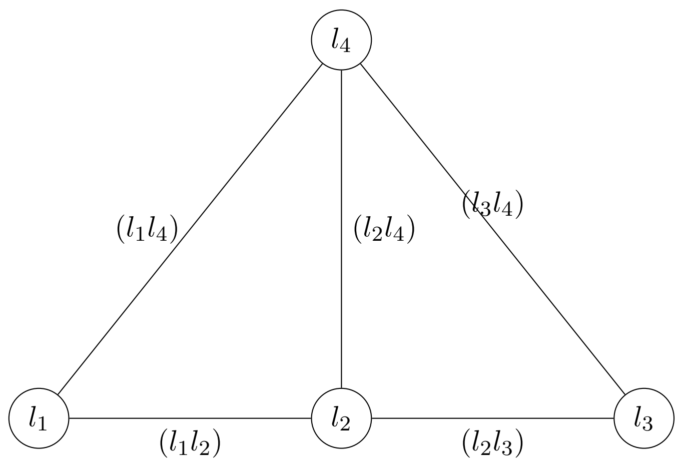


Figure 2: Complete CNVSG $\mathbb{G}$(p).

Definition 12. *Strong complex neutrosophic vague soft graph (SCNVSG) are defined as CNVSG $G = (G^*, S, K, P)$ of G^* = (V, E) if*

$$\eth^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min\{\eth^-_{S(p)}(l_i), \eth^-_{S(p)}(l_j)\}e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\eth^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min\{\eth^+_{S(p)}(l_i), \eth^+_{S(p)}(l_j)\}e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$

$$\Im^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min\{\Im^-_{S(p)}(l_i), \Im^-_{S(p)}(l_j)\}e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}}$$
$$\Im^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min\{\Im^+_{S(p)}(l_i), \Im^+_{S(p)}(l_j)\}e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}}$$

$$\wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max\{\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)\}e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$

$$\wp^{+}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max\{\wp^{+}_{S(p)}(l_i), \wp^{+}_{S(p)}(l_j)\}e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$

Example 3. *Assume $V = \{l_1, l_2, l_3\}$ and $E = \{l_1l_2, l_2l_3, l_1l_3\}$ be a graph G^*. Let (S, P) be a strong complex neutrosophic vague soft set (SCNVSS) over V and let $P = (p_1, p_2)$ be a set of parameters. The strong complex neutrosophic vague approximation function $S : P \to P(V)$ is defined by*

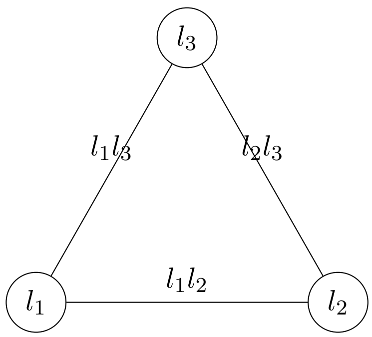
$$S(p_1) = \begin{cases} \{l_1, (.30e^{i\pi(1.5)}, .70e^{i\pi(1.6)}, .60e^{i\pi(1.4)})^-, (.40e^{i\pi(1.6)}, .80e^{i\pi(1.7)}, .60e^{i\pi(1.7)})^+\} \\ \{l_2, (.20e^{i\pi(1.3)}, .60e^{i\pi(1.7)}, .70e^{i\pi(1.4)})^-, (.30e^{i\pi(1.4)}, .80e^{i\pi(1.6)}, .50e^{i\pi(1.5)})^+\} \\ \{l_3, (.10e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .80e^{i\pi(1.6)})^-, (.30e^{i\pi(1.7)}, .70e^{i\pi(1.6)}, .70e^{i\pi(1.4)})^+\} \end{cases}$$

$$S(p_2) = \begin{cases} \{l_1, (.10e^{i\pi(1.7)}, .50e^{i\pi(1.4)}, .80e^{i\pi(1.6)})^-, (.20e^{i\pi(1.6)}, .70e^{i\pi(1.5)}, .60e^{i\pi(1.5)})^+\} \\ \{l_2, (.20e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .80e^{i\pi(1.7)})^-, (.20e^{i\pi(1.6)}, .60e^{i\pi(1.7)}, .70e^{i\pi(1.8)})^+\} \\ \{l_3, (.20e^{i\pi(1.2)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.4)})^-, (.40e^{i\pi(1.8)}, .80e^{i\pi(1.6)}, .50e^{i\pi(1.5)})^+\} \end{cases}$$

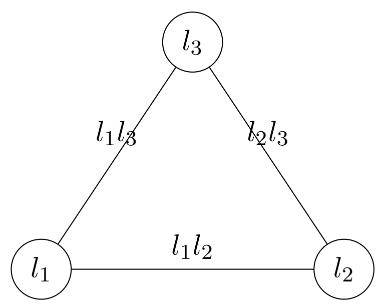
Let (K, P) be an SCNVSS over E and let $P = (p_1, p_2)$ be a set of parameters. The strong complex neutrosophic vague approximation function $K : P \to P(E)$ is defined by

$$K(p_1) = \begin{cases} \{l_1l_2, (.20e^{i\pi(1.3)}, .60e^{i\pi(1.6)}, .70e^{i\pi(1.4)})^-, (.30e^{i\pi(1.4)}, .80e^{i\pi(1.6)}, .60e^{i\pi(1.7)})^+\} \\ \{l_2l_3, (.10e^{i\pi(1.3)}, .60e^{i\pi(1.5)}, .80e^{i\pi(1.6)})^-, (.30e^{i\pi(1.4)}, .70e^{i\pi(1.6)}, .70e^{i\pi(1.5)})^+\} \\ \{l_1l_3, (.10e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .80e^{i\pi(1.6)})^-, (.30e^{i\pi(1.6)}, .70e^{i\pi(1.6)}, .70e^{i\pi(1.7)})^+\} \end{cases}$$

$$K(p_2) = \begin{cases} \{l_1l_2, (.10e^{i\pi(1.6)}, .50e^{i\pi(1.3)}, .80e^{i\pi(1.7)})^-, (.20e^{i\pi(1.6)}, .60e^{i\pi(1.5)}, .70e^{i\pi(1.8)})^+\} \\ \{l_2l_3, (.20e^{i\pi(1.2)}, .50e^{i\pi(1.3)}, .80e^{i\pi(1.7)})^-, (.20e^{i\pi(1.6)}, .60e^{i\pi(1.6)}, .70e^{i\pi(1.8)})^+\} \\ \{l_1l_3, (.10e^{i\pi(1.2)}, .50e^{i\pi(1.3)}, .80e^{i\pi(1.6)})^-, (.20e^{i\pi(1.6)}, .70e^{i\pi(1.5)}, .60e^{i\pi(1.5)})^+\} \end{cases}$$



$\mathbb{G}(p_1)$



$\mathbb{G}(p_2)$

Figure 3: SCNVSG $G = \{\mathbb{G}(p_1), \mathbb{G}(p_2)\}$.

Definition 13. *The complement of an CNVSG $G = (G^*, S, K, P)$ on G^* is an CNVSG G^* where*

1. $\bar{S} = S$

2.
$$\bar{\eth}^-_{S(p)}(l_i)e^{iA_{S(p)}(l_i)} = \eth^-_{S(p)}(l_i)e^{iA_{S(p)}(l_i)}, \bar{\eth}^+_{S(p)}(l_i)e^{iA_{S(p)}(l_i)} = \eth^+_{S(p)}(l_i)e^{iA_{S(p)}(l_i)}$$
$$\bar{\Im}^-_{S(p)}(l_i)e^{iB_{S(p)}(l_i)} = \Im^-_{S(p)}(l_i)e^{iB_{S(p)}(l_i)}, \bar{\Im}^+_{S(p)}(l_i)e^{iB_{S(p)}(l_i)} = \Im^+_{S(p)}(l_i)e^{iB_{S(p)}(l_i)}$$
$$\bar{\wp}^-_{S(p)}(l_i)e^{iC_{S(p)}(l_i)} = \wp^-_{S(p)}(l_i)e^{iC_{S(p)}(l_i)}, \bar{\wp}^+_{S(p)}(l_i)e^{iC_{S(p)}(l_i)} = \wp^+_{S(p)}(l_i)e^{iC_{S(p)}(l_i)}$$

3.
$$\bar{\eth}^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min[\eth^-_{S(p)}(l_i), \eth^-_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}} - \eth^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}$$
$$\bar{\eth}^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min[\eth^+_{S(p)}(l_i), \eth^+_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}} - \eth^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}$$
$$\bar{\Im}^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min[\Im^-_{S(p)}(l_i), \Im^-_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i),BS(l_j)\}} - \Im^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}$$
$$\bar{\Im}^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min[\Im^+_{S(p)}(l_i), \Im^+_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i),B_{S(p)}(l_j)\}} - \Im^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}$$
$$\bar{\wp}^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \max[\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i),C_{S(p)}(l_j)\}} - \wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}$$
$$\bar{\wp}^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max[\wp^+_{S(p)}(l_i), \wp^+_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i),C_{S(p)}(l_j)\}} - \wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}$$

for all $l_il_j \in E$ and $p \in P$

Definition 14. *For a CNVSG on G^*, let $G = (G^*, S, K, P)$. Then, based on the description above, $\bar{\bar{G}}$ is given by the complement of the complement of the complex neutrosophic vague soft graph $\bar{\bar{G}} = (\bar{\bar{V}}, \bar{\bar{E}})$ on G^*, where*
$\bar{\bar{V}} = V$ *and*

$$\bar{\bar{\eth}}^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min[\eth^-_{S(p)}(l_i), \eth^-_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}} - \eth^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}$$
$$\bar{\bar{\eth}}^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min[\eth^+_{S(p)}(l_i), \eth^+_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}} - \eth^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}$$
$$\bar{\bar{\Im}}^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min[\Im^-_{S(p)}(l_i), \Im^-_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i),B_{S(p)}(l_j)\}} - \Im^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}$$
$$\bar{\bar{\Im}}^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min[\Im^+_{S(p)}(l_i), \Im^+_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i),B_{S(p)}(l_j)\}} - \Im^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}$$
$$\bar{\bar{\wp}}^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \max[\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i),C_{S(p)}(l_j)\}} - \wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}$$
$$\bar{\bar{\wp}}^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max[\wp^+_{S(p)}(l_i), \wp^+_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i),C_{S(p)}(l_j)\}} - \wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}$$

and

$$\bar{\bar{\eth}}^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \eth^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}, \bar{\bar{\eth}}^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \eth^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}$$

$$\bar{\bar{\Im}}^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \Im^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}, \bar{\bar{\Im}}^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \Im^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}$$

$$\bar{\bar{\wp}}^{-}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \wp^{-}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}, \bar{\bar{\wp}}^{+}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \wp^{+}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}$$

where the complex neutrosophic vague relation on V is denoted by E. Since $\bar{\bar{G}}$ is an SCNVSG, $G \subseteq \bar{\bar{G}}$ is also an SCNVSG.

Proposition 1. *If G is the complement of the complement of SCNVSG G, then G is an SCNVSG ($G = \bar{\bar{G}}$).*

Proof: *Let SCNVSG G.*

Let (S, P) be a SCNVSS over V and let $P = p$ be a set of parameters. The strong complex neutrosophic vague approximation function $S : P \to P(V)$ is defined by

$$S(p) = \begin{cases} \{l_1, (.30e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .50e^{i\pi(1.5)})^{-}, (.50e^{i\pi(1.2)}, .70e^{i\pi(1.5)}, .40e^{i\pi(1.4)})^{+}\} \\ \{l_2, (.20e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .70e^{i\pi(1.8)})^{-}, (.40e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .60e^{i\pi(1.6)})^{+}\} \\ \{l_3, (.10e^{i\pi(1.4)}, .40e^{i\pi(1.6)}, .80e^{i\pi(1.7)})^{-}, (.20e^{i\pi(1.1)}, .40e^{i\pi(1.7)}, .70e^{i\pi(1.6)})^{+}\} \\ \{l_4, (.30e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .60e^{i\pi(1.9)})^{-}, (.40e^{i\pi(1.3)}, .80e^{i\pi(1.4)}, .50e^{i\pi(1.7)})^{+}\} \end{cases}$$

Let (K, P) be a SCNVSS over E and let $P = p$ be a set of parameters. The strong complex neutrosophic vague approximation function $K : P \to P(E)$ is defined by

$$K(p) = \begin{cases} \{l_1l_4, (.30e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .60e^{i\pi(1.9)})^{-}, (.40e^{i\pi(1.2)}, .70e^{i\pi(1.4)}, .50e^{i\pi(1.7)})^{+}\} \\ \{l_2l_3, (.10e^{i\pi(1.4)}, .40e^{i\pi(1.3)}, .80e^{i\pi(1.8)})^{-}, (.20e^{i\pi(1.1)}, .40e^{i\pi(1.4)}, .70e^{i\pi(1.6)})^{+}\} \\ \{l_2l_4, (.20e^{i\pi(1.4)}, .70e^{i\pi(1.3)}, .60e^{i\pi(1.8)})^{-}, (.40e^{i\pi(1.3)}, .50e^{i\pi(1.4)}, .60e^{i\pi(1.7)})^{+}\} \end{cases}$$

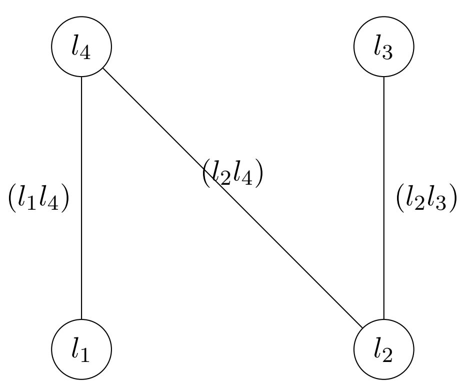


Figure 4: *G* ***- SCNVSG.***

Take the complement for a strong complex neutrosophic vague soft graph.

Let (S, P) *be a complement of SCNVSS over* V *and let* $P = p$ *be a set of parameters. The complement of the strong complex neutrosophic vague approximation function* $S : P \to P(V)$ *is defined by*

$$S(p) = \begin{cases} \{l_1, (.30e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .50e^{i\pi(1.5)})^-, (.50e^{i\pi(1.2)}, .70e^{i\pi(1.5)}, .40e^{i\pi(1.4)})^+\} \\ \{l_2, (.20e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .70e^{i\pi(1.8)})^-, (.40e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .60e^{i\pi(1.6)})^+\} \\ \{l_3, (.10e^{i\pi(1.4)}, .40e^{i\pi(1.6)}, .80e^{i\pi(1.7)})^-, (.20e^{i\pi(1.1)}, .40e^{i\pi(1.7)}, .70e^{i\pi(1.6)})^+\} \\ \{l_4, (.30e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .60e^{i\pi(1.9)})^-, (.40e^{i\pi(1.3)}, .80e^{i\pi(1.4)}, .50e^{i\pi(1.7)})^+\} \end{cases}$$

Let (K, P) *be a complement of SCNVSS over* E *and let* $P = p$ *be a set of parameters. The complement of the strong complex neutrosophic vague approximation function* $K : P \to P(E)$ *is defined by*

$$K(p) = \begin{cases} \{l_1l_2, (.20e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .70e^{i\pi(1.8)})^-, (.40e^{i\pi(1.2)}, .50e^{i\pi(1.4)}, .60e^{i\pi(1.6)})^+\} \\ \{l_1l_3, (.10e^{i\pi(1.3)}, .40e^{i\pi(1.4)}, .80e^{i\pi(1.7)})^-, (.20e^{i\pi(1.1)}, .40e^{i\pi(1.5)}, .70e^{i\pi(1.6)})^+\} \\ \{l_3l_4, (.10e^{i\pi(1.4)}, .40e^{i\pi(1.5)}, .80e^{i\pi(1.9)})^-, (.20e^{i\pi(1.1)}, .40e^{i\pi(1.4)}, .70e^{i\pi(1.7)})^+\} \end{cases}$$

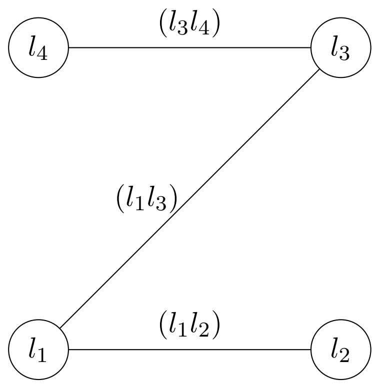


Figure 5: $\bar{G}-$***Complement of SCNVSG.***

Take the complement of the complement of a strong complex neutrosophic vague soft graph.

Let (S, P) *be a complement of the complement of an SCNVSS over* V *and let* $P = p$ *be a set of parameters. The complement of the complement of a strong complex neutrosophic vague approximation function* $S : P \to P(V)$ *is defined by*

$$S(p) = \begin{cases} \{l_1, (.30e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .50e^{i\pi(1.5)})^-, (.50e^{i\pi(1.2)}, .70e^{i\pi(1.5)}, .40e^{i\pi(1.4)})^+\} \\ \{l_2, (.20e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .70e^{i\pi(1.8)})^-, (.40e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .60e^{i\pi(1.6)})^+\} \\ \{l_3, (.10e^{i\pi(1.4)}, .40e^{i\pi(1.6)}, .80e^{i\pi(1.7)})^-, (.20e^{i\pi(1.1)}, .40e^{i\pi(1.7)}, .70e^{i\pi(1.6)})^+\} \\ \{l_4, (.30e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .60e^{i\pi(1.9)})^-, (.40e^{i\pi(1.3)}, .80e^{i\pi(1.4)}, .50e^{i\pi(1.7)})^+\} \end{cases}$$

Let (K, P) be a complement of the complement of an SCNVSS over E and let $P = p$ be a set of parameters. The complement of the complement of a strong complex neutrosophic vague approximation function $K : P \to P(E)$ is defined by

$$K(p) = \begin{cases} \{l_1l_4, (.30e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .60e^{i\pi(1.9)})^-, (.40e^{i\pi(1.2)}, .70e^{i\pi(1.4)}, .50e^{i\pi(1.7)})^+\} \\ \{l_2l_3, (.10e^{i\pi(1.4)}, .40e^{i\pi(1.3)}, .80e^{i\pi(1.8)})^-, (.20e^{i\pi(1.1)}, .40e^{i\pi(1.4)}, .70e^{i\pi(1.6)})^+\} \\ \{l_2l_4, (.20e^{i\pi(1.4)}, .70e^{i\pi(1.3)}, .60e^{i\pi(1.8)})^-, (.40e^{i\pi(1.3)}, .50e^{i\pi(1.4)}, .60e^{i\pi(1.7)})^+\} \end{cases}$$

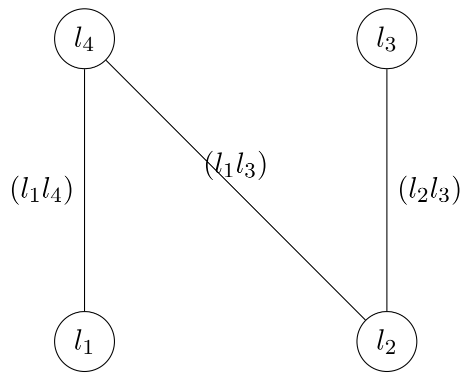


Figure 6: $\bar{\bar{G}}$ ***- Complement of the complement of an SCNVSG.***

Hence, $G = \bar{\bar{G}}$.

Example 4. *Consider a CNVSG G^* =(V, E). Take a look at an SCNVSG G as shown in figure 7, the complement of SCNVSG as shown in figure 8 and the complement of the complement of SCNVSG as shown in 9, which shows it clearly.*

Let strong complex neutrosophic vague soft graph G.

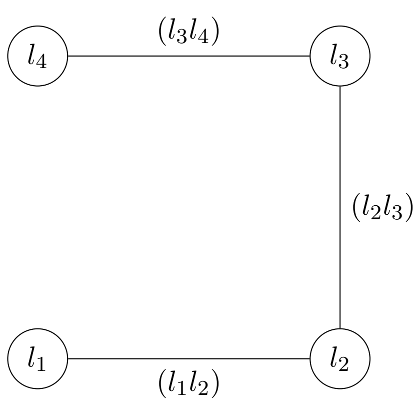


Figure 7: G ***- SCNVSG.***

Vertices:

$$S(p) = \begin{cases} \{l_1, (.60e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .30e^{i\pi(1.5)})^-, (.70e^{i\pi(1.2)}, .70e^{i\pi(1.5)}, .20e^{i\pi(1.4)})^+\} \\ \{l_2, (.70e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .20e^{i\pi(1.8)})^-, (.80e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.6)})^+\} \\ \{l_3, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.6)}, .50e^{i\pi(1.7)})^-, (.60e^{i\pi(1.1)}, .40e^{i\pi(1.7)}, .30e^{i\pi(1.6)})^+\} \\ \{l_4, (.60e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .40e^{i\pi(1.9)})^-, (.80e^{i\pi(1.3)}, .80e^{i\pi(1.4)}, .10e^{i\pi(1.7)})^+\} \end{cases}$$

Edges:

$$K(p) = \begin{cases} \{l_1l_2, (.60e^{i\pi(1.3)}, .60e^{i\pi(1.3)}, .30e^{i\pi(1.8)})^-, (.70e^{i\pi(1.2)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.6)})^+\} \\ \{l_2l_3, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.3)}, .50e^{i\pi(1.8)})^-, (.60e^{i\pi(1.1)}, .40e^{i\pi(1.4)}, .30e^{i\pi(1.6)})^+\} \\ \{l_3l_4, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.5)}, .50e^{i\pi(1.9)})^-, (.60e^{i\pi(1.1)}, .40e^{i\pi(1.4)}, .30e^{i\pi(1.7)})^+\} \end{cases}$$

Take the complement for a strong complex neutrosophic vague soft graph.

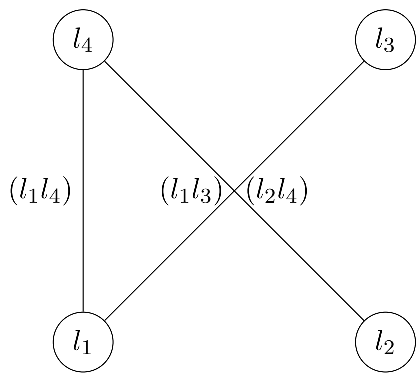


Figure 8: $\bar{G}-$***Complement of SCNVSG.***

Vertices:

$$S(p) = \begin{cases} \{l_1, (.60e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .30e^{i\pi(1.5)})^-, (.70e^{i\pi(1.2)}, .70e^{i\pi(1.5)}, .20e^{i\pi(1.4)})^+\} \\ \{l_2, (.70e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .20e^{i\pi(1.8)})^-, (.80e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.6)})^+\} \\ \{l_3, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.6)}, .50e^{i\pi(1.7)})^-, (.60e^{i\pi(1.1)}, .40e^{i\pi(1.7)}, .30e^{i\pi(1.6)})^+\} \\ \{l_4, (.60e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .40e^{i\pi(1.9)})^-, (.80e^{i\pi(1.3)}, .80e^{i\pi(1.4)}, .10e^{i\pi(1.7)})^+\} \end{cases}$$

Edges:

$$K(p) = \begin{cases} \{l_1l_3, (.50e^{i\pi(1.3)}, .40e^{i\pi(1.4)}, .50e^{i\pi(1.7)})^-, (.60e^{i\pi(1.1)}, .40e^{i\pi(1.5)}, .30e^{i\pi(1.6)})^+\} \\ \{l_2l_4, (.60e^{i\pi(1.4)}, .70e^{i\pi(1.3)}, .40e^{i\pi(1.9)})^-, (.80e^{i\pi(1.3)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.7)})^+\} \\ \{l_1l_4, (.60e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .40e^{i\pi(1.9)})^-, (.70e^{i\pi(1.2)}, .70e^{i\pi(1.4)}, .20e^{i\pi(1.7)})^+\} \end{cases}$$

Take the complement of the complement of a strong complex neutrosophic vague soft graph.

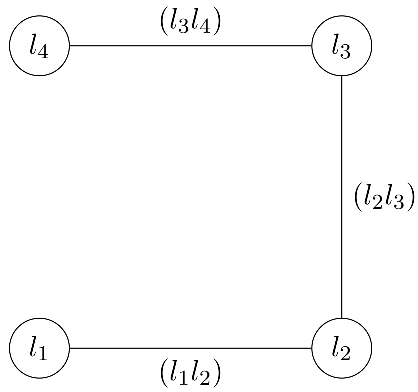


Figure 9: $\bar{\bar{G}}$ ***- Complement of the complement of an SCNVSG.***

Vertices:

$$S(p) = \begin{cases} \{l_1, (.60e^{i\pi(1.3)}, .60e^{i\pi(1.4)}, .30e^{i\pi(1.5)})^-, (.70e^{i\pi(1.2)}, .70e^{i\pi(1.5)}, .20e^{i\pi(1.4)})^+\} \\ \{l_2, (.70e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .20e^{i\pi(1.8)})^-, (.80e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.6)})^+\} \\ \{l_3, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.6)}, .50e^{i\pi(1.7)})^-, (.60e^{i\pi(1.1)}, .40e^{i\pi(1.7)}, .30e^{i\pi(1.6)})^+\} \\ \{l_4, (.60e^{i\pi(1.4)}, .70e^{i\pi(1.5)}, .40e^{i\pi(1.9)})^-, (.80e^{i\pi(1.3)}, .80e^{i\pi(1.4)}, .10e^{i\pi(1.7)})^+\} \end{cases}$$

Edges:

$$K(p) = \begin{cases} \{l_1l_2, (.60e^{i\pi(1.3)}, .60e^{i\pi(1.3)}, .30e^{i\pi(1.8)})^-, (.70e^{i\pi(1.2)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.6)})^+\} \\ \{l_2l_3, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.3)}, .50e^{i\pi(1.8)})^-, (.60e^{i\pi(1.1)}, .40e^{i\pi(1.4)}, .30e^{i\pi(1.6)})^+\} \\ \{l_3l_4, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.5)}, .50e^{i\pi(1.9)})^-, (.60e^{i\pi(1.1)}, .40e^{i\pi(1.4)}, .30e^{i\pi(1.7)})^+\} \end{cases}$$

Hence, G is self-complementary.

Definition 15. *A SCNVSG G is said to be* self-complementary *if it is isomorphic to its complement, denoted by* $\bar{G}$. *In this context,* $\bar{G}$ *represents the complement of the CNVSG G.*

Proposition 2. *Let* $G = (G^*, S, K, P)$ *be a strong complex neutrosophic vague soft graph if*

$$\eth^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min[\eth^-_{S(p)}(l_i), \eth^-_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\eth^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min[\eth^+_{S(p)}(l_i), \eth^+_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\Im^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min[\Im^-_{S(p)}(l_i), \Im^-_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), BS(l_j)\}}$$
$$\Im^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min[\Im^+_{S(p)}(l_i), \Im^+_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}}$$
$$\wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max[\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$
$$\wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max[\wp^+_{S(p)}(l_i), \wp^+_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$

for all $l_il_j \in V$ and $p \in P$.
Then G is self-complementary.

Proof: *Let $G = (G^*, S, K, P)$ be a SCNVSG such that*

$$\eth^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min[\eth^-_{S(p)}(l_i), \eth^-_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\eth^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \min[\eth^+_{S(p)}(l_i), \eth^+_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\Im^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min[\Im^-_{S(p)}(l_i), \Im^-_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), BS(l_j)\}}$$
$$\Im^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \min[\Im^+_{S(p)}(l_i), \Im^+_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}}$$
$$\wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max[\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$
$$\wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \max[\wp^+_{S(p)}(l_i), \wp^+_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$

for all $l_il_j \in V$ and $p \in P$. Then $G \approx \bar{G}$ under the identity map $I : V \to V$.
Hence, G is self-complementary.

Proposition 3. *Consider the SCNVSG $G = (G^*, S, K, P)$. Then*

$$\sum_{l_i \neq l_j} \eth^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \min[\eth^-_{S(p)}(l_i), \eth^-_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \eth^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \min[\eth^+_{S(p)}(l_i), \eth^+_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \Im^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \min[\Im^-_{S(p)}(l_i), \Im^-_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), BS(l_j)\}}$$
$$\sum_{l_i \neq l_j} \Im^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \min[\Im^+_{S(p)}(l_i), \Im^+_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \max[\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \max[\wp^+_{S(p)}(l_i), \wp^+_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$

Proof: *If G is a complex neutrosophic vague soft graph self-complementary. Then, an isomorphism $f : V_1 \to V_2$ that satisfies*

$$\bar{\eth}^-_{S(p)}(f(l_i))e^{iA_{S(p)}(f(l_i))} = \eth^-_{S(p)}(f(l_i))e^{iA_{S(p)}(f(l_i))} = \eth^-_{S(p)}(l_i)e^{iA_{S(p)}(l_i)}$$
$$\bar{\eth}^+_{S(p)}(f(l_i))e^{iA_{S(p)}(f(l_i))} = \eth^+_{S(p)}(f(l_i))e^{iA_{S(p)}(f(l_i))} = \eth^+_{S(p)}(l_i)e^{iA_{S(p)}(l_i)}$$
$$\bar{\Im}^-_{S(p)}(f(l_i))e^{iB_{S(p)}(f(l_i))} = \Im^-_{S(p)}(f(l_i))e^{iB_{S(p)}(f(l_i))} = \Im^-_{S(p)}(l_i)e^{iB_{S(p)}(l_i)}$$
$$\bar{\Im}^+_{S(p)}(f(l_i))e^{iB_{S(p)}(f(l_i))} = \Im^+_{S(p)}(f(l_i))e^{iB_{S(p)}(f(l_i))} = \Im^+_{S(p)}(l_i)e^{iB_{S(p)}(l_i)}$$

$$\bar{\wp}^{-}_{S(p)}(f(l_i))e^{iC_{S(p)}(f(l_i))} = \wp^{-}_{S(p)}(f(l_i))e^{iC_{S(p)}(f(l_i))} = \wp^{-}_{S(p)}(l_i)e^{iC_{S(p)}(l_i)}$$
$$\bar{\wp}^{+}_{S(p)}(f(l_i))e^{iC_{S(p)}(f(l_i))} = \wp^{+}_{S(p)}(f(l_i))e^{iC_{S(p)}(f(l_i))} = \wp^{+}_{S(p)}(l_i)e^{iC_{S(p)}(l_i)}$$

and

$$\bar{\eth}^{-}_{K(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))} = \eth^{-}_{K(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))} = \eth^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}$$
$$\bar{\eth}^{+}_{K(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))} = \eth^{+}_{K(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))} = \eth^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}$$
$$\bar{\Im}^{-}_{K(p)}(f(l_i)f(l_j))e^{iB_{K(p)}(f(l_i)f(l_j))} = \Im^{-}_{K(p)}(f(l_i)f(l_j))e^{iB_{K(p)}(f(l_i)f(l_j))} = \Im^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}$$
$$\bar{\Im}^{+}_{K(p)}(f(l_i)f(l_j))e^{iB_{K(p)}(f(l_i)f(l_j))} = \Im^{+}_{K(p)}(f(l_i)f(l_j))e^{iB_{K(p)}(f(l_i)f(l_j))} = \Im^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}$$
$$\bar{\wp}^{-}_{K(p)}(f(l_i)f(l_j))e^{iC_{K(p)}(f(l_i)f(l_j))} = \wp^{-}_{K(p)}(f(l_i)f(l_j))e^{iC_{K(p)}(f(l_i)f(l_j))} = \wp^{-}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}$$
$$\bar{\wp}^{+}_{K(p)}(f(l_i)f(l_j))e^{iC_{K(p)}(f(l_i)f(l_j))} = \wp^{+}_{K(p)}(f(l_i)f(l_j))e^{iC_{K(p)}(f(l_i)f(l_j))} = \wp^{+}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}$$

for all $(l_il_j) \in E_1$

we have

$$\bar{\eth}^{-}_{K(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))} = \min[\bar{\eth}^{-}_{S(p)}(f(l_i)), \bar{\eth}^{-}_{S(p)}(f(l_j))]e^{i\min\{A_{S(p)}(f(l_i)),A_{S(p)}(f(l_j))\}} - \eth^{-}_{K(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))}$$

i.e

$$\eth^{-}_{K(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))} = \min[\eth^{-}_{S(p)}(f(l_i)), \eth^{-}_{S(p)}(f(l_j))]e^{i\min\{A_{S(p)}(f(l_i)),A_{S(p)}(f(l_j))\}} - \eth^{-}_{K(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))}$$

$$\eth^{-}_{K(p)}(l_il_j))e^{iA_{K(p)}(l_il_j)} = \min[\eth^{-}_{S(p)}(l_i), \eth^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}} - \eth^{-}_{K(p)}(l_il_j))e^{iA_{K(p)}(l_il_j)}$$

that is

$$\sum_{l_i\neq l_j} \eth^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} + \sum_{l_i\neq l_j} \eth^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \sum_{l_i\neq l_j} \min[\eth^{-}_{S(p)}(l_i), \eth^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}}$$

similarly,

$$\sum_{l_i\neq l_j} \eth^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} + \sum_{l_i\neq l_j} \eth^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = \sum_{l_i\neq l_j} \min[\eth^{+}_{S(p)}(l_i), \eth^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i),A_{S(p)}(l_j)\}}$$

$$\sum_{l_i\neq l_j} \Im^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} + \sum_{l_i\neq l_j} \Im^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \sum_{l_i\neq l_j} \min[\Im^{-}_{S(p)}(l_i), \Im^{-}_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i),B_{S(p)}(l_j)\}}$$

$$\sum_{l_i\neq l_j} \Im^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} + \sum_{l_i\neq l_j} \Im^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = \sum_{l_i\neq l_j} \min[\Im^{+}_{S(p)}(l_i), \Im^{+}_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i),B_{S(p)}(l_j)\}}$$

$$\sum_{l_i \neq l_j} \wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} + \sum_{l_i \neq l_j} \wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \sum_{l_i \neq l_j} \max[\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} + \sum_{l_i \neq l_j} \wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = \sum_{l_i \neq l_j} \max[\wp^+_{S(p)}(l_i), \wp^+_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$

Then

$$\sum_{l_i \neq l_j} \eth^-_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \min[\eth^-_{S(p)}(l_i), \eth^-_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \eth^+_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \min[\eth^+_{S(p)}(l_i), \eth^+_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \Im^-_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \min[\Im^-_{S(p)}(l_i), \Im^-_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), BS(l_j)\}}$$
$$\sum_{l_i \neq l_j} \Im^+_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \min[\Im^+_{S(p)}(l_i), \Im^+_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \wp^-_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \max[\wp^-_{S(p)}(l_i), \wp^-_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$
$$\sum_{l_i \neq l_j} \wp^+_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} = 1/2 \sum_{l_i \neq l_j} \max[\wp^+_{S(p)}(l_i), \wp^+_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}$$

Proposition 4. *Consider the complement of SCNVSG $\bar{G}_1 \approx \bar{G}_2$ (isomorphism) and G_1 and G_2.*

Proof: *There exists a bijection $f : V_1 \to V_2$ such that the graphs G_1 and G_2 are isomorphic under this mapping.*

$$\eth^-_{S_1(p)}(l_i)e^{iA_{S_1(p)}(l_i)} = \eth^-_{S_2(p)}(f(l_i))e^{iA_{S_2(p)}(f(l_i))}, \eth^+_{S_1(p)}(l_i)e^{iA_{S_1(p)}(l_i)} = \eth^+_{S_2(p)}(f(l_i))e^{iA_{S_2(p)}(f(l_i))}$$
$$\Im^-_{S_1(p)}(l_i)e^{iB_{S_1(p)}(l_i)} = \Im^-_{S_2(p)}(f(l_i))e^{iB_{S_2(p)}(f(l_i))}, \Im^+_{S_1(p)}(l_i)e^{iB_{S_1(p)}(l_i)} = \Im^+_{S_2(p)}(f(l_i))e^{iB_{S_2(p)}(f(l_i))}$$
$$\wp^-_{S_1(p)}(l_i)e^{iC_{S_1(p)}(l_i)} = \wp^-_{S_2(p)}(f(l_i))e^{iC_{S_2(p)}(f(l_i))}, \wp^+_{S_1(p)}(l_i)e^{iC_{S_1(p)}(l_i)} = \wp^+_{S_2(p)}(f(l_i))e^{iC_{S_2(p)}(f(l_i))}$$

for all $l_i \in S_1$

and

$$\eth^-_{K_1(p)}(l_il_j)e^{iA_{K_1(p)}(l_il_j)} = \eth^-_{K_2(p)}(f(l_i)f(l_j))e^{iA_{K_2(p)}(f(l_i)f(l_j))},$$
$$\eth^+_{K_1(p)}(l_il_j)e^{iA_{K_1(p)}(l_il_j)} = \eth^+_{K_2(p)}(f(l_i)f(l_j))e^{iA_{K_2(p)}(f(l_i)f(l_j))}$$
$$\Im^-_{K_1(p)}(l_il_j)e^{iB_{K_1(p)}(l_il_j)} = \Im^-_{K_2(p)}(f(l_i)f(l_j))e^{iB_{K_2(p)}(f(l_i)f(l_j))},$$
$$\Im^+_{K_1(p)}(l_il_j)e^{iB_{K_1(p)}(l_il_j)} = \Im^+_{K_2(p)}(f(l_i)f(l_j))e^{iB_{K_2(p)}(f(l_i)f(l_j))}$$
$$\wp^-_{K_1(p)}(l_il_j)e^{iC_{K_1(p)}(l_il_j)} = \wp^-_{K_2(p)}(f(l_i)f(l_j))e^{iC_{K_2(p)}(f(l_i)f(l_j))},$$
$$\wp^+_{K_1(p)}(l_il_j)e^{iC_{K_1(p)}(l_il_j)} = \wp^+_{K_2(p)}(f(l_i)f(l_j))e^{iC_{K_2(p)}(f(l_i)f(l_j))}$$

for all $l_il_j \in K$ *and* $p \in P$

we have

$$\begin{aligned}\bar{\eth}^{-}_{K_1(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\eth^{-}_{S_1(p)}(l_i), \eth^{-}_{S_1(p)}(l_j)]e^{i\min\{A_{S_1(p)}(l_i), A_{S(p)}(l_j)\}} - \eth^{-}_{K_1(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} \\ &= \min[\eth^{-}_{S_2(p)}(f(l_i)), \eth^{-}_{S_2(p)}(f(l_j))]e^{i\min\{A_{S_2(p)}(f(l_i)), A_{S(p)}(f(l_j))\}} \\ &\quad - \eth^{-}_{K_2(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))} \\ &= \bar{\eth}^{-}_{K_2(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))}\end{aligned}$$

$$\begin{aligned}\bar{\Im}^{-}_{K_1(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} &= \min[\Im^{-}_{S_1(p)}(l_i), \Im^{-}_{S_1(p)}(l_j)]e^{i\min\{B_{S_1(p)}(l_i), B_{S(p)}(l_j)\}} - \Im^{-}_{K_1(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} \\ &= \min[\Im^{-}_{S_2(p)}(f(l_i)), \Im^{-}_{S_2(p)}(f(l_j))]e^{i\min\{B_{S_2(p)}(f(l_i)), B_{S(p)}(f(l_j))\}} \\ &\quad - \Im^{-}_{K_2(p)}(f(l_i)f(l_j))e^{iB_{K(p)}(f(l_i)f(l_j))} \\ &= \bar{\Im}^{-}_{K_2(p)}(f(l_i)f(l_j))e^{iB_{K(p)}(f(l_i)f(l_j))}\end{aligned}$$

$$\begin{aligned}\bar{\wp}^{-}_{K_1(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} &= \min[\wp^{-}_{S_1(p)}(l_i), \wp^{-}_{S_1(p)}(l_j)]e^{i\min\{C_{S_1(p)}(l_i), C_{S(p)}(l_j)\}} - \wp^{-}_{K_1(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} \\ &= \min[\wp^{-}_{S_2(p)}(f(l_i)), \wp^{-}_{S_2(p)}(f(l_j))]e^{i\min\{C_{S_2(p)}(f(l_i)), C_{S(p)}(f(l_j))\}} \\ &\quad - \wp^{-}_{K_2(p)}(f(l_i)f(l_j))e^{iC_{K(p)}(f(l_i)f(l_j))} \\ &= \bar{\wp}^{-}_{K_2(p)}(f(l_i)f(l_j))e^{iC_{K(p)}(f(l_i)f(l_j))}\end{aligned}$$

$$\begin{aligned}\bar{\eth}^{+}_{K_1(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\eth^{+}_{S_1(p)}(l_i), \eth^{-}_{S_1(p)}(l_j)]e^{i\min\{A_{S_1(p)}(l_i), A_{S(p)}(l_j)\}} - \eth^{+}_{K_1(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} \\ &= \min[\eth^{+}_{S_2(p)}(f(l_i)), \eth^{-}_{S_2(p)}(f(l_j))]e^{i\min\{A_{S_2(p)}(f(l_i)), A_{S(p)}(f(l_j))\}} \\ &\quad - \eth^{+}_{K_2(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))} \\ &= \bar{\eth}^{+}_{K_2(p)}(f(l_i)f(l_j))e^{iA_{K(p)}(f(l_i)f(l_j))}\end{aligned}$$

$$\begin{aligned}\bar{\Im}^{+}_{K_1(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} &= \min[\Im^{+}_{S_1(p)}(l_i), \Im^{+}_{S_1(p)}(l_j)]e^{i\min\{B_{S_1(p)}(l_i), B_{S(p)}(l_j)\}} - \Im^{+}_{K_1(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} \\ &= \min[\Im^{+}_{S_2(p)}(f(l_i)), \Im^{+}_{S_2(p)}(f(l_j))]e^{i\min\{B_{S_2(p)}(f(l_i)), B_{S(p)}(f(l_j))\}} \\ &\quad - \Im^{+}_{K_2(p)}(f(l_i)f(l_j))e^{iB_{K(p)}(f(l_i)f(l_j))} \\ &= \bar{\Im}^{+}_{K_2(p)}(f(l_i)f(l_j))e^{iB_{K(p)}(f(l_i)f(l_j))}\end{aligned}$$

$$\begin{aligned}\bar{\wp}^{+}_{K_1(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} &= \min[\wp^{-}_{S_1(p)}(l_i), \wp^{+}_{S_1(p)}(l_j)]e^{i\min\{C_{S_1(p)}(l_i), C_{S(p)}(l_j)\}} - \wp^{+}_{K_1(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} \\ &= \min[\wp^{-}_{S_2(p)}(f(l_i)), \wp^{+}_{S_2(p)}(f(l_j))]e^{i\min\{C_{S_2(p)}(f(l_i)), C_{S(p)}(f(l_j))\}} \\ &\qquad - \wp^{+}_{K_2(p)}(f(l_i)f(l_j))e^{iC_{K(p)}(f(l_i)f(l_j))} \\ &= \bar{\wp}^{+}_{K_2(p)}(f(l_i)f(l_j))e^{iC_{K(p)}(f(l_i)f(l_j))}\end{aligned}$$

for all $(l_il_j) \in k$ *and* $p \in P$

Hence $\bar{G}_1 \approx \bar{G}_2$.

Definition 16. *Let* $G = (G^*, S, K, P)$ *be a complete complex neutrosophic vague soft graph, where* $G^* = (V, E)$. *The* complement *of* G, *denoted by* $\bar{G} = (\bar{S}, \bar{K})$, *is defined as another complex neutrosophic vague soft graph over the same underlying structure* $G^* = (\bar{V}, \bar{E})$, *where:*

(1) *The vertex set remains unchanged, i.e.,* $\bar{V} = V$.

(2) *The complex neutrosophic vague soft values on vertices are preserved:*

$$\begin{aligned}\bar{\eth}^{-}_{S(p)}(l_i)e^{iA_{S(p)}(l_i)} &= \eth^{-}_{S(p)}(l_i)e^{iA_{S(p)}(l_i)}, \quad \bar{\eth}^{+}_{S(p)}(l_i)e^{iA_{S(p)}(l_i)} = \eth^{+}_{S(p)}(l_i)e^{iA_{S(p)}(l_i)}, \\ \bar{\Im}^{-}_{S(p)}(l_i)e^{iB_{S(p)}(l_i)} &= \Im^{-}_{S(p)}(l_i)e^{iB_{S(p)}(l_i)}, \quad \bar{\Im}^{+}_{S(p)}(l_i)e^{iB_{S(p)}(l_i)} = \Im^{+}_{S(p)}(l_i)e^{iB_{S(p)}(l_i)}, \\ \bar{\wp}^{-}_{S(p)}(l_i)e^{iC_{S(p)}(l_i)} &= \wp^{-}_{S(p)}(l_i)e^{iC_{S(p)}(l_i)}, \quad \bar{\wp}^{+}_{S(p)}(l_i)e^{iC_{S(p)}(l_i)} = \wp^{+}_{S(p)}(l_i)e^{iC_{S(p)}(l_i)}.\end{aligned}$$

(3) *The complex neutrosophic vague soft edge functions of the complement* $\bar{K}$ *are defined as:*

$$\begin{aligned}\bar{\eth}^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\eth^{-}_{S(p)}(l_i), \eth^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} - \eth^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}, \\ \bar{\eth}^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\eth^{+}_{S(p)}(l_i), \eth^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} - \eth^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}, \\ \bar{\Im}^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} &= \min[\Im^{-}_{S(p)}(l_i), \Im^{-}_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}} - \Im^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}, \\ \bar{\Im}^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} &= \min[\Im^{+}_{S(p)}(l_i), \Im^{+}_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}} - \Im^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}, \\ \bar{\wp}^{-}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} &= \max[\wp^{-}_{S(p)}(l_i), \wp^{-}_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}} - \wp^{-}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)}, \\ \bar{\wp}^{+}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} &= \max[\wp^{+}_{S(p)}(l_i), \wp^{+}_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}} - \wp^{+}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)},\end{aligned}$$

for all $l_il_j \in E$ *and for each* $p \in P$.

Proposition 5. *Consider the complement of a complete complex neutrosophic vague soft graph that contains no edges. In this case, if* G *is a complete graph, then its complement* $\bar{G}$ *contains no edges, i.e., the edge set of* $\bar{G}$ *is empty.*

Proof: *Consider the graph $G = (G^*, S, K, P)$, which represents the complement of a complete complex neutrosophic vague soft graph.*

$$\begin{aligned}
\bar{\eth}^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\eth^{-}_{S(p)}(l_i), \eth^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} \\
\bar{\eth}^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\eth^{+}_{S(p)}(l_i), \eth^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} \\
\bar{\Im}^{-}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} &= \min[\Im^{-}_{S(p)}(l_i), \Im^{-}_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}} \\
\bar{\Im}^{+}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)} &= \min[\Im^{+}_{S(p)}(l_i), \Im^{+}_{S(p)}(l_j)]e^{i\min\{B_{S(p)}(l_i), B_{S(p)}(l_j)\}} \\
\bar{\wp}^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \max[\wp^{-}_{S(p)}(l_i), \wp^{-}_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}} \\
\bar{\wp}^{+}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)} &= \max[\wp^{+}_{S(p)}(l_i), \wp^{+}_{S(p)}(l_j)]e^{i\max\{C_{S(p)}(l_i), C_{S(p)}(l_j)\}}
\end{aligned}$$

for all $l_il_j \in V$ and $p \in P$
Hence in $\bar{G}$,

$$\begin{aligned}
\bar{\eth}^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\eth^{-}_{S(p)}(l_i), \eth^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} - \eth^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} \\
&= \min[\eth^{-}_{S(p)}(l_i), \eth^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} \\
&\qquad - \min[\eth^{-}_{S(p)}(l_i), \eth^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} \\
&= 0, \forall i, j, ..., n
\end{aligned}$$

$$\begin{aligned}
\bar{\eth}^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\eth^{+}_{S(p)}(l_i), \eth^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} - \eth^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} \\
&= \min[\eth^{+}_{S(p)}(l_i), \eth^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} \\
&\qquad - \min[\eth^{+}_{S(p)}(l_i), \eth^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} \\
&= 0, \forall i, j, ..., n
\end{aligned}$$

$$\begin{aligned}
\bar{\Im}^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\Im^{-}_{S(p)}(l_i), \Im^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} - \Im^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} \\
&= \min[\Im^{-}_{S(p)}(l_i), \Im^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} \\
&\qquad - \min[\Im^{-}_{S(p)}(l_i), \Im^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} \\
&= 0, \forall i, j, ..., n
\end{aligned}$$

$$\begin{aligned}\bar{\Im}^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\Im^{+}_{S(p)}(l_i), \Im^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} - \Im^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}\\ &= \min[\Im^{+}_{S(p)}(l_i), \Im^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}\\ &\quad - \min[\Im^{+}_{S(p)}(l_i), \Im^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}\\ &= 0, \forall i, j, ..., n\end{aligned}$$

$$\begin{aligned}\bar{\wp}^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\wp^{-}_{S(p)}(l_i), \wp^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} - \wp^{-}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}\\ &= \min[\wp^{-}_{S(p)}(l_i), \wp^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}\\ &\quad - \min[\wp^{-}_{S(p)}(l_i), \wp^{-}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}\\ &= 0, \forall i, j, ..., n\end{aligned}$$

$$\begin{aligned}\bar{\wp}^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)} &= \min[\wp^{+}_{S(p)}(l_i), \wp^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}} - \wp^{+}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}\\ &= \min[\wp^{+}_{S(p)}(l_i), \wp^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}\\ &\quad - \min[\wp^{+}_{S(p)}(l_i), \wp^{+}_{S(p)}(l_j)]e^{i\min\{A_{S(p)}(l_i), A_{S(p)}(l_j)\}}\\ &= 0, \forall i, j, ..., n\end{aligned}$$

Thus

$(\bar{\eth}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}, \bar{\Im}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}, \bar{\wp}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)})^{-} = (0,0,0)^{-}$
$(\bar{\eth}_{K(p)}(l_il_j)e^{iA_{K(p)}(l_il_j)}, \bar{\Im}_{K(p)}(l_il_j)e^{iB_{K(p)}(l_il_j)}, \bar{\wp}_{K(p)}(l_il_j)e^{iC_{K(p)}(l_il_j)})^{+} = (0,0,0)^{+}$

Consequently, if G represents a complete complex neutrosophic vague soft graph, then the edge set of its complement, denoted by $\bar{G}$, is void.

4. **Conclusion**

This work focuses on the innovative concept of complex neutrosophic vague soft graphs. Additionally, it explores several fundamental properties of strong, complete, complementary, and self-complementary types of complex neutrosophic vague soft graphs. The introduced notions are further elucidated through pertinent examples. Future work will emphasize operations related to complex neutrosophic vague soft graphs, particularly their regular forms and other variants. The framework of complex neutrosophic vague soft graphs holds potential for diverse applications, such as social network analysis, recommendation systems, traffic modeling, and transportation systems.

Acknowledgement: The authors would like to thank the editor-in-chief and anonymous reviewers for their valuable comments and suggestions. This research was supported by the University Grants Commission (UGC) of India under the scheme CSIR-UGC JRF with reference number 231610081608. The first author gratefully acknowledges the financial support provided by UGC.

References

[1] Ali, M.; Smarandache, F. Complex neutrosophic set. *Neural Comput. Appl.* **2017**, *28*, 1817–1834.

[2] Alkhazaleh, S. Neutrosophic vague set theory. *Crit. Rev.* **2015**, *10*, 29–39.

[3] Al-Quran, A.; Hassan, N. Neutrosophic vague soft expert set theory. *J. Intell. Fuzzy Syst.* **2016**, *30*(6), 3691–3702.

[4] Akram, M.; Nawaz, S. On fuzzy soft graphs. *Ital. J. Pure Appl. Math.* **2015**, *34*, 497–514.

[5] Akram, M.; Nawaz, S. Operation on soft graphs. *Fuzzy Inf. Eng.* **2015**, *7*, 423–449.

[6] Akram, M.; Shahzadi, S. Intuitionistic fuzzy soft graphs with applications. *J. Appl. Math. Comput.* **2017**, *55*, 369–397.

[7] Akram, M.; Shahzadi, S. Neutrosophic soft graphs with applications. *J. Intell. Fuzzy Syst.* **2017**, *32*(1), 841–858.

[8] Gau, W.L.; Buehrer, D.J. Vague sets. *IEEE Trans. Syst. Man Cybern.* **1993**, *23*(2), 610–614.

[9] Yaqoob, N.; Akram, M. Complex neutrosophic graph. *Bull. Comput. Appl. Math.* **2018**, *6*(2), ISSN: 2244-8659.

[10] Nguyen, H.T.; Kandel, A.; Kreinovich, V. Complex fuzzy sets: Towards new foundations. *IEEE Trans. Fuzzy Syst.* **2000**, *2*, 1045–1048.

[11] Broumi, S.; Talea, M.; Bakali, A.; Smarandache, F. Single valued neutrosophic graph. *Neutrosophic Sets Syst.* **2016**, ISSN: 2149-1402.

[12] Broumi, S.; Ali, M.; Smarandache, F. Complex neutrosophic soft set. In *Proceedings of the IEEE Conference on Fuzzy Systems*, Naples, Italy, 2017.

[13] Hussain, S.; Hussain, R.J.; Smarandache, F. On neutrosophic vague graphs. *Neutrosophic Sets Syst.* **2019**, *28*.

[14] Hussain, S.; Hussain, R.J.; Muhiuddin, G.; Anitha, P. Operations on neutrosophic vague soft graphs. *Neutrosophic Sets Syst.* **2022**, *51*, 254–268.

[15] Smarandache, F. Neutrosophic set – a generalization of the intuitionistic fuzzy set. In *Proceedings of the IEEE International Conference on Granular Computing*, 2006, pp. 38–42.

[16] Suresh, R.; Veeramani, V.; Thamizharasi, R. Innovative study on complex fuzzy soft graph and its properties. *IAENG Int. J. Appl. Math.* **2024**, *54*, 2041–2048.

[17] Sudhakar, V.J.; Ali, M.; Vinoth, D. Interval valued signed neutrosophic graphs. *Int. J. Math. Arch.* **2016**, *9*(9), 35–43.

[18] Sudhakar, V.J.; Ali, M.; Vinoth, D.; Sathishkumar, G. Self-centered interval valued signed neutrosophic graphs. *Int. J. Math. Arch.* **2016**, *9*(9), 44–5.

[19] Sudhakar, V.J.; Navaneethakumar, V.; Jayaprakasam, S. Interval-valued regular neutrosophic graphs. *Int. J. Sci. Humanit.* **2019**, *5*(2), 76–85.

[20] Sudhakar, V.J.; Navaneethakumar, V.; Yuvaraj. Self-centered interval valued neutrosophic graphs. *Int. J. Sci. Humanit.* **2019**, *5*(2), 86–102.

[21] Sudhakar, V.J.; Jagadeeswari, L. Bipolar interval valued neutrosophic signed graphs. *Malaya J. Matematik* **2021**, *9*(1), 1086–1094.

[22] Zadeh, L.A. Fuzzy sets. *Inform. Control* **1965**, *8*(3), 338–353.

Received: Dec. 9, 2024. Accepted: June 24, 2025