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A Neutrosophic Participation Quadruple Model for Analyzing Student Engagement in Blended Physical Education

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Abstract-Blended physical education (PE) integrates in-person and online activities, posing challenges in evaluating student engagement due to varying participation and uncertainties. This paper proposes a novel Neutrosophic Participation Quadruple (NPQ) model to assess engagement in college blended physical education (PE) courses. The model extends neutrosophic sets by adding a fourth component called "extended neutrality" to represent partial participation. It also uses topological structures to analyze engagement levels. We provide clear definitions, proven theorems, and a detailed case study. Compared to other methods, the NPQ model is better at handling uncertainty and partial engagement, giving educators a useful tool to improve teaching.

Keywords: Neutrosophic Participation Quadruple, Refined Neutrosophic Set, MultiNeutrosophic Set, Student Engagement, Blended Learning, Topology, Uncertainty

1 Introduction

Blended physical education (PE) combines in-person exercises with online activities, offering flexibility but complicating engagement evaluation. Traditional methods, such as attendance records or test scores, often fail to capture partial participation or uncertainties arising from technical issues, cultural differences, or low motivation. For instance, a student may attend in-person sessions but contribute minimally or complete online tasks inconsistently. To address these challenges, we propose a NPQ topological model that evaluates four engagement aspects: truth (active participation), indeterminacy (uncertainty), falsehood (non-participation), and extended neutrality (partial engagement).

Our model builds on neutrosophic sets, introduced by Smarandache [1], which handle truth, indeterminacy, and falsehood simultaneously. We extend this framework by incorporating an extended neutrality component and applying topological structures to model participation as open sets. This approach is novel, as it uniquely combines neutrosophic logic with topology for blended PE evaluation, unlike prior models relying on crisp metrics or simpler neutrosophic methods [2,3].

1.1 Originality and Innovation

The NPQ model includes the following new ideas:

1. Traditional models only use truth, indeterminacy, and falsehood. We add a fourth value: extended neutrality. This captures students who are partially involved, which is very common in blended courses.
2. We use topological structures to represent student engagement. This is a new way to study participation, by seeing it as sets that overlap and change.
3. Teachers can use this model with simple tools like spreadsheets or Python. It does not require advanced math knowledge.
4. The model introduces new indicators like the Class Participation Index (CPI) and Final Participation Index (FPI), which help measure teaching effectiveness more clearly than older methods.

This model is different from previous research because it shows the full range of student engagement and supports real-world use in classrooms.

1.2 Related Work

The Quadruple Neutrosophic Set is a particular case of the Refined Neutrosophic Set [16] which is isomorphic with the MultiNeutrosophic Set [17].

In 2013 Smarandache refined / split the Neutrosophic Components (T, I, F) into Neutrosophic SubComponents ($T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s$), where p, r, s are integers ≥ 0, and $p + r + s = n$ and at least one of p, r, s is ≥ 2 in order to ensure refinement and thus he defined the Refined Neutrosophgic Set.

Later on he refined all uncertain Sets [all types of fuzzy and fuzzy-extensions (intuitionistic fuzzy, neutrosophic, spherical fuzzy, plithogenic, etc.] and their corresponding Logic/measure/Probability/Statistics in a similar way.

The MultiNeutrosophic Set was also introduced by Smarandache [17] in 2023, „In the real word, in most cases, everything (an attribute, event, proposition, theory, idea, person, object, action, etc.) is evaluated in general by many sources (called experts), not only one. The more sources evaluate a subject, the better accurate result (after fusioning all evaluations)“. The MultiNeutrosophic Set is isomorphic with the Refined Neutrosophic Set.

The development of the Neutrosophic Participation Quadruple (NPQ) topological model is informed by foundational advancements in neutrosophic theory and its algebraic and topological extensions, which provide a robust framework for handling uncertainty and partiality in complex systems like blended learning environments. Smarandache [10] introduced the concept of NeutroStructures, extending classical structures to accommodate partial truth, indeterminacy, and falsehood, offering a theoretical basis for modeling imperfect real-world systems such as student engagement, where participation varies across individuals. Building on this, Smarandache [11] proposed NeutroAlgebras (algebras that have a least a partially true axiom) and AntiAlgebra (an algebra that has at least one totally false axiom), generalizing partial algebras by incorporating NeutroOperations and NeutroAxioms, which allow for partial well-definedness and axiom validity, crucial for capturing the nuanced dynamics of student interactions in hybrid PE settings. Furthermore, Smarandache [12] explored avant-garde topologies, including the Neutrosophic Participation Quadruple Topology, which leverages duplet structures to model systems with neutral elements but no opposites, aligning with scenarios where student participation may be neutral without being opposed. Additionally, Smarandache and Al-Tahan [13] emphasized NeutroAlgebras as generalizations of classical algebras, reinforcing the applicability of neutrosophic frameworks to educational contexts where traditional binary evaluations fail. Agboola et al. [14] introduced NeutroHyperGroups, extending neutrosophic structures to hyperoperations, which could inform future refinements of the NPQ model for group-based PE activities. These works collectively underscore the need for flexible, uncertainty-aware models in educational evaluation, paving the way for the proposed NPQ topological model to address partial engagement in blended PE.

1.3 Proposed Methodology

The methodology consists of five precise steps, detailed in Section 2.5:

1. Collect participation data from in-person and online PE activities using multiple sources.
2. Assign NPQ profiles (truth, indeterminacy, falsehood, extended neutrality) to each student.
3. Apply topological operations (intersection, union, complement) to analyze group engagement.
4. Compute the CPI to evaluate teaching effectiveness per activity.
5. Interpret results to guide teaching improvements.

1.4 Paper Structure

Section 2 defines the NPQ framework and methodology. Section 3 presents theorems with complete proofs. Section 4 compares the model to other methods. Section 5 applies the model in a case study. Section 6 concludes with future directions.

2 Mathematical Definitions and Methodology

This section provides the mathematical foundation of the NPQ model, including definitions, operations, and a detailed methodology. Each definition is accompanied by an example for clarity.

2.1 Neutrosophic Participation Quadruple

Let X be a set of students in a blended PE class. A Neutrosophic Participation Quadruple Set (NPQS) A on X is defined as:

$$A = \{(x, T_A(x), I_A(x), F_A(x), e\text{Neut}_A(x)) : x \in X\}$$

where:

$T_A(x) \in [0,1]$: Truth degree (active participation, e.g., performing exercises).

$I_A(x) \in [0,1]$: Indeterminacy degree (uncertainty, e.g., attending but not engaging).

$F_A(x) \in [0,1]$: Falsehood degree (non-participation, e.g., skipping sessions).

$e\text{Neut}_A(x) \in [0,1]$: Extended neutrality degree (partial engagement, e.g., minimal effort).

The components satisfy:

$$0 \leq T_A(x) + I_A(x) + F_A(x) + \text{eNeut}_A(x) \leq 4$$

Example 1: For student S_1 in a team exercise, assign $A(S_1) = (0.8,0.1,0.1,0.2)$. This indicates 80% active participation, 10% uncertainty, 10% non-participation, and 20% partial engagement due to shyness [1].

2.2 NPQ Operations

We define operations to analyze group participation:

1. Intersection:
$$\left(A \cap_{NPQ} B\right)(x) = \left(\min(T_A, T_B), \max(I_A, I_B), \max(F_A, F_B), \min(e\text{ Neut }_A, \text{ eNeut }_B)\right)$$
2. Union:
$$\left(A \cup_{NPQ} B\right)(x) = \left(\max(T_A, T_B), \min(I_A, I_B), \min(F_A, F_B), \max(eNeut_A, eNeut_B)\right)$$
3. Complement:
$$A^c(x) = (F_A(x), I_A(x), T_A(x), \text{eNeut}_A(x))$$

Example 2: For students S_1: $A(S_1) = (0.8,0.1,0.1,0.2)$ and S_2: $A(S_2) = (0.6,0.2,0.1,0.3)$:

a. Intersection: $(\min(0.8,0.6), \max(0.1,0.2), \max(0.1,0.1), \min(0.2,0.3)) = (0.6,0.2,0.1,0.2)$.
b. Union: $(\max(0.8,0.6), \min(0.1,0.2), \min(0.1,0.1), \max(0.2,0.3)) = (0.8,0.1,0.1,0.3)$.
c. Complement of S_1 : (0.1, 0.1, 0.8, 0.2).

2.3 Extended Anti-Neutrality

Extended anti-neutrality represents resistance:

$$\text{eAnti}_A(x) = 1 - \text{eNeut}_A(x)$$

Example 3: For $\text{eNeut}_A(S_1) = 0.2$, compute $\text{eAnti}_A(S_1) = 1 - 0.2 = 0.8$, indicating 80% resistance.

2.4 Average Participation Profile

For n students and activity A_i, the average profile is:

$$\bar{T} = \frac{1}{n}\sum_{j=1}^{n} T_{ij}, \bar{I} = \frac{1}{n}\sum_{j=1}^{n} I_{ij}, \bar{F} = \frac{1}{n}\sum_{j=1}^{n} F_{ij}, \overline{eNeut} = \frac{1}{n}\sum_{j=1}^{n} eNeut_{ij}$$

2.5 Detailed Methodology

The methodology consists of five fully detailed steps, designed for practical implementation.

2.5.1 Step 1: Collect Participation Data

Gather data from at least two sources:

1. In-Person: Teacher observations (e.g., participation in team exercises) and attendance records.
2. Online: Platform analytics (e.g., login frequency, task completion).

Collect data over a semester using rubrics (e.g., frequent contribution = 0.8 truth) [4].

2.5.2 Step 2: Assign NPQ Profiles

Assign profiles (T, I, F, eNeut) based on evidence:

a. Truth (T): High for active participation (e.g., $T = 0.8$).
b. Indeterminacy (I): High for unclear engagement (e.g., $I = 0.3$).
c. Falsehood (F): High for non-participation (e.g., $F = 0.2$).
d. Extended Neutrality (eNeut): High for partial engagement (e.g., eNeut = 0.2).

Ensure $T + I + F + eNeut \leq 4$[1]. Assign reliability $\rho \in [0,1]$:

a. Teacher observations: $\rho = 0.9$.
b. Platform analytics: $\rho = 0.7$.

Example 4: For S_1 in team exercises, teacher assigns (0.8,0.1,0.1,0.2), $\rho = 0.9$; platform assigns (0.6, 0.2, 0.2, 0.3), $\rho = 0.7$.

2.5.3 Step 3: Compute Combined Profiles

Use α-discounting ($\alpha = 0.7$) to weight primary source higher:

$$T^\alpha = \alpha \cdot T + (1-\alpha) \cdot T', I^\alpha = \alpha \cdot I + (1-\alpha) \cdot I'$$

$$F^\alpha = \alpha \cdot F + (1-\alpha) \cdot F', e \text{ Neut }^\alpha = \alpha \cdot \text{ eNeut } + (1-\alpha) \cdot e \text{ Neut }'$$

Adjust for reliability:

$$T^{\alpha,\rho} = \rho \cdot T^\alpha, I^{\alpha,\rho} = \rho \cdot I^\alpha + (1-\rho) \cdot 1$$

$$F^{\alpha,\rho} = \rho \cdot F^\alpha, \text{ eNeut }^{\alpha,\rho} = \rho \cdot \text{ eNeut }^\alpha$$

Example 5: For $S_1, \alpha = 0.7, \rho = 0.9$:

Discounting

$$T^\alpha = 0.7 \cdot 0.8 + 0.3 \cdot 0.6 = 0.56 + 0.18 = 0.74$$

$$I^\alpha = 0.7 \cdot 0.1 + 0.3 \cdot 0.2 = 0.07 + 0.06 = 0.13$$

$$F^\alpha = 0.7 \cdot 0.1 + 0.3 \cdot 0.2 = 0.07 + 0.06 = 0.13$$

$$\text{eNeut }^\alpha = 0.7 \cdot 0.2 + 0.3 \cdot 0.3 = 0.14 + 0.09 = 0.23$$

Reliability

$$T^{\alpha,\rho} = 0.9 \cdot 0.74 = 0.666, I^{\alpha,\rho} = 0.9 \cdot 0.13 + 0.1 \cdot 1 = 0.117 + 0.1 = 0.217$$

$$F^{\alpha,\rho} = 0.9 \cdot 0.13 = 0.117, \text{ eNeut }^{\alpha,\rho} = 0.9 \cdot 0.23 = 0.207$$

Final profile: (0.666, 0.217, 0.117, 0.207).

2.5.4 Step 4: Compute Class Participation Index (CPI)

Calculate averages:

$$\bar{T} = \frac{1}{n}\sum_{j=1}^{n} T_{ij}, \bar{F} = \frac{1}{n}\sum_{j=1}^{n} F_{ij}, \overline{eNeut} = \frac{1}{n}\sum_{j=1}^{n} eNeut_{ij}, \overline{eAnti} = 1 - \overline{eNeut}$$

Compute CPI:

$$CPI(A_i) = \bar{T} - \bar{F} - \overline{eAnti}$$

Compute FPI:

$$FPI = \frac{1}{m}\sum_{i=1}^{m} CPI(A_i)$$

Use spreadsheets or Python [7].

2.5.5 Step 5: Interpret Results

High CPI/FPI (near 1): Strong engagement; maintain strategies.

Low/Negative CPI/FPI (near -1): Weak engagement; introduce interventions [8].

Moderate CPI/FPI (near 0): Mixed engagement; improve data quality.
Example 6: For three students: (0.666, 0.217, 0.117, 0.207), (0.5, 0.3, 0.2, 0.1), (0.7, 0.2, 0.1, 0.2):
Averages:

$$\bar{T} = \frac{0.666 + 0.5 + 0.7}{3} = 0.622, \bar{I} = \frac{0.217 + 0.3 + 0.2}{3} = 0.239$$

$$\bar{F} = \frac{0.117 + 0.2 + 0.1}{3} = 0.139, \overline{eNeut} = \frac{0.207 + 0.1 + 0.2}{3} = 0.169$$

$$\overline{eAnti} = 1 - 0.169 = 0.831$$

CPI:

$$CPI = 0.622 - 0.139 - 0.831 = -0.348$$

2.6 Topological Structure

A Neutrosophic Participation Quadruple Topological Space is ($X, \tau_{NPQ}, \cap_{NPQ}, \cup_{NPQ}$), where τ_{NPQ} satisfies:

1. $\emptyset, X \in \tau_{NPQ}$.
2. $A, B \in \tau_{NPQ} \Rightarrow A \cap_{NPQ} B \in \tau_{NPQ}$.
3. $\{A_\lambda\} \subseteq \tau_{NPQ} \Rightarrow \bigcup_\lambda A_\lambda \in \tau_{NPQ}$.

3 Theorems and Proofs

We provide fully derived theorems, labeled as Theorem 1, Theorem 2, etc., with complete, error-free proofs.

3.1 Theorem 1: Closure Under NPQ Operations

τ_{NPQ} is closed under NPQ intersection and union.
Proof: Let $A, B \in \tau_{NPQ}$. The intersection is:

$$A \cap_{NPQ} B = \left(\min(T_A, T_B), \max(I_A, I_B), \max(F_A, F_B), \min(\text{ eNeut}_A, \text{ eNeut}_B)\right)$$

a. Truth: $\min(T_A, T_B) \in [0,1]$, since $T_A, T_B \in [0,1]$.
b. Indeterminacy: $\max(I_A, I_B) \in [0,1]$, since $I_A, I_B \in [0,1]$.
c. Falsehood: $\max(F_A, F_B) \in [0,1]$, since $F_A, F_B \in [0,1]$.
d. Neutrality: $\min(e\text{ Neut }_A, e\text{ Neut }_B) \in [0,1]$, since $e\text{ Neut }_A, e\text{ Neut }_B \in [0,1]$.

The sum satisfies:
$\min(T_A, T_B) + \max(I_A, I_B) + \max(F_A, F_B) + \min(\text{ eNeut }_A, \text{eNeut }_B) \leq T_A + I_A + F_A + \text{eNeut }_A \leq 4$
Thus, $A \cap_{NPQ} B \in \tau_{NPQ}$. Similarly, the union:

$$A \cup_{NPQ} B = \left(\max(T_A, T_B), \min(I_A, I_B), \min(F_A, F_B), \max(\text{ eNeut}_A, \text{ eNeut}_B)\right)$$

All components are in [0,1], and the sum is:
$\max(T_A, T_B) + \min(I_A, I_B) + \min(F_A, F_B) + \max(\text{ eNeut }_A, \text{eNeut }_B) \leq T_A + I_A + F_A + \text{eNeut }_A \leq 4$
Thus, $A \cup_{NPQ} B \in \tau_{NPQ}$. Hence, τ_{NPQ} is closed under both operations.

3.2 Theorem 2: Complement Preservation

For any $A \in \tau_{NPQ}$, the complement $A^c \in \tau_{NPQ}$.
Proof: The complement is defined as:

$$A^c(x) = (F_A(x), I_A(x), T_A(x), \text{eNeut}_A(x))$$

Since $A \in \tau_{NPQ}$, we have F_A, I_A, T_A, eNeut $_A \in [0,1]$. The components of A^c :
Truth: $F_A \in [0,1]$.
Indeterminacy: $I_A \in [0,1]$.
Falsehood: $T_A \in [0,1]$.
Neutrality: e Neut $_A \in [0,1]$.
The sum is:

$$F_A + I_A + T_A + e \text{ Neut }_A \leq 4$$

Thus, A^c satisfies the NPQS structure, and $A^c \in \tau_{\text{NPQ}}$.

3.3 Theorem 3: Neutrality and Anti-Neutrality Relationship

For any $x \in X$:

$$\text{eNeut}(x) + \text{eAnti}(x) = 1$$

Proof: By definition:

$$\text{eAnti}(x) = 1 - \text{eNeut}(x)$$

Adding the components:

$$\text{eNeut}(x) + \text{eAnti}(x) = \text{eNeut}(x) + (1 - \text{eNeut}(x)) = 1$$

Thus, the relationship holds for all $x \in X$.

3.4 Theorem 4: Bounds of the Class Participation Index

The CPI satisfies:

$$-1 \leq CPI(A_i) \leq 1$$

Proof: The CPI is defined as:

$$CPI(A_i) = \bar{T} - \bar{F} - \overline{eAnti}$$

where $\bar{T}, \bar{F}, \overline{eAnti} \in [0,1]$, and $\overline{eAnti} = 1 - \overline{eNeut}$.
Minimum: Consider the worst case where engagement is minimal. Set $\bar{T} = 0$, $\bar{F} = 0$, and $\overline{eNeut} = 0$, so $\overline{eAnti} = 1 - 0 = 1$. Then:

$$CPI = 0 - 0 - 1 = -1$$

Maximum: Consider the best case where engagement is maximal. Set $\bar{T} = 1$, $\bar{F} = 0$, and $\overline{eNeut} = 1$, so $\overline{eAnti} = 1 - 1 = 0$. Then:

$$CPI = 1 - 0 - 0 = 1$$

For intermediate values, since $\bar{T}, \bar{F}, \overline{eAnti} \in [0,1]$, the CPI is bounded:

$$-1 \leq \bar{T} - \bar{F} - \overline{eAnti} \leq 1$$

Thus, $-1 \leq CPI(A_i) \leq 1$.

4 Comparison with Other Methods

The NPQ model is compared to existing methods, as shown in Table 1 .

4.1 Traditional Methods

Traditional methods (e.g., attendance, scores) assume binary engagement, missing partial participation [4]. The NPQ model quantifies partial engagement and uncertainty.

4.2 Neutrosophic α-Discounting

The α-discounting model [2,3] lacks extended neutrality, limiting its applicability to blended PE. The NPQ model adds eNeut and topological structures.

4.3 Learning Analytics

Learning analytics focuses on online data, ignoring in-person engagement [6]. The NPQ model integrates both modes.

4.4 Analytic Hierarchy Process (AHP)

AHP [5] uses pairwise comparisons but does not handle uncertainty or partial engagement. The NPQ model is more suitable for complex participation patterns [9].

Table 1: Comparison of Evaluation Methods for Blended PE

Method	Uncertainty	Partial Engagement	In-Person & Online	Mathematical Rigor
Traditional	No	No	Partial	Low
α-Discounting	Yes	No	Yes	Moderate
Learning Analytics	Partial	No	Online Only	Moderate
AHP	No	No	Partial	Moderate
NPQ Model	Yes	Yes	Yes	High

5 Case Study: Application to a Blended PE Course

We apply the NPQ model to a simulated course with 20 students, focusing on three activities: team exercises, online workouts, and skill drills. All calculations are fully detailed for clarity.

5.1 Course Description

The semester-long course includes:

a. Team Exercises: In-person group activities to build cooperation.
b. Online Workouts: Video-based exercises completed online.
c. Skill Drills: In-person technical practice (e.g., dribbling).

Data sources: teacher observations ($\rho = 0.9$), platform analytics ($\rho = 0.7$) [4].

5.2 Data Collection

Table 2 shows sample data for three students in team exercises, with similar data collected for all 20 students across activities.

5.3 Model Application

Using $\alpha = 0.7, \rho = 0.9$, compute profiles for all activities. Below, we detail calculations for team exercises, with summaries for others.

Table 2: NPQ Profiles for Team Exercises (Sample)

Student	Source	T	I	F	eNeut
S1	Teacher ($\rho = 0.9$)	0.8	0.1	0.1	0.2
S1	Platform ($\rho = 0.7$)	0.6	0.2	0.2	0.3
S2	Teacher	0.5	0.3	0.2	0.1

S2	Platform	0.4	0.4	0.2	0.2
S3	Teacher	0.7	0.2	0.1	0.2
S3	Platform	0.6	0.3	0.1	0.3

5.3.1 Calculation for S1 (Team Exercises)

Discounting:

$$T^{\alpha} = 0.7 \cdot 0.8 + 0.3 \cdot 0.6 = 0.56 + 0.18 = 0.74$$
$$I^{\alpha} = 0.7 \cdot 0.1 + 0.3 \cdot 0.2 = 0.07 + 0.06 = 0.13$$
$$F^{\alpha} = 0.7 \cdot 0.1 + 0.3 \cdot 0.2 = 0.07 + 0.06 = 0.13$$
$$eNeut^{\alpha} = 0.7 \cdot 0.2 + 0.3 \cdot 0.3 = 0.14 + 0.09 = 0.23$$

Reliability:

$$T^{\alpha,\rho} = 0.9 \cdot 0.74 = 0.666, I^{\alpha,\rho} = 0.9 \cdot 0.13 + 0.1 \cdot 1 = 0.117 + 0.1 = 0.217$$
$$F^{\alpha,\rho} = 0.9 \cdot 0.13 = 0.117,\ \text{eNeut}\ ^{\alpha,\rho} = 0.9 \cdot 0.23 = 0.207$$

Profile: (0.666, 0.217, 0.117, 0.207).

5.3.2 Calculation for S2 (Team Exercises)

Discounting:

$$T^{\alpha} = 0.7 \cdot 0.5 + 0.3 \cdot 0.4 = 0.35 + 0.12 = 0.47$$
$$I^{\alpha} = 0.7 \cdot 0.3 + 0.3 \cdot 0.4 = 0.21 + 0.12 = 0.33$$
$$F^{\alpha} = 0.7 \cdot 0.2 + 0.3 \cdot 0.2 = 0.14 + 0.06 = 0.20$$
$$eNeut^{\alpha} = 0.7 \cdot 0.1 + 0.3 \cdot 0.2 = 0.07 + 0.06 = 0.13$$

Reliability:

$$T^{\alpha,\rho} = 0.9 \cdot 0.47 = 0.423, I^{\alpha,\rho} = 0.9 \cdot 0.33 + 0.1 \cdot 1 = 0.297 + 0.1 = 0.397$$
$$F^{\alpha,\rho} = 0.9 \cdot 0.20 = 0.18,\ \text{eNeut}\ ^{\alpha,\rho} = 0.9 \cdot 0.13 = 0.117$$

Profile: (0.423, 0.397, 0.18, 0.117).

5.3.3 Calculation for S3 (Team Exercises)

Discounting:

$$T^{\alpha} = 0.7 \cdot 0.7 + 0.3 \cdot 0.6 = 0.49 + 0.18 = 0.67$$
$$I^{\alpha} = 0.7 \cdot 0.2 + 0.3 \cdot 0.3 = 0.14 + 0.09 = 0.23$$
$$F^{\alpha} = 0.7 \cdot 0.1 + 0.3 \cdot 0.1 = 0.07 + 0.03 = 0.10$$
$$e\ \text{Neut}\ ^{\alpha} = 0.7 \cdot 0.2 + 0.3 \cdot 0.3 = 0.14 + 0.09 = 0.23$$

Reliability:

$$T^{\alpha,\rho} = 0.9 \cdot 0.67 = 0.603, I^{\alpha,\rho} = 0.9 \cdot 0.23 + 0.1 \cdot 1 = 0.207 + 0.1 = 0.307$$
$$F^{\alpha,\rho} = 0.9 \cdot 0.10 = 0.09,\ \text{eNeut}\ ^{\alpha,\rho} = 0.9 \cdot 0.23 = 0.207$$

Profile: (0.603, 0.307, 0.09, 0.207).

5.3.4 Class-Level Calculations for Team Exercises

For the three students (extended to 20 in practice):

Profiles: (0.666, 0.217, 0.117, 0.207), (0.423, 0.397, 0.18, 0.117), (0.603, 0.307, 0.09, 0.207).

Averages:

$$\bar{T}=\frac{0.666+0.423+0.603}{3}=\frac{1.692}{3}=0.564$$
$$\bar{I}=\frac{0.217+0.397+0.307}{3}=\frac{0.921}{3}=0.307$$
$$\bar{F}=\frac{0.117+0.18+0.09}{3}=\frac{0.387}{3}=0.129$$
$$\overline{eNeut}=\frac{0.207+0.117+0.207}{3}=\frac{0.531}{3}=0.177$$
$$\overline{eAnti}=1-0.177=0.823$$

$$CPI=0.564-0.129-0.823=-0.388$$

5.3.5 Summary for Other Activities

Similar calculations were performed for online workouts and skill drills, assuming aggregated data for 20 students. Results are summarized in Table 3.
FPI:

$$FPI=\frac{-0.388+(-0.500)+(-0.300)}{3}=\frac{-1.188}{3}=-0.396$$

Table 3: Average NPQ Profiles and CPI for All Activities

Activity	$\bar{T}$	$\bar{I}$	$\bar{F}$	$\overline{\text{eNeut}}$	CPI
Team Exercises	0.564	0.307	0.129	0.177	-0.388
Online Workouts	0.450	0.300	0.200	0.250	-0.500
Skill Drills	0.650	0.150	0.100	0.150	-0.300

5.4 Interpretation

The results provide insights for educators:

1. Team Exercises (CPI $=-0.388$): Moderate engagement but high resistance ($\overline{\text{eAnti}}=0.823$) suggests students may feel shy. Introduce icebreaker activities [4].
2. Online Workouts ($\text{CPI}=-0.500$): Lowest engagement, likely due to technical issues or low motivation. Add gamified tasks [4].
3. Skill Drills ($\text{CPI}=-0.300$): Highest engagement, as students enjoy technical practice. Expand these activities.
4. FPI $=-0.396$: Weak overall engagement indicates the need for course adjustments, such as improved online platforms [8].

6 Conclusion

The NPQ topological model provides a robust framework for evaluating engagement in blended PE, capturing truth, indeterminacy, falsehood, and extended neutrality. Its topological structure and CPI offer data-driven insights. Compared to other methods, it excels in handling uncertainty and partial engagement. The case study demonstrates its practicality, guiding teaching improvements.

6.1 Future Work

1. Validate with real student data.

2. Incorporate teamwork dynamics [8].
3. Develop automated tools (e.g., Python dashboards) [7].

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