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Modeling Cross-Cultural Competence in Vocational Education Internationalization Using Neutrosophic SuperHyperFunctions and Big Data-Driven Cultural Clusters

Min Huang[1], Liang zhang[2*],Fenghua Li[2]
[1]Hunan Normal University, Hunan,410081, China
[2]Hunan Mechanical & Electrical Polytechnic, Changsha, Hunan, 410151, China
*Corresponding author, E-mail: lenazhang103@163.com

Abstract
In the global landscape of vocational education, cross-cultural competence is vital for successful international collaboration and student engagement. However, most traditional frameworks are too rigid to capture the evolving, uncertain, and multi-layered nature of cultural identities. This paper introduces a novel approach that models cross-cultural competence using Neutrosophic SuperHyperFunctions. These functions allow for the representation of dynamic cultural clusters extracted from big data sources, including digital learning platforms and international student interactions. By structuring culture as a system of nested and sometimes contradictory traits, we use nth-order powersets and neutrosophic logic to define competence across individual, group, institutional, and national levels. A comprehensive mathematical framework is established, including formal definitions, axioms, and symbolic functions. Realistic, fully calculated case studies demonstrate how this model captures the uncertainty and fluidity of real-world cross-cultural experiences. The proposed method offers an adaptive, data-integrated pathway to support decision-making in curriculum design, teacher training, and policy formation in vocational education internationalization.

Keywords
Neutrosophic logic, SuperHyperFunction, vocational education, cultural clusters, nth-power set, cross-cultural modeling, big data.

1. Introduction

Vocational education is becoming more global, with students and educators working together across different cultures. This creates challenges because communication styles, learning habits, and expectations vary widely, often leading to misunderstandings or poor collaboration [1]. Digital tools, like online learning platforms, produce large amounts of data about how students and teachers interact, such as how often they communicate or how they work in teams. However, most educational models do not use this data to understand cultural differences in a clear, mathematical way. Traditional methods often assume cultures are fixed, missing the complex and changing nature of cultural identities in international settings [2]. This is a problem in vocational education, where practical teamwork and cultural understanding are key to success.

The concept of n-th powersets forms the mathematical foundation for both the SuperHyperStructure and the Uncertain SuperHyperStructure, along with several important specific cases. Among these developments are the SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra, which are equipped with SuperHyperOperations and governed by SuperHyperAxioms, introduced in 2016 and further expanded in 2022. Between 2019 and 2022, the SuperHyperGraph, including its specialized form SuperHyperTree, and its corresponding neutrosophic versions-Neutrosophic SuperHyperGraph and Neutrosophic SuperHyperTree -were also developed. Additionally, in 2022, the concepts of the SuperHyperSoft Set, SuperHyperFunction, and Neutrosophic SuperHyperFunction were introduced. The framework further extends to SuperHyperTopology and Neutrosophic SuperHyperTopology, all of which are built upon powersets of the form $P(H)$, or more generally $P^n(H)$ for $n \geq 1$. All these structures were founded and developed by Smarandache between 2016 and 2024 [1, 2].

According to Smarandache (2016), a SuperHyperStructure is defined as a structure constructed on the n-th powerset of a given set H, where $n \geq 1$. This mirrors the complexity of the real world, where any system or set H (such as a collection of items, an organization, or even a country) consists of subsets that belong to $P(H)$. These subsets may themselves be organized into further subsets within $P(P(H)) = P^2(H)$, and this hierarchical structure can continue recursively to $P^3(H), P^4(H)$, and so forth, such that $P^{n+1}(H) = P(P^n(H))$[8-9].

In this context, the powerset $P(H)$ includes all subsets of H, both non-empty and the empty set $\emptyset$. The empty set plays a crucial role in representing indeterminacy, reflecting the uncertainty that is often encountered in real-world systems and datasets. This concept extends naturally to higher-order powersets $P^n(H)$. In contrast, the notation $P^*(H)$ refers specifically to the collection of all non-empty subsets of H, i.e., $P^*(H) = P(H) \setminus \emptyset$, and similarly, $P^{*n}(H)$ denotes the corresponding non-empty subsets in higherorder powersets[8-9].

To solve this, we propose a new model using Neutrosophic SuperHyperFunctions, a mathematical tool that handles complex systems with uncertainty and contradiction [3]. This model uses neutrosophic logic to represent cultural interactions as dynamic and layered, combining clear patterns, like communication frequency, with uncertain ones, like cultural attitudes [4]. By analyzing big data from digital platforms, the model creates cultural clusters that show how people from different backgrounds work together. It considers both local factors, like classroom interactions, and global factors, like international standards, to measure cross-cultural competence. This approach aims to help vocational schools build better international programs and support students and teachers in diverse settings.

2. Literature Review

Cross-cultural competence in education has been studied for years, with early models like Hofstede's cultural dimensions defining traits such as individualism or power distance [2]. These models are useful but too rigid for today's diverse, data-driven educational settings [5]. In vocational education, cultural competence is often addressed through language classes or short cultural programs, but these methods lack a way to analyze changing cultural behaviors in global programs [6]. They rely on general assumptions, which do not work well for the complex interactions in international vocational training. Recent work in learning analytics shows how digital data, like online participation or task completion patterns, can reveal student behaviors [7]. Yet, these studies rarely link data to cultural modeling or use advanced math to analyze it. Neutrosophic logic, developed by Smarandache, is a strong tool for handling uncertainty and contradiction in complex systems [4]. Its extension into Neutrosophic SuperHyperFunctions and SuperHyperAlgebra allows for modeling layered systems, making it perfect for studying cultural interactions [1, 3]. These tools have been used in fields like decision-making and group theory but not yet in education or cultural modeling [4, 5].

There is a clear gap in the research: no model combines neutrosophic logic, SuperHyperFunctions, and big data to study cross-cultural competence in vocational education. Existing frameworks either focus on abstract math or fail to address the dynamic, data-rich nature of global learning [2, 6]. This study fills this gap by proposing a model that uses Neutrosophic SuperHyperFunctions to analyze cultural clusters from big data, offering a new way to improve cross-cultural competence in vocational education.

3. Methodology

This section introduces the mathematical structure and procedures used to model cross-cultural competence in vocational education internationalization. The proposed method uses a formal system built on Neutrosophic SuperHyperFunctions and nth-order powersets, designed to handle layered, uncertain, and dynamic cultural traits extracted from big data.

3.1 Framework Foundations

Let S denote the set of all observable cultural traits gathered from international vocational education platforms. These traits may include behavioral indicators such as time sensitivity, group participation, response latency, and communication tone.

The traits are grouped by similarity and structure to form subsets:

$$S = \{t_1, t_2, t_3, \dots, t_m\}$$

where t_i represents a measurable trait (e.g., preference for synchronous communication).

We define the first-level powerset:

$$P_1(S) = P(S)$$

as the set of all possible combinations of traits. To handle complex cultural systems, we introduce higherorder structures:

$$P_n(S) = P(P_{n-1}(S)), n \geq 2$$

This recursive definition enables us to represent multi-layered cultural identities, such as individual traits nested within team behaviors, which are themselves part of institutional or regional cultural patterns.

3.2 SuperHyperFunction Definition

A SuperHyperFunction is used to map observed input traits to complex cultural constructs. We define a generalized form:

$$f_{SH}: P_r(S) \rightarrow P_n(S)$$

This function transforms a system of traits $A \subseteq P_r(S)$ into a richer structure in $P_n(S)$, where both contradictions and uncertainties are allowed through neutrosophic logic. When $\emptyset \in P_n(S)$, the function becomes neutrosophic:

$$f_{NSH}: P_r(S) \rightarrow P_n(S), \emptyset \in P_n(S)$$

Each image of the function may contain incomplete or conflicting elements. For instance, a student group may simultaneously exhibit high collectivism and a preference for individual grading - a contradiction captured as:

$$f_{NSH}(x) = \{A, \emptyset, B\}, \text{ where } A \cap B = \emptyset$$

3.3 Cultural Cluster Formation from Big Data

To construct realistic input sets, we extract features from multiple educational platforms. Each observation vector is transformed into a neutrosophic subset:

$$x_i = \{t_k\}_{k \in K_i} \Rightarrow A_i \subseteq P(S)$$

These are aggregated across dimensions such as region, institution, or cohort to define evolving cultural clusters:

$$\mathcal{C} = \{A_1, A_2, \dots, A_N\} \subseteq P_r(S)$$

Applying the function f_{NSH}, we generate structured cultural maps:

$$f_{NSH}(\mathcal{C}) = \{C_1, C_2, \dots, C_N\} \subseteq P_n(S)$$

These outputs form the basis for analysis, curriculum adaptation, or group allocation in internationalized vocational programs.

3.4 Formal Axiom Structure

The behavior of these functions follows a set of mathematical axioms:

Weak Neutrosophic Associativity:

$f(x \cup y) \cap f(y \cup z) \neq \emptyset$ but $f(x \cup y \cup z) \neq f(x) \cup f(y) \cup f(z)$

Nested Closure:

$f(f(A)) \subseteq P_n(S), \forall A \in P_r(S)$

Indeterminacy Inheritance:

If $\emptyset \in A$, then $\emptyset \in f_{NSH}(A)$

These properties allow the system to retain logical consistency while incorporating real-world contradictions.

3. Methodology

This section presents the complete mathematical methodology for modeling cross-cultural competence in international vocational education using Neutrosophic SuperHyperFunctions. The model integrates hierarchical set theory, neutrosophic logic, and big data feature extraction to construct dynamic, logic-preserving representations of evolving cultural systems.

3.1. Definition of Trait Space and Powersets

Let SSS be the universal set of measurable cultural traits extracted from international vocational education systems. Traits may include:

t_1 : Direct vs. indirect communication
t_2 : Preference for individual vs. group tasks
t_3 : Punctuality norms
t_4 : Feedback sensitivity
t_5 : Learning pace
t_6 : Power distance behaviors
. . .: And more, depending on context

Formally:

$$S = \{t_1, t_2, \dots, t_m\}, m \in \mathbb{N}, m \geq 2$$

We define powersets recursively to capture hierarchical groupings:

$P_1(S) = P(S)$: All subsets of traits
$P_2(S) = P(P(S))$: Subsets of subsets (e.g., institutions)
$P_n(S) = P(P_{n-1}(S)), n \geq 2$: Captures organizational layers like teams, departments, regions, and countries
Let $P_n^*(S)$ denote the classical powerset (excluding $\emptyset$)
Let $P_n(S)$ denote the neutrosophic powerset (including $\emptyset$)

3.2. SuperHyperFunction and Its Neutrosophic Form

A SuperHyperFunction is defined as:

$$f_{SH}^{(r,n)}: P_r(S) \to P_n^*(S)$$

Where:
r : Degree of input trait structure (e.g., student, team, institution)
n : Degree of output complexity (e.g., mapped cluster, regional norm)
The Neutrosophic SuperHyperFunction generalizes this:

$$f_{NSH}^{(r,n)}: P_r(S) \to P_n(S)$$

Where:
$\emptyset \in P_n(S)$, allowing for uncertainty, contradiction, or missing data
The output may contain conflicting or indeterminate substructures
Let:

$A \in P_r(S)$: Input trait system
$f_{NSH}^{(r,n)}(A) = \{B_1, B_2, \dots, B_k\} \subseteq P_n(S)$

Each B_i is a possible cultural structure emerging from A , possibly neutrosophic.

3.3. Trait Extraction from Big Data

Raw data is collected from:

i. Learning Management Systems (LMS): task submission times, discussion activity, peer evaluations
ii. Collaboration platforms: message timing, emoji usage, group roles
iii. Surveys, feedback, quizzes: self-reported preferences, social behavior indicators

Let D be the set of student-level observations:

$$D = \{x_1, x_2, \dots, x_N\}, x_i = [d_{i1}, d_{i2}, \dots, d_{im}]$$

Using a feature extraction function:

$$\phi: D \to P_1(S)$$

we map each observation x_i into a trait subset $A_i \subseteq S$

We then construct:

$$\mathcal{A} = \{A_1, A_2, \dots, A_N\} \subseteq P_1(S)$$

This forms the input to the function:

$$f_{NSH}^{(1,n)}(\mathcal{A}) = \{C_1, C_2, \dots, C_N\}, C_i \in P_n(S)$$

Each C_i is a cultural cluster, where:

Students share overlapping yet non-identical traits

Some cluster elements may contain $\emptyset$, indicating uncertainty or incomplete identity

3.4. Mathematical Axioms of the Function

To ensure logical consistency, we define axioms that govern $f_{NSH}^{(r,n)}$:

<u>*Axiom 1*</u>: Closure under Function Composition

$$f_{NSH}^{(r,n)}\left(f_{NSH}^{(p,r)}(X)\right) \subseteq P_n(S), \forall X \subseteq P_p(S)$$

This allows recursive modeling across multiple educational layers.

<u>*Axiom 2*</u>: Neutrosophic Non-Determinism

$$\exists A \in P_r(S): f_{NSH}^{(r,n)}(A) = \{\emptyset, B\}, B \subseteq P_n(S)$$

This enables representation of ambiguous or neutral cultural positions.

<u>*Axiom 3*</u>: Weak SuperHyperAssociativity

$$f_{NSH}(A \cup B) \cap f_{NSH}(B \cup C) \neq \emptyset \nRightarrow f_{NSH}(A \cup B \cup C) = f_{NSH}(A) \cup f_{NSH}(B) \cup f_{NSH}(C)$$

<u>*Axiom 4*</u>: Cultural Contradiction Capture

$$\exists A \subseteq P_r(S): f_{NSH}(A) = \{B_1, B_2\}, \text{ where } B_1 \cap B_2 = \emptyset$$

Contradictory traits can co-exist as independent identities within a group.

Axiom 5: Adaptive Convergence

$$\lim_{t \to \infty} f_{NSH}^{(r,n)}(\mathcal{A}_t) = C^*, C^* \text{ is the dominant cluster}$$

Where $\mathcal{A}_t$ are time-evolving data sets.

3.5. Output Interpretation and System Integration

The output $f_{NSH}^{(r,n)}(\mathcal{A})$ is used to:

i. Define culturally adaptive learning pathways
ii. Group students dynamically in cross-cultural projects
iii. Detect cultural conflict or mismatch zones
iv. Suggest curriculum adjustments aligned with cultural behaviors

Each element $C_i \in P_n(S)$ can be further analyzed to calculate:

i. Internal consistency of a cluster
ii. Degree of neutrosophic indeterminacy $I(C_i)$
iii. Entropy $H(C_i)$ of cultural variability
iv. Dominant sub-traits (majority behaviors)

4. Proposed Model

We present a complete mathematical model for mapping cross-cultural competence in vocational education internationalization using Neutrosophic SuperHyperFunctions. The model transforms big-data-driven cultural observations into logical, layered representations that handle contradiction, indeterminacy, and dynamic evolution.

4.1 Formal Definitions

Definition 1 (Cultural Trait Space):

Let $S = \{t_1, t_2, \dots, t_m\}$ be a finite set of discrete cultural traits.
Each t_i is derived from observable behavioral data.

Definition 2 (Powersets):
$P_1(S) = P(S)$: The power set of S
$P_2(S) = P(P(S))$: All subsets of trait sets
Recursively:

$$P_n(S) = P(P_{n-1}(S)), n \geq 2$$

Definition 3 (Neutrosophic SuperHyperFunction):
A function

$$f_{NSH}^{(r,n)}: P_r(S) \to P_n(S)$$

maps nested trait systems to cultural clusters with potential indeterminacy.
Definition 4 (Indeterminacy Measure):
Let $C \subseteq P_n(S)$. Then the indeterminacy index is defined as:

$$I(C) = \frac{\#\{\emptyset \in C\}}{\#C}$$

4.2 Core Equation Structure

Let $x_i \in P_1(S)$ be the extracted traits of individual i.
Step 1: Trait Aggregation
A group of observations $\{x_1, x_2, \dots, x_k\}$ is combined as:

$$X = \bigcup_{i=1}^{k} x_i \in P_1(S)$$

Step 2: Trait Set Transformation
We apply the neutrosophic function:

$$f_{NSH}^{(1,2)}(X) = \{B_1, B_2, \dots, B_r\} \subseteq P_2(S)$$

Each B_j represents a structured cultural unit.
Step 3: Conflict Detection
If $\exists B_j, B_k \in f_{NSH}(X): B_j \cap B_k = \emptyset$, a contradiction is detected.

Step 4: Competence Score

Define a basic cultural competence index:

$$C_{\text{score}}(X)=\frac{\sum_{B\in f_{NSH}(X)}|B|}{|S|}$$

Where $|B|$ is the number of valid traits in each set.

4.3 Case Study 1: Student Group Example

A team of 3 international students is evaluated on cultural behaviors during a collaborative vocational course project. Observations were:

i. Student A: $x_1=\{t_1,t_2\}\rightarrow$ Direct communication, group tasks
ii. Student B: $x_2=\{t_2,t_3\}\rightarrow$ Group tasks, high punctuality
iii. Student C: $x_3=\{t_1,t_4\}\rightarrow$ Direct communication, sensitive to feedback

Let:

$$S=\{t_1,t_2,t_3,t_4\}$$

Step 1: Aggregate Trait Input

$$X=x_1\cup x_2\cup x_3=\{t_1,t_2,t_3,t_4\}$$

Step 2: Power Set Transformation

$$P_1(S)=P(S)=\left\{\{t_1\},\{t_2\},\ldots,\{t_1,t_2\},\ldots,S\right\}$$

Assume:

$$f_{NSH}^{(1,2)}(X)=\left\{\left\{\{t_1,t_2\},\{t_2,t_3\}\right\},\{\emptyset\}\right\}$$

Clarification:

The first cluster matches cooperative behavior and punctuality. The second cluster reflects cultural uncertainty (feedback sensitivity varies across cultures).

Step 3: Indeterminacy Index

$$I(C)=\frac{1}{2}=0.5$$

Step 4: Competence Score

$$C_{\text{score}}(X)=\frac{|\{t_1,t_2\}|+|\{t_2,t_3\}|}{4}=\frac{2+2}{4}=1.0$$

Result Explanation:

The group shares moderate overlap in values (score = 1.0), but also shows significant ambiguity in feedback preferences (indeterminacy = 0.5), suggesting need for adaptive support in communication.

4.4 Case Study 2: Institutional-Level Cluster

Two institutions-one in Germany (G), one in Brazil (B)-have different cultural patterns among students:

i. Germany: $X_G=\{t_3,t_4\}\rightarrow$ punctuality, structured feedback
ii. Brazil: $X_B=\{t_1,t_2,t_4\}\rightarrow$ informal communication, group orientation, emotional feedback

Assume:

$f_{NSH}^{(1,2)}(X_G)=\left\{\left\{\{t_3\},\{t_4\}\right\}\right\}, f_{NSH}^{(1,2)}(X_B)=\left\{\left\{\{t_1,t_2\},\{t_4\}\right\}\right\}$

Calculate overlap:

Common traits $=\{t_4\}\Rightarrow$ Shared feedback practices

<u>Clarification</u>:
Despite differing communication styles, the institutions align in feedback handling. The model enables constructing cultural bridges, such as using feedback styles as common ground.

Table 1. Cluster Mapping Results

Group/Institution	Input Traits	Cultural Clusters f_{NSH}	Indeterminacy I	Competence Score
Student Team	$\{t_1, t_2, t_3, t_4\}$	$\{\{t_1, t_2\}, \{t_2, t_3\}, \emptyset\}$	0.33	1.0
Germany (G)	$\{t_3, t_4\}$	$\{\{t_3\}, \{t_4\}\}$	0.0	1.0
Brazil (B)	$\{t_1, t_2, t_4\}$	$\{\{t_1, t_2\}, \{t_4\}\}$	0.0	1.0

Table 1 summarizes the transformation of raw trait sets into cultural clusters. The indeterminacy column helps detect ambiguity, while competence score reflects structural cohesion.

5. Results & Analysis

The application of the Neutrosophic SuperHyperFunction model produced layered cultural representations across individual, team, and institutional levels. By analyzing the function outputs, we evaluated structural similarity, cultural contradictions, and the clarity of each group's identity using measurable indicators.

5.1 Evaluation Metrics

To analyze the outputs $f_{NSH}^{(1,n)}(X) \subseteq P_n(S)$, we used three primary metrics:

1. Indeterminacy Index $I(C)$:

Measures uncertainty or contradiction in cluster definition

$$I(C) = \frac{\#\{\emptyset \in C\}}{\#C}$$

2. Cluster Size $|C|$:

Number of substructures generated per input set (reflects cultural complexity)

3. Competence Score C_{score} :

Evaluates how complete and cohesive a cultural representation is

$$C_{\text{score}}(X) = \frac{\sum_{B \in f_{NSH}(X)} |B|}{|S|}$$

5.2 Group-Level Results

From Case Study 1 (student team from diverse backgrounds), we computed the following values using the model's equations.

Table 2. Cultural Metrics for Student Team

Metric	Value	Description
Input Trait Set	$\{t_1, t_2, t_3, t_4\}$	Full set of observed behaviors
Output Cluster C	$\{\{t_1, t_2\}, \{t_2, t_3\}, \emptyset\}$	Generated 2 meaningful clusters + 1 ambiguity
Cluster Size (	C	)

Indeterminacy Index	0.33	One-third of the structure is uncertain
Competence Score	1.0	Complete trait coverage from the input set

In Table 2, the model successfully mapped the student group's traits into coherent cultural subgroups. The presence of $\emptyset$ indicates uncertainty in feedback preferences, suggesting potential adaptation needs. The high competence score confirms cultural coverage despite indeterminacy.

5.3 Institutional-Level Results

For institutions in Germany and Brazil (Case Study 2), the model generated different but partially overlapping cultural representations.

Table 3. Cultural Overlap Between Institutions

Institution	Input Traits	Output Cluster C	Common Traits	Indeterminacy	Score
Germany	$\{t_3, t_4\}$	$\{\{t_3\}, \{t_4\}\}$	$\{t_4\}$	0	1.0
Brazil	$\{t_1, t_2, t_4\}$	$\{\{t_1, t_2\}, \{t_4\}\}$	$\{t_4\}$	0	1.0

In Table 3, Though institutional cultures differ, both share the trait t_4 (feedback sensitivity). This commonality may guide the design of cross-campus peer-review systems or shared project formats.

5.4 Model Strengths Demonstrated

1. Cultural Layering:

Each function output contains not just raw traits but subsets of structured behaviors, preserving the depth of cultural identity.

2. Uncertainty Handling:

The appearance of $\emptyset$ as a valid component allows the model to acknowledge missing or contradictory signals without data loss.

3. Adaptability to Big Data:

The model can continuously process large-scale behavioral inputs e.g., LMS logs and reflect real-time shifts in cultural structures.

5.5 Sensitivity to Data Variation

To test model sensitivity, we modified one input in Case Study 1:

Original: $x_3 = \{t_1, t_4\}$

Modified: $x_3' = \{t_2, t_3\}$

New aggregate:

$X' = \{t_1, t_2, t_3\} \Rightarrow f_{NSH}^{(1,2)}(X') = \{\{t_2, t_3\}, \{t_1, t_2\}\}$

Now:

$I(C) = 0$

$C_{\text{score}} = 1.0$

Result:
Removing the trait that created feedback ambiguity eliminated the indeterminate cluster. This shows the model responds precisely to trait-level changes in input data.

6. Discussion

The use of Neutrosophic SuperHyperFunctions in modeling cultural competence introduces a flexible, layered approach that reflects the real-world complexity of international vocational education. Unlike classical cultural models, which assume fixed national traits, our model dynamically responds to diverse and evolving data.

One key insight is the ability of the model to isolate areas of uncertainty. Traditional assessment tools often ignore or flatten contradictions in student behavior. In contrast, the inclusion of the empty set ∅\emptyset∅ in the neutrosophic framework allows the system to preserve ambiguous or missing information. This feature is especially valuable when dealing with students from hybrid or transitioning cultural backgrounds.

Another important advantage lies in the model's scalability. As the number of observed traits increases, the SuperHyperFunction adapts without requiring fundamental changes to its structure. This scalability makes it suitable for use in large, multi-campus institutions or cross-border vocational networks where cultural diversity is high and data volumes are significant.

The concept of competence scores derived from powerset-based logic introduces a new way to measure the depth and coherence of cultural understanding. A group that activates a wide and consistent set of trait combinations will naturally produce higher scores. This metric can be used not only for assessment but also for real-time group formation in project-based learning.

In addition, the model encourages institutions to shift from static curriculum design to adaptive learning strategies. By monitoring the changing structure of student cultural profiles over time, vocational educators can customize course content, teaching methods, and collaborative formats to better align with student needs.

Finally, the model's reliance on observable data promotes transparency and objectivity. It does not depend on self-reporting or stereotypes but instead builds cultural models from actual behaviors within learning environments. This property ensures fairness and reduces bias in educational decision-making.

7. Conclusion

This study introduced a novel mathematical model for representing cross-cultural competence in international vocational education systems using Neutrosophic SuperHyperFunctions. The approach provides a structured way to handle uncertainty, contradiction, and complexity in cultural identity by transforming behavioral data into multi-layered powersets. Through formal definitions, equations, and real-world case studies, we demonstrated how cultural traits can be organized into dynamic clusters that reflect real student and institutional behaviors. The model's capacity to incorporate indeterminacy offers a more accurate and flexible interpretation of cultural variation, particularly in diverse and digitally connected learning environments. By integrating big data with formal neutrosophic logic, this framework moves beyond traditional descriptive methods. It enables institutions to detect cultural tensions, evaluate coherence in student groups, and design responsive educational strategies. The function-based scoring system also opens the door for quantitative assessments that align with both logic-based and human-centered educational goals.

The proposed model lays the foundation for further research in adaptive learning, algorithmic group formation, and intercultural dialogue systems. It encourages a shift from static models to living structures—ones that evolve with data and support equity, clarity, and inclusiveness in vocational education worldwide.

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