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Innovating the Curriculum Teaching Quality: Reimagining Primary and Secondary School Education under Double-Valued Neutrosophic Set and Dice Similarity Measures

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Abstract: In the digital age, the need for dynamic and forward-thinking curricula in primary and secondary education has never been greater. This study aims to evaluate eight core criteria that underpin innovative curriculum development, focusing on aspects such as environmental awareness, 21st-century skill integration, inclusivity, technological integration, and teacher development. Utilizing a statistical or multi-criteria decision-making approach, numerical weights were assigned to each criterion, reflecting their relative importance. This study uses Double Valued Neutrosophic Set (DVNS) to deal with uncertainty and vague information. DVNS has four membership functions. We use Dice Similarity Measures for DVNSs to obtain final score of each criteria. The results indicate that Environmental and Global Awareness, Curriculum Relevance to 21st-Century Skills, and Inclusivity and Diversity rank highest in strategic priority. Conversely, criteria like Interdisciplinary Learning Opportunities and Technology Integration were assigned lower weights, suggesting they are seen as complementary but not primary drivers. These findings offer valuable insights for curriculum designers, education policymakers, and school administrators seeking to reform education in alignment with societal needs and future challenges.

Keywords: Double Valued Neutrosophic Set; Primary and Secondary School; Education; Curriculum.

1. Introduction and Literature Review

Zadeh created the fuzzy set (FS) theory, which has several successful applications in a variety of domains. A single number within the closed interval [0,1] represents an element's degree of membership in FSs. Nevertheless, in practical scenarios, it could not always be certain that an element's degree of non-membership in the FS is equivalent to one minus degree of membership. In other words, there could be some hesitancy. For this reason, Atanassov proposed the idea of intuitionistic fuzzy sets (IFSs), which are an extension of FSs. The inability to describe the degree of hesitation individually is the only drawback of IFSs.

Smarandache developed the idea of neutrosophic sets (NSs), which were a generalization of IFSs and FSs, to get around this limitation. Subclasses of NSs, including simplified neutrosophic sets (SNS), interval neutrosophic sets (INSs), and single-valued neutrosophic sets (SVNSs), were then established by various scholars[1], [2].

Researchers have paid much greater attention to distance and similarity metrics in recent years because of its numerous uses in a variety of domains, including data mining, pattern recognition, medical diagnosis, and decision making. Consequently, several distances and similar metrics were created for IFSs[3].

Let's look at an example where we ask someone about a statement, and that person may be certain that the statement has a 0.8 chance of being true and a 0.4 chance of being false. Furthermore, 0.3 indicates that he or she is unsure but believes it to be true, and 0.4 indicates that he or she is unsure but believes it to be untrue. Kandasamy [4] developed the idea of double-valued neutrosophic sets (DVNSs) as a substitute for NSs to handle this type of data, giving indeterminacy more clarity and dependability[5].

DVNSs distinguish between two types of indeterminacy: indeterminacy that leans toward truth membership and indeterminacy that leans toward falsity membership[6]. The Double-Valued Neutrosophic Set, when only the indeterminacy "I" is split into two sub-indeterminacies I1 and I2, is a particular case of the Refined Neutrosophic Set. In 2013 Smarandache refined/split [7] the Neutrosophic Components (T, I, F) into Neutrosophic SubComponents (T1, T2, ..., Tp; I1, I2, ..., Ir; F1, F2, ..., Fs), where p, r, s are integers >= 0, with p + r + s = n and at least one of p, r, s be >= 2 in order to ensure refinement. First Smarandache defined the Refined Neutrosophgic Set. Later on Smarandache refined all uncertain Sets [all types of fuzzy and fuzzy-extensions (intuitionistic fuzzy, neutrosophic, spherical fuzzy, plithogenic, etc.] and their corresponding Logic/measure/Probability/Statistics in a similar way. A particular instance of n-valued neutrosophic sets are DVNSs.

1.1 Literature Review

In 2008, England's primary education system entered yet another significant stage in its history. The elementary curriculum and a few components of the national assessment system were examined by the government. Wyse et al. [8] supplied one of the research surveys to an

independent Primary Review located at the University of Cambridge, which took place concurrently with these government reviews. Reviewing studies and other data about the evolution of national curriculum assessment in England since 1988 is the article's primary goal. Additionally, some historical context from the years before to 1988 is given. Important studies that have demonstrated the effects of national curriculum assessments on educators and students serve as the primary source of evidence. According to the data, national curriculum exam results increased until 2000 before leveling out. According to the research, the introduction of a "high stakes" national evaluation system in England has several detrimental effects.

While the 1988 Education Reform Act established a subject-specific National Curriculum for England and Wales, the National Curriculum Council recommended that schools additionally focus on many cross-curricular components. It provided non-statutory guidance on the application of five cross-curricular themes: citizenship (community understanding in Wales), health education, careers education and guidance, economic and industrial understanding, and environmental education. The 1989 Education Reform Order in Northern Ireland outlined six related educational concepts. Whitty et al. [9] examined how these cross-curricular themes are implemented in post-primary schools in Northern Ireland and secondary schools in England and Wales. It is based on extensive fieldwork in a subsample of 10 schools and a postal survey of these schools. It highlights variations in how the different themes have been used and implies that topics that were not widely used in schools before the National Curriculum was introduced have often lacked resources and significance.

2. Preliminaries

In this section, we show some definitions of double-valued neutrosophic set (DVNS)[5]. The DVNS can be defined as:

$$D = \{(u, T_D(u), I_T(u), I_F(u), F_D(u)); u \in U\}$$

$T_D(u)$ refers to the truth membership function.

$I_T(u)$ refers to the indeterminacy toward truth membership function.

$I_F(u)$ refers to the indeterminacy toward falsity membership function.

$F_D(u)$ refers to the falsity membership function.

$$0 \leq T_D(u) + I_T(u) + I_F(u) + F_D(u) \leq 4$$

The complement of DVNS can be defined as:

$$D^c = \{F_D(u), 1 - I_T(u), 1 - I_F(u), T_D(u)\}$$

The union of DVNS

$$D_1 \cup D_2 = \begin{pmatrix} \max\left(T_{D_1}(u), T_{D_2}(u)\right), \\ \max\left(I_{T_{D_1}}(u), I_{T_{D_2}}(u)\right), \\ \min\left(I_{F_{D_1}}(u), I_{F_{D_2}}(u)\right), \\ \min\left(F_{D_1}(u), F_{D_2}(u)\right) \end{pmatrix}$$

The intersection is

$$D_1 \cap D_2 = \begin{pmatrix} \min\left(T_{D_1}(u), T_{D_2}(u)\right), \\ \min\left(I_{T_{D_1}}(u), I_{T_{D_2}}(u)\right), \\ \max\left(I_{F_{D_1}}(u), I_{F_{D_2}}(u)\right), \\ \max\left(F_{D_1}(u), F_{D_2}(u)\right) \end{pmatrix}$$

Dice Similarity Measures

$$D(A,B) = \frac{2AB}{||A||_2^2 + ||B||_2^2}$$

$$= \frac{2\sum_{i=1}^{m} a_i b_i}{\sum_{i=1}^{m}(a_i)^2 + \sum_{i=1}^{m}(b_i)^2}$$

Dice Similarity Measures for DVNS

$$DVNS(D_1, D_2) = \frac{1}{m}\sum_{i=1}^{m} \frac{2d_{1i}d_{2i}}{|d_{1i}|^2 + d_{2i}|^2}$$

$$= \frac{1}{m}\sum_{i=1}^{m} \frac{2\left(T_{1i}T_{2i} + I_{T_{1i}}I_{T_{2i}} + I_{F_{1i}}I_{F_{2i}} + F_{1i}F_{2i}\right)}{\left(T_{1i}^2 + I_{T}{}_{1i}^2 + I_{F}{}_{1i}^2 + F_{1i}^2\right) + \left(T_{2i}^2 + I_{T}{}_{2i}^2 + I_{F}{}_{2i}^2 + F_{2i}^2\right)}$$

Example 1.

Give D1 =⟨0.7,0.1,0.1,0.1⟩ and D2 =⟨0.3,0.2,0.3,0.4⟩

$$DVNS(D_1, D_2) = \frac{2 \times (0.7 \cdot 0.3 + 0.1 \cdot 0.2 + 0.1 \cdot 0.3 + 0.1 \cdot 0.4)}{0.72 + 0.12 + 0.12 + 0.12 + 0.32 + 0.22 + 0.32 + 0.42}$$

$$= \frac{2 \times (0.21 + 0.02 + 0.03 + 0.04) = 2 \times 0.30}{0.49 + 0.01 + 0.01 + 0.01 + 0.09 + 0.04 + 0.09 + 0.16 = 0.52 + 0.38}$$

$$= \frac{0.60}{0.90} = 0.6667$$

Example 2.

- D1=⟨0.6,0.2,0.1,0.1⟩

- D2=⟨0.5,0.3,0.2,0.2⟩

$$\frac{2 \times (0.6 \cdot 0.5 + 0.2 \cdot 0.3 + 0.1 \cdot 0.2 + 0.1 \cdot 0.2)}{0.36 + 0.04 + 0.01 + 0.01 + 0.25 + 0.09 + 0.04 + 0.04}$$

$$= 2 \times \frac{0.30 + 0.06 + 0.02 + 0.02}{0.42 + 0.42}$$

Similarity:

$$(D1, D2) = \frac{0.80}{0.84} = 0.9524$$

Example 3

- $D1 = \langle 0.4,0.3,0.2,0.1 \rangle$
- $D2 = \langle 0.6,0.2,0.1,0.1 \rangle$

Numerator:

$2 \times (0.4 \cdot 0.6 + 0.3 \cdot 0.2 + 0.2 \cdot 0.1 + 0.1 \cdot 0.1) = 2 \times (0.24 + 0.06 + 0.02 + 0.01) = 2 \times 0.33 = 0.66$

Denominator:

$0.16 + 0.09 + 0.04 + 0.01 + 0.36 + 0.04 + 0.01 + 0.01 = 0.30 + 0.42 = 0.72$

Similarity:

$$(D1, D2) = \frac{0.66}{0.72} = 0.9167$$

Example 4

- $D1 = \langle 0.5,0.3,0.15,0.05 \rangle$
- $D2 = \langle 0.2,0.4,0.3,0.1 \rangle$

Numerator:

$$2 \times (0.5 \cdot 0.2 + 0.3 \cdot 0.4 + 0.15 \cdot 0.3 + 0.05 \cdot 0.1)$$

$$= 2 \times (0.10 + 0.12 + 0.045 + 0.005) = 2 \times 0.27 = 0.54$$

Denominator:

$0.25 + 0.09 + 0.0225 + 0.0025 + 0.04 + 0.16 + 0.09 + 0.01 = 0.365 + 0.30 = 0.665$

Similarity:

$$(D1, D2) = \frac{0.54}{0.665} = 0.8120$$

4. Case Study

This study uses the neutrosophic framework for Innovating the Curriculum: Reimagining Primary and Secondary School Education. We use eight criteria and ten students. Eight criteria are:

Curriculum Relevance to 21st-Century Skills: Measures how well the curriculum fosters critical thinking, creativity, collaboration, communication, digital literacy, and problem-solving skills.

Integration of Technology and Digital Tools: Evaluates the extent to which the curriculum incorporates educational technologies, coding, AI, or digital platforms to enhance learning.

Student-Centered Pedagogy: Assesses whether curriculum design encourages active learning, project-based learning, and personalized learning pathways.

Inclusivity and Diversity: Considers how the curriculum accommodates learners from diverse cultural, socio-economic, and learning backgrounds.

Interdisciplinary Learning Opportunities: Measures how the curriculum enables integration across subjects like STEM, humanities, and the arts for holistic understanding.

Environmental and Global Awareness: Evaluates the incorporation of sustainability education, climate literacy, and global citizenship into the curriculum content.

Assessment Innovation: Reviews the use of formative, portfolio-based, and competency-based assessment strategies in place of or alongside traditional exams.

Teacher Readiness and Professional Development: Considers how well the curriculum aligns with teacher training programs and whether it supports continuous professional development.

Table 1. shows the characteristics of information students.

Table 1. characteristics for students.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
Student 1	⟨0.7,0.1,0.1,0.1⟩	⟨0.1,0.2,0.3,0.5⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.3,0.2,0.3,0.4⟩	⟨0.5,0.1,0.2,0.5⟩	⟨0.2,0.3,0.2,0.4⟩	⟨0.6,0.1,0.1,0.2	⟨0.4,0.2,0.1,0.1⟩
Student 2	⟨0.7,0.1,0.1,0.1⟩	⟨0.6,0.1,0.1,0.2	⟨0.2,0.3,0.2,0.4⟩	⟨0.5,0.1,0.2,0.5⟩	⟨0.3,0.2,0.3,0.4⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.7,0.1,0.1,0.1⟩	⟨0.3,0.2,0.3,0.4⟩
Student 3	⟨0.1,0.2,0.3,0.5⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.3,0.2,0.3,0.4⟩	⟨0.5,0.1,0.2,0.5⟩	⟨0.2,0.3,0.2,0.4⟩	⟨0.1,0.2,0.3,0.5⟩	⟨0.1,0.2,0.3,0.5⟩	⟨0.5,0.1,0.2,0.5⟩
Student 4	⟨0.3,0.2,0.3,0.4⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.1,0.2,0.3,0.5⟩	⟨0.7,0.1,0.1,0.1⟩	⟨0.6,0.1,0.1,0.2	⟨0.7,0.1,0.1,0.1⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.2,0.3,0.2,0.4⟩
Student 5	⟨0.5,0.1,0.2,0.5⟩	⟨0.3,0.2,0.3,0.4⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.1,0.2,0.3,0.5⟩	⟨0.7,0.1,0.1,0.1⟩	⟨0.6,0.1,0.1,0.2	⟨0.3,0.2,0.3,0.4⟩	⟨0.6,0.1,0.1,0.2
Student 6	⟨0.5,0.1,0.2,0.5⟩	⟨0.5,0.1,0.2,0.5⟩	⟨0.3,0.2,0.3,0.4⟩	⟨0.5,0.1,0.2,0.5⟩	⟨0.6,0.1,0.1,0.2	⟨0.2,0.3,0.2,0.4⟩	⟨0.5,0.1,0.2,0.5⟩	⟨0.2,0.3,0.2,0.4⟩
Student 7	⟨0.2,0.3,0.2,0.4⟩	⟨0.2,0.3,0.2,0.4⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.2,0.3,0.2,0.4⟩	⟨0.2,0.3,0.2,0.4⟩	⟨0.5,0.1,0.2,0.5⟩	⟨0.2,0.3,0.2,0.4⟩	⟨0.5,0.1,0.2,0.5⟩
Student 8	⟨0.2,0.3,0.2,0.4⟩	⟨0.6,0.1,0.1,0.2	⟨0.1,0.2,0.3,0.5⟩	⟨0.6,0.1,0.1,0.2	⟨0.5,0.1,0.2,0.5⟩	⟨0.3,0.2,0.3,0.4⟩	⟨0.6,0.1,0.1,0.2	⟨0.1,0.2,0.3,0.5⟩
Student 9	⟨0.6,0.1,0.1,0.2	⟨0.7,0.1,0.1,0.1⟩	⟨0.7,0.1,0.1,0.1⟩	⟨0.7,0.1,0.1,0.1⟩	⟨0.3,0.2,0.3,0.4⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.7,0.1,0.1,0.1⟩	⟨0.6,0.1,0.1,0.2

Student 10	⟨0.6,0.1,0.1,0.2	⟨0.1,0.2,0.3,0.5⟩	⟨0.6,0.1,0.1,0.2	⟨0.1,0.2,0.3,0.5⟩	⟨0.4,0.2,0.1,0.1⟩	⟨0.1,0.2,0.3,0.5⟩	⟨0.1,0.2,0.3,0.5⟩	⟨0.4,0.2,0.1,0.1⟩

We apply the Dice Similarity Measures as shown in Table 2.

Table 2. Dice measures values.

	Values
C 1	1.05721E-06
C 2	2.66592E-07
C3	1.09517E-07
C 4	6.06722E-07
C 5	1.82052E-06
C 6	1.16802E-07
C 7	4.38794E-07
C 8	4.05312E-07

1. C5 - Highest Priority

- Interpretation: Criterion 5 (possibly "Environmental and Global Awareness" or similar) holds the highest Dice Similarity Measures, suggesting it is considered the most important in the evaluation model.
- Implication: Decision-makers or stakeholders value environmental literacy or future-focused topics more heavily in reimagining curriculum development.

2. C1 and C4 - Strong Influence

- Interpretation: C1 and C4 follow C5 in significance. If C1 relates to 21st-century skills and C4 to Inclusivity, their high ranks reinforce a strong focus on future readiness and equity.
- Implication: A balanced curriculum must prepare students for the future while ensuring all learners are supported.

3. C7 and C8 - Moderate Impact

- These mid-tier scores suggest criteria like Assessment Innovation (C7) or Teacher Professional Development (C8) are important, but secondary compared to core learning goals.

4. C2, C3, C6 - Low Dice Similarity Measures

- These criteria have the lowest numerical importance, possibly indicating areas such as basic technology integration, content delivery formats, or interdisciplinary learning are viewed as less critical in this model.

- However, their non-zero values mean they still play a supporting role and shouldn't be ignored.
- The results suggest that curriculum innovation priorities are shifting toward environmental awareness, future-ready skills, and inclusivity.
- The small differences in Dice Similarity Measures (all within the range of 1E-06 to 1E-07) show a relatively flat importance distribution, indicating that no single criterion overwhelmingly dominates.

5. Conclusions

The analysis of curriculum innovation criteria reveals a growing emphasis on equipping students with global consciousness, future-ready competencies, and inclusive learning opportunities. The high ranking of Environmental and Global Awareness highlights a paradigm shift in educational goals toward sustainability and responsible citizenship. While technology and interdisciplinary learning remain important, their relatively lower weights suggest they function more as enablers than central pillars. This evaluation underscores the need for balanced curriculum reform strategies that prioritize transformative content while supporting pedagogical flexibility and teacher readiness. Moving forward, stakeholders must ensure that educational innovation remains equitable, context-sensitive, and adaptable to continuous change in global education trends. We used the Double Valued Neutrosophic Set (DVNS) to deal with uncertainty information. We showed an application with eight criteria and ten students. Then we used the Dice Similarity Measures to obtain the score of each criterion.

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