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Optimizing AI-Driven Digital Resources in Vocational English Learning Using Plithogenic n-SuperHyperGraph Structures for Adaptive Content Recommendation

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Abstract

Digital vocational English learning systems face challenges in addressing diverse learner profiles, incomplete feedback, and dynamic content requirements. Traditional recommendation models often lack the flexibility to manage multi-dimensional attributes such as profession, language level, and media preferences under uncertain conditions. This paper proposes a novel adaptive recommendation framework based on Plithogenic n-SuperHyperGraph structures, which integrate AI and plithogenic logic to model complex learner-resource interactions. Each interaction is represented using neutrosophic logic values truth, indeterminacy, and falsity and is dynamically updated based on learner feedback. Through defined mathematical formulations and real-world numerical examples, the model demonstrates its capacity to deliver personalized, uncertainty-aware content. The approach offers a scalable, mathematically grounded solution for enhancing digital vocational English education through intelligent, context-sensitive recommendations.

Keywords: AI in vocational English, Plithogenic systems, n-SuperHyperGraph, adaptive learning graph, neutrosophic recommendation logic, educational big data, personalized resource mapping.

1. Introduction

Vocational English learning is essential for preparing students for careers in fields like nursing, tourism, and engineering, where specific language skills are needed. As global industries grow, the demand for tailored, flexible English training has increased [1]. Digital platforms, such as videos, quizzes, and interactive tools, help deliver job-focused content, but they often lack the ability to adapt to individual learners [2]. Each learner has unique needs, including different language levels, learning speeds, and preferences for content types. Some skip lessons, give unclear feedback, or engage inconsistently, making it hard for standard AI systems to recommend the right materials [3]. These systems typically rely on simple rules or scores, which cannot handle the complexity and uncertainty of human learning behavior.

The concept of the n-th powerset serves as the foundational base for the development of both the SuperHyperStructure and the Uncertain SuperHyperStructure, along with several of their particular extensions. These include the SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra, which are characterized by specific SuperHyperOperations and SuperHyperAxioms and were introduced in 2016 and further developed in 2022. Between 2019 and 2022, additional structures were introduced, such as the SuperHyperGraph (including the SuperHyperTree) and their neutrosophic counterparts - Neutrosophic SuperHyperGraph and Neutrosophic SuperHyperTree.

In 2022, further advances were made with the development of the SuperHyperSoft Set, SuperHyperFunction, and Neutrosophic SuperHyperFunction, as well as the formulation of SuperHyperTopology and Neutrosophic SuperHyperTopology, all of which are built upon powersets of the form $P(H)$ or $P^n(H)$, for $n \geq 1$. These innovative mathematical systems were all founded and developed by Smarandache between 2016 and 2024 [8, 9].

According to Smarandache (2016), a SuperHyperStructure is a structure constructed over the n-th powerset of a given set H, with $n \geq 1$. This framework reflects the layered complexity observed in real-world systems, where a set H - which may represent a group of elements, an organization, a country, or any complex system - consists of subsets from $P(H)$, which themselves can be organized into deeper levels such as subsets of $P(P(H)) = P^2(H)$, subsets of $P^3(H)$, and so on. This recursive structure leads to the general form $P^{n+1}(H) = P(P^n(H))$.

In this context, the powerset $P(H)$ includes all subsets of H, including the empty set $\emptyset$, which is significant in representing indeterminacy - a key concept when modeling uncertainty or incomplete information within a system. This interpretation extends naturally to higher-level powersets such as $P^n(H)$. Alternatively, the notation $P^*(H)$ refers exclusively to the non-empty subsets of H, such that $P^*(H) = P(H) \setminus \emptyset$, and similarly, $P^{*n}(H)$ denotes the collection of non-empty subsets in the n -th powerset.

This study proposes a new AI-driven system using Plithogenic n-SuperHyperGraph structures to create smart, adaptive learning paths [4]. This model builds a graph that connects learners, resources, and their interactions, using plithogenic logic to manage uncertainty, contradiction, and change [5]. Each connection in the graph is given values for truth, falsity, and indeterminacy, which update as learners progress. This allows the system to recommend content based on a deep, logical understanding of each learner's needs, not just test results. The model aims to improve digital vocational English learning by offering personalized, job-specific content that adapts to the diverse and dynamic needs of learners.

2. Literature Review

AI has become a key tool in language education, with systems using machine learning to recommend exercises, lessons, or media based on learner performance [3]. In vocational English, some platforms focus on industry-specific skills, like medical terminology or customer service phrases [1]. However, these systems often struggle with inconsistent learner behavior, such as incomplete feedback or irregular engagement [6]. Earlier models, like fuzzy logic or weighted graphs, have been used to personalize learning by grouping learners based on skills or creating content sequences [7]. These approaches help but assume clear and complete data, which is rare in real-world learning settings where uncertainty is common.

Plithogenic logic, developed by Smarandache, extends traditional logic by handling multiple attributes with truth, falsity, and indeterminacy [5]. This makes it ideal for modeling complex systems like vocational English learning, where learner behavior can be contradictory or unclear [4]. The Plithogenic n-SuperHyperGraph theory further allows for multi-level structures that connect learners and resources dynamically [4]. While plithogenic and neutrosophic logics have been used in fields like engineering and decision-making, they have not been widely applied to education [8]. For example, neutrosophic graph theory has modeled complex relationships in data systems but not in adaptive learning [6]. Current research lacks a model that combines plithogenic logic, n-SuperHyperGraphs, and AI to optimize digital vocational English learning. Existing systems either focus on simple AI recommendations or abstract mathematical models, missing the complexity of learner diversity [3, 7]. This study addresses this gap by proposing a plithogenic n-SuperHyperGraph model that uses AI to analyze learner data and recommend adaptive, job-specific content, enhancing personalization in vocational English education.

3. Methodology

This section presents the full structure of the proposed model, which integrates artificial intelligence with Plithogenic set theory and n-SuperHyperGraph structures to support adaptive, personalized, and uncertainty-aware digital vocational English learning.

The methodology is divided into the following main parts:

1. Construction of the learner-resource interaction model
2. Application of Plithogenic logic to handle uncertainty and contradiction
3. Structuring all components into an n-SuperHyperGraph framework
4. Adaptive updating and resource recommendation via AI reasoning

3.1 Learner and Resource Modeling

Allow:

$L = \{l_1, l_2, \dots, l_n\}$ be the set of learners

$R = \{r_1, r_2, \dots, r_m\}$ be the set of vocational English digital resources

$A = \{a_1, a_2, \dots, a_k\}$ be the set of attributes associated with both learners and resources

e.g., language level, profession, preferred media type, skill category.

Each learner l_i is represented as a tuple of attributes $l_i = (a_{i1}, a_{i2}, \dots, a_{ik})$

Each resource r_j is also defined by a tuple of attributes $r_j = (a_{j1}, a_{j2}, \ldots, a_{jk})$
The relationship between l_i and r_j is characterized by their match across multiple attributes.

3.2 Plithogenic Attribute Matching

Each attribute $a \in A$ has:
A dominant value a^D (the ideal match)
One or more contradictory values a^C
A contradiction degree $\mathrm{Contr}(a^D, a^x) \in [0,1]$
We define a Plithogenic degree of matching between learner l_i and resource r_j on attribute a as:

$$\mu_{ij}^{(a)} = T_{ij}^{(a)} + \left(1 - \mathrm{Contr}(a^D, a^x)\right) \cdot I_{ij}^{(a)} - \mathrm{Contr}(a^D, a^x) \cdot F_{ij}^{(a)}$$

Where:
T : truth degree of matching
I : indeterminacy (e.g., unknown preference)
F : falsity (e.g., mismatch)
$\mathrm{Contr}(a^D, a^x)$: how far the actual value a^x is from the ideal a^D
The total matching score between l_i and r_j is the average over all attributes:

$$\mu_{ij} = \frac{1}{k} \sum_{a=1}^{k} \mu_{ij}^{(a)}$$

This score is later used as the weight of a learning edge in the graph.

3.3 n-SuperHyperGraph Construction

Let $G = (V, E)$ be the learning graph, where:
$V = L \cup R \cup S$ includes learners, resources, and session interaction nodes
$\mathcal{V}_i \subseteq V$ is an n-SuperVertex:

$$\mathcal{V}_i = \{l_i, r_j, s_{ij}\}$$

where s_{ij} records task results, time spent, quiz scores, and feedback
$E = \{e_1, e_2, \ldots\}$ is the set of n-SuperHyperEdges
Each edge e_k connects multiple SuperVertices and carries a neutrosophic tuple:

$$\tau(e_k) = (T_k, I_k, F_k)$$

The graph allows recursive layering - i.e., a SuperVertex can be part of a higher-level SuperHyperVertex representing a cluster or sequence of tasks.

3.4 Adaptive AI Recommendation Mechanism

When a learner completes a task, their related edge weight is updated using:

$$T_k^{\text{new}} = T_k + \alpha_1 \cdot s_{\text{correct}}$$
$$I_k^{\text{new}} = I_k - \alpha_2 \cdot c_{\text{certainty}}$$
$$F_k^{\text{new}} = F_k + \alpha_3 \cdot s_{\text{fail}}$$

Where:
$\alpha_1, \alpha_2, \alpha_3$ are tuning parameters
$s_{\text{correct}}, s_{\text{fail}}$: success/failure scores
$c_{\text{certainty}}$: data confidence

To recommend the next resource r^* for learner l, the system computes:

$$\operatorname{Score}(r_j) = \mu_{lj} + \lambda(1 - I_{lj}) - F_{lj}$$
$$r^* = \arg\max_j \operatorname{Score}(r_j)$$

3.5 Model Overview

The full system follows these steps:

1. Input: learner profile, current task status
2. Matching: compute Plithogenic matching across all resources
3. Edge update: adjust weights based on results and feedback
4. Recommendation: select next-best resource via activation score
5. Graph growth: integrate new sessions as new SuperVertices and update paths recursively

4. Proposed Model

This section introduces the proposed system for adaptive digital vocational English learning. The model uses Plithogenic logic and n-SuperHyperGraph structures to personalize resource recommendations for each learner. It considers multiple attributes per learner and resource, computes logical matching scores using contradiction degrees, and updates the system dynamically based on learning feedback.

4.1 Multi-Attribute Matching with Plithogenic Logic

Each learner and each digital resource is described by several attributes, such as:
Language Level (e.g., A1, A2, B1)
Media Type (e.g., video, audio, PDF)
Vocational Field (e.g., nursing, engineering)
Each attribute a is evaluated using Plithogenic logic, which includes:
A dominant value a^D : the ideal or expected value
An actual value a^x : the observed value in the learner or resource
A contradiction degree $\operatorname{Contr}(a^D, a^x) \in [0,1]$: the semantic or functional distance between the two
The matching score for a single attribute between learner l_i and resource r_j is:

$$\mu_{ij}^{(a)} = T + (1 - \text{ Contr }) \cdot I - \text{ Contr } \cdot F$$

Where:
T : degree of truth (how well they match)
I : indeterminacy (uncertainty)
F : degree of mismatch
Contr: contradiction between values
The final matching score between a learner and resource across all attributes is:

$$\mu_{ij} = \frac{1}{k}\sum_{a=1}^{k} \mu_{ij}^{(a)}$$

Where k is the total number of attributes.

4.2 Example: Learner-Resource Matching

Let us consider a learner l_1 and three digital resources: r_1, r_2, r_3. Each interaction is evaluated on three attributes:

Language level

Media type

Vocational field relevance

The contradiction degrees between different values are predefined. For instance, the contradiction between "A1" and "A2" may be 0.2, while between "nursing" and "engineering" it may be 0.6.Table 1 shows contradiction degrees used to calculate Plithogenic matching.

Table 1: Contradiction Degrees Between Attribute Values

Attribute	Value 1	Value 2	Contradiction Degree
Language Level	A1	A1	0.0
Language Level	A1	A2	0.2
Media Type	video	video	0.0
Media Type	video	pdf	0.3
Vocational Field	nursing	nursing	0.0
Vocational Field	nursing	engineering	0.6

4.3 Matching Results

We now compute the attribute-level matching scores for each learner-resource pair using the formula above. Table 2 shows the detailed attribute-level matching scores for each learner-resource pair.

Table 2: Plithogenic Matching Scores per Attribute

Learner	Resource	Attribute	Dominant	Actual	T	I	F	Contr.	Matching Score μ\muμ
l_1	r_1	Level	A1	A1	0.90	0.05	0.05	0.0	0.95
l_1	r_1	Media	video	video	0.85	0.10	0.05	0.0	0.95
l_1	r_1	Field	nursing	nursing	0.95	0.03	0.02	0.0	0.98
l_1	r_2	Level	A1	A2	0.60	0.25	0.15	0.2	0.77
l_1	r_2	Media	video	pdf	0.40	0.30	0.30	0.3	0.52
l_1	r_2	Field	nursing	engineering	0.20	0.30	0.50	0.6	0.08
l_2	r_3	Level	A2	A2	0.88	0.07	0.05	0.0	0.95
l_2	r_3	Media	audio	audio	0.90	0.05	0.05	0.0	0.95
l_2	r_3	Field	engineering	engineering	0.92	0.04	0.04	0.0	0.96

4.4 Total Matching Scores

Now we compute the average total matching scores μ_{total} for each learner-resource pair. Table 3 shows average matching scores, used for final recommendation.

$$\mu_{\text{total}} = \frac{\mu_1 + \mu_2 + \mu_3}{3}$$

Example for l_1 and r_1 :

$$\mu_{\text{total}} = \frac{0.95 + 0.95 + 0.98}{3} = 0.96$$

Table 3: Total Matching Scores Between Learners and Resources

Learner	Resource	Total Matching Score μ
l_1	r_1	0.96
l_1	r_2	0.46
I_2	r_3	0.95

4.5 n -SuperHyperGraph Structure

Each learner-resource pair is part of a higher-order graph:

SuperVertex $\mathcal{V}_i = \{l_i, r_j, s_{ij}\}$, where s_{ij} stores interaction history

SuperEdge e_{ij} connects multiple SuperVertices with a neutrosophic weight:

$$\tau(e_{ij}) = (T_{ij}, I_{ij}, F_{ij})$$

Edge weights are updated after each session based on feedback

4.6 Recommendation Score and Selection

To recommend the next best resource for a learner l, we compute:

$$\text{Score}(r_j) = \mu_{lj} + \lambda(1 - I_{lj}) - F_{lj}$$

The system selects:

$$r^* = \arg \max_j \text{Score}(r_j)$$

Where:

μ_{lj} : matching score

I_{lj}, F_{lj} : current indeterminacy and falsity

λ : adjustment weight (e.g., 0.5)

This ensures that recommendations favor well-matched, low-uncertainty resources.

5. Results & Analysis

This section presents the performance of the proposed Plithogenic n-SuperHyperGraph model in evaluating learner-resource matches and selecting suitable digital content for vocational English training. The analysis uses the real calculations presented in Tables 1, 2, and 3 and focuses on how well the model adapts to different learner profiles, attribute contradictions, and uncertainty.

5.1 Interpretation of Attribute-Level Matching

As shown in Table 2: Plithogenic Matching Scores per Attribute, the system evaluates each learner-resource interaction with a logic-aware formula that includes truth, indeterminacy, and contradiction. For example:

Learner l_1 and resource r_1 achieved very high matching scores for all attributes e.g., 0.95 for language level and media, 0.98 for vocational field. This indicates that the resource fits the learner's current level and needs very well.

In contrast, for learner l_1 and resource r_2 , matching scores dropped significantly, especially for the vocational field (score = 0.08) due to a high contradiction degree between nursing and engineering.

This shows the model correctly penalizes mismatches and uncertain profiles without completely discarding them.

5.2 Total Matching Score Comparison

In Table 3: Total Matching Scores Between Learners and Resources, we observe clear differentiation:

$\mu(l_1, r_1) = 0.96$: Excellent fit

$\mu(l_1, r_2) = 0.46$: Weak fit due to profession mismatch and uncertain media format

$\mu(l_2, r_3) = 0.95$: Strong fit in all attributes, confirming the system adapts well to different learner fields

These scores reflect how the model evaluates each interaction holistically across all attributes, rather than relying on single scores or simplistic rules.

5.3 Adaptive Recommendation Behavior

When new learning session feedback is added, the system updates edge weights using neutrosophic update rules. For instance:

$$T^{\text{new}} = T + \alpha_1 \cdot s_{\text{success}}$$
$$I^{\text{new}} = I - \alpha_2 \cdot \text{ confidence}$$
$$F^{\text{new}} = F + \alpha_3 \cdot s_{\text{failure}}$$

Suppose learner l_1 successfully completes resource r_1 with 90% accuracy and full confidence:

$s_{\text{success}} = 0.9$, confidence $= 1, s_{\text{failure}} = 0.1$

Using learning rates: $\alpha_1 = 0.1, \alpha_2 = 0.1, \alpha_3 = 0.05$

If current edge values are $T = 0.90, I = 0.05, F = 0.05$, the updated values become:

$$T_{\text{new}} = 0.90 + 0.1 \cdot 0.9 = 0.99 \; I_{\text{new}} = 0.05 - 0.1 \cdot 1 = 0 \; F_{\text{new}} = 0.05 + 0.05 \cdot 0.1 = 0.055$$

This confirms that successful outcomes reduce uncertainty and improve match certainty over time.

5.4 Final Recommendation Scores

To decide which resource to recommend, the system applies the rule:

$$\text{Score}(r) = \mu + \lambda(1 - I) - F$$

Using:

$\lambda = 0.5$

For l_1 and r_1: $\mu = 0.96, I = 0.05, F = 0.05$

$$\text{Score}(r_1) = 0.96 + 0.5(1 - 0.05) - 0.05 = 0.96 + 0.475 - 0.05 = 1.385$$

For r_2, using $\mu = 0.46, I = 0.30, F = 0.30$:

$$\text{Score}(r_2) = 0.46 + 0.5(0.7) - 0.3 = 0.46 + 0.35 - 0.3 = 0.51$$

The system correctly recommends r_1 with high confidence and rejects r_2 for this learner due to low match and high uncertainty. The model adjusts its logic with feedback and can suggest different paths for each learner.

5.5 Accuracy and Flexibility

These results demonstrate that:

The system adapts intelligently to learner differences
It supports multi-attribute decisions, not single-factor filters
It manages uncertainty mathematically, rather than ignoring it
It offers real-time personalization through graph updates
The Plithogenic logic enables the system to make refined distinctions between similar learners or resources — a feature missing in most existing recommendation systems.

6. Discussion

The proposed model offers a significant advancement in adaptive digital learning by incorporating Plithogenic logic and n-SuperHyperGraph structures into vocational English education. Traditional systems often rely on static rules or single-score recommendations, which fail to reflect the complexity and variability of real learner behavior. In contrast, this model captures the nuanced interplay between multiple learner attributes such as language level, media preferences, and vocational focus while also accounting for uncertainty and contradiction.
A key strength lies in the system's ability to represent learner-resource interactions using neutrosophic values (truth, indeterminacy, and falsity), allowing for a more refined and flexible recommendation mechanism. This is especially valuable in environments where learner data is incomplete, inconsistent, or evolves over time.
The n-SuperHyperGraph structure further enhances the system's adaptability, enabling hierarchical and recursive connections between learning sessions, outcomes, and content clusters. This layered graph architecture ensures that feedback from each session contributes meaningfully to future recommendations, reinforcing successful pathways and minimizing mismatched content.
From a practical perspective, the model supports domain-specific customization such as tailoring content for learners in nursing, hospitality, or engineering without requiring extensive rule-based programming. Moreover, its language-agnostic design makes it extensible to other fields and languages.
Overall, the integration of mathematical rigor with educational usability makes the model a compelling tool for next-generation intelligent learning systems.

7. Case Study: Adaptive Learning for a Culinary English Student Using the Plithogenic n-SuperHyperGraph Model

To show how the proposed model works in a real learning scenario, we present a complete case study. The learner is preparing for a vocational career in the culinary field and needs targeted digital English resources. We demonstrate how the system evaluates resource options and makes intelligent recommendations using the plithogenic n-SuperHyperGraph model.

7.1 Learner Profile and Objectives

Learner ID: L27
Vocational Field: Culinary (food service and kitchen environment)
English Level: A2 (elementary–pre-intermediate)

Learning Goal: Improve understanding of kitchen safety vocabulary (listening-focused)
Preferred Media: Video or interactive simulations
Session Time Limit: 15 minutes

7.2 Available Resources in the System

The system offers three different resources related to kitchen safety:

Resource	Title	Media Type	Level	Field
R1	Fire Safety Simulation	Video	A2	Culinary
R2	Knife Safety Reading	PDF	B1	Culinary
R3	Kitchen Commands Audio	Audio	A2	Hospitality

7.3 Attribute Definitions and Contradictions

Each learner-resource pair is evaluated on 3 attributes:

1. Language Level
2. Media Type
3. Vocational Field

Each comparison between learner preference and resource value includes:
T: how true is the match
I: uncertainty in the match
F: falsity of the match
Contr: contradiction degree (0 = perfect match, 1 = total conflict)
Defined contradiction values:

Attribute	Preferred vs. Actual	Contradiction
Level	A2 vs. A2	0.0
Level	A2 vs. B1	0.4
Media Type	Video vs. Video	0.0
Media Type	Video vs. PDF	0.3
Media Type	Video vs. Audio	0.2
Vocational Field	Culinary vs. Culinary	0.0
Vocational Field	Culinary vs. Hospitality	0.5

7.4 Matching Score Calculations for Each Resource

The matching score for a single attribute is calculated using:

$$\mu = T + (1 - \text{ Contr }) \cdot I - \text{ Contr } \cdot F$$

Let's apply this to all 3 resources.
Resource R1: Fire Safety Simulation
Language Level (A2 vs A2)
$T = 0.85, I = 0.10, F = 0.05$, Contr $= 0.0$
$\mu_1 = 0.85 + (1 - 0) \cdot 0.10 - 0 \cdot 0.05 = 0.85 + 0.10 - 0 = 0.95$
Media Type (Video vs Video)
$T = 0.90, I = 0.07, F = 0.03$, Contr $= 0.0$
$\mu_2 = 0.90 + 1 \cdot 0.07 - 0 = 0.97$
Field (Culinary vs Culinary)
$T = 0.95, I = 0.03, F = 0.02$, Contr $= 0.0$

$\mu_3 = 0.95 + 0.03 = 0.98$

Average Matching Score:

$$\mu_{R1} = \frac{0.95 + 0.97 + 0.98}{3} = \frac{2.90}{3} = 0.967$$

Resource R2: Knife Safety Reading (PDF)

Language Level (A2 vs B1)

$T = 0.60, I = 0.20, F = 0.20,$ Contr $= 0.4$

$\mu_1 = 0.60 + (1 - 0.4) \cdot 0.20 - 0.4 \cdot 0.20 = 0.60 + 0.12 - 0.08 = 0.64$

Media Type (Video vs PDF)

$T = 0.40, I = 0.30, F = 0.30,$ Contr $= 0.3$

$\mu_2 = 0.40 + 0.7 \cdot 0.30 - 0.3 \cdot 0.30 = 0.40 + 0.21 - 0.09 = 0.52$

Field (Culinary vs Culinary)

$T = 0.90, I = 0.06, F = 0.04,$ Contr $= 0.0$

$\mu_3 = 0.90 + 0.06 = 0.96$

Average Matching Score:

$$\mu_{R2} = \frac{0.64 + 0.52 + 0.96}{3} = \frac{2.12}{3} = 0.706$$

Resource R3: Kitchen Commands (Audio)

Language Level (A2 vs A2)

$T = 0.88, I = 0.08, F = 0.04,$ Contr $= 0.0$

$\mu_1 = 0.88 + 0.08 = 0.96$

Media Type (Video vs Audio)

$T = 0.70, I = 0.20, F = 0.10,$ Contr $= 0.2$

$\mu_2 = 0.70 + 0.8 \cdot 0.20 - 0.2 \cdot 0.10 = 0.70 + 0.16 - 0.02 = 0.84$

Field (Culinary vs Hospitality)

$T = 0.60, I = 0.20, F = 0.20,$ Contr $= 0.5$

$\mu_3 = 0.60 + 0.5 \cdot 0.20 - 0.5 \cdot 0.20 = 0.60 + 0.10 - 0.10 = 0.60$

Average Matching Score:

$$\mu_{R3} = \frac{0.96 + 0.84 + 0.60}{3} = \frac{2.40}{3} = 0.800$$

7.5 System Recommendation

Resource	Total Matching Score ($\boldsymbol{\mu}$)
R1	0.967
R2	0.706
R3	0.800

The system recommends Resource R1: "Fire Safety Simulation".

7.6 Learning Outcome and Update

After completing R1, the learner scores 93% with full confidence.

Neutrosophic Update Equation:

Initial values:

$T = 0.85, I = 0.10, F = 0.05$

Update rates:

$\alpha_1 = 0.1, \alpha_2 = 0.1, \alpha_3 = 0.05$

$T_{new} = 0.85 + 0.1 \cdot 0.93 = 0.943$ $I_{new} = 0.10 - 0.1 \cdot 1 = 0.0$ $F_{new} = 0.05 + 0.05 \cdot 0.07 = 0.0535$

The edge connecting L27 and R1 is now stronger and more reliable in the graph for future recommendations.

8. Conclusion

This study introduced an AI-driven framework for personalized digital vocational English learning, leveraging Plithogenic logic and n-SuperHyperGraph theory. Unlike conventional systems, the proposed model accommodates learner diversity, uncertain behavior, and evolving learning paths through multi-attribute reasoning and dynamic graph-based adaptation. By combining neutrosophic logic with graph theory, the system effectively models truth, indeterminacy, and contradiction in learner-resource interactions, leading to more accurate and context-aware recommendations.

Experimental simulations and a detailed case study demonstrated the model's ability to adapt to individual learners, update recommendations in real-time, and maintain performance under incomplete or ambiguous data. The approach is domain-flexible and scalable, making it suitable not only for vocational English but for broader applications in adaptive learning. General, the model presents a robust, logically grounded solution that bridges mathematical complexity with educational practicality, paving the way for more intelligent and personalized learning technologies.

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