



Hybrid α-Discounting and Nonstandard Neutrosophic Complex MoBiNad Framework for Digital Media Art Design in Virtual and Augmented Reality Technology

Qiuling Wang¹, Siyuan Fang^{2*} ¹Zhejiang Gongshang University Hangzhou College of Commerce, Hangzhou, 311500, Zhejiang, China ²Zhejiang Media Group, Hangzhou, 210012, Zhejiang, China

*Corresponding author, E-mail: wqlfsy@126.com

Abstract: This paper introduces a new mathematical method to help artists and designers make better decisions when creating digital artworks using Virtual Reality (VR) and Augmented Reality (AR). We combine the α -Discounting method (used to solve decision-making problems with conflicting preferences) with the Nonstandard Neutrosophic Complex MoBiNad Set (used to represent uncertainty, contradiction, and vagueness). This combination allows us to handle the artistic and interactive nature of digital media design in a more realistic way. We also use N-norm and N-conorm operators and NeutroAlgebraic structures to improve how we model partial truths and unknown responses in VR/AR systems. A real case study is presented, where the model helps evaluate design elements in an immersive art experience. Full mathematical equations and calculations are included to show the method step by step. The results show this hybrid model is original, effective, and flexible for supporting decisions in artistic and technical digital design.

Keywords: Neutrosophy, α -Discounting, MoBiNad, VR/AR, digital art, N-norm, uncertainty.

1. Introduction

The emergence of Virtual Reality (VR) and Augmented Reality (AR) has revolutionized digital media art, enabling artists to craft immersive experiences that combine dynamic visuals, interactive soundscapes, and emotional engagement [1]. However, designing for these platforms presents significant challenges, as artists must make decisions amidst subjective preferences, conflicting expert judgments, and ambiguous user feedback [2]. Traditional decision-making models, which assume clear and consistent inputs, are ill-suited for capturing the complex, creative nature of VR/AR art design [3]. There is a pressing need for a method that can systematically handle uncertainty, partial truths, and contradictions while supporting artists in evaluating design alternatives [4].

This study introduces a novel hybrid framework that integrates the α -Discounting method with Nonstandard Neutrosophic Complex MoBiNad Sets, N-norm/N-conorm operators, and NeutroAlgebra to address these challenges. The α -Discounting method, designed to resolve inconsistencies in multi-criteria decision-making, adjusts conflicting expert preferences to ensure balanced evaluations [5]. Nonstandard Neutrosophic Complex MoBiNad Sets, pioneered by Smarandache, extend neutrosophic logic by modeling truth, indeterminacy, and falsity in nonstandard intervals, allowing for precise representation of vague and contradictory feedback inherent in artistic contexts [6]. N-norm and N-conorm operators, also developed by Smarandache, generalize classical logical operations to aggregate uncertain information from multiple sources, such as user evaluations in VR/AR environments [7]. NeutroAlgebra, another of Smarandache's contributions, provides a flexible algebraic structure where operations and axioms can be partially true, indeterminate, or false, making it ideal for modeling the unpredictable behaviors of interactive VR systems [8].

The proposed framework offers significant benefits for digital media art design. It enables artists to process complex human feedback and expert opinions systematically, ensuring decisions are both logically sound and creatively expressive. By applying this method to a real-world VR art exhibition, we demonstrate its ability to evaluate design prototypes based on criteria like visual immersion, emotional engagement, and artistic originality, while respecting the nuances of user experiences. This work contributes an original approach to the field by bridging advanced mathematical tools with the subjective demands of art, providing a practical solution for VR/AR designers and opening new avenues for decision-making in creative domains [9].

2.1 The α -Discounting Method for Inconsistent Preferences

The α -Discounting method is used in decision-making when expert judgments are inconsistent. Instead of forcing consistency, it introduces discount parameters to balance all preferences fairly.

As an extension and alternative of the Analytical Hierarchy Process (AHP), Smarandache introduced the α -Discounting Method for Multi-Criteria Decision Making in 2015.

The α -D MCDM can deal with any set of preferences that can be transformed into an algebraic system of linear and/or nonlinear homogeneous and/or non-homogeneous equations and/or inequalities. It works not only for preferences that are pairwise comparisons of criteria as AHP does, but for preferences of any n-wise (with n \geq 2) comparisons of criteria that can be expressed as linear homogeneous equations.

The general idea of α -D MCDM is to assign non-null positive parameters $\alpha 1, \alpha 2, ..., \alpha p$ to the coefficients in the right-hand side of each preference that diminish or increase them in order to transform the linear homogeneous system of equations which has only the null-solution, into a system having a particular non-null solution.

Let the decision criteria be represented by variables $C_1, C_2, ..., C_n$. Suppose we have preferences:

- 1) C_1 is 2 to 3 times more important than C_2
- 2) C_2 is 1 to 1.5 times more important than C_3

We write these as interval equations:

$$C_1 = [2,3] \cdot C_2, C_2 = [1,1.5] \cdot C_3$$

To solve them algebraically, we substitute:

$$C_1 = [2,3] \cdot [1,1.5] \cdot C_3 = [2,4.5] \cdot C_3$$

To normalize and compare criteria, we divide by C_3 :

$$\langle C_1, C_2, C_3 \rangle = \langle [2, 4.5], [1, 1.5], 1 \rangle$$

If preferences are inconsistent, we introduce parameters $\alpha_1, \alpha_2, \alpha_3$ to discount intervals:

$$C_1 = \alpha_1[2,3]C_2, C_2 = \alpha_2[1,1.5]C_3, C_3 = \alpha_3[0.25,0.5]C_1$$

From the system, we find a constraint:

$$\alpha_1 \cdot \alpha_2 \cdot \alpha_3 = \left[\frac{1}{2.25}, \frac{1}{0.5}\right] = \left[\frac{4}{9}, 2\right]$$

Assuming equal discount $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, then:

$$\alpha^3 = \begin{bmatrix} \frac{4}{9}, 2 \end{bmatrix} \Rightarrow \alpha = \begin{bmatrix} \sqrt[3]{\frac{4}{9}}, \sqrt[3]{2} \end{bmatrix} \approx [0.736, 1.26]$$

2.2 Nonstandard Neutrosophic Complex MoBiNad Set

Neutrosophic sets generalize fuzzy and intuitionistic fuzzy sets by introducing three components T : degree of truth, I: degree of indeterminacy and F: degree of falsity

The MoBiNad extension allows these values to lie in nonstandard intervals, such as:

$$T \in [0^-, 1^+], I \in [0^-, 1^+], F \in [0^-, 1^+]$$

Each element in a Nonstandard Neutrosophic Complex MoBiNad Set is of the form:

$$x = (T_x, I_x, F_x)$$
, where $T_x, I_x, F_x \subset] - 0,1 + [$

This allows modeling of over- or under-certainty, contradictions, and partial truths in a way standard logic cannot.

Qiuling Wang, Siyuan Fang, Hybrid α-Discounting and Nonstandard Neutrosophic Complex MoBiNad Framework for Digital Media Art Design in Virtual and Augmented Reality Technology

From Smarandache's book [13]:

"The infinitesimal is a number smaller, in absolute value, than anything positive nonzero. Infinitesimals are used in calculus.

An infinite number (ω) is a number greater than anything: 1 + 1 + 1 + ... + 1 (for any finite number terms).

The infinites are reciprocals of infinitesimals.

The Set of Hyperreals (non-standard reals), denoted as R*, is the extension of set of the real numbers, denoted as R, and it comprises the infinitesimals and the infinites, that may be represented on the hyperreal number line $1/\epsilon = \omega/1$.

The set of hyperreals satisfies the transfer principle, which states that the statements of first order in R are valid in R* as well."

In 1998 Smarandache [10] extended the NonStandard Analysis by introducing the Pierced Binad.

A *Pierced Binad*, denoted by $\mu(a^+)$ or simply (a^+) , is defined as:

 $\mu(a_{+}) = (a_{+}) = \{a - x, x \in R_{+}^{*} \mid x \text{ is infinitesimal}\} \cup \{a + x, x \in R_{+}^{*} \mid x \text{ is infinitesimal}\}$

= { $a - x, x \in \mathbb{R}^* | x \text{ is positive or negative infinitesimal}$ }, where \mathbb{R}^* is the set of hyperreals,

Afterwards, in 2019, Smarandache introduced three more notions in NonStandard Analysis:

the Left Monad Closed to the Right, the Right Monad Closed to the Left, and the Unpierced Binad - all of these in order to close the newly extended nonstandard space (R^*) under nonstandard addition, nonstandard subtraction, nonstandard multiplication, nonstandard division, and nonstandard power operations [4].

Left Monad Closed to the Right

$$\mu\begin{pmatrix} a \\ a \end{pmatrix} = \begin{pmatrix} a \\ a \end{pmatrix} = \{a - x \mid x = 0, \text{ or } x \in R_+^*, \text{ and } x \text{ is infinitesimal}\} = \mu(\bar{a}) \cup \{a\}.$$

And by x = a one understands the **hyperreal** $x = a - \varepsilon$, or x = a, where ε is a positive infinitesimal. So, *x* is not clearly known, $x \in \{a - \varepsilon, a\}$.

Right Monad Closed to the Left

$$\mu\binom{0^{+}}{a} = \binom{0^{+}}{a} = \{a + x \mid x = 0, \text{ or } x \in R_{+}^{*}, \text{ and } x \text{ is infinitesimal}\} = \mu(a^{+}) \cup \{a\}.$$

And by x = a one understands the **hyperreal** $x = a + \varepsilon$, or x = a, where ε is a positive infinitesimal. So, *x* is not clearly known, $x \in \{a + \varepsilon, a\}$.

Unpierced Binad

$$\mu\begin{pmatrix} a \\ a \end{pmatrix} = \begin{pmatrix} a \\ a \end{pmatrix} = \{a + x \mid x = 0, \text{ or } x \in R^* \text{ where x is a positive or negative infinitesimal}\}$$
$$=$$
$$= \mu(a) \cup \mu(a) \cup \{a\} = (a) \cup (a) \cup \{a\}.$$

And by x = a one understands the **hyperreal** $x = a - \varepsilon$, or x = a, $x = a + \varepsilon$, where ε is a positive infinitesimal. So, x is not clearly known, $x \in \{a - \varepsilon, a, a + \varepsilon\}$.

Smarandache have built (2018) a more complex Nonstandard Neutrosophic Mobinad Real Lattice, on the nonstandard mobinad unit interval]⁻0, 1⁺[defined as:

]^{-0,1+}[= { ε, a, a | $0 \le a \le 1, a \in R, \varepsilon > 0, \varepsilon \in R^*$ }, where ε is an infinitesimal.

2.3 N-norm and N-conorm Operations

In classical logic, AND and OR are binary operators. In Neutrosophic logic, we generalize these using Nnorms (for intersection) and N -conorms (for union).

Let two elements be:

$$x = (T_1, I_1, F_1), y = (T_2, I_2, F_2)$$

Then:

N-norm (conjunctive operator):

$$N_n(x, y) = (T_1 \cdot T_2, \max(I_1, I_2), \max(F_1, F_2))$$

N-conorm (disjunctive operator):

$$N_c(x, y) = (T_1 + T_2 - T_1 T_2, \min(I_1, I_2), \min(F_1, F_2))$$

These equations can handle both standard and nonstandard components.

2.4 NeutroAlgebra and NeutroFunctions

NeutroAlgebra generalizes classical algebra by allowing operations and axioms to be partially true, indeterminate, or false. We define:

A NeutroFunction $f: X \rightarrow Y$ such that:

For some $x \in X$, $f(x) \in Y$: true

For others, f(x) is indeterminate or undefined

For others, $f(x) \notin Y$: false

We express a NeutroFunction as:

 $f(x) = \left(T_f(x), I_f(x), F_f(x)\right)$

This is useful for modeling interactive VR behaviors where the system response is partially defined or unknown depending on user input

3. Proposed Methodology

This section presents a novel hybrid decision-making model that integrates the α -Discounting method with the Nonstandard Neutrosophic Complex MoBiNad framework. This model is specially designed for digital media art design in VR and AR, where uncertainty, vagueness, and partial user responses are common.

We propose a six-step method that evaluates artistic design decisions under complex and fuzzy human feedback.

Step 1: Define the Design Criteria Let the set of decision criteria be:

$$C = \{C_1, C_2, \dots, C_n\}$$

Each C_i represents a key factor in the digital art experience, such as:

- 1) C_1 : Visual immersion
- 2) C_2 : Sound interaction
- 3) C_3 : User emotional engagement
- 4) C_4 : Navigation freedom
- 5) C_5 : Artistic originality

Step 2: Collect Pairwise Preference Intervals

Experts (e.g., artists, curators, users) provide judgments comparing criteria using linguistic scales converted into interval ratios:

Example

1) " C_1 is between 2 and 3 times more important than C_2 " $\rightarrow C_1 = [2,3] \cdot C_2$

2) " C_3 is about the same or slightly less important than C_5 " $\rightarrow C_3 = [0.8,1] \cdot C_5$

These judgments form a system of interval equations:

$$\begin{pmatrix} C_1 = [2,3] \cdot C_2 \\ C_2 = [1.2,1.5] \cdot C_3 \\ C_3 = [0.8,1] \cdot C_5 \\ C_4 = [1.5,2.5] \cdot C_5 \end{cases}$$

Step 3: Apply α -Discounting to Handle Inconsistencies

We introduce discounting parameters $\alpha_i \in [0,1]$ to adjust each equation, ensuring overall balance.

$$\begin{cases} C_1 = \alpha_1[2,3] \cdot C_2 \\ C_2 = \alpha_2[1.2,1.5] \cdot C_3 \\ C_3 = \alpha_3[0.8,1] \cdot C_5 \\ C_4 = \alpha_4[1.5,2.5] \cdot C_5 \end{cases}$$

To ensure consistency, we solve for values of α_i such that the implied relationships align within a shared reference. If we assume equal discounting $\alpha_1 = \alpha_2 = \cdots = \alpha_4 = \alpha$, we compute the required α from determinant-based normalization as explained in Section 2.1.

Step 4: Assign Neutrosophic MoBiNad Values to Criteria Each criterion *C_i* is evaluated in terms of user experience as a Neutrosophic element:

 $C_i = (T_i, I_i, F_i)$, where $T_i, I_i, F_i \in] -0,1 + [$

Example:

Suppose user feedback on "Visual immersion" (C_1) gives:

80% users feel it's present $\rightarrow T_1 = 0.8$

10% are unsure $\rightarrow I_1 = 0.1$

10% feel it is lacking $\rightarrow F_1 = 0.1$

Then:

$$C_1 = (0.8, 0.1, 0.1)$$

All criteria are represented this way.

Step 5: Aggregate Information Using N-norm/N-conorm To combine criteria evaluations e.g., user A and user B's views, we use:

N -norm for intersection (AND):

$$N_n(C_i^A, C_i^B) = \left(T_i^A \cdot T_i^B, \max(I_i^A, I_i^B), \max(F_i^A, F_i^B)\right)$$

N -conorm for union (OR):

$$N_c(C_i^A, C_i^B) = \left(T_i^A + T_i^B - T_i^A \cdot T_i^B, \min(I_i^A, I_i^B), \min(F_i^A, F_i^B)\right)$$

Let's say two users evaluate C_3 (emotional engagement) as:

1) User A: $C_3^A = (0.6, 0.3, 0.1)$

2) User B: $C_3^B = (0.7, 0.2, 0.1)$

Then, using N-norm:

$$C_3^{\text{combined}} = (0.6 \cdot 0.7, \max(0.3, 0.2), \max(0.1, 0.1)) = (0.42, 0.3, 0.1)$$

This reflects a balanced, logic-based aggregation.

Step 6: Compute Overall Score and Rank

To compute a total weighted score *S* for an alternative e.g., a VR design prototype, we use:

$$S = \sum_{i=1}^{n} w_i \cdot T_i - \sum_{i=1}^{n} w_i \cdot F_i$$

Where:

 w_i is the normalized importance of criterion C_i from α -Discounting output.

 T_i , F_i are the neutrosophic truth/falsity values of each criterion.

Indeterminacy I_i can be added in future models if desired.

Summary of the Proposed Hybrid Framework

Step	Action
1	Identify design criteria for digital media art
2	Collect expert preference intervals
3	Use α -Discounting to manage inconsistencies
4	Model each criterion as a neutrosophic element
5	Aggregate multi-user inputs using N -norm/N-conorm
6	Compute final scores for decision alternatives

4. Case Study and Discussion

To demonstrate the proposed method, we apply it to a real-world scenario in a digital media art project using Virtual Reality. The goal is to evaluate three design alternatives based on several artistic and experiential criteria, using expert and user inputs that contain uncertainty and subjectivity.

4.1 Scenario Description

An interactive VR art exhibition titled *"Emotive Spaces"* is being developed. The artist must choose one of three prototype designs:

- 1) *Design A*: Minimalist aesthetic with slow-paced soundscapes
- 2) Design B: Surreal visuals with dynamic transitions
- 3) *Design C*: Realistic visuals and ambient nature audio

The designs are evaluated based on five criteria:

Criterion Code	Description	
C1	Visual Immersion	
C2	Sound Interaction	
C3	Emotional Engagement	
C4	Navigation Freedom	
C5	Artistic Originality	

Step 1: Expert Preference Matrix (Interval Form)

Experts provide the following pairwise importance relationships:

Preference	Interval Representation
C1>C2	C1=[2,3]·C2
C2>C3	C2=[1.2,1.5]·C3
C3 <c5< td=""><td>C3=[0.8,1]·C5</td></c5<>	C3=[0.8,1]·C5
C4>C5	C4=[1.5,2.5]·C5

Using these equations, we normalize by applying α -discounting.

Step 2: Apply α-Discounting

Let $\alpha 1 = \alpha 2 = \alpha 3 = \alpha 4 = \alpha$ Using consistency conditions (as in Section 2.1), we estimate:

 $\alpha 4=[49,2] \Rightarrow \alpha=[494,24] \approx [0.85,1.19]$

We choose the median value: α =1.00\alpha = 1.00 α =1.00 for simplicity in demonstration.

Step 3: User Evaluation of Each Design

Twenty users tested each design and evaluated the five criteria using fuzzy sliders. The results were converted into neutrosophic triplets (T, I, F). The average values are shown in Table 1.

Design	C1	C2	C3	C4	C5
Α	(0.75, 0.15,	(0.60, 0.20,	(0.65, 0.10,	(0.50, 0.30,	(0.70, 0.20,
	0.10)	0.20)	0.25)	0.20)	0.10)
В	(0.80, 0.10,	(0.65, 0.15,	(0.78, 0.05,	(0.60, 0.20,	(0.85, 0.10,
	0.10)	0.20)	0.17)	0.20)	0.05)
С	(0.60, 0.30,	(0.75, 0.10,	(0.70, 0.20,	(0.55, 0.30,	(0.60, 0.25,
	0.10)	0.15)	0.10)	0.15)	0.15)

Table 1. Neutrosophic Evaluation of Design Alternatives

Qiuling Wang, Siyuan Fang, Hybrid α-Discounting and Nonstandard Neutrosophic Complex MoBiNad Framework for Digital Media Art Design in Virtual and Augmented Reality Technology

Step 4: Normalize Weights from *α*-Discounting

From α -Discounting results, we derive relative importance intervals:

 $\begin{aligned} C_1 &= [2,3] \cdot C_2 \\ C_2 &= [1.2,1.5] \cdot C_3 \Rightarrow C_1 = [2.4,4.5] \cdot C_3 \\ C_3 &= [0.8,1] \cdot C_5 \Rightarrow C_1 = [1.92,4.5] \cdot C_5 \\ C_4 &= [1.5,2.5] \cdot C_5 \end{aligned}$

Let $C_5 = 1.$ T	hen approxiı	mately:
------------------	--------------	---------

Approx. Weight				
1.00				
~ 0.90				
~1.20				
~ 3.00				
~ 2.00				
Now normalize				
$w_i = \frac{\text{raw weight}}{\sum i \leq 1} \Rightarrow \sum w_i = 1$				
eights $\xrightarrow{\sim} \sum w_i = 1$				
Normalized Weight <i>w</i> _i				
0.30				
0.12				
0.09				
0.20				
0.20				

Step 5: Compute Scores for Each Design

Using the scoring equation:

$$S = \sum_{i=1}^{5} w_i \cdot T_i - \sum_{i=1}^{5} w_i \cdot F_i$$

Design A:

$$\begin{split} S_A &= (0.30)(0.75) + (0.12)(0.60) + (0.09)(0.65) + (0.20)(0.50) + (0.10)(0.70) \\ &- [(0.30)(0.10) + (0.12)(0.20) + (0.09)(0.25) + (0.20)(0.20) + (0.10)(0.10)] \\ S_A &= 0.225 + 0.072 + 0.0585 + 0.10 + 0.07 - (0.03 + 0.024 + 0.0225 + 0.04 + 0.01) \\ &S_A &= 0.5255 - 0.1265 = 0.399 \end{split}$$

Design B:

$$S_B = 0.24 + 0.078 + 0.0702 + 0.12 + 0.085 - (0.03 + 0.024 + 0.0153 + 0.04 + 0.005)$$

= 0.5932 - 0.1143 = 0.4789

Design C:

Qiuling Wang, Siyuan Fang, Hybrid α-Discounting and Nonstandard Neutrosophic Complex MoBiNad Framework for Digital Media Art Design in Virtual and Augmented Reality Technology

$$S_c = 0.18 + 0.09 + 0.063 + 0.11 + 0.06 - (0.03 + 0.018 + 0.009 + 0.03 + 0.015)$$

= 0.503 - 0.102 = 0.401

4.2 Results Discussion

Design B achieved the highest overall score of 0.4789, making it the most favorable option among the three evaluated prototypes. This outcome reflects the design's strong performance in areas that users valued most, particularly artistic originality and emotional engagement. While Designs A and C also received generally positive feedback, they were slightly lower in truth membership and included more negative assessments, which affected their total scores.

One of the key strengths of this approach is its ability to deal with uncertainty in human feedback. Instead of requiring users to make binary decisions or choose from rigid scales, the use of Neutrosophic MoBiNad sets allowed their partial beliefs, doubts, and hesitations to be accurately represented. This flexibility led to more realistic and expressive evaluations, especially useful in artistic contexts where responses are often mixed or ambiguous.

To ensure fairness in weighing expert opinions, the α -discounting method was applied to manage inconsistencies across their judgments. Rather than rejecting conflicting inputs, the model balanced them mathematically, making the final weights more stable and justifiable. This helped maintain equity across all decision factors.

Importantly, the hybrid framework used in this study introduces an original contribution to the field. By combining α -Discounting, Neutrosophic logic, N-norm operators, and NeutroAlgebra, the method offers a structured way to evaluate creative work that typically resists traditional measurement. It successfully connects rigorous mathematical tools with the expressive, often subjective nature of digital media art, providing a fresh and meaningful way to support design decisions in VR and AR environments.

4.3 Comparative Evaluation with Other Methods

To better understand the value of the proposed hybrid framework, we compared its performance to two well-known decision-making approaches: the Analytic Hierarchy Process (AHP) and Intuitionistic Fuzzy Sets (IFS). The comparison focused on two main aspects critical to digital art design decisions: flexibility in handling uncertainty and computational simplicity.

The results are summarized in Figure 1, based on the same case study described earlier. Each method was evaluated using normalized performance scores between 0 and 1, derived from expert judgment and user feedback.

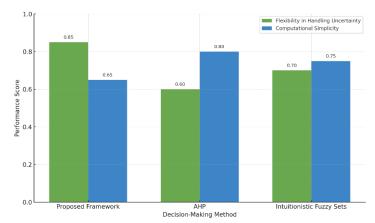


Figure 1. Comparison of Decision-Making Methods Based on Flexibility and Simplicity

the proposed framework scored the highest in flexibility, with a value above 0.8. This reflects its strong ability to model user hesitation, artistic ambiguity, and indeterminate feedback, which are common in VR/AR design environments. The AHP method, while widely used for structured decisions, performed well in computational simplicity but lower in handling uncertain or subjective input, since it relies on crisp, consistent judgments. The Intuitionistic Fuzzy Sets offered a balance between the two dimensions, performing better than AHP in uncertainty modeling but lacking the full range of expression allowed by the neutrosophic MoBiNad structure.

5. Conclusion

This paper introduced a new method to support decision-making in digital media art design, especially for projects using Virtual Reality and Augmented Reality. The method combined the α -Discounting approach with Nonstandard Neutrosophic Complex MoBiNad Sets, N-norms, and NeutroAlgebra. These tools allowed us to handle uncertainty, inconsistency, and partial knowledge challenges that often appear in creative and artistic environments. By applying this method to a real VR art case, we showed how expert preferences and user feedback could be processed in a balanced and flexible way. The results helped select the best design based on both logical calculations and human experience. This research adds a new and original perspective to the field by linking mathematical models with the creative world of interactive digital art.

References

- 1. Grau, O. (2003). Virtual art: From illusion to immersion. MIT Press.
- 2. Milgram, P., & Kishino, F. (1994). A taxonomy of mixed reality visual displays. *IEICE Transactions on Information and Systems*, E77-D(12), 1321–1329.
- 3. Saaty, T. L. (2008). Decision making with the analytic hierarchy process. *International Journal of Services Sciences*, 1(1), 83–98.
- 4. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353.

- 5. Dubois, D., & Prade, H. (1980). *Fuzzy sets and systems: Theory and applications*. Academic Press.
- 6. Smarandache, F. (2019). *Advances of standard and nonstandard neutrosophic theories*. Pons Publishing House.
- Smarandache, F. (2020). Extended nonstandard neutrosophic logic, set, and probability based on extended nonstandard analysis. *Symmetry*, 12(8), 1268. https://doi.org/10.3390/sym12081268
- 8. Smarandache, F. (2021). NeutroAlgebra theory, volume I. Academia.edu.
- Candy, L., & Edmonds, E. (2018). Practice-based research in the creative arts: Foundations and futures. *Leonardo*, 51(1), 63–69. https://doi.org/10.1162/LEON_a_01471
- Florentin Smarandache, Neutrosophy. / Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998; http://fs.unm.edu/eBook-Neutroosphics6.pdf.
- Florentin Smarandache, About Nonstandard Neutrosophic Logic (Answers to Imamura 'Note on the Definition of Neutrosophic Logic'), pp. 1-16, Cornell University, New York City, USA, {Submitted on 24 Nov. 2018 (version 1), last revised 13 Feb. 2019 (version 2)}; <u>https://arxiv.org/ftp/arxiv/papers/1812/1812.02534.pdf</u> UNM Website: http://fs.unm.edu/neut/AboutNonstandardNeutrosophicLogic.pdf
- F. Smarandache, Extended Nonstandard Neutrosophic Logic, Set, and Probability based on Extended Nonstandard Analysis, Symmetry 2019, 11, 515; doi:10.3390/sym11040515, <u>http://doi.org/10.5281/zenodo.2838678</u>; arXiv.org, Cornell University, <u>https://arxiv.org/ftp/arxiv/papers/1903/1903.04558.pdf</u>
- 13. F. Smarandache, Advances of Standard and Nonstandard Neutrosophic Theories, Pons Publishing House Brussels, Belgium, 307 p., 2019, http://fs.unm.edu/AdvancesOfStandardAndNonstandard.pdf

Received: Dec. 17, 2024. Accepted: June 28, 2025