Decision-Making Modeling in Agro-Food Systems Using Type-2 Interval-Valued Linguistic Complex Neutrosophic Sets

R. K. Saini[1*], F. Smarandache[2], Moh. Kasim[3] and Ashik Ahirwar[4]

[1,3,4] Department of Mathematical Sciences and Computer Applications,

Bundelkhand University, Jhansi, Uttar Pradesh, India

[2]Department of Mathematics, University of New Mexico,
705 Gurley Avenue, Gallup, NM 87301, USA

*Correspondence: prof.rksaini@bujhansi.ac.in (011-09412322576)

Abstract: Neutrosophic sets are a strong mathematical framework for representing decision-making uncertainty, ambiguity, and indeterminacy. They have three separate membership functions: truth (T), falsity (F), and indeterminacy. Neutrosophic sets and their extensions, such as complex neutrosophic sets, interval neutrosophic sets, and interval-valued complex neutrosophic sets, provide a versatile foundation for addressing multidimensional uncertainties in real-world applications [1]. However, numerical numbers for membership degrees sometimes fail to reflect decision-makers' subjective language preferences. Linguistic variables have been incorporated into the neutrosophic framework to convert qualitative assessments (e.g., "high risk," "moderate yield") into more structured, quantitative representations, often utilizing interval-valued or complex-number formats. This study presents the Type-2 Interval-Valued Linguistic Complex Neutrosophic Set, a new model. This enhanced extension enhances the flexibility and precision of agro-food choice analysis. It utilizes interval-valued linguistic terms to model truth, indeterminacy, and falsity (e.g. $[T^L, T^U] = [0.7, 0.9]$ "high yield stability"). Interval complex membership functions incorporating phase angles (e.g., $[e^{i\theta_L}, e^{i\theta_U}]$) are employed to represent spatiotemporal or contextual variations (e.g., seasonal droughts), while type-2 fuzzy logic is used to capture hierarchical uncertainties in linguistic evaluations. These elements provide a comprehensive and adaptable solution to the inherent ambiguity and complexity of real-world agriculture and food system decision-making.

Keywords: Interval complex neutrosophic sets, linguistic modeling, Type-2 fuzzy logic, TOPSIS, drought resilience, Bundelkhand.

1. Introduction

Various decision-making procedures have utilized neutrosophic sets (NS), as demonstrated in studies [2–8]. To better handle complex and practical decision-making scenarios, extensions such as interval neutrosophic sets (INS) and complex neutrosophic sets (CNS) have been proposed. Wang et al. [9] introduced the concept of INS, in which the truth, indeterminacy, and falsity membership values are expressed as intervals rather than precise real numbers, allowing for a more flexible representation of uncertainty. To further address the inherent vagueness, incompleteness, indeterminacy, and variability present in periodic data, Ali and Smarandache [10] proposed CNS, which extends both complex fuzzy sets and complex intuitionistic fuzzy sets. These advanced models have proven effective in handling complex decision-making problems [7].

Building upon this foundation, Ali et al. [11] recently introduced interval valued complex neutrosophic sets (IVCNS), which integrate the structures of both CNS and INS in a unified framework. Their work established several algebraic operations for IVCNS and applied these principles to decision-making tasks, such as selecting green suppliers. The research highlighted that IVCNS, when equipped with robust ranking mechanisms based on score, accuracy, and certainty functions, offers a practical approach for solving real-world decision problems, as further illustrated by Ye [12]. In many real-life applications, decision-makers often face vague and imprecise information best captured through linguistic variables rather than exact neutrosophic membership values, as noted in [13]. These developments mark significant progress in modeling uncertainty and enhancing the effectiveness of neutrosophic decision-making frameworks.

The use of linguistic variables in decision-making has long been recognized as a practical approach for modeling uncertainty and imprecision. In the realm of multi-criteria decision-making (MCDM), linguistic neutrosophic environments provide a robust framework for capturing the nuanced relationships among attributes, as demonstrated in [13]. Fang and Ye [14] introduced the concept of linguistic neutrosophic numbers, in which the degrees of truth, indeterminacy, and falsity are independently described using linguistic terms, thereby facilitating more flexible group decision-making processes. Expanding on this, Ma et al. [15] developed interval neutrosophic linguistic numbers (INLNs) to address treatment selection problems through interval neutrosophic linguistic MCDM methods. Numerous additional applications of these models can be found in [4, 16–27].

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has also seen significant adaptation within neutrosophic frameworks. Sahin and Yiğider [12] extended TOPSIS to single-valued neutrosophic environments, while Chen and Hwang proposed a modified version to handle varied MCDM scenarios [14]. Applications of neutrosophic TOPSIS include medical diagnosis for predicting diabetic patients, as explored in [15, 16]. Furthermore, decision-making methods based on soft sets, fuzzy sets, and intuitionistic fuzzy sets have been employed in [17–19]. An advanced TOPSIS approach using single-valued neutrosophic soft sets for expert-based multi-attribute decision-making has been proposed in [20]. Saqlain et al. [21, 22] further extended the generalized fuzzy TOPSIS method using an accuracy function.

The concept of NS sets, introduced by Maji [23], and their extensions, including neutrosophic soft matrices and related decision-making frameworks, have been detailed in [24–27]. Elhassouny and Smarandache [28] proposed a simplified neutrosophic TOPSIS method using single-valued neutrosophic information, while Saqlain et al. presented a generalized neutrosophic TOPSIS model within a neutrosophic hyper soft set environment [21]. Despite the success of these models, many do not explicitly account for temporal aspects in observation data, a limitation in dynamic decision contexts, as discussed in [2, 11, 18, 23, 33, 41, 42].

Real-world decision-making problems often involve uncertain, heterogeneous, inconsistent, and time-sensitive data. Traditional fuzzy set or classical neutrosophic models may fall short in adequately representing such complex scenarios. Therefore, integrating linguistic variables with IVCNS offers a more robust and comprehensive framework. This approach effectively captures the full spectrum of uncertainty and imprecision inherent in modern decision support systems, enabling more accurate, context-aware solutions.

In this paper, we introduce novel concepts under the framework of type-2 interval valued complex neutrosophic set (T2IVLCNS), which offer enhanced flexibility and adaptability for real-world applications compared to their earlier counterparts, as motivated by reference-based analysis. Leveraging the foundational principles of T2IVLCNS, we incorporate key set-theoretic operations, such as complement, union, and intersection, to construct weighted evaluation models. These models are applied to assess candidate alternatives against decision criteria, classify crops based on multiple attributes, determine the relative significance of criteria, and formulate a score function to facilitate the ranking of crop alternatives in a structured decision-making process.

1.1 Motivation of the Research Work

The Bundelkhand region, spanning parts of Uttar Pradesh and Madhya Pradesh, is frequently affected by extreme climatic variability, particularly recurrent droughts, which result in water scarcity, reduced agricultural productivity, and severe socio-economic stress on the farming community. In such ecologically fragile zones, selecting crops that are resilient, resource-efficient, and low-maintenance is crucial for achieving sustainable agricultural practices and enhancing rural livelihoods.

Traditional decision-making models often fall short in such environments due to the inherent uncertainty, vagueness, and complexity involved in evaluating agricultural alternatives under diverse environmental constraints. NS, as a generalization of fuzzy and intuitionistic fuzzy sets, has emerged as a robust mathematical framework for modeling uncertainty, ambiguity, and indeterminacy in complex decision-making problems, by incorporating three independent membership degrees truth (T), indeterminacy (I), and falsity (F). NS and its extensions offer a comprehensive approach to representing imprecise and inconsistent information. However, conventional neutrosophic models assign fixed numerical values to membership degrees, which often fail to capture the subjective and linguistic nature of human evaluations (e.g., “moderate resistance” or “high adaptability”). To address this limitation, the proposed research introduces a novel decision-making approach using the T2IVLCNS. This advanced framework enhances flexibility and realism by incorporating interval-valued linguistic terms to express the nuanced judgments of decision-makers, embedding complex-valued membership functions with phase components to model contextual or spatiotemporal variability (e.g., effects of seasonal droughts) and employing Type-2 fuzzy logic to manage higher-order uncertainties within linguistic evaluations. The T2IVLCNS framework is applied to the problem of identifying the most suitable crop for drought-prone areas of Bundelkhand among six potential alternatives as

- A_1: Peanut (Mungfali)
- A_2: Soybean (Beans)
- A_3: Sesame (Til)
- A_4: Pearl Millet (Bajra)
- A_5: Green Gram (Moong Dal)
- A_6: Black Gram (Urad Dal)

These alternatives are assessed against five critical evaluation criterias as

- C_1: Adaptability to Diverse Environments
- C_2: Climate Resilience
- C_3: Pest Resistance
- C_4: Growth with Minimal Resources
- C_5: Low Input & Maintenance Requirements

The T2IVLCNS model captures the linguistic preferences and uncertainty inherent in expert evaluations, allowing for a holistic and scientifically grounded selection of the optimal crop. This research not only contributes to precision agriculture in challenging environments like Bundelkhand but also demonstrates the efficacy of advanced neutrosophic models in MCDM for real-world problems characterized by indeterminate and vague data.

1.2 Objective of the Investigation

The primary objective of this investigation is to develop and apply an advanced MCDM framework based on T2IVLCNS for selecting the most suitable crop for cultivation in the drought-prone Bundelkhand region. The specific objectives of the study are:-

1. To identify and define a set of critical evaluation criteria, such as adaptability, climate resilience, pest resistance, resource efficiency, and input requirements that influence crop suitability in drought-affected regions.
2. To incorporate expert knowledge and linguistic judgments in evaluating multiple crop alternatives using interval-valued and complex linguistic terms that reflect real-world uncertainty, vagueness, and indeterminacy.
3. To construct a novel T2IVLCNS-based decision model that integrates:-
   (α) Interval-valued linguistic evaluations for expressing degrees of truth, falsity, and indeterminacy;
   (β) Complex neutrosophic membership values to model contextual variability (e.g., seasonal drought impacts);
   (χ) Type-2 fuzzy logic to manage hierarchical and higher-order uncertainties.
4. To apply the proposed T2IVLCNS model for comparative analysis and ranking of six drought-tolerant crops (Peanut, Soybean, Sesame, Pearl Millet, Green Gram, and Black Gram) in the Bundelkhand context.
5. To validate the robustness and applicability of the T2IVLCNS framework in agro-food decision analysis and demonstrate its superiority over traditional decision-making methods.
6. To provide actionable insights and recommendations to policymakers, agricultural planners, and farmers for sustainable crop selection and resource allocation in ecologically vulnerable regions like Bundelkhand.

1.3 Related Work of the Study

Several researchers have explored mathematical models to support decision-making in agriculture under uncertainty, particularly in regions facing climatic challenges such as drought. The following areas provide a foundation for the current investigation:

(i) Neutrosophic Sets and Decision-Making
(ii) Fuzzy and Neutrosophic MCDM Models in Agriculture
(iii) Linguistic Variables in Agro-Decision Models
(iv) Complex Neutrosophic and Hybrid Decision Frameworks
(v) Crop Suitability in Drought-Prone Areas

1.4 Research Gap

Despite these developments, no existing study comprehensively integrates T2IVLCNS with multi-criteria crop selection in drought-affected agro-regions. The proposed work fills this gap by introducing a robust hybrid model that can capture expert knowledge in linguistically rich, uncertain, and multidimensional environments.

2. Preliminaries

Definition 2.1. [48] (Type-1 Single valued Linguistic Complex Neutrosophic Set [T1SVLCNS]): Let $\aleph$ be a universe of discourse and a complex neutrosophic set Λ included Λ in $\aleph$. Let $\tilde{S} = \{\tilde{s}_1, \tilde{s}_2, \tilde{s}_3, ..., \tilde{s}_n,$ for $2 \le n < \infty$ be a set of totally ordered labels (a classical min/max operators work on $\tilde{S}$), with $\tilde{s}_i < \tilde{s}_j$ for $i < j$

where $i,j \in \{1,2,3,.....,n\}$. Let $\tilde{R} = \{[\tilde{s}_i, \tilde{s}_j],\ \tilde{s}_i, \tilde{s}_j \in \tilde{S},\ i < j\}$ be a set of label intervals. A T1SVLCNS is a set $\Lambda \subset \aleph$ such that each element x in Λ has linguistic degree of complex truth membership $T_\Lambda(x) \in \tilde{S} \times \tilde{S}$, a linguistic degree of complex indeterminate membership $I_\Lambda(x) \in \tilde{S} \times \tilde{S}$, and a linguistic degree of complex falsity membership $F_\Lambda(x) \in \tilde{S} \times \tilde{S}$ and $\tilde{s}_{\theta(x)} \in \tilde{S}$. A T1SVLCNS set Λ can be written as $\Lambda = \left\langle x, \left[\tilde{s}_{\theta(x)}, T_\Lambda(x), I_\Lambda(x), F_\Lambda(x) \right] \right\rangle$, where $T_\Lambda(x) = T_{1\Lambda}(x).\ e^{i.T_{2\Lambda}(x)}$, $I_\Lambda(x) = I_{1\Lambda}(x).\ e^{i.I_{2\Lambda}(x)}$, and $F_\Lambda(x) = F_{1\Lambda}(x).\ e^{i.F_{2\Lambda}(x)}$ in which $T_\Lambda(x)$ is representing linguistic amplitude truth membership, and $e^{i.T_{2\Lambda}(x)}$ is denoting the linguistic phase truth membership function, $I_\Lambda(x)$ refers to linguistic amplitude indeterminate membership and $e^{i.I_{2\Lambda}(x)}$ denoting linguistic phase indeterminate membership and $F_\Lambda(x)$ is representing the linguistic amplitude falsity membership and $e^{i.F_{2\Lambda}(x)}$ indicating the linguistic phase falsehood membership function such that

$$3 * \tilde{s}_1 \le \left\langle \min\left(\{T_{1\Lambda}(x)\} + \{I_{1\Lambda}(x)\} + \{F_{1\Lambda}(x)\}\right), \max\left(\{T_{1\Lambda}(x)\} + \{I_{1\Lambda}(x)\} + \{F_{1\Lambda}(x)\}\right) \right\rangle \le 3 * \tilde{s}_n,$$

$$3 * \tilde{s}_1 \le \left\langle \min\left(\{T_{2\Lambda}(x)\} + \{I_{2\Lambda}(x)\} + \{F_{2\Lambda}(x)\}\right), \max\left(\{T_{2\Lambda}(x)\} + \{I_{2\Lambda}(x)\} + \{F_{2\Lambda}(x)\}\right) \right\rangle \le 3 * \tilde{s}_n,$$

Definition 2.2.[48]

Let $\aleph$ be a universe of discourse and a complex neutrosophic set Λ included Λ in $\aleph$. Let $\tilde{S} = \{\tilde{s}_1, \tilde{s}_2, \tilde{s}_3, ..., \tilde{s}_n,$ for $2 \le n < \infty$ be a set of ordered labels with $\tilde{s}_i < \tilde{s}_j$ for $i < j$ where $i,j \in \{1,2,3,.....,n\}$. Let $\tilde{R} = \{[\tilde{s}_i, \tilde{s}_j],\ \tilde{s}_i, \tilde{s}_j \in \tilde{S},\ i < j\}$ be a collection of label intervals. A T2SVLCNS is a set $\Lambda \subset \aleph$ such that each element x in Λ has linguistic degree of complex truth membership $T_\Lambda(x) \in \tilde{R}$ and a linguistic degree of complex indeterminate membership $I_\Lambda(x) \in \tilde{R}$, and a linguistic degree of complex falsity membership $F_\Lambda(x) \in \tilde{R}$ and $\tilde{s}_{\theta(x)} \in \tilde{S}$. A T2SVLCNS set Λ can be written as $\Lambda = \left\langle x, \left[\tilde{s}_{\theta(x)}, T_\Lambda(x), I_\Lambda(x), F_\Lambda(x) \right] \right\rangle$, and $T_\Lambda(x) = T_{1\Lambda}(x).\ e^{i.T_{2\Lambda}(x)}$, $I_\Lambda(x) = I_{1\Lambda}(x).\ e^{i.I_{2\Lambda}(x)}$ $F_\Lambda(x) = F_{1\Lambda}(x).\ e^{i.F_{2\Lambda}(x)}$, where $T_\Lambda(x)$ is representing linguistic amplitude truth membership and $e^{i.T_{2\Lambda}(x)}$ is denoting the linguistic phase truth membership function, $I_\Lambda(x)$ refers to linguistic amplitude indeterminate membership and $e^{i.I_{2\Lambda}(x)}$ indicates linguistic phase indeterminate membership. Further, $F_\Lambda(x)$ is called the linguistic amplitude falsity membership and $e^{i.F_{2\Lambda}(x)}$ is said to be the linguistic phase falsehood membership function while $0 \le T_{\Lambda}(x)+I_{\Lambda}(x)+F_{\Lambda}(x) \le 3.$ Here we proposed T2IVLCNS as follows:

Definition 2.3

Let X be universe of discourse and Λ be a CNS defined on X. Let $\Lambda = \{\tilde{s}_1, \tilde{s}_2, \tilde{s}_3, ...\tilde{s}_n, \text{ for } 2 \le n \le \infty\}$ be a collection of single value, linguistic markers, where $\tilde{s}_1 < \tilde{s}_2 < \tilde{s}_3 < ..... < \tilde{s}_n$ and they are the qualitative values of a linguistic variables. The linguistic relation of order $\tilde{s}_i < \tilde{s}_j$, for $i < j$ means that label $\tilde{s}_i$ is less important than label $\tilde{s}_j$. Let $\tilde{R} = \{[\tilde{s}_1, \tilde{s}_2]; \tilde{s}_i, \tilde{s}_j \in \tilde{S}, \text{ for } i < j\}$ be a set of label intervals. An T2IVLCNS is a set $\Lambda \subset X$ defined as $\Lambda = \left\langle x, \left[\tilde{s}_{(\theta)}, T_{\Lambda}(x), I_{\Lambda}(x), F_{\Lambda}(x)\right]\right\rangle$ such that each element x in Λ has linguistic degree of complex interval-truth membership $T_{\Lambda}(x) \in \tilde{R} \times \tilde{R}$, a linguistic degree of complex interval-indeterminate membership $I_{\Lambda}(x) \in \tilde{R} \times \tilde{R}$ and a linguistic degree of complex interval-falsity membership $F_{\Lambda}(x) \in \tilde{R} \times \tilde{R}$ and $\tilde{s}_{(\theta)} \in \Lambda$, where $T_{\Lambda}(x) = [\inf r_{\Lambda}(x). \sup r_{\Lambda}(x)].e^{i[\inf \omega_{\Lambda}(x), \sup \omega_{\Lambda}(x)]}$,

$$I_{\Lambda}(x) = [\inf k_{\Lambda}(x). \sup k_{\Lambda}(x)].e^{i[\inf \rho_{\Lambda}(x), \sup \rho_{\Lambda}(x)]},$$

$$F_{\Lambda}(x) = [\inf t_{\Lambda}(x). \sup t_{\Lambda}(x)].e^{i[\inf \sigma_{\Lambda}(x), \sup \sigma_{\Lambda}(x)]}.$$

The term $[\inf r_{\Lambda}(x). \sup r_{\Lambda}(x)]$ representing linguistic interval-amplitude truth membership and the term $[\inf \omega_{\Lambda}(x), \sup \omega_{\Lambda}(x)]$ is denoting the linguistic interval-phase truth membership function. The term $[\inf k_{\Lambda}(x). \sup k_{\Lambda}(x)]$ representing linguistic interval-amplitude indeterminacy membership and $[\inf \rho_{\Lambda}(x), \sup \rho_{\Lambda}(x)]$ is denoting the linguistic interval-phase indeterminacy membership function. Further the term $[\inf t_{\Lambda}(x). \sup t_{\Lambda}(x)]$ representing linguistic interval-amplitude falsity membership and $[\inf \sigma_{\Lambda}(x), \sup \sigma_{\Lambda}(x)]$ is denoting the linguistic interval-phase falsehood membership function. For smooth computation the T2IVLCNS represented

$$[\inf r_{\Lambda}(x). \sup r_{\Lambda}(x)] = \left[r_{\Lambda}^{L}(x), r_{\Lambda}^{U}(x)\right] = \left[r_{\Lambda}^{L}, r_{\Lambda}^{U}\right], \ [\inf k_{\Lambda}(x). \sup k_{\Lambda}(x)] = \left[k_{\Lambda}^{L}(x), k_{\Lambda}^{U}(x)\right] = \left[k_{\Lambda}^{L}, k_{\Lambda}^{U}\right],$$

$$[\inf t_{\Lambda}(x). \sup t_{\Lambda}(x)] = \left[t_{\Lambda}^{L}(x), t_{\Lambda}^{U}(x)\right] = \left[t_{\Lambda}^{L}, t_{\Lambda}^{U}\right], \ [\inf \omega_{\Lambda}(x), \sup \omega_{\Lambda}(x)] = \left[\omega_{\Lambda}^{L}(x), \omega_{\Lambda}^{U}(x)\right] = \left[\omega_{\Lambda}^{L}, \omega_{\Lambda}^{U}\right],$$

$$[\inf \rho_{\Lambda}(x), \sup \rho_{\Lambda}(x)] = \left[\rho_{\Lambda}^{L}(x), \rho_{\Lambda}^{U}(x)\right] = \left[\rho_{\Lambda}^{L}, \rho_{\Lambda}^{U}\right], \ [\inf \sigma_{\Lambda}(x), \sup \sigma_{\Lambda}(x)] = \left[\sigma_{\Lambda}^{L}(x), \sigma_{\Lambda}^{U}(x)\right] = \left[\sigma_{\Lambda}^{L}, \sigma_{\Lambda}^{U}\right].$$

So that $\Lambda = \left\langle x, \left[\tilde{s}_{(\theta)}, \left[r_{\Lambda}^{L}, r_{\Lambda}^{U}\right].e^{i\left[\omega_{\Lambda}^{L}, \omega_{\Lambda}^{U}\right]}, \left[k_{\Lambda}^{L}, k_{\Lambda}^{U}\right].e^{i\left[\rho_{\Lambda}^{L}, \rho_{\Lambda}^{U}\right]}, \left[t_{\Lambda}^{L}, t_{\Lambda}^{U}\right].e^{i\left[\sigma_{\Lambda}^{L}, \sigma_{\Lambda}^{U}\right]}\right]\right\rangle$.

Definition 2.4.

Let $\Lambda = \left\langle x, \left[\tilde{s}_{\theta_{\Lambda}(x)}, \left[r_{\Lambda}^{L}, r_{\Lambda}^{U}\right].e^{i\left[\omega_{\Lambda}^{L}, \omega_{\Lambda}^{U}\right]}, \left[k_{\Lambda}^{L}, k_{\Lambda}^{U}\right].e^{i\left[\rho_{\Lambda}^{L}, \rho_{\Lambda}^{U}\right]}, \left[t_{\Lambda}^{L}, t_{\Lambda}^{U}\right].e^{i\left[\sigma_{\Lambda}^{L}, \sigma_{\Lambda}^{U}\right]}\right]\right\rangle$ and $\Sigma = \left\langle x, \left[\tilde{s}_{\theta_{\Sigma}(x)}, \left[r_{\Sigma}^{L}, r_{\Sigma}^{U}\right].e^{i\left[\omega_{\Sigma}^{L}, \omega_{\Sigma}^{U}\right]}, \left[k_{\Sigma}^{L}, k_{\Sigma}^{U}\right].e^{i\left[\rho_{\Sigma}^{L}, \rho_{\Sigma}^{U}\right]}, \left[t_{\Sigma}^{L}, t_{\Sigma}^{U}\right].e^{i\left[\sigma_{\Sigma}^{L}, \sigma_{\Sigma}^{U}\right]}\right]\right\rangle$ be two T2IVLCNS respectively. Then

Union: $\Lambda \bigcup \Sigma = \left\langle x, \left[\tilde{s}_{\theta_{\Lambda\cup\Sigma}(x)}, T_{\Lambda\cup\Sigma}(x), I_{\Lambda\cup\Sigma}(x), F_{\Lambda\cup\Sigma}(x)\right]\right\rangle$, here $\tilde{s}_{\theta_{\Lambda\cup\Sigma}(x)} = \vee\left(\tilde{s}_{\theta_{\Lambda}(x)}, \tilde{s}_{\theta_{\Sigma}(x)}\right)$

$T_{\Lambda\cup\Sigma}(x) = \left[r_{\Lambda\cup\Sigma}^{L}, r_{\Lambda\cup\Sigma}^{U}\right].e^{i\left[\omega_{\Lambda\cup\Sigma}^{L}, \omega_{\Lambda\cup\Sigma}^{U}\right]}$, where $r_{\Lambda\cup\Sigma}^{L} = \vee\left(r_{\Lambda}^{L}, r_{\Sigma}^{L}\right), \ r_{\Lambda\cup\Sigma}^{U} = \vee\left(r_{\Lambda}^{U}, r_{\Sigma}^{U}\right),$

$\omega_{\Lambda\cup\Sigma}^{L} = \vee\left(\omega_{\Lambda}^{L}, \omega_{\Sigma}^{L}\right), \ \omega_{\Lambda\cup\Sigma}^{U} = \vee\left(\omega_{\Lambda}^{U}, \omega_{\Sigma}^{U}\right)$

$I_{\Lambda\cup\Sigma}(x) = \left[k_{\Lambda\cup\Sigma}^{L}, k_{\Lambda\cup\Sigma}^{U}\right].e^{i\left[\rho_{\Lambda\cup\Sigma}^{L}, \rho_{\Lambda\cup\Sigma}^{U}\right]},$ $k_{\Lambda\cup\Sigma}^{L} = \wedge\left(k_{\Lambda}^{L}, k_{\Sigma}^{L}\right), k_{\Lambda\cup\Sigma}^{U} = \wedge\left(k_{\Lambda}^{U}, k_{\Sigma}^{U}\right),$

$\rho_{\Lambda\cup\Sigma}^{L} = \wedge\left(\rho_{\Lambda}^{L}, \rho_{\Sigma}^{L}\right), \rho_{\Lambda\cup\Sigma}^{U} = \wedge\left(\rho_{\Lambda}^{U}, \rho_{\Sigma}^{U}\right)$

$F_{\Lambda\cup\Sigma}(x) = \left[t_{\Lambda\cup\Sigma}^{L}, t_{\Lambda\cup\Sigma}^{U}\right].e^{i\left[\sigma_{\tilde{S}\cup\tilde{P}}^{L}, \sigma_{\tilde{S}\cup\tilde{P}}^{U}\right]},$ $t_{\Lambda\cup\Sigma}^{L} = \wedge\left(t_{\Lambda}^{L}, t_{\Sigma}^{L}\right), t_{\Lambda\cup\Sigma}^{U} = \wedge\left(t_{\Lambda}^{U}, t_{\Sigma}^{U}\right),$

$\sigma_{\Lambda\cup\Sigma}^{L} = \wedge\left(\sigma_{\Lambda}^{L}, \sigma_{\Sigma}^{L}\right), \sigma_{\Lambda\cup\Sigma}^{U} = \wedge\left(\sigma_{\Lambda}^{U}, \sigma_{\Sigma}^{U}\right)$

for all $x \in \mathrm{X}$. The symbols $\vee, \wedge$ represents maximize and minimize operators, respectively.

Intersection: $\Lambda \bigcap \Sigma = \left\langle x, \left[\tilde{s}_{\theta_{\Lambda\cap\Sigma}(x)}, T_{\Lambda\cap\Sigma}(x), I_{\Lambda\cap\Sigma}(x), F_{\Lambda\cap\Sigma}(x)\right]\right\rangle$, where $\tilde{s}_{\theta_{\Lambda\cap\Sigma}(x)} = \wedge\left(\tilde{s}_{\theta_{\Lambda}(x)}, \tilde{s}_{\theta_{\Sigma}(x)}\right)$

$T_{\Lambda\cap\Sigma}(x) = \left[r_{\Lambda\cap\Sigma}^{L}, r_{\Lambda\cap\Sigma}^{U}\right].e^{i\left[\omega_{\Lambda\cap\Sigma}^{L}, \omega_{\Lambda\cap\Sigma}^{U}\right]},$ $r_{\Lambda\cap\Sigma}^{L} = \wedge\left(r_{\Lambda}^{L}, r_{\Sigma}^{L}\right), \ r_{\Lambda\cap\Sigma}^{U} = \wedge\left(r_{\Lambda}^{U}, r_{\Sigma}^{U}\right),$

$\omega_{\Lambda\cap\Sigma}^{L} = \wedge\left(\omega_{\Lambda}^{L}, \omega_{\Sigma}^{L}\right), \ \omega_{\Lambda\cap\Sigma}^{U} = \wedge\left(\omega_{\Lambda}^{U}, \omega_{\Sigma}^{U}\right)$

$I_{\Lambda\cap\Sigma}(x) = \left[k_{\Lambda\cap\Sigma}^{L}, k_{\Lambda\cap\Sigma}^{U}\right].e^{i\left[\rho_{\Lambda\cap\Sigma}^{L}, \rho_{\Lambda\cap\Sigma}^{U}\right]},$ $k_{\Lambda\cap\Sigma}^{L} = \vee\left(k_{\Lambda}^{L}, k_{\Sigma}^{L}\right), k_{\Lambda\cap\Sigma}^{U} = \vee\left(k_{\Lambda}^{U}, k_{\Sigma}^{U}\right),$

$\rho_{\Lambda\cap\Sigma}^{L} = \vee\left(\rho_{\Lambda}^{L}, \rho_{\Sigma}^{L}\right), \rho_{\Lambda\cap\Sigma}^{U} = \vee\left(\rho_{\Lambda}^{U}, \rho_{\Sigma}^{U}\right)$

$F_{\Lambda\cap\Sigma}(x) = \left[t_{\Lambda\cap\Sigma}^{L}, t_{\Lambda\cap\Sigma}^{U}\right].e^{i\left[\sigma_{\Lambda\cap\Sigma}^{L}, \sigma_{\Lambda\cap\Sigma}^{U}\right]},$ $t_{\Lambda\cap\Sigma}^{L} = \vee\left(t_{\Lambda}^{L}, t_{\Sigma}^{L}\right), t_{\Lambda\cap\Sigma}^{U} = \vee\left(t_{\Lambda}^{U}, t_{\Sigma}^{U}\right),$

$\sigma_{\Lambda\cap\Sigma}^{L} = \vee\left(\sigma_{\Lambda}^{L}, \sigma_{\Sigma}^{L}\right), \sigma_{\Lambda\cap\Sigma}^{U} = \vee\left(\sigma_{\Lambda}^{U}, \sigma_{\Sigma}^{U}\right)$

for all $x \in \mathrm{X}$. The symbols $\vee, \wedge$ represents maximize and minimize operators, respectively.

Proposition 2.1

Let $\Lambda = \left\langle x, \left[\tilde{s}_{\theta_\Lambda(x)}, \left[r_\Lambda^L, r_\Lambda^U \right].e^{i\left[\omega_\Lambda^L, \omega_\Lambda^U\right]}, \left[k_\Lambda^L, k_\Lambda^U \right].e^{i\left[\rho_\Lambda^L, \rho_\Lambda^U\right]}, \left[t_\Lambda^L, t_\Lambda^U \right].e^{i\left[\sigma_\Lambda^L, \sigma_\Lambda^U\right]} \right] \right\rangle$

and $\Sigma = \left\langle x, \left[\tilde{s}_{\theta_\Sigma(x)}, \left[r_\Sigma^L, r_\Sigma^U \right].e^{i\left[\omega_\Sigma^L, \omega_\Sigma^U\right]}, \left[k_\Sigma^L, k_\Sigma^U \right].e^{i\left[\rho_\Sigma^L, \rho_\Sigma^U\right]}, \left[t_\Sigma^L, t_\Sigma^U \right].e^{i\left[\sigma_\Sigma^L, \sigma_\Sigma^U\right]} \right] \right\rangle$ be two T2IVLCNS respectively. Then $\Lambda \bigcup \Sigma = \Sigma \bigcup \Lambda,\ \Lambda \bigcap \Sigma = \Sigma \bigcap \Lambda,\ \Lambda \bigcup \Lambda = \Lambda,\ \Lambda \bigcap \Lambda = \Lambda.$

Proposition 2.2

Let $\Lambda = \left\langle x, \left[\tilde{s}_{\theta_\Lambda(x)}, \left[r_\Lambda^L, r_\Lambda^U \right].e^{i\left[\omega_\Lambda^L, \omega_\Lambda^U\right]}, \left[k_\Lambda^L, k_\Lambda^U \right].e^{i\left[\rho_\Lambda^L, \rho_\Lambda^U\right]}, \left[t_\Lambda^L, t_\Lambda^U \right].e^{i\left[\sigma_\Lambda^L, \sigma_\Lambda^U\right]} \right] \right\rangle$

$$\Sigma = \left\langle x, \left[\tilde{s}_{\theta_\Sigma(x)}, \left[r_\Sigma^L, r_\Sigma^U \right].e^{i\left[\omega_\Sigma^L, \omega_\Sigma^U\right]}, \left[k_\Sigma^L, k_\Sigma^U \right].e^{i\left[\rho_\Sigma^L, \rho_\Sigma^U\right]}, \left[t_\Sigma^L, t_\Sigma^U \right].e^{i\left[\sigma_\Sigma^L, \sigma_\Sigma^U\right]} \right] \right\rangle$$

and be three T2IVLCNS respectively. Then

$$\Lambda \bigcup \left(\Sigma \bigcup \Omega \right) = \left(\Sigma \bigcup \Lambda \right) \bigcup \Omega,\ \ \Lambda \bigcap \left(\Sigma \bigcap \Omega \right) = \left(\Sigma \bigcap \Lambda \right) \bigcap \Omega,$$

$$\Lambda \bigcup \left(\Sigma \bigcap \Omega \right) = \left(\Lambda \bigcup \Sigma \right) \bigcap \left(\Lambda \bigcup \Omega \right), \quad \Lambda \bigcup \left(\Lambda \bigcap \Lambda \right) = \Lambda$$

$$\Lambda \bigcap \left(\Sigma \bigcup \Omega \right) = \left(\Lambda \bigcap \Sigma \right) \bigcup \left(\Lambda \bigcap \Omega \right), \quad \Lambda \bigcap \left(\Lambda \bigcup \Lambda \right) = \Lambda.$$

Note: The union of two T2IVLCNS i.e. $\Lambda \bigcup \Sigma$ is the minimum set comprising together Λ and Σ. The intersection of two T2IVLCNS i.e. $\Lambda \bigcap \Sigma$ is the leading one enclosed in Λ and Σ. Let Π be the power set of all T2IVLCNS then, $\left(\Pi, \bigcup, \bigcap \right)$ forms a distributive lattice.

3. Hamming and Euclidian distances

Definition 3.1

Let $\Lambda = \left\langle x, \left[\tilde{s}_{\theta_\Lambda(x)}, \left[r_\Lambda^L, r_\Lambda^U \right].e^{i\left[\omega_\Lambda^L, \omega_\Lambda^U\right]}, \left[k_\Lambda^L, k_\Lambda^U \right].e^{i\left[\rho_\Lambda^L, \rho_\Lambda^U\right]}, \left[t_\Lambda^L, t_\Lambda^U \right].e^{i\left[\sigma_\Lambda^L, \sigma_\Lambda^U\right]} \right] \right\rangle$ and

$\Sigma = \left\langle x, \left[\tilde{s}_{\theta_\Sigma(x)}, \left[r_\Sigma^L, r_\Sigma^U \right].e^{i\left[\omega_\Sigma^L, \omega_\Sigma^U\right]}, \left[k_\Sigma^L, k_\Sigma^U \right].e^{i\left[\rho_\Sigma^L, \rho_\Sigma^U\right]}, \left[t_\Sigma^L, t_\Sigma^U \right].e^{i\left[\sigma_\Sigma^L, \sigma_\Sigma^U\right]} \right] \right\rangle$ be two T2IVLCNS respectively.

Then
$$d_H^a(\Lambda, \Sigma) = \frac{1}{6(n-1)} \begin{bmatrix} \left| \tilde{s}_{\theta_\Lambda(x)} \times r_\Lambda^L - \tilde{s}_{\theta_\Sigma(x)} \times r_\Sigma^L \right| + \left| \tilde{s}_{\theta_\Lambda(x)} \times r_\Lambda^U - \tilde{s}_{\theta_\Sigma(x)} \times r_\Sigma^U \right| + \\ \left| \tilde{s}_{\theta_\Lambda(x)} \times k_\Lambda^L - \tilde{s}_{\theta_\Sigma(x)} \times k_\Sigma^L \right| + \left| \tilde{s}_{\theta_\Lambda(x)} \times k_\Lambda^U - \tilde{s}_{\theta_\Sigma(x)} \times k_\Sigma^U \right| + \\ \left| \tilde{s}_{\theta_\Lambda(x)} \times t_\Lambda^L - \tilde{s}_{\theta_\Sigma(x)} \times t_\Sigma^L \right| + \left| \tilde{s}_{\theta_\Lambda(x)} \times t_\Lambda^U - \tilde{s}_{\theta_\Sigma(x)} \times t_\Sigma^U \right| \end{bmatrix} \tag{1}$$

$$d_H^p(\Lambda, \Sigma) = \left[\left| \omega_\Lambda^L - \omega_\Sigma^L \right| + \left| \omega_\Lambda^U - \omega_\Sigma^U \right| + \left| \rho_\Lambda^L - \rho_\Sigma^L \right| + \left| \rho_\Lambda^U - \rho_\Sigma^U \right| + \left| \sigma_\Lambda^L - \sigma_\Sigma^L \right| + \left| \sigma_\Lambda^U - \sigma_\Sigma^U \right| \right] \tag{2}$$

$$d_E^a(\Lambda, \Sigma) = \sqrt{ \frac{1}{6(n-1)} \begin{bmatrix} \left(\tilde{s}_{\theta_\Lambda(x)} \times r_\Lambda^L - \tilde{s}_{\theta_\Sigma(x)} \times r_\Sigma^L \right)^2 + \left(\tilde{s}_{\theta_\Lambda(x)} \times r_\Lambda^U - \tilde{s}_{\theta_\Sigma(x)} \times r_\Sigma^U \right)^2 + \\ \left(\tilde{s}_{\theta_\Lambda(x)} \times k_\Lambda^L - \tilde{s}_{\theta_\Sigma(x)} \times k_\Sigma^L \right)^2 + \left(\tilde{s}_{\theta_\Lambda(x)} \times k_\Lambda^U - \tilde{s}_{\theta_\Sigma(x)} \times k_\Sigma^U \right)^2 + \\ \left(\tilde{s}_{\theta_\Lambda(x)} \times t_\Lambda^L - \tilde{s}_{\theta_\Sigma(x)} \times t_\Sigma^L \right)^2 + \left(\tilde{s}_{\theta_\Lambda(x)} \times t_\Lambda^U - \tilde{s}_{\theta_\Sigma(x)} \times t_\Sigma^U \right)^2 \end{bmatrix} } \tag{3}$$

$$d_E^p(\Lambda, \Sigma) = \sqrt{ \left[\left(\omega_\Lambda^L - \omega_\Sigma^L \right)^2 + \left(\omega_\Lambda^U - \omega_\Sigma^U \right)^2 + \left(\rho_\Lambda^L - \rho_\Sigma^L \right)^2 + \left(\rho_\Lambda^U - \rho_\Sigma^U \right)^2 \left(\sigma_\Lambda^L - \sigma_\Sigma^L \right)^2 + \left(\sigma_\Lambda^U - \sigma_\Sigma^U \right)^2 \right] } \tag{4}$$

4. Operational Rules of T2IVLCNS

Definition 4.1

Let $\Lambda=\left\langle x,\left[\tilde{s}_{\theta_\Lambda(x)},\left[r_\Lambda^L,r_\Lambda^U\right].e^{i\left[\omega_\Lambda^L,\omega_\Lambda^U\right]},\left[k_\Lambda^L,k_\Lambda^U\right].e^{i\left[\rho_\Lambda^L,\rho_\Lambda^U\right]},\left[t_\Lambda^L,t_\Lambda^U\right].e^{i\left[\sigma_\Lambda^L,\sigma_\Lambda^U\right]}\right]\right\rangle$ and $\Sigma=\left\langle x,\left[\tilde{s}_{\theta_\Sigma(x)},\left[r_\Sigma^L,r_\Sigma^U\right].e^{i\left[\omega_\Sigma^L,\omega_\Sigma^U\right]},\left[k_\Sigma^L,k_\Sigma^U\right].e^{i\left[\rho_\Sigma^L,\rho_\Sigma^U\right]},\left[t_\Sigma^L,t_\Sigma^U\right].e^{i\left[\sigma_\Sigma^L,\sigma_\Sigma^U\right]}\right]\right\rangle$ be two T2IVLCNS respectively. Then the operational rules of T2IVLCNS are illustrated as:

(i) Product: $\Lambda\otimes\Sigma=\left\langle x,\left[\tilde{s}_{\theta_{\Lambda\otimes\Sigma}(x)},T_{\Lambda\otimes\Sigma}(x),I_{\Lambda\otimes\Sigma}(x),F_{\Lambda\otimes\Sigma}(x)\right]\right\rangle$, where $\tilde{s}_{\theta_{\Lambda\otimes\Sigma}(x)}=\tilde{s}_{\theta_\Lambda(x)}.\tilde{s}_{\theta_\Sigma(x)}$ and

$$T_{\Lambda\otimes\Sigma}(x)=\left[T_{\Lambda\otimes\Sigma}^L(x),T_{\Lambda\otimes\Sigma}^U(x)\right]=\left[\left(r_\Lambda^L.r_\Sigma^L\right).e^{i\left(\omega_\Lambda^L.\omega_\Sigma^L\right)},\left(r_\Lambda^U.r_\Sigma^U\right).e^{i\left(\omega_\Lambda^U.\omega_\Sigma^U\right)}\right],$$

$$I_{\Lambda\otimes\Sigma}(x)=\left[I_{\Lambda\otimes\Sigma}^L(x),I_{\Lambda\otimes\Sigma}^U(x)\right]=\left[\left(k_\Lambda^L.k_\Sigma^L\right).e^{i\left(\rho_\Lambda^L.\rho_\Sigma^L\right)},\left(k_\Lambda^U.k_\Sigma^U\right).e^{i\left(\rho_\Lambda^U.\rho_\Sigma^U\right)}\right],$$

$$F_{\Lambda\otimes\Sigma}(x)=\left[F_{\Lambda\otimes\Sigma}^L(x),F_{\Lambda\otimes\Sigma}^U(x)\right]=\left[\left(t_\Lambda^L.t_\Sigma^L\right).e^{i\left(\sigma_\Lambda^L.\sigma_\Sigma^L\right)},\left(t_\Lambda^U.t_\Sigma^U\right).e^{i\left(\sigma_\Lambda^U.\sigma_\Sigma^U\right)}\right],$$

(ii) Addition: $\Lambda\oplus\Sigma=\left\langle x,\left[\tilde{s}_{\theta_{\Lambda\oplus\Sigma}(x)},T_{\Lambda\oplus\Sigma}(x),I_{\Lambda\oplus\Sigma}(x),F_{\Lambda\oplus\Sigma}(x)\right]\right\rangle$, where $\tilde{s}_{\theta_{\Lambda\oplus\Sigma}(x)}=\tilde{s}_{\theta_\Lambda(x)}+\tilde{s}_{\theta_\Sigma(x)}$ and

$$T_{\Lambda\oplus\Sigma}(x)=\left[T_{\Lambda\otimes\Sigma}^L(x),T_{\Lambda\otimes\Sigma}^U(x)\right]=\left[\left(r_\Lambda^L+r_\Sigma^L-r_\Lambda^L.r_\Sigma^L\right).e^{i\left(\omega_\Lambda^L+\omega_\Sigma^L\right)},\left(r_\Lambda^U+r_\Sigma^U-r_\Lambda^U.r_\Sigma^U\right).e^{i\left(\omega_\Lambda^U+\omega_\Sigma^U\right)}\right],$$

$$I_{\Lambda\otimes\Sigma}(x)=\left[I_{\Lambda\otimes\Sigma}^L(x),I_{\Lambda\otimes\Sigma}^U(x)\right]=\left[\left(k_\Lambda^L+k_\Sigma^L\right).e^{i\left(\rho_\Lambda^L+\rho_\Sigma^L\right)},\left(k_\Lambda^U+k_\Sigma^U\right).e^{i\left(\rho_\Lambda^U.\rho_\Sigma^U\right)}\right],$$

$$F_{\Lambda\otimes\Sigma}(x)=\left[F_{\Lambda\otimes\Sigma}^L(x),F_{\Lambda\otimes\Sigma}^U(x)\right]=\left[\left(t_\Lambda^L+t_\Sigma^L\right).e^{i\left(\sigma_\Lambda^L+\sigma_\Sigma^L\right)},\left(t_\Lambda^U+t_\Sigma^U\right).e^{i\left(\sigma_\Lambda^U+\sigma_\Sigma^U\right)}\right],$$

(iii) Scalar Multiplication: For all $\lambda>0$, $\lambda\Lambda=\left\langle x,\left[\tilde{s}_{\theta_{\lambda\Lambda}(x)},T_{\lambda\Lambda}(x),I_{\lambda\Lambda}(x),F_{\lambda\Lambda}(x)\right]\right\rangle$, $\tilde{s}_{\theta_{\lambda\Lambda}(x)}=\lambda\tilde{s}_{\theta_\Lambda(x)}$

$$T_{\lambda\Lambda}(x)=\left[T_{\lambda\Lambda}^L(x),T_{\lambda\Lambda}^U(x)\right]=\left[\left(1\text{-}\left(1\text{-}r_\Lambda^L\right)^\lambda\right).e^{i\lambda\omega_\Lambda^L},\left(1\text{-}\left(1\text{-}r_\Lambda^U\right)^\lambda\right).e^{i\lambda\omega_\Lambda^U}\right],$$

$$I_{\lambda\Lambda}(x)=\left[I_{\lambda\Lambda}^L(x),I_{\lambda\Lambda}^U(x)\right]=\left[\left(1\text{-}\left(1\text{-}k_\Lambda^L\right)^\lambda\right).e^{i\lambda\rho_\Lambda^L},\left(1\text{-}\left(1\text{-}k_\Lambda^U\right)^\lambda\right).e^{i\lambda\rho_\Lambda^U}\right],$$

$$F_{\lambda\Lambda}(x)=\left[F_{\lambda\Lambda}^L(x),F_{\lambda\Lambda}^U(x)\right]=\left[\left(1\text{-}\left(1\text{-}t_\Lambda^L\right)^\lambda\right).e^{i\lambda\sigma_\Lambda^L},\left(1\text{-}\left(1\text{-}t_\Lambda^U\right)^\lambda\right).e^{i\lambda\sigma_\Lambda^U}\right],$$

Proposition 4.1

Let $\Lambda=\left\langle x,\left[\tilde{s}_{\theta_\Lambda(x)},\left[r_\Lambda^L,r_\Lambda^U\right].e^{i\left[\omega_\Lambda^L,\omega_\Lambda^U\right]},\left[k_\Lambda^L,k_\Lambda^U\right].e^{i\left[\rho_\Lambda^L,\rho_\Lambda^U\right]},\left[t_\Lambda^L,t_\Lambda^U\right].e^{i\left[\sigma_\Lambda^L,\sigma_\Lambda^U\right]}\right]\right\rangle$ and $\Sigma=\left\langle x,\left[\tilde{s}_{\theta_\Sigma(x)},\left[r_\Sigma^L,r_\Sigma^U\right].e^{i\left[\omega_\Sigma^L,\omega_\Sigma^U\right]},\left[k_\Sigma^L,k_\Sigma^U\right].e^{i\left[\rho_\Sigma^L,\rho_\Sigma^U\right]},\left[t_\Sigma^L,t_\Sigma^U\right].e^{i\left[\sigma_\Sigma^L,\sigma_\Sigma^U\right]}\right]\right\rangle$ be two T2IVLCNS respectively. We have

$\Lambda\otimes\Sigma=\Sigma\otimes\Lambda$, $\Lambda\oplus\Sigma=\Sigma\oplus\Lambda$, $\lambda\left(\Lambda\otimes\Sigma\right)=\lambda\left(\Sigma\otimes\Lambda\right)$, $\left(\lambda_1\otimes\lambda_2\right)\Lambda=\lambda_1\Lambda\otimes\lambda_2\Lambda.$

5. Generalized TOPSIS Model for T2IVLCNS

For the generalized TOPSIS model for IVLCNS, we suppose that a team of h DM's D_q for $q=1,......,h$ is accountable for assessing t alternatives $A_m\left(m=1,......,t\right)$ under the n selection criteria $C_p\left(p=1,......,n\right)$. The following steps are used for the proposed generalized TOPSIS technique.

Step I: Aggregate Ratings of Alternatives versus Criteria

Let $\Lambda_{mpq}=\left\langle x,\left[\left(\tilde{s}_{\theta_\Lambda(x)}\right)_{mpq},\left(T_\Lambda(x)\right)_{mpq},\left(I_\Lambda(x)\right)_{mpq},\left(F_\Lambda(x)\right)_{mpq}\right]\right\rangle$ be the suitability assessment allocated to A_a alternative by D_d decision makers for criteria C_c where

$$\left(T_\Lambda(x)\right)_{mpq}=\left[\left(T_\Lambda(x)\right)_{mpq}^L,\left(T_\Lambda(x)\right)_{mpq}^U\right]=\left[\left(r_\Lambda^L\right)_{mpq},\left(r_\Lambda^U\right)_{mpq}\right].e^{i\left[\left(\omega_\Lambda^L\right)_{mpq},\left(\omega_\Lambda^U\right)_{mpq}\right]}$$

$$\left(I_\Lambda(x)\right)_{mpq}=\left[\left(I_\Lambda(x)\right)_{mpq}^L,\left(I_\Lambda(x)\right)_{mpq}^U\right]=\left[\left(k_\Lambda^L\right)_{mpq},\left(k_\Lambda^U\right)_{mpq}\right].e^{i\left[\left(\rho_\Lambda^L\right)_{mpq},\left(\rho_\Lambda^U\right)_{mpq}\right]}$$

$$\left(F_\Lambda(x)\right)_{mpq}=\left[\left(F_\Lambda(x)\right)_{mpq}^L,\left(F_\Lambda(x)\right)_{mpq}^U\right]=\left[\left(t_\Lambda^L\right)_{mpq},\left(t_\Lambda^U\right)_{mpq}\right].e^{i\left[\left(\sigma_\Lambda^L\right)_{mpq},\left(\sigma_\Lambda^U\right)_{mpq}\right]}$$

for $m=1,......,t,\ \ p=1,......,n$ and $q=1,......,h.$ Using the operational rules for T2IVLCNS, the average suitability $\Lambda_{mp}=\left\langle x,\left[\left(\tilde{s}_{\theta_\Lambda(x)}\right)_{mp},\left(T_\Lambda(x)\right)_{mp},\left(I_\Lambda(x)\right)_{mp},\left(F_\Lambda(x)\right)_{mp}\right]\right\rangle$ can be evaluated as follows:

$$\left(T_\Lambda(x)\right)_{mp}=\left[\left(T_\Lambda(x)\right)_{mp}^L,\left(T_\Lambda(x)\right)_{mp}^U\right]=\left[\left(1-\left(\prod_{q=1}^{h}\left(1-r_\Lambda^L\right)_{mpq}\right)^{\frac{1}{h}}\right),\left(1-\left(\prod_{q=1}^{h}\left(1-r_\Lambda^U\right)_{mpq}\right)^{\frac{1}{h}}\right)\right].e^{i\left[\frac{1}{h}\sum_{q=1}^{h}\left(\omega_\Lambda^L\right)_{mq},\frac{1}{h}\sum_{q=1}^{h}\left(\omega_\Lambda^U\right)_{mq}\right]}$$

$$\left(I_\Lambda(x)\right)_{mp}=\left[\left(I_\Lambda(x)\right)_{mp}^L,\left(I_\Lambda(x)\right)_{mp}^U\right]=\left[\left(\prod_{q=1}^{h}\left(k_\Lambda^L\right)_{mpq}\right)^{\frac{1}{h}},\left(\prod_{q=1}^{h}\left(k_\Lambda^U\right)_{mpq}\right)^{\frac{1}{h}}\right].e^{i\left[\frac{1}{h}\sum_{q=1}^{h}\left(\rho_\Lambda^L\right)_{mq},\frac{1}{h}\sum_{q=1}^{h}\left(\rho_\Lambda^U\right)_{mq}\right]}$$

$$\left(F_\Lambda(x)\right)_{mp}=\left[\left(F_\Lambda(x)\right)_{mp}^L,\left(F_\Lambda(x)\right)_{mp}^U\right]=\left[\left(\prod_{q=1}^{h}\left(t_\Lambda^L\right)_{mpq}\right)^{\frac{1}{h}},\left(\prod_{q=1}^{h}\left(t_\Lambda^U\right)_{mpq}\right)^{\frac{1}{h}}\right].e^{i\left[\frac{1}{h}\sum_{q=1}^{h}\left(\sigma_\Lambda^L\right)_{mq},\frac{1}{h}\sum_{q=1}^{h}\left(\sigma_\Lambda^U\right)_{mq}\right]} \tag{5}$$

Step II: Aggregate the Importance Weights

Let $w_{pq}=\left\langle x,\left[\left(\tilde{s}_{\theta_\Lambda(x)}\right)_{pq},\left(T_\Lambda(x)\right)_{pq},\left(I_\Lambda(x)\right)_{pq},\left(F_\Lambda(x)\right)_{pq}\right]\right\rangle$ be the weight allocated by decision makers D_d to criterion C_c, where $\left(T_\Lambda(x)\right)_{pq}=\left[\left(T_\Lambda(x)\right)_{pq}^L,\left(T_\Lambda(x)\right)_{pq}^U\right]=\left[\left(r_\Lambda^L\right)_{pq},\left(r_\Lambda^U\right)_{pq}\right].e^{i\left[\left(\omega_\Lambda^L\right)_{pq},\left(\omega_\Lambda^U\right)_{pq}\right]}$

$$\left(I_\Lambda(x)\right)_{pq}=\left[\left(I_\Lambda(x)\right)_{pq}^L,\left(I_\Lambda(x)\right)_{pq}^U\right]=\left[\left(k_\Lambda^L\right)_{pq},\left(k_\Lambda^U\right)_{pq}\right].e^{i\left[\left(\rho_\Lambda^L\right)_{pq},\left(\rho_\Lambda^U\right)_{pq}\right]}$$

$$\left(F_\Lambda(x)\right)_{pq}=\left[\left(F_\Lambda(x)\right)_{pq}^L,\left(F_\Lambda(x)\right)_{pq}^U\right]=\left[\left(t_\Lambda^L\right)_{pq},\left(t_\Lambda^U\right)_{pq}\right].e^{i\left[\left(\sigma_\Lambda^L\right)_{pq},\left(\sigma_\Lambda^U\right)_{pq}\right]}$$

for $p=1,......,n$ and $q=1,......,h.$ Using the operational rules of the T2IVLCNS, the average weight $w_p=\left\langle x,\left[\left(\tilde{s}_{\theta_\Lambda(x)}\right)_p,\left(T_\Lambda(x)\right)_p,\left(I_\Lambda(x)\right)_p,\left(F_\Lambda(x)\right)_p\right]\right\rangle$ can be evaluated as flows:

$$w_p = \frac{w_{p1} \oplus w_{p2} \oplus w_{p3} \oplus .... \oplus w_{ph}}{h}, \tag{6}$$

Where
$$\left(T_{\Lambda}(x)\right)_p = \left[1-\left(1-\sum_{q=1}^{h}\left(r_{\Lambda}^{L}\right)_{pq}\right)^{\frac{1}{h}}, 1-\left(1-\sum_{q=1}^{h}\left(r_{\Lambda}^{U}\right)_{pq}\right)^{\frac{1}{h}}\right].e^{i\left[\frac{1}{h}\sum_{q=1}^{h}\left(\omega_{\Lambda}^{L}\right)_q, \frac{1}{h}\sum_{q=1}^{h}\left(\omega_{\Lambda}^{U}\right)_q\right]}$$

$$\left(I_{\Lambda}(x)\right)_p = \left[\left(\sum_{q=1}^{h}\left(k_{\Lambda}^{L}\right)_{pq}\right)^{\frac{1}{h}}, \left(\sum_{q=1}^{h}\left(k_{\Lambda}^{U}\right)_{pq}\right)^{\frac{1}{h}}\right].e^{i\left[\frac{1}{h}\sum_{q=1}^{h}\left(\rho_{\Lambda}^{L}\right)_q, \frac{1}{h}\sum_{q=1}^{h}\left(\rho_{\Lambda}^{U}\right)_q\right]}$$

$$\left(F_{\Lambda}(x)\right)_p = \left[\left(\sum_{q=1}^{h}\left(t_{\Lambda}^{L}\right)_{pq}\right)^{\frac{1}{h}}, \left(\sum_{q=1}^{h}\left(t_{\Lambda}^{U}\right)_{pq}\right)^{\frac{1}{h}}\right].e^{i\left[\frac{1}{h}\sum_{q=1}^{h}\left(\sigma_{\Lambda}^{L}\right)_q, \frac{1}{h}\sum_{q=1}^{h}\left(\sigma_{\Lambda}^{U}\right)_q\right]}$$

Step III. Aggregate the Weighted Ratings of Alternatives versus Criteria

The operations rules of T2IVLCNS for weighted ratings of alternatives can be evaluated as follows:

$$G_m = \frac{1}{n}\sum_{p=1}^{n} x_{mp} * w_p, \quad m = 1.2.....t, \quad p = 1.2.....n. \tag{7}$$

Step IV. Calculation of A^+, A^-, d_i^+, d_i^-

The positive-ideal solution (FPIS, A^+) and fuzzy negative ideal solution (FNIS, A^-) are obtained as follows:

$$A^+ = \left\langle x, \left\{\max\left\{(\tilde{s}_{\theta_{\Lambda}(x)})_{mpq}, (\tilde{s}_{\theta_{\Lambda}(x)})_{pq}\right\}, [1,1].e^{i\max\left(\left[\omega_{mpq}^{L}, \omega_{pq}^{L}(x), \omega_{mpq}^{U}(x).\omega_{pq}^{U}(x)\right]\right)}, [0,0], [0,0]\right\}\right\rangle \tag{8}$$

$$A^- = \left\langle x, \left\{\begin{array}{l}\max\left\{(\tilde{s}_{\theta_{\Lambda}(x)})_{mpq}, (\tilde{s}_{\theta_{\Lambda}(x)})_{pq}\right\}, [0,0], [1,1].e^{i\max\left(\left[\rho_{mpq}^{L}.\rho_{pq}^{L}(x), \rho_{mpq}^{U}(x).\rho_{pq}^{U}(x)\right]\right)}, \\ [1,1].e^{i\max\left(\left[\sigma_{mpq}^{L}.\sigma_{pq}^{L}(x), \sigma_{mpq}^{U}(x)..\sigma_{pq}^{U}(x)\right]\right)}\end{array}\right\}\right\rangle \tag{9}$$

The amplitude terms and the phase terms using the distances of each alternative $A_a, a = 1,.....,t$ from A^+ and A^- are calculated as

$$d_m^{a^+} = \sqrt{\left(G_m^a - A^{a^+}\right)^2}, d_m^{a^-} = \sqrt{\left(G_m^a - A^{a^-}\right)^2}, \ d_m^{p^+} = \sqrt{\left(G_m^a - A^{p^+}\right)^2}, \ d_m^{p^-} = \sqrt{\left(G_m^a - A^{p^-}\right)^2}$$

where $d_m^{a^+}, d_m^{a^-}$ characterizes the shortest distances of the candidate A_m and $d_m^{p^+}, d_m^{p^-}$ characterizes the farthest distance of candidate A_m.

Step V. The Closeness Coefficient

To define the classification order of all candidates, the closeness coefficients for the amplitude and the phase terms of every candidate are defined as

$$CC_i^a = \frac{d_i^{a^-}}{d_i^{a^+} + d_i^{a^-}} \tag{10}$$

$$CC_i^a = \frac{d_i^{p^-}}{d_i^{p^+} + d_i^{p^-}} \tag{11}$$

Higher value of the closeness coefficient depute that a candidate is near to PIS and farther from NIS together. Let Λ and Σ be any two T2IVLCNS's. Then, by classification method it is cleared that:

If $CC^a_\Lambda > CC^a_\Sigma$ then $\Lambda > \Sigma$

If $CC^a_\Lambda = CC^a_\Sigma$ and $CC^p_\Lambda > CC^p_\Sigma$ then $\Lambda > \Sigma$

If $CC^a_\Lambda = CC^a_\Sigma$ and $CC^p_\Lambda = CC^p_\Sigma$ then $\Lambda = \Sigma$.

6. Application of Crop Selection under the Proposed Generalized TOPSIS Method

In this section we applied the generalized TOPSIS technique for T2IVLCNS of crop selection in Bundelkhand region of Uttar Pradesh, India which is one of the most agricultural food compatible states of India. Assume that farmer needs to choose an alternative crop option. A group of four DMs, i.e., D_1, D_2, D_3 and D_4 were asked to proceed to their own evaluation for the significance weights of selection criteria and the ratings of six potential alternatives in a distinct manner. Based on the discussion with the group members, five selection criteria are considered, including as in table 1

Table 1: Criteria for Evaluating Crop Significance

Code	Evaluation Criteria	Significance Description
C_1	Adaptability to varied Environments	Crop adaptability to soil and weather.
C_2	Climate Resilience	Crop resilience under harsh weather conditions
C_3	Pest Resistance	Crop pest and disease resistance
C_4	Growth with Minimal Resources	Crop survival and output with inadequate water, nutrients, or inputs
C_5	Minimal Input and Maintenance	Low labor, fertilizer, and operating costs make growing easy

The computational process for evaluating crop suitability proceeds as four DMs collaboratively assessed the suitability of six potential Kharif crops, considered as alternatives. These alternatives are

- **A_1** = Peanut (*Mungfali*)
- **A_2** = Soybean (*Beans*)
- **A_3** = Sesame (*Til*)
- **A_4** = Pearl Millet (*Bajra*)
- **A_5** = Green Gram (*Moong Dal*)
- **A_6** = Black Gram (*Urad Dal*)

Each DM assigned linguistic evaluations based on pre-defined criteria to determine the overall preference and suitability ranking of these alternatives versus the criteria using the T2IVLCNS

$$\tilde{s}_{\theta(x)} = \left\{ \tilde{s}_{\theta_{VB}(x)} = VB, \tilde{s}_{\theta_B(x)} = B, \tilde{s}_{\theta_M(x)} = M, \tilde{s}_{\theta_G(x)} = G, \tilde{s}_{\theta_{VG}(x)} = VG \right\}$$

Table 2: Alternatives and Linguistic Terms

	$\tilde{s}_{\theta_\Lambda(x)}$	r^L_Λ	r^U_Λ	ω^L_Λ	ω^U_Λ	k^L_Λ	k^U_Λ	ρ^L_Λ	ρ^U_Λ	t^L_Λ	t^U_Λ	σ^L_Λ	σ^U_Λ

$\tilde{s}_{\theta_{VB}(x)} = VB$	1	0.1	0.2	0.4	0.5	0.5	0.6	0.3	0.5	0.7	0.8	0.2	0.4
$\tilde{s}_{\theta_{B}(x)} = B$	2	0.3	0.4	0.5	0.6	0.5	0.6	0.3	0.5	0.4	0.5	0.3	0.4
$\tilde{s}_{\theta_{M}(x)} = M$	3	0.4	0.5	0.3	0.7	0.4	0.6	0.4	0.5	0.3	0.5	0.3	0.5
$\tilde{s}_{\theta_{G}(x)} = G$	4	0.5	0.7	0.6	0.7	0.4	0.5	0.4	0.6	0.3	0.4	0.4	0.5
$\tilde{s}_{\theta_{VG}(x)} = VG$	5	0.7	0.9	0.8	0.9	0.3	0.4	0.6	0.7	0.2	0.3	0.7	0.8

Table 3 represents the aggregation ratings of crops against the criteria of six alternative A_1, A_2, A_3, A_4, A_5, A_6 versus five criteria C_1, C_2, C_3, C_4, C_5 from four D_1, D_2, D_3, D_4, DMs using the T2IVLCNS.

Table 3: Aggregation Ratings of Crops against the Criteria

Decision Makers DM_1	Criteria				
Alternative (Crops)	C_1	C_2	C_3	C_4	C_5
A_1	VG	G	VG	VG	VG
A_2	G	G	G	VG	G
A_3	G	VG	M	VG	G
A_4	VG	M	VG	G	VG
A_5	VG	VG	G	G	VG
A_6	G	G	VG	M	G
Decision Makers DM_2					
A_1	M	M	G	G	M
A_2	G	G	VG	G	G
A_3	M	M	M	G	M
A_4	VG	VG	VG	M	VG
A_5	M	M	G	M	M
A_6	M	G	M	M	M
Decision Makers DM_3					
A_1	M	G	VG	M	M
A_2	M	M	G	G	M
A_3	M	M	M	G	M
A_4	VG	VG	VG	G	VG
A_5	M	G	M	M	M
A_6	M	M	G	M	M
Decision Makers DM_4					
A_1	VG	G	VG	VG	VG
A_2	G	VG	M	M	G
A_3	M	G	VG	M	M
A_4	G	G	G	VG	G
A_5	VG	G	VG	G	VG

A_6	VG	VG	G	G	VG
Decision Makers DM_5					
A_1	VG	G	VG	VG	VG
A_2	G	VG	M	VG	G
A_3	VG	G	VG	G	VG
A_4	G	G	G	VG	G
A_5	VG	VG	G	G	VG
A_6	G	G	VG	M	G

Using Eq. (5), the aggregated ratings of the crop versus the criteria from the DMs are shown at the last column of Table 4.

Table 4: Aggregated Ratings of Crops Versus Criteria

C_1		$\tilde{s}_{\theta_\Lambda(x)}$	r^L_Λ	r^U_Λ	ω^L_Λ	ω^U_Λ	k^L_Λ	k^U_Λ	ρ^L_Λ	ρ^U_Λ	t^L_Λ	t^U_Λ	σ^L_Λ	σ^U_Λ
	A_1	4.750	0.779	0.881	0.775	0.875	0.238	0.341	0.775	0.875	0.221	0.322	0.675	0.775
	A_2	4.250	0.729	0.832	0.725	0.825	0.336	0.440	0.725	0.825	0.271	0.372	0.625	0.725
	A_3	4.250	0.722	0.832	0.750	0.850	0.283	0.405	0.750	0.850	0.263	0.366	0.650	0.750
	A_4	4.500	0.722	0.832	0.725	0.825	0.283	0.405	0.725	0.825	0.263	0.366	0.625	0.725
	A_5	4.500	0.755	0.859	0.750	0.850	0.283	0.387	0.750	0.850	0.245	0.346	0.650	0.750
	A_6	4.000	0.692	0.800	0.700	0.800	0.336	0.461	0.700	0.800	0.291	0.394	0.600	0.700
C_2	A_1	3.500	0.613	0.717	0.650	0.750	0.400	0.548	0.650	0.750	0.346	0.447	0.550	0.650
	A_2	4.250	0.729	0.832	0.725	0.825	0.336	0.440	0.725	0.825	0.271	0.372	0.625	0.725
	A_3	3.250	0.560	0.664	0.625	0.725	0.400	0.573	0.625	0.725	0.372	0.473	0.525	0.625
	A_4	4.500	0.779	0.881	0.750	0.850	0.238	0.357	0.750	0.850	0.238	0.341	0.650	0.750
	A_5	3.250	0.560	0.664	0.625	0.725	0.400	0.573	0.625	0.725	0.372	0.473	0.525	0.625
	A_6	3.250	0.613	0.717	0.625	0.725	0.400	0.573	0.625	0.725	0.372	0.473	0.525	0.625
C_3	A_1	3.750	0.650	0.762	0.675	0.775	0.336	0.482	0.675	0.775	0.313	0.416	0.575	0.675
	A_2	3.500	0.613	0.717	0.650	0.750	0.400	0.548	0.650	0.750	0.346	0.447	0.550	0.650
	A_3	3.250	0.560	0.664	0.625	0.725	0.400	0.573	0.625	0.725	0.372	0.473	0.525	0.625
	A_4	4.750	0.779	0.881	0.775	0.875	0.238	0.341	0.775	0.875	0.221	0.322	0.675	0.775
	A_5	3.250	0.560	0.664	0.625	0.725	0.400	0.573	0.625	0.725	0.372	0.473	0.525	0.625
	A_6	3.250	0.560	0.664	0.625	0.725	0.400	0.573	0.625	0.725	0.372	0.473	0.525	0.625

C_4	A_1	4.750	0.779	0.881	0.775	0.875	0.238	0.341	0.775	0.875	0.221	0.322	0.675	0.775
	A_2	3.750	0.650	0.762	0.675	0.775	0.336	0.482	0.675	0.775	0.313	0.416	0.575	0.675
	A_3	3.750	0.650	0.762	0.675	0.775	0.336	0.482	0.675	0.775	0.313	0.416	0.575	0.675
	A_4	4.250	0.729	0.832	0.725	0.825	0.336	0.440	0.725	0.825	0.271	0.372	0.625	0.725
	A_5	4.500	0.755	0.859	0.750	0.850	0.283	0.387	0.750	0.850	0.245	0.346	0.650	0.750
	A_6	4.500	0.755	0.859	0.750	0.850	0.283	0.387	0.750	0.850	0.245	0.346	0.650	0.750
C_5	A_1	4.750	0.779	0.881	0.775	0.875	0.238	0.341	0.775	0.875	0.221	0.322	0.675	0.775
	A_2	4.250	0.755	0.859	0.725	0.825	0.283	0.405	0.725	0.825	0.263	0.366	0.625	0.725
	A_3	4.500	0.755	0.859	0.750	0.850	0.283	0.387	0.750	0.850	0.245	0.346	0.650	0.750
	A_4	4.250	0.729	0.832	0.725	0.825	0.336	0.440	0.725	0.825	0.271	0.372	0.625	0.725
	A_5	4.500	0.755	0.859	0.750	0.850	0.283	0.387	0.750	0.850	0.245	0.346	0.650	0.750
	A_6	4.000	0.692	0.800	0.700	0.800	0.336	0.461	0.700	0.800	0.291	0.394	0.600	0.700

After establishing the crop assortment criteria, the DMs were invited to assess the relative importance of each criterion by assigning significance levels using IVLCNS.

$$\theta = \{\theta_1 = \mathrm{UI}, \theta_2 = \mathrm{OI}, \theta_3 = \mathrm{I}, \theta_4 = \mathrm{VI}, \theta_5 = \mathrm{AI}\},$$

Table 5: Relative Importance of each Criterion using IVLCNS

	θ	r^L_Λ	r^U_Λ	ω^L_Λ	ω^U_Λ	k^L_Λ	k^U_Λ	ρ^L_Λ	ρ^U_Λ	t^L_Λ	t^U_Λ	σ^L_Λ	σ^U_Λ
UI = Unimportant	1	0.1	0.2	0.4	0.5	0.6	0.7	0.4	0.5	0.6	0.7	0.3	0.4
OI = Ordinary Important	2	0.2	0.4	0.5	0.6	0.5	0.6	0.5	0.6	0.6	0.7	0.4	0.5
I = Important	3	0.4	0.6	0.6	0.7	0.4	0.6	0.6	0.7	0.4	0.5	0.5	0.6
VI = Very Important	4	0.6	0.8	0.7	0.8	0.4	0.5	0.7	0.8	0.3	0.4	0.6	0.7
AI = Absolute Important	5	0.7	0.9	0.8	0.9	0.2	0.3	0.8	0.9	0.2	0.3	0.7	0.8

The following table 6 shows the significance weights of the five criteria from the four DMs and table 7 shows the aggregate weighted ratings of alternatives versus criteria

Table 6: Important Aggregated Weights of the Criteria

Decision Makers→ Criteria↓	D_1	D_2	D_3	D_4

C_1	AI	VI	AI	I
C_2	VI	I	VI	OI
C_3	I	I	I	VI
C_4	OI	VI	I	I
C_5	I	OI	I	I

Table 7: Aggregated Weights Ratings

$\tilde{s}_{\theta_\Lambda(x)}$	r^L_Λ	r^U_Λ	ω^L_Λ	ω^U_Λ	k^L_Λ	k^U_Λ	ρ^L_Λ	ρ^U_Λ	t^L_Λ	t^U_Λ	σ^L_Λ	σ^U_Λ
4.250	0.617	0.832	0.725	0.825	0.283	0.405	0.725	0.825	0.263	0.366	0.625	0.725
3.250	0.474	0.687	0.625	0.725	0.423	0.548	0.625	0.725	0.383	0.486	0.525	0.625
3.250	0.355	0.557	0.625	0.725	0.400	0.573	0.625	0.725	0.372	0.473	0.525	0.625
3.000	0.417	0.628	0.600	0.700	0.423	0.573	0.600	0.700	0.412	0.514	0.500	0.600
2.750	0.355	0.557	0.575	0.675	0.423	0.600	0.575	0.675	0.443	0.544	0.475	0.575

Table 8 presents the weighted performance ratings of each crop alternative, computed using Equation (7). These ratings reflect the aggregated assessments provided by the DMs, incorporating both the significance weights of the evaluation criteria and the linguistic preferences assigned to each alternative. The results provide a comprehensive overview of how each crop performs across the defined criteria within the IVLCN environment.

Table 8: Aggregation of weighted Ratings of Alternative versus Criteria

Alternative	$\theta.\tilde{s}_{\theta(x)}$	r^L_Λ	r^U_Λ	ω^L_Λ	ω^U_Λ	k^L_Λ	k^U_Λ	ρ^L_Λ	ρ^U_Λ	t^L_Λ	t^U_Λ	σ^L_Λ	σ^U_Λ
A_1	14.850	0.284	0.557	0.441	0.584	0.135	0.242	0.315	0.475	0.092	0.165	0.292	0.410
A_2	13.213	0.247	0.497	0.397	0.533	0.151	0.263	0.271	0.445	0.107	0.184	0.239	0.347
A_3	13.225	0.263	0.521	0.414	0.568	0.140	0.257	0.301	0.448	0.096	0.178	0.266	0.392
A_4	13.700	0.273	0.536	0.422	0.580	0.137	0.255	0.312	0.453	0.093	0.177	0.276	0.406
A_5	14.013	0.276	0.546	0.432	0.573	0.138	0.246	0.309	0.467	0.098	0.174	0.284	0.403
A_6	12.063	0.230	0.466	0.375	0.526	0.151	0.276	0.271	0.427	0.114	0.205	0.220	0.347

7. Calculation of A^+, A^-, d^+ and d^-

As illustrated in Table 4, the distance of each crop alternative from the fuzzy positive ideal solution (A^+) and the fuzzy negative ideal solution (A^-), for both amplitude and phase terms, is computed using Equations (8) to (13). These equations facilitate the evaluation of how close or far each alternative is from the optimal (A^+) and least preferred (A^-) conditions based on the IVLCN framework. Specifically, Equations (8) and (9) are used to determine the Euclidean distance between each alternative and the ideal solutions in terms of the amplitude component, which represents the magnitude of membership, indeterminacy, and non-membership degrees. These distances are critical in identifying the relative performance of each alternative with respect to the best and worst possible scenarios under uncertain and imprecise environments.

Table 9: According to table 2

	$\tilde{s}_{\theta_\Lambda(x)}$	r^L_Λ	r^U_Λ	ω^L_Λ	ω^U_Λ	k^L_Λ	k^U_Λ	ρ^L_Λ	ρ^U_Λ	t^L_Λ	t^U_Λ	σ^L_Λ	σ^U_Λ

Max R	5	0	0	0.8	0.9	0	0	0	0	0	0	0	0
Min R	1	0	0	0	0	0	0	0.6	0.7	0	0	0.7	0.8

Table 10: According to table 5

	$\tilde{s}_{\theta_\Lambda(x)}$	r_Λ^L	r_Λ^U	ω_Λ^L	ω_Λ^U	k_Λ^L	k_Λ^U	ρ_Λ^L	ρ_Λ^U	t_Λ^L	t_Λ^U	σ_Λ^L	σ_Λ^U
Max w	5	0	0	0.8	0.9	0	0	0	0	0	0	0	0
Min w	1	0	0	0	0	0	0	0.8	0.9	0	0	0.8	0.9

Table 11: Weighted Assessment of Each Crop

	Ω	r_Λ^L	r_Λ^U	ω_Λ^L	ω_Λ^U	k_Λ^L	k_Λ^U	ρ_Λ^L	ρ_Λ^U	t_Λ^L	t_Λ^U	σ_Λ^L	σ_Λ^U
A^+	5	1	1	0.64	0.81	0	0	0	0	0	0	0	0
A^-	1	0	0	0	0	1	1	0.48	0.63	1	1	0.49	0.64

Thus $$A^+ = \left\langle \left(\Omega_5, \left([1,1] e^{j[0.64,0.81]}, [0,0] e^{j[0,0]}, [0,0] e^{j[0,0]} \right) \right) \right\rangle$$

$$A^- = \left\langle \left(\Omega_1, \left([0,0] e^{j[0,0]}, [1,1] e^{j[0.48,0.63]}, [1,1] e^{j[0.49,0.64]} \right) \right) \right\rangle$$

Table 12: The Distance of every Alternative from A^+ and A^-

Crops	**Amplitude terms**		**Phase terms**	
	d_a^+	d_a^+	d_p^+	d_p^+
A_1: Peanut (Mungfali)	1.2257	2.0016	0.8171	0.8243
A_2: Soybean (Beans)	1.109	1.6205	0.7646	0.8178
A_3: Sesame (Til)	1.0804	1.6827	0.7906	0.8194
A_4: Pearl Millet (Bajra)	1.1236	1.7902	0.8021	0.8218
A_5: Green Gram (Moong Dal)	1.1496	1.8541	0.8097	0.8182
A_6: Black Gram (Urad Dal)	1.0751	1.4094	0.7586	0.8129

8. Obtain the Closeness Coefficient

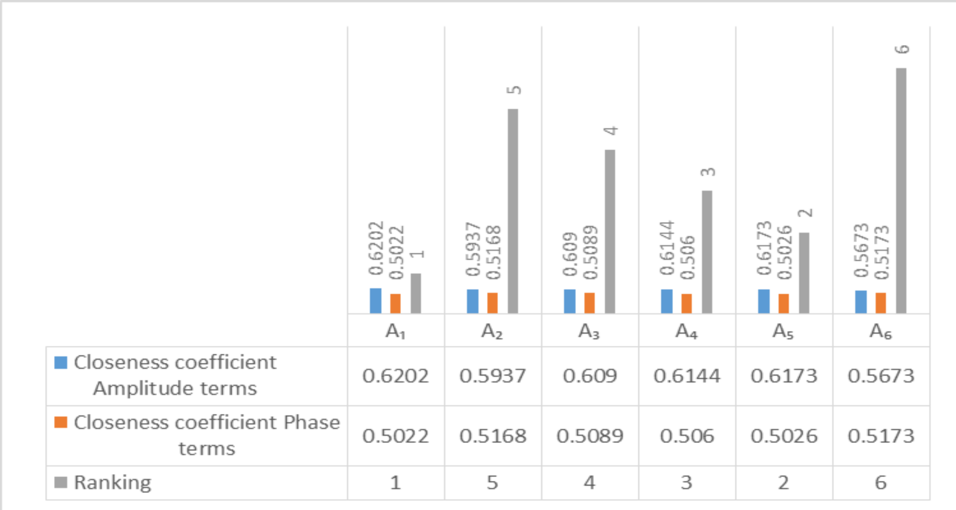
The closeness coefficient (CC) for each alternative is calculated using Equations (14) and (15), which measure the relative proximity of each alternative to the positive ideal solution (A^+) and its remoteness from the negative ideal solution (A^-). These values reflect the overall performance of each crop alternative in a fuzzy neutrosophic environment. As illustrated in Table 13, the closeness coefficient values are derived for all alternatives. Based on the computed CC values, the alternatives are ranked in descending order i.e. $A_1 > A_5 > A_4 > A_3 > A_2 > A_6$ where a higher closeness coefficient indicates better suitability. Accordingly, the final ranking order of the six crops is established, and it is concluded that crop peanut is the most preferred option for the farmers in Bundelkhand region among the evaluated other crops.

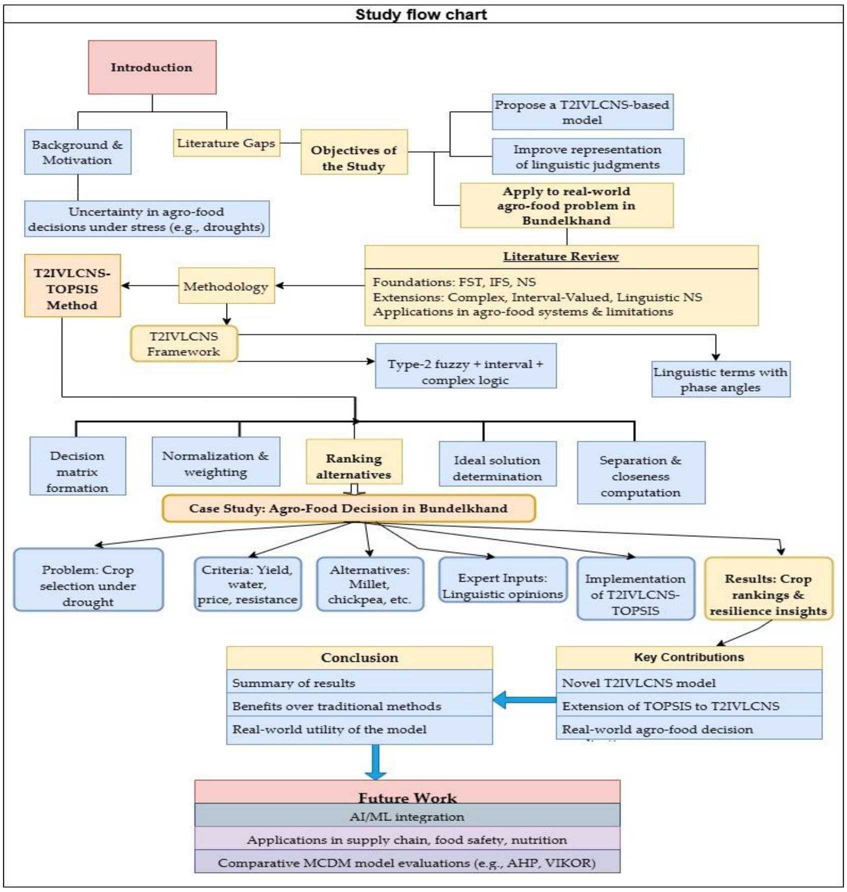
Table-13: Closeness coefficient of Crops

Crops	**Closeness coefficient**		**Ranking**
	Amplitude terms	**Phase terms**	
A_1	0.6202	0.5022	1
A_2	0.5937	0.5168	5

A_3	0.6090	0.5089	4
A_4	0.6144	0.5060	3
A_5	0.6173	0.5026	2
A_6	0.5673	0.5173	6

Table 14: Ranking of Crops





9. Conclusion

This paper introduces innovative methodologies within the Type-2 Interval-Valued Linguistic Complex Neutrosophic Set (T2IVLCNS) framework, offering greater flexibility and adaptability for real-world decision-making compared to existing models. A key contribution of this study is the development and application of a generalized TOPSIS method tailored to the T2IVLCNS environment, enhancing its robustness in handling uncertainty and linguistic imprecision. The research showcases its practical relevance through the classification of agro-food alternatives against multiple criteria, demonstrating the framework's effectiveness in multi-criteria decision-making scenarios. A significant application focuses on crop selection for the drought-prone Bundelkhand region of Uttar Pradesh, India, highlighting the model's utility in addressing complex, context-specific agricultural challenges. These contributions underscore the potential of the proposed approach in advancing decision support systems within uncertain and linguistically complex environments.

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