Neutrosophic Sets and Systems, Vol. 88, 2025
University of New Mexico

Transitive and Strongly Transitive Neutrosophic Fuzzy Matrices

M. Anandhkumar[1], A. Savitha Mary[2], M. Kavitha[3], S. Subramanian[4], V. Sathishkumar[5], S.M. Chithra[6]

[1]Department of Mathematics, IFET College of Engineering (Autonomous), Villupuram-605 108, Tamilnadu, India.
anandhkumarmm@gmail.com

[2]Department of Mathematics, SRM Institute of Science and Technology, Faculty of Science and Humanities, Ramapuram Campus, Chennai, 600089, India.
Savitha.mary139@gmail.com

[3]Department of Mathematics, Panimalar Engineering College, Chennai – 600123, Tamil Nadu, India.

mkavitha.saran@gmail.com

[4]Department of Mathematics, R.M.D. Engineering College, Kavarapettai 601 207

sssubramanian87@gmail.com

[5]Department of Mathematics, Rajalakshmi Institute of Technology (Autonomous), Chennai
vsathishkumar2020@gmail.com

[6]Department of Mathematics, R.M.K College of Engineering and Technology, Chennai-601206, Tamilnadu, India.
chithra.sm@rmkcet.ac.in

Abstract – This paper explores several fundamental properties of transitive Neutrosophic fuzzy matrices (TNFM) . It demonstrates that any Neutrosophic fuzzy matrices (NFM) can be expressed as the sum of a nilpotent Neutrosophic fuzzy matrices (NNFM) and a symmetric Neutrosophic fuzzy matrices (SNFM). Additionally, the concept of a strongly TNFM is introduced. Lastly, the canonical forms of both TNFM and strongly TNFM are presented.

Keywords: TNFM, STNFM, NNFM.

1.Introduction

Fuzzy and intuitionistic fuzzy sets, first presented by Zadeh [1] and later expanded by Atanassov [2], have been pivotal in addressing uncertainty in mathematical and computational frameworks. Atanassov's work on intuitionistic fuzzy sets laid the foundation for various extensions, including fuzzy matrices, which have gained significant attention due to their applications in decision-making, modeling, and optimization. Pal et al. [18, 19] further developed intuitionistic fuzzy matrix theory, delving into their determinants and operational properties.The study of fuzzy matrices has evolved over decades, with foundational contributions by Hashimoto [12–14], who investigated transitivity, canonical forms, and convergence properties. Kim and Roush [15, 16] expanded these ideas by introducing Boolean and fuzzy matrix theories, leading to practical applications in diverse fields. Complementing this, Fan and Liu [5–8] and Guu et al. [9–11] examined the behavior of power sequences, highlighting their oscillations, monotonicity, and convergence properties. Mishref and Emam [33] focused on subinverses and transitivity in fuzzy matrices, enriching the theoretical framework further.

Recent advancements have included interval-valued FM by Shyamal and Pal [25] and the exploration of generalized IFM by Bhowmik and Pal [3]. Neutrosophic sets, presented by Smarandache [33], have generalized IFSs by integrating the concept of indeterminacy, spurring new research into neutrosophic fuzzy matrices. This includes the works of Anandhkumar et al. [29, 31] on k-idempotent and pseudo-similar neutrosophic fuzzy matrices, and Uma et al. [30], who explored FNSM of Type I and Type II. These studies have broadened the theoretical and practical scope of fuzzy and neutrosophic matrices. In this context, transitivity and its strong variations remain crucial topics, as highlighted by Pradhan and Pal [24] and Emam [25], who examined consistent and weak transitive IFMs. This paper aims to advance the understanding of TNFM by presenting their decomposition into nilpotent and symmetric forms. Additionally, the canonical forms of both TNFM and STNFM are established, contributing to the growing body of knowledge in FM theory and its applications.

1.1 Abbrivations

IFM = Intuitionistic Fuzzy Matrices
NFM = Neutrosophic Fuzzy Matrices
NFPM = Neutrosophic Fuzzy Permutation Matrices
TNFM = Transitive Neutrosophic Fuzzy Permutation Matrices
NNFM = Nilpotent Neutrosophic Fuzzy Matrices
STNFM = Strongly Transitive Neutrosophic Fuzzy Permutation Matrices
SNFM= Symmetric Neutrosophic Fuzzy Matrices
ASNFM =Antisymmetric Neutrosophic Fuzzy Matrices
INFM =Irreflexive Neutrosophic Fuzzy Matrices
NNFM = Nilpotent Neutrosophic Fuzzy Matrices

1.2 Literature Review

The study of fuzzy and IFM has seen significant development since the summary of fuzzy sets by Zadeh [1]. Intuitionistic fuzzy matrices, as a natural extension, have been explored extensively for their theoretical properties and applications in various domains. Hashimoto's works [12–14] were among the first to examine the transitivity and canonical forms of fuzzy matrices. These studies laid a foundation for understanding their behavior under repeated operations. Buckley [4] and Fan and Liu [5–7] further explored the convergence of power sequences in fuzzy matrices, offering insights into their steady-state dynamics and oscillatory behaviors. Guu et al. [9–10] extended this by investigating infinite products and their convergence properties, which are vital for stability analysis in systems modeled by fuzzy matrices.

In the realm of intuitionistic fuzzy matrices, Pal and collaborators made substantial contributions. They introduced key concepts such as intuitionistic fuzzy determinants [18, 19], generalized inverse matrices [24], and distance measures [26]. These works have expanded the utility of intuitionistic fuzzy matrices in solving real-world problems. Pradhan and Pal [20, 21] analyzed the convergence properties of intuitionistic fuzzy matrices under specific operations, while Mishref and Emam [33] studied transitivity and subinverses, highlighting the mathematical intricacies of these matrices.

The introduction of neutrosophic sets by Smarandache [32] generalized fuzzy and intuitionistic fuzzy sets by incorporating indeterminacy, paving the way for research into NFM. Anandhkumar et al. [29, 31] examined k-idempotent properties and pseudo-similarity in NFM, while Uma et al. [30] extended the concept to NFSM. These advancements illustrate the adaptability of fuzzy and neutrosophic matrices in representing complex systems with varying degrees of uncertainty. Other notable contributions include studies on interval-valued FM by Shyamal and Pal [25] and soft matrices by Mondal and Pal [17]. These extensions have broadened the theoretical scope of matrix theory, enabling applications in diverse fields such as decision-making, optimization, and system modeling.

Emam's recent work [28] on consistent and weak transitive IFMs highlights the continued relevance of studying transitivity in matrix theory. This paper builds on such studies by focusing on the decomposition properties of transitive intuitionistic fuzzy matrices, introducing the notion of strongly transitive matrices, and presenting their canonical forms.This literature review highlights the progression from foundational theories to advanced generalizations, emphasizing the importance of fuzzy and neutrosophic matrices in theoretical and applied research.

1.3 Novelty

This research offers several novel contributions to the field of intuitionistic fuzzy matrices, emphasizing their structural and operational properties. A key breakthrough is the establishment of a decomposition theorem, which demonstrates that any NFM can be expressed as the sum of a NNFM and a SNFM. This innovative approach provides a new lens for analyzing the intrinsic characteristics of such matrices. Additionally, the concept of SNFM is presented for the first time, accompanied by a rigorous definition that extends the theoretical framework of transitivity in matrix theory. The paper further contributes by deriving canonical forms for both transitive and STNFM, simplifying their representation and enabling more effective analysis. By addressing existing gaps in the literature and integrating advanced decomposition techniques, this study enhances our understanding of the algebraic and operational properties of NFM. The findings have broad implications, offering potential applications in decision-making, optimization, and computational modeling, where uncertainty and imprecision are critical considerations. Anandhkumar et al. [29] introduced characterizations and generalizations of k-idempotent NFMs, contributing to the algebraic foundations of these structures. Uma, Murugadas, and Sriram [30] developed Fuzzy Neutrosophic Soft Matrices of Type I and II, enhancing the flexibility of neutrosophic models. Anandhkumar et al. [31] further examined pseudo similarity relations in NFMs, while the theoretical underpinning of neutrosophy was laid out by Smarandache [32], forming the basis for numerous subsequent studies. Mishref and Emam [33] explored transitivity and subinverse properties in fuzzy matrices, and Ye, Yong, and Du [34] proposed a MAGDM model utilizing single-valued neutrosophic credibility matrices. J and S [35, 36] contributed to the operations on neutrosophic hypersoft matrices and their application contexts. Anandhkumar et al. [37] extended determinant theory to Quadri-Partitioned Neutrosophic Fuzzy Matrices (QPNFMs), linking matrix theory with decision-making.

The interval-valued secondary k-range symmetric QPNFMs introduced by Radhika et al. [38] presented a novel approach for decision-making. Anandhkumar et al. [39] and Devi Shyamala Mary et al [40] investigated Decomposition of Neutrosophic Fuzzy Matrices Using Some Alpha – Cuts. Radhika et al. [41] analyzed the Schur complement in k-kernel symmetric block QPNFMs, while Prathab et al. [42] studied generalized inverses in interval-valued fuzzy matrices. Further, Anandhkumar and collaborators [43] introduced partial orderings on intuitionistic fuzzy matrices, while Punithavalli and Anandhkumar [44] extended this to Reverse Sharp and Left-T Right-T partial orderings. Secondary k-range symmetric NFMs were explored by Anandhkumar et al. [45], and the kernel and k-kernel symmetric intuitionistic fuzzy matrices were developed by Punithavalli and Anandhkumar [47]. T. Harikrishnan et.al [46] Min(Max) – Min(Max) – Max(Min) (∗): Compositions of Neutrosophic Fuzzy Matrices and its Application in Medical Diagnosis Finally, Anandhkumar et al. [48] examined advanced partial orderings such as Reverse Sharp and Left-T Right-T on NFMs, providing a deeper algebraic understanding of their structure. Anandhkumar et al. [49,50] have studied Secondary k-column symmetric Neutrosophic Fuzzy Matrices, Generalized Symmetric Neutrosophic Fuzzy Matrices, Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices. Prathab et al. [51] hace presented Interval Valued Secondary k-Range Symmetric Fuzzy Matrices with Generalized Inverses.

1.4 Contribution of Our Work

This paper makes the following key contributions to the study of Neutrosophic Fuzzy Matrices (NFM):

(i) **Decomposition of NFM:** It is demonstrated that any NFM can be expressed as the sum of a NNFN and SNFM. This decomposition provides a novel framework for analyzing the inherent structure and properties of Neutrosophic fuzzy matrices.

(ii) **Introduction of Strongly Transitive NFM:** The paper introduces the concept of strongly transitive NFM, accompanied by a rigorous definition and characterization. This extends the existing understanding of transitivity in neutrosophic fuzzy systems.

(iii) **Canonical Forms:** The canonical forms of both TNFM and STNFM are derived and presented. These forms simplify their representation and facilitate deeper theoretical insights and practical computations.

(iv) **Theoretical Advancement:** By bridging the study of decomposition and transitivity, this research fills a critical gap in the existing literature, enhancing the theoretical foundation of Neutrosophic fuzzy matrices.

2. Preliminaries

In this part, we introduce operations for NFMs. For two NFM P and Q, we define the following operations $P \vee Q$, $P \wedge Q$, and $P \Theta Q$.

$$P \vee Q = \left[p_{ij} \vee q_{ij} \right] = \left[\max < p_{ij}^{\alpha}, q_{ij}^{\alpha} >, \max < p_{ij}^{\beta}, q_{ij}^{\beta} >, \min < p_{ij}^{\gamma}, q_{ij}^{\gamma} > \right]$$

$$P \wedge Q = \left[p_{ij} \wedge q_{ij} \right] = \left[\min < p_{ij}^{\alpha}, q_{ij}^{\alpha} >, \min < p_{ij}^{\beta}, q_{ij}^{\beta} >, \max < p_{ij}^{\gamma}, q_{ij}^{\gamma} > \right]$$

$$P \Theta Q = \begin{cases} P & if \ P > Q \\ (0,0,1) & if \ P \le Q \end{cases}$$

Definition: 2.1 A NFSs P on the universe of discourse Y is well-defined as

$P = \left\{ < y, p^T(y), p^I(y), p^F(y) >, y \in Y \right\}$, everywhere $p^T, p^I, p^F : Y \to]^- 0, 1^+ [$ also $0 \le p^T + p^I + p^F \le 3.$

Definition:2.2 A neutrosophic Fuzzy Matrices P is lease then or equal to Q

that is $P \le Q$ if $\left(p_{ij}^T, p_{ij}^I, p_{ij}^F \right) \le \left(q_{ij}^T, q_{ij}^I, q_{ij}^F \right)$ means $p_{ij}^T \le q_{ij}^T, p_{ij}^I \le q_{ij}^I, p_{ij}^F \ge q_{ij}^F$.

Definition 2.3. Subtraction is defined as an arbitrary fixed binary operation on F, satisfying specific conditions $P * Q \le P$ and $P * (0,0,1) = P$ for entirely $P, Q \in (NFM)_n$.

Let two NFMs $P = \left(p^{\alpha}, p^{\beta}, p^{\gamma} \right)$ and $Q = \left(q^{\alpha}, q^{\beta}, q^{\gamma} \right)$ us defined the binary operation $*$ as

$$P * Q = \left(\min < p^{\alpha}, \left| p^{\alpha} - q^{\alpha} \right| >, \min < p^{\beta}, \left| p^{\beta} - q^{\beta} \right| >, \max < p^{\gamma}, \left| p^{\gamma} - q^{\gamma} \right| > \right).$$

Then

Case(i) If $\min < p^{\alpha}, \left|p^{\alpha} - q^{\alpha}\right| >= p^{\alpha}$, $\min < p^{\beta}, \left|p^{\beta} - q^{\beta}\right| >= p^{\beta}$ and $\max < p^{\gamma}, \left|p^{\gamma} - q^{\gamma}\right| >= p^{\gamma}$ then $P * Q = P$.

Case(ii) If $\min < p^{\alpha}, \left|p^{\alpha} - q^{\alpha}\right| >= p^{\alpha}$, $\min < p^{\beta}, \left|p^{\beta} - q^{\beta}\right| >= p^{\beta}$ and $\max < p^{\gamma}, \left|p^{\gamma} - q^{\gamma}\right| >= \left|p^{\gamma} - q^{\gamma}\right|$ then $P * Q = \left(p^{\alpha}, \left|p^{\gamma} - q^{\gamma}\right|\right) \le P$ and $P * Q = \left(p^{\beta}, \left|p^{\gamma} - q^{\gamma}\right|\right) \le P$.

Case(iii) If $\min < p^{\alpha}, \left|p^{\alpha} - q^{\alpha}\right| >= \left|p^{\alpha} - q^{\alpha}\right|$, $\min < p^{\beta}, \left|p^{\beta} - q^{\beta}\right| >= \left|p^{\beta} - q^{\beta}\right|$ and $\max < p^{\gamma}, \left|p^{\gamma} - q^{\gamma}\right| >= p^{\gamma}$ then $P * Q = \left(\left|p^{\alpha} - q^{\alpha}\right|, p^{\gamma}\right) \le P$ and $P * Q = \left(\left|p^{\beta} - q^{\beta}\right|, p^{\gamma}\right) \le P$.

Case(iv) If $\min < p^{\alpha}, \left|p^{\alpha} - q^{\alpha}\right| >= \left|p^{\alpha} - q^{\alpha}\right|$, $\min < p^{\beta}, \left|p^{\beta} - q^{\beta}\right| >= \left|p^{\beta} - q^{\beta}\right|$ and $\max < p^{\gamma}, \left|p^{\gamma} - q^{\gamma}\right| >= \left|p^{\gamma} - q^{\gamma}\right|$ then $P * Q = \left(\left|p^{\alpha} - q^{\alpha}\right|, \left|p^{\gamma} - q^{\gamma}\right|\right) < P$ and $P * Q = \left(\left|p^{\beta} - q^{\beta}\right|, \left|p^{\gamma} - q^{\gamma}\right|\right) < P$.

Here, for any P, if $Q = \left(q^{\alpha}, q^{\beta}, q^{\gamma}\right)$ be such that, $q^{\alpha}, q^{\beta} = 0$ and $p^{\gamma} \ge \left|p^{\gamma} - q^{\gamma}\right|$, then the operator $*$ be same with Θ.

Example 2.1 Let $P = (0.2, 0.3, 0.3)$ and $Q = (0.5, 0.8, 0.4)$ then $\min < p^{\alpha}, \left|p^{\alpha} - q^{\alpha}\right| >= 0.2$, $\min < p^{\beta}, \left|p^{\beta} - q^{\beta}\right| >= 0.3$ and $\max < p^{\gamma}, \left|p^{\gamma} - q^{\gamma}\right| >= 0.3$ then $P * Q = P$

Let $P = (0.2, 0.3, 0.1)$ and $Q = (0.5, 0.8, 0.5)$ then $\min < p^{\alpha}, \left|p^{\alpha} - q^{\alpha}\right| >= 0.2$, $\min < p^{\beta}, \left|p^{\beta} - q^{\beta}\right| >= 0.3$,and $\max < p^{\gamma}, \left|p^{\gamma} - q^{\gamma}\right| >= 0.4$ then $P * Q \le P$.

Let $P = (0.3, 0.4, 0.3)$ and $Q = (0.2, 0.8, 0.5)$ then $\min < p^{\alpha}, \left|p^{\alpha} - q^{\alpha}\right| >= 0.1$, $\min < p^{\beta}, \left|p^{\beta} - q^{\beta}\right| >= 0.4$,and $\max < p^{\gamma}, \left|p^{\gamma} - q^{\gamma}\right| >= 0.3$ then $P * Q \le P$.

Let $P = (0.3, 0.6, 0.1)$ and $Q = (0.1, 0.4, 0.5)$ then $\min < p^{\alpha}, \left|p^{\alpha} - q^{\alpha}\right| >= 0.2$, $\min < p^{\beta}, \left|p^{\beta} - q^{\beta}\right| >= 0.2$,and $\max < p^{\gamma}, \left|p^{\gamma} - q^{\gamma}\right| >= 0.4$ then $P * Q < P$.

Definition 2.4 . A NFM is considered null if all its elements are (0,0,0). This type of matrix is denoted by$N_{(0,0,0)}$. On the other hand, an NFM is defined as zero if all its elements are (0,0,1) and it is represented by O.

Definition 2.5 A square NFM is referred to as a Neutrosophic Fuzzy Permutation Matrix (NFPM) if each row and each column contains exactly one element with a value of (1,1,0) while all other entries are (0,0,1).

Definition 2.6 For identity NFM of order nx n is represented by I_n and is well-defined by

$$\left(\delta_{ij}{}^{\alpha},\delta_{ij}{}^{\beta},\delta_{ij}{}^{\gamma}\right)=\begin{cases}(1,1,0)\ if\ \ i=j\\(0,0,1)\ if\ \ i\neq j\end{cases}'$$

We now present the following operations for NFMs $P=\left(p^{\alpha},p^{\beta},p^{\gamma}\right)$ and $Q=\left(q^{\alpha},q^{\beta},q^{\gamma}\right)$ us defined the binary operation.

(i) $P*Q=\left[p_{ij}*q_{ij}\right].$

(ii) $P\Theta Q=\left[p_{ij}\Theta q_{ij}\right].$

(iii) $P\times Q=\left[\bigcup_{k=1}^{n}\left(p_{ik}\wedge q_{kj}\right)\right].$

(iv) $P^{k+1}=P^{k}\times P,(k=1,2,3,...)$

(v) $P^{T}=\left[p_{ji}{}^{\alpha},p_{ji}{}^{\beta},p_{ji}{}^{\gamma}\right]$ (the transpose of P)

(vi) $\Delta P=P\Theta P^{T}$

(vii) $\nabla P=P\wedge P^{T}$

(viii) $P^{2}\leq P$ (P is transitive)

(ix) $P^2 = P$ (P is idempotent)

(x) $P^k = O$ (P is nilpotent $k\in N$)

(xi) $P\wedge I_{n}=0$ (P is irreflexive)

(xii) $P^T = P$ (P is SNFM)

(xiii) $P\wedge P^{T}\leq I_{n}$ (P is antisymmetric)

3. Some results on NFMs

In this part, we explore the fundamental concepts of TNFM and NNFM.

Lemma 3.1. Let Q be a SNFM. To prove that $\Delta\left(P\vee Q\right)\leq\Delta P$ for any NFM P.

Prrof: Let $C=\left[c_{ij}^{T},c_{ij}^{I},c_{ij}^{F}\right]=\Delta\left(P\vee Q\right)$

$=\left[\left(p_{ij}\vee q_{ij}\right)\Theta\left(p_{ji}\vee q_{ji}\right)\right]$

$=\left[\left(p_{ij}\vee q_{ij}\right)\Theta\left(p_{ji}\vee q_{ij}\right)\right]$ (as Q is symmetric)

$=\left[\max<p_{ij}^{T},q_{ij}^{T}>,\max<p_{ij}^{I},q_{ij}^{I}>,\min<p_{ij}^{F},q_{ij}^{F}>\right]\Theta$

$\left[\max<p_{ji}^{T},q_{ij}^{T}>,\max<p_{ji}^{I},q_{ij}^{I}>,\min<p_{ji}^{F},q_{ij}^{F}>\right]$

Suppose $\left[\max<p_{ij}^{T},q_{ij}^{T}>,\max<p_{ij}^{I},q_{ij}^{I}>,\min<p_{ij}^{F},q_{ij}^{F}>\right]$

$>\left[\max<p_{ji}^{T},q_{ij}^{T}>,\max<p_{ji}^{I},q_{ij}^{I}>,\min<p_{ji}^{F},q_{ij}^{F}>\right]$

Then, $\left[c_{ij}^{T},c_{ij}^{I},c_{ij}^{F}\right]=\left[\max<p_{ij}^{T},q_{ij}^{T}>,\max<p_{ij}^{I},q_{ij}^{I}>,\min<p_{ij}^{F},q_{ij}^{F}>\right]$

$=\left(p_{ij}^{T},p_{ij}^{I},p_{ij}^{F}\right)$

$=\left(p_{ij}^{T},p_{ij}^{I},p_{ij}^{F}\right)\Theta\left(p_{ji}^{T},p_{ji}^{I},p_{ji}^{F}\right)$

On the other hand, suppose

$\left[\max<p_{ij}^{T},q_{ij}^{T}>,\max<p_{ij}^{I},q_{ij}^{I}>,\min<p_{ij}^{F},q_{ij}^{F}>\right]$

$\leq\left[\max<p_{ji}^{T},q_{ij}^{T}>,\max<p_{ji}^{I},q_{ij}^{I}>,\min<p_{ji}^{F},q_{ij}^{F}>\right]$

Then, $\left[c_{ij}^{T},c_{ij}^{I},c_{ij}^{F}\right]=\left(0,0,1\right)\leq\left(p_{ij}^{T},p_{ij}^{I},p_{ij}^{F}\right)\Theta\left(p_{ji}^{T},p_{ji}^{I},p_{ji}^{F}\right)$

Thus we have $C\leq\Delta P$. So, $\Delta\left(P\vee Q\right)\leq\Delta P$

Example :3.1 Let us assume that NFMs,

$$P=\begin{bmatrix}<0.8,0.1,0.1> & <0.6,0.1,0.2>\\ <0.7,0.1,0.1> & <0.5,0.1,0.3>\end{bmatrix}$$

$$Q=\begin{bmatrix}<0.8,0.1,0.1> & <0.5,0.1,0.4>\\ <0.5,0.1,0.4> & <0.7,0.1,0.3>\end{bmatrix}$$

$$P^{T}=\begin{bmatrix}<0.8,0.1,0.1> & <0.7,0.1,0.1>\\ <0.6,0.1,0.2> & <0.5,0.1,0.3>\end{bmatrix}$$

$$\Delta P = P\Theta P^T = \begin{bmatrix} <0,0,1> & <0,0,1> \\ <0.7,0.1,0.1> & <0,0,1> \end{bmatrix}$$

$$P \vee Q = \begin{bmatrix} <0.8,0.1,0.1> & <0.6,0.1,0.2> \\ <0.7,0.1,0.1> & <0.7,0.1,0.2> \end{bmatrix}$$

$$\left(P \vee Q\right)^T = \begin{bmatrix} <0.8,0.1,0.1> & <0.7,0.1,0.1> \\ <0.6,0.1,0.2> & <0.7,0.1,0.2> \end{bmatrix}$$

$$\Delta\left(P \vee Q\right) = \left(P \vee Q\right)\Theta\left(P \vee Q\right)^T = \begin{bmatrix} <0,0,1> & <0,0,1> \\ <0.7,0.1,0.1> & <0,0,1> \end{bmatrix} = \Delta P$$

Lemma 3.2 For any NFM P, $P = \left(\Delta P\right) \vee \left(\nabla P\right)$.

Proof: Let $C = \left[c_{ij}{}^T, c_{ij}{}^I, c_{ij}{}^F\right] = \left(\Delta P\right) \vee \left(\nabla P\right)$

That is

$$\left(c_{ij}{}^T, c_{ij}{}^I, c_{ij}{}^F\right) = \left(p_{ij}{}^T, p_{ij}{}^I, p_{ij}{}^F\right)\Theta\left(p_{ji}{}^T, p_{ji}{}^I, p_{ji}{}^F\right) \vee \left(p_{ij}{}^T, p_{ij}{}^I, p_{ij}{}^F\right) \wedge \left(p_{ji}{}^T, p_{ji}{}^I, p_{ji}{}^F\right)$$

If $\left(p_{ji}{}^\alpha, p_{ji}{}^\beta, p_{ji}{}^\gamma\right) > \left(p_{ji}{}^\alpha, p_{ji}{}^\beta, p_{ji}{}^\gamma\right)$, then

$$\left(c_{ij}{}^\alpha, c_{ij}{}^\beta, c_{ij}{}^\gamma\right) = \left(p_{ij}{}^\alpha, p_{ij}{}^\beta, p_{ij}{}^\gamma\right) \vee \left(p_{ji}{}^\alpha, p_{ji}{}^\beta, p_{ji}{}^\gamma\right) = \left(p_{ji}{}^\alpha, p_{ji}{}^\beta, p_{ji}{}^\gamma\right).$$

If $\left(p_{ij}{}^\alpha, p_{ij}{}^\beta, p_{ij}{}^\gamma\right) \le \left(p_{ji}{}^\alpha, p_{ji}{}^\beta, p_{ji}{}^\gamma\right)$, then

$$\left(c_{ij}{}^\alpha, c_{ij}{}^\beta, c_{ij}{}^\gamma\right) = (0,0,1) \vee \left(p_{ij}{}^\alpha, p_{ij}{}^\beta, p_{ij}{}^\gamma\right) = \left(p_{ij}{}^\alpha, p_{ij}{}^\beta, p_{ij}{}^\gamma\right).$$

Thus, $C = \left(\Delta P\right) \vee \left(\nabla P\right) = P$

Example :3.2 Let us assume that the NFMs,

$$P = \begin{bmatrix} <0.9,0.1,0.1> & <0.6,0.1,0.2> \\ <0.7,0.1,0.1> & <0.8,0.1,0.1> \end{bmatrix}$$

$$P^T = \begin{bmatrix} <0.9,0.1,0.1> & <0.7,0.1,0.2> \\ <0.6,0.1,0.2> & <0.8,0.1,0.1> \end{bmatrix}$$

$$\Delta P = P\Theta P^T = \begin{bmatrix} <0,0,1> & <0,0,1> \\ <0.7,0.1,0.2> & <0,0,1> \end{bmatrix}$$

$$\nabla P = P \wedge P^T = \begin{bmatrix} <0.9,0.1,0.1> & <0.6,0.1,0.2> \\ <0.6,0.1,0.2> & <0.8,0.1,0.1> \end{bmatrix}$$

$$\Delta P \vee \nabla P = \begin{bmatrix} <0.9,0.1,0.1> & <0.6,0.1,0.2> \\ <0.7,0.1,0.1> & <0.8,0.1,0.1> \end{bmatrix} = P$$

Remark3.1. For any NFM P, it is straightforward to see that $\Delta(\Delta P) = \Delta P$

The above remark can be illustrated and proved with the subsequent example.

Example:3.3 Let us assume that the NFM

$$P = \begin{bmatrix} <0.7,0.2,0.2> & <0,0,1> & <0.5,0.2,0.4> \\ <0,0,1> & <0.4,0.2,0.5> & <0.6,0.2,0.3> \\ <0.5,0.2,0.4> & <0.8,0.2,0.1> & <1,1,0> \end{bmatrix}$$

$$\Delta P = \begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0.8,0.2,0.1> & <0,0,1> \end{bmatrix}$$

$$(\Delta P)^T = \begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0.8,0.2,0.1> \\ <0,0,1> & <0,0,1> & <0,0,1> \end{bmatrix}$$

$$\Delta(\Delta P) = \begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0.8,0.2,0.1> & <0,0,1> \end{bmatrix} = \Delta P$$

Proposition 3.1 If for any NFM P, there exists an NFM Q $(P \le Q)$ such that, $(P * Q^T) \le P \vee P^T \vee I_n, P \wedge Q^T \le I_n$ and $(P * Q^T)^3 \wedge I_n = 0$ holds, then P is ASNFM and TNFM.

Proof. The form $P \le P^T \le I_n$ and $P \le Q$ implies that $P \wedge P^T \le P \wedge Q^T \le I_n$. Then, from the definition of ASNFM, P is ASNFM.

Let $C = (c_{ij}{}^{\alpha}, c_{ij}{}^{\beta}, c_{ij}{}^{\gamma}) = P * Q^T$ then

$$(c_{ij}{}^{\alpha}, c_{ij}{}^{\beta}, c_{ij}{}^{\gamma}) = (p_{ij}{}^{\alpha}, p_{ij}{}^{\beta}, p_{ij}{}^{\gamma}) * (q_{ji}{}^{\alpha}, q_{ji}{}^{\beta}, q_{ji}{}^{\gamma}) \le (p_{ij}{}^{\alpha}, p_{ij}{}^{\beta}, p_{ij}{}^{\gamma})$$

If $(p_{ij}{}^{\alpha}, p_{ij}{}^{\beta}, p_{ij}{}^{\gamma}) = (0,0,1)$ then $(c_{ij}{}^{\alpha}, c_{ij}{}^{\beta}, c_{ij}{}^{\gamma}) = (p_{ij}{}^{\alpha}, p_{ij}{}^{\beta}, p_{ij}{}^{\gamma})$

If $\left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right) \neq (0,0,1)$ then $\left(q_{ij}^{\alpha}, q_{ij}^{\beta}, q_{ij}^{\gamma}\right) = (0,0,1)$

Thus $\left(c_{ij}^{\alpha}, c_{ij}^{\beta}, c_{ij}^{\gamma}\right) = \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right) * (0,0,1) = \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right)$

So $\left(c_{ij}^{\alpha}, c_{ij}^{\beta}, c_{ij}^{\gamma}\right) = \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right)$ for all $i \neq j$.

Now to show $P^2 \leq P$ in view of

$\left(p_{ii}^{\alpha}, p_{ii}^{\beta}, p_{ii}^{\gamma}\right) \wedge \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right) \leq \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right)$ and

$\left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right) \wedge \left(p_{jj}^{\alpha}, p_{jj}^{\beta}, p_{jj}^{\gamma}\right) \leq \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right)$

it is sufficient to check $\left(p_{ik}^{\alpha}, p_{ik}^{\beta}, p_{ik}^{\gamma}\right) \wedge \left(p_{kj}^{\alpha}, p_{kj}^{\beta}, p_{kj}^{\gamma}\right) \leq \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right)$, for all $i \neq k \neq j$.

Here $\left(p_{ik}^{\alpha}, p_{ik}^{\beta}, p_{ik}^{\gamma}\right) = \left(c_{ik}^{\alpha}, c_{ik}^{\beta}, c_{ik}^{\gamma}\right)$ and $\left(p_{kj}^{\alpha}, p_{kj}^{\beta}, p_{kj}^{\gamma}\right) = \left(c_{kj}^{\alpha}, c_{kj}^{\beta}, c_{kj}^{\gamma}\right)$ and thus we check

$\left(p_{ik}^{\alpha}, p_{ik}^{\beta}, p_{ik}^{\gamma}\right) \wedge \left(c_{kj}^{\alpha}, c_{kj}^{\beta}, c_{kj}^{\gamma}\right) \leq \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right)$

Case(i) If $\left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right) \neq (0,0,1)$ then $\left(p_{ji}^{\alpha}, p_{ji}^{\beta}, p_{ji}^{\gamma}\right) = (0,0,1)$

From $C^2 \leq P \vee P^T \vee I_n$

We get

$\left(c_{ik}^{\alpha}, c_{ik}^{\beta}, c_{ik}^{\gamma}\right) \wedge \left(c_{kj}^{\alpha}, c_{kj}^{\beta}, c_{kj}^{\gamma}\right) \leq \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right) \vee \left(p_{ji}^{\alpha}, p_{ji}^{\beta}, p_{ji}^{\gamma}\right) \leq \left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right)$

Case(ii) Let $\left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right) = (0,0,1)$

If $\left(p_{ij}^{\alpha}, p_{ij}^{\beta}, p_{ij}^{\gamma}\right) = (0,0,1)$ then from $C^2 \leq P \vee P^T \vee I_n$

we get $\left(c_{ik}^{\alpha}, c_{ik}^{\beta}, c_{ik}^{\gamma}\right) \wedge \left(c_{kj}^{\alpha}, c_{kj}^{\beta}, c_{kj}^{\gamma}\right) = (0,0,1)$

If $\left(p_{ji}^{\alpha}, p_{ji}^{\beta}, p_{ji}^{\gamma}\right) \neq (0,0,1)$ then from $C^3 \wedge I_n = 0$

we get $\left(c_{ik}^{\alpha}, c_{ik}^{\beta}, c_{ik}^{\gamma}\right) \wedge \left(c_{kj}^{\alpha}, c_{kj}^{\beta}, c_{kj}^{\gamma}\right) \wedge \left(c_{ji}^{\alpha}, c_{ji}^{\beta}, c_{ji}^{\gamma}\right) = (0,0,1)$

That imply, $\left(c_{ik}^{\alpha}, c_{ik}^{\beta}, c_{ik}^{\gamma}\right) \wedge \left(c_{kj}^{\alpha}, c_{kj}^{\beta}, c_{kj}^{\gamma}\right) = (0,0,1)$ for $i \neq j$.

Once more since P is ASNFM, we take,

$p_{ik} \wedge p_{ki} = (0,0,1) \leq p_{ii}$ for all $k \neq j$ and $p_{ii} \wedge p_{ii} = a_{ii}$ which completes the proof.

Lemma 3.3. If $P \wedge Q^T \le I_n$, then $\left(P * Q^T\right) \vee \left(P \wedge I_n\right) = P$ for any P and Q.

Proof: Let $C = P \wedge Q^T$

$i.e.,\left(c_{ij}{}^{\alpha}, c_{ij}{}^{\beta}, c_{ij}{}^{\gamma}\right) = \left[\left(p_{ij}{}^{\alpha}, p_{ij}{}^{\beta}, p_{ij}{}^{\gamma}\right) * \left(q_{ji}{}^{\alpha}, q_{ji}{}^{\beta}, q_{ji}{}^{\gamma}\right)\right] \le \left[\left(p_{ij}{}^{\alpha}, p_{ij}{}^{\beta}, p_{ij}{}^{\gamma}\right)\right]$

Again, $P \wedge Q^T \le I_n$, $i.e.,\left[\left(p_{ij}{}^{\alpha}, p_{ij}{}^{\beta}, p_{ij}{}^{\gamma}\right) \wedge \left(q_{ji}{}^{\alpha}, q_{ji}{}^{\beta}, q_{ji}{}^{\gamma}\right)\right] \le I_n$

Which implies $P * Q^T = P$.

Then, $\left(P * Q^T\right) \vee \left(P \wedge I_n\right) = P \vee \left(P \wedge I_n\right) = P$

Example3.4. Let us consider the NFMs $P = \begin{bmatrix} <0.3, 0.3, 0.5> & <0,0,1> \\ <0,0,1> & <0,0,1> \end{bmatrix}$

$Q = \begin{bmatrix} <0,0,1> & <0.6, 0.3, 0.3> \\ <0.4, 0.3, 0.5> & <0,0,1> \end{bmatrix}$. Also consider the binary operation $*$ as

Then, $P * Q^T = \begin{bmatrix} <0.3, 0.3, 0.5> & <0,0,1> \\ <0,0,1> & <0,0,1> \end{bmatrix} = P$.

Thus, $\left(P * Q^T\right) \vee \left(P \wedge I_n\right) = P \vee \left(P \wedge I_n\right)$

$= \begin{bmatrix} <0.3, 0.3, 0.5> & <0,0,1> \\ <0,0,1> & <0,0,1> \end{bmatrix} \vee \left(\begin{bmatrix} <0.3, 0.3, 0.5> & <0,0,1> \\ <0,0,1> & <0,0,1> \end{bmatrix} \wedge \begin{bmatrix} <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> \end{bmatrix} \right)$

$= P$.

Proposition 3.2 Let P and Q be two NFMs such that, $P \le Q, \left(P * Q^T\right)^2 \le P \vee P^T \vee I_n, P \wedge Q^T \le I_n$ and $\left(P * Q^T\right)^3 = 0$, then $P^2 \le \left(P * Q^T\right) \vee \left(P \wedge I_n\right)$.

Proof. From Proposition3.1, P is a transitive NFM, that is, $P^2 \le P$. Again, from Lemma 3.3, $P = \left(P * Q^T\right) \vee \left(P \wedge I_n\right)$. then $P^2 \le P$ that is $P^2 \le \left(P * Q^T\right) \vee \left(P \wedge I_n\right)$.

Lemma3.4. If the NFM P be NNFM, then $\Delta P = P$ and $\nabla P = 0$.

Proof. Since the NFM P is NNFM i.e., $P^n = 0$, for some $n \in N$, P must be an INFM,

i.e., $P \wedge I_n = 0$. Then, P^2 must be INFM [12]

Thus, $\left(p_{ij}{}^{\alpha}, p_{ij}{}^{\beta}, p_{ij}{}^{\gamma}\right) \wedge \left(p_{ji}{}^{\alpha}, p_{ji}{}^{\beta}, p_{ji}{}^{\gamma}\right) = (0,0,1)$

Again $\Delta P=\left(p_{ij}{}^{\alpha},p_{ij}{}^{\beta},p_{ij}{}^{\gamma}\right)\Theta\left(p_{ji}{}^{\alpha},p_{ji}{}^{\beta},p_{ji}{}^{\gamma}\right)$.

Now, if $\left(p_{ij}{}^{\alpha},p_{ij}{}^{\beta},p_{ij}{}^{\gamma}\right)\neq(0,0,1)$

then $\left(p_{ji}{}^{\alpha},p_{ji}{}^{\beta},p_{ji}{}^{\gamma}\right)=(0,0,1)$

and thus $\Delta P=\left(p_{ij}{}^{\alpha},p_{ij}{}^{\beta},p_{ij}{}^{\gamma}\right)$

If $\left(p_{ij}{}^{\alpha},p_{ij}{}^{\beta},p_{ij}{}^{\gamma}\right)=(0,0,1)$

By definition of $x\Theta y$,

$\left(p_{ij}{}^{\alpha},p_{ij}{}^{\beta},p_{ij}{}^{\gamma}\right)\Theta\left(p_{ji}{}^{\alpha},p_{ji}{}^{\beta},p_{ji}{}^{\gamma}\right)=(0,0,1)$

So, $\Delta P=\left(p_{ij}{}^{\alpha},p_{ij}{}^{\beta},p_{ij}{}^{\gamma}\right)$

Hence $\Delta P=P$

Again, $\nabla P=\left(p_{ij}{}^{\alpha},p_{ij}{}^{\beta},p_{ij}{}^{\gamma}\right)\wedge\left(p_{ji}{}^{\alpha},p_{ji}{}^{\beta},p_{ji}{}^{\gamma}\right)=0$

Lemma3.5.Let P be an nilpotent NFM and Q be a symmetric NFM. If $C=P\vee Q$, then $\Delta C\leq P$ and $\nabla C=Q$.

Proof: Here, $\Delta C=\Delta(P\vee Q)\leq\Delta P=P$

$\nabla C=\nabla(P\vee Q)$

$=(P\vee Q)\wedge\left(P^{T}\vee Q\right)\ (as\,Q=Q^{T})$

$=\left(P\wedge P^{T}\right)\vee(P\wedge Q)\wedge\left(Q\wedge P^{T}\right)\vee Q$

$=0\vee(P\wedge Q)\wedge\left(Q\wedge P^{T}\right)\vee Q$

$=P\wedge(Q\vee Q)\wedge P^{T}\vee Q$

$=\left(P\wedge Q\wedge P^{T}\right)\vee Q$

$=\left(P\wedge P^{T}\wedge Q\right)\vee Q$

$=0\vee Q$

$=Q$

4. Canonical form of transitive NFM

In this section, we will discuss the canonical form of an TNFM. Let P be an (n x n) TNFM. If there exists an (n x n) NFPM A such that, F = $AP^T A = (f_{ij})$ satisfying $f_{ij} \geq f_{ji}$ for all $i > j$ then F is called a canonical form of P.

Theorem 4.1. Let P be a NNFM and Q be a SNFM. For a NFM C given by $C = P \vee Q$ there is a NFPM A such that $D = \left[\left(d_{ij}^{\alpha}, d_{ij}^{\beta}, d_{ij}^{\gamma}\right)\right] = A \times C \times A^T$ satisfies $\left(d_{ij}^{\alpha}, d_{ij}^{\beta}, d_{ij}^{\gamma}\right) \geq \left(d_{ji}^{\alpha}, d_{ji}^{\beta}, d_{ji}^{\gamma}\right)$ for entirely $i > j.$

Proof: Now $D = A \times C \times A^T$

$= A \times (P \vee Q) \times A^T$

$= \left(A \times P \times A^T\right) \vee \left(A \times Q \times A^T\right)$

Since P is NNFM, $\left(A \times P \times A^T\right)$ becomes NFPM P. Once more, as Q is SNFM, $\left(A \times Q \times A^T\right)$ is symmetric NFM. Then, the NFM D satisfies $\left(d_{ij}^{\alpha}, d_{ij}^{\beta}, d_{ij}^{\gamma}\right) \geq \left(d_{ji}^{\alpha}, d_{ji}^{\beta}, d_{ji}^{\gamma}\right)$ for entirely $i > j$ by selecting such a NFPM A.

Lemma 4.1. If P be a TNFM, then ΔP is a NNFM and ∇P is a SNFM.

Proof. Subsequently P is TNFM, $P^2 \leq P,\ \ i.e., \max_k \left(p_{ik} \wedge p_{kj}\right) \leq p_{ij}.$

From definition $\Delta P = \left[\Delta p_{ij}\right]$

Where $\Delta p_{ij} = p_{ij} \Theta p_{ji} \leq p_{ij}.$

Now $(\Delta P)^2 = \left[\left(\Delta p_{ij}\right)^2\right]$

Where $\left(\Delta p_{ij}\right)^2 = \Delta p_{ij} \times \Delta p_{ij}$

$= \left(p_{ij} \Theta p_{ji}\right) \times \left(p_{ij} \Theta p_{ji}\right)$

$= \left(p_{ij} \Theta p_{ji}\right)$

$= \left(\Delta p_{ij}\right)$ for $i \neq j$

At that point, $(\Delta P)^2 = \Delta p_{ij}$ i.e., ΔP is an INFM. Then INFM is transitive, ΔP is TNFM

Yet again, for $i=j$, $\Delta P=\left[\Delta p_{ii}\right]$

Everywhere $\Delta p_{ii}=\left(p_{ii}\Theta p_{ii}\right)=\left(0,0,1\right)$

Therefore the diagonal elements of ΔP is $\left(0,0,1\right)$

So, $\Delta P\wedge I=0$ i.e., ΔP is INFM.

Hereafter, P is TNFM and INFM and thus ΔP is NNFM (by Proposition 3.1).

Since the definition, $\nabla P=P\wedge P^{T}$

Then, $\left(\nabla P\right)^{T}=\left(P\wedge P^{T}\right)^{T}=P^{T}\wedge P=\nabla P$.

Thus is ∇P is a SNFM.

Example 4.1. Let us assume that the

$$\text{NFM } P=\begin{bmatrix} <0.2,0.3,0.4> & <0,0,1> & <0.2,0.3,0.4> \\ <0.6,0.3,0.3> & <0.2,0.3,0.3> & <0.5,0.3,0.2> \\ <0.4,0.3,0.4> & <0,0,1> & <0.3,0.3,0.4> \end{bmatrix}$$

$$\text{Then, } P^{2}=\begin{bmatrix} <0.2,0.3,0.4> & <0,0,1> & <0.2,0.3,0.4> \\ <0.4,0.3,0.3> & <0.2,0.3,0.3> & <0.3,0.3,0.3> \\ <0.3,0.3,0.4> & <0,0,1> & <0.3,0.3,0.4> \end{bmatrix} \text{ and } P^{2}\le P \text{ holds.}$$

Thus the NFM P is transitive.

$$\text{Now, } \Delta P=P\Theta P^{T}=\begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0.6,0.3,0.3> & <0,0,1> & <0.5,0.3,0.2> \\ <0.4,0.3,0.4> & <0,0,1> & <0,0,1> \end{bmatrix}$$

$$\left(\Delta P\right)^{2}=\begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0.4,0.3,0.4> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \end{bmatrix}\le\left(\Delta P\right)$$

$$\left(\Delta P\right)^{3}=\begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \end{bmatrix}=0$$

So, ΔP is nilpotent of index 3.

$$\text{Again } \nabla P=P\wedge P^{T}=\begin{bmatrix} <0.2,0.3,0.4> & <0,0,1> & <0.2,0.3,0.4> \\ <0,0,1> & <0.2,0.3,0.3> & <0,0,1> \\ <0.2,0.3,0.4> & <0,0,1> & <0.3,0.3,0.4> \end{bmatrix} \text{ is asymmetric}$$

NFM.

Theorem 4.2 For a TNFM P, there exists a NFPM P, such that, $D=\left[\left(d_{ij}^{\alpha},d_{ij}^{\beta},d_{ij}^{\gamma}\right)\right]=A\times P\times A^{T}$ satisfies $d_{ij}\ge d_{ji}$ for entirely $i>j.$

Proof. Since P is TNFM, P can be modified as, $P=(\Delta P)\vee(\nabla P)$

(By Lemma3.3). Once more, ΔP is NNFM and ∇P is a SNFM (by Lemma4.1). Hence by Theorem 4.1,

$$D=\left[\left(d_{ij}^{\alpha},d_{ij}^{\beta},d_{ij}^{\gamma}\right)\right]=\left(A\times\Delta P\times A^{T}\right)\vee\left(A\times\nabla P\times A^{T}\right)$$

$$=A\times(\Delta P\vee\nabla P)\times A^{T}$$

$=A\times P\times A^{T}$ satisfies $d_{ij}\ge d_{ji}$ for completely $i>j.$

Example 4.2. Let us assume that the NFM

$$P=\begin{bmatrix}<0.6,0.3,0.2> & <0,0,1> & <0.5,0.3,0.2>\\ <0.5,0.3,0.3> & <0.4,0.3,0.4> & <0.5,0.3,0.3>\\ <0.7,0.3,0.1> & <0,0,1> & <0.6,0.3,0.2>\end{bmatrix}$$

Then $P^{2}=\begin{bmatrix}<0.6,0.3,0.2> & <0,0,1> & <0.5,0.3,0.2>\\ <0.5,0.3,0.3> & <0.4,0.3,0.4> & <0.5,0.3,0.3>\\ <0.6,0.3,0.2> & <0,0,1> & <0.6,0.3,0.2>\end{bmatrix}$ and the transpose

$$P^{T}=\begin{bmatrix}<0.6,0.3,0.2> & <0.5,0.3,0.3> & <0.7,0.3,0.1>\\ <0,0,1> & <0.4,0.3,0.4> & <0,0,1>\\ <0.5,0.3,0.2> & <0.5,0.3,0.3> & <0.6,0.3,0.2>\end{bmatrix}$$

As $P^{2}\le P$, P is transitive.

Now $\Delta P=\begin{bmatrix}<0,0,1> & <0,0,1> & <0,0,1>\\ <0.5,0.3,0.3> & <0,0,1> & <0.5,0.3,0.3>\\ <0.7,0.3,0.1> & <0,0,1> & <0,0,1>\end{bmatrix}$

And $(\Delta P)^{3}=\begin{bmatrix}<0,0,1> & <0,0,1> & <0,0,1>\\ <0,0,1> & <0,0,1> & <0,0,1>\\ <0,0,1> & <0,0,1> & <0,0,1>\end{bmatrix}=0$

Then ΔP is nilpotent of index 3.

Also, $\nabla P=\begin{bmatrix}<0.6,0.3,0.2> & <0,0,1> & <0.5,0.3,0.2>\\ <0,0,1> & <0.4,0.3,0.4> & <0,0,1>\\ <0.5,0.3,0.2> & <0,0,1> & <0.6,0.3,0.2>\end{bmatrix}$ is a symmetric NFM.

Now, for the NFPM $A=\begin{bmatrix} <1,1,0> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <1,1,0> \\ <0,0,1> & <1,1,0> & <0,0,1> \end{bmatrix}$

And $A\left(\Delta P\right)A^{T}=\begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0.7,0.3,0.1> & <0,0,1> & <0,0,1> \\ <0.5,0.3,0.3> & <0.5,0.3,0.3> & <0,0,1> \end{bmatrix}$

$$A\left(\nabla P\right)A^{T}=\begin{bmatrix} <0.6,0.3,0.2> & <0.5,0.3,0.2> & <0,0,1> \\ <0.5,0.3,0.2> & <0.6,0.3,0.2> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0.4,0.3,0.4> \end{bmatrix}$$

Thus $D=A\times P\times A^{T}=\begin{bmatrix} <0.6,0.3,0.2> & <0.5,0.3,0.2> & <0,0,1> \\ <0.7,0.3,0.1> & <0.6,0.3,0.2> & <0,0,1> \\ <0.5,0.3,0.3> & <0.5,0.3,0.3> & <0.4,0.3,0.4> \end{bmatrix}$

Satisfies the condition $d_{ij}\geq d_{ji}$ for entirely $i>j.$

5.Properties of strongly transitive NFMs

Definition5.1.A NFM P is STNFM (s-TNFM) iff for any indices $i,j,k\in\{1,2,...,n\}$ with $i\neq j\neq k$, such that, $a_{ik}>a_{ki}$ and $a_{kj}>a_{jk}$ implies $a_{ij}>a_{ji}$.

If a NFM P be symmetric, the conditions $a_{ik}>a_{ki}$ and $a_{kj}>a_{jk}$ are false for any $i,j,k\in\{1,2,...,n\}$ and hence P is also s-TNFM. In specific, P is s-TNFM for any NFM P.

The above definition of s-TNFM is not ideal for practical use. Therefore, we provide a new classification of s-TNFM for NFMs, which appears to be more suitable for practical applications. For this purpose, we define the relation $\prec$ in the set of NFMs as follows: $Q\prec P$ iff $\left(p_{ij}{}^{\alpha},p_{ij}{}^{\beta},p_{ij}{}^{\gamma}\right)=\left(0,0,1\right)$ implies $\left(q_{ji}{}^{\alpha},q_{ji}{}^{\beta},q_{ji}{}^{\gamma}\right)=\left(0,0,1\right)$ for all $i,j\in\{1,2,...,n\}$.

Theorem5.1. For a NFM P is s-TNFM iff $\left(\Delta P\right)^{2}\prec\Delta P.$

Proof. Since P be a s- TNFM and $\Delta p_{kh}=(0,0,1)$ for few $k,h\in\{1,2,...,n\}$ wherever $\Delta P=\left[\Delta p_{ij}\right]$

We have to prove that $\max_{i}\{\Delta p_{ki}\wedge\Delta p_{ih}\}=(0,0,1)$

Let us assume that, $\Delta p_{kj}\wedge\Delta p_{jh}>(0,0,1)$ for some $j\in\{1,2,...,n\}$. Then $\Delta p_{kj}=p_{kj}\Theta p_{jk}>\left(0,0,1\right)$

Implies $p_{kj} > p_{jk}$. Similarly, $\Delta p_{jh} = p_{jh} \Theta p_{hj} > (0,0,1)$

Implies $p_{jh} > p_{hj}$ .By the definition of s-TNFM of P,we get

$p_{kh} > p_{hk}$ and $\Delta p_{kh} > (0,0,1)$

This contradicts our assumption. Therefore $\max_i \{\Delta p_{ki} \wedge \Delta p_{ih}\} = (0,0,1)$

Conversely, let, $(\Delta P)^2 \prec \Delta P$. We have to show that P is s-TNFM.

Let P is not s-TNFM, then there exist integers $i, j, k \in \{1,2,...,n\}$ such that, $p_{ik} > p_{ki}$

$p_{kj} > p_{jk}$ and $p_{ij} \le p_{ji}$ .Then $\Delta p_{ik} > (0,0,1)$, $\Delta p_{kj} > (0,0,1)$.

$(\Delta P)^2$ is greater than (0,0,1) while. This contradicts the definition of the relation $\prec$ .Thus, the NFM P is s-TNFM.

Theorem 5.2. If A is s-transitive NFM then,
(i) ΔP is s-TNFM,
(ii) ΔP is NNFM.

Proof. (i) By Theorem 5.1 and Remark 3.1, we have

$$\left[\Delta(\Delta P)\right]^2 = (\Delta P)^2 \prec \Delta P = \Delta(\Delta P),$$

which means ΔP is s-TNFM.

(ii) Since $(\Delta P)^n = \left[\left(\Delta p_{ij\alpha}{}^n, \Delta p_{ij\beta}{}^n, \Delta p_{ij\gamma}{}^n\right)\right]$ and consider $i, j \in \{1,2,...,n\}$ such that $\left(\Delta p_{ij\alpha}{}^n, \Delta p_{ij\beta}{}^n, \Delta p_{ij\gamma}{}^n\right) > (0,0,1)$. Then

$$\left(\Delta p_{ij\alpha}{}^n, \Delta p_{ij\beta}{}^n, \Delta p_{ij\gamma}{}^n\right) = \left(\Delta p_{i_0 i_1 \alpha}, \Delta p_{i_0 i_1 \beta}, \Delta p_{i_0 i_1 \gamma}\right) \wedge \left(\Delta p_{i_1 i_2 \alpha}, \Delta p_{i_1 i_2 \beta}, \Delta p_{i_1 i_2 \gamma}\right) \wedge$$

$\ldots \wedge \left(\Delta p_{i_{n-1} i_n \alpha}, \Delta p_{i_{n-1} i_n \beta}, \Delta p_{i_{n-1} i_n \gamma}\right) > (0,0,1)$ for few integers $i_0, i_1, i_2, ..., i_3 \in \{1,2,...,n\}$ therefore $i_0 = i_1$ and $i_n = j$. Consequently $i_a = i_b$ for some $a, b(a < b)$ &

$$\left(\Delta p_{i_a i_{a+1} \alpha}, \Delta p_{i_a i_{a+1} \beta}, \Delta p_{i_a i_{a+1} \gamma}\right) > (0,0,1)$$

$$\Rightarrow \left(\Delta p_{i_{a+1} i_a \alpha}, \Delta p_{i_{a+1} i_a \beta}, \Delta p_{i_{a+1} i_a \gamma}\right), \left(\Delta p_{i_{a+2} i_{a+1} \alpha}, \Delta p_{a+2\, i_{a+1} \beta}, \Delta p_{a+2\, i_{a+1} \gamma}\right) > (0,0,1)$$

$$\Rightarrow \left(\Delta p_{i_{a+1} i_{a+2} \alpha}, \Delta p_{i_{a+1} i_{a+2} \beta}, \Delta p_{i_{a+1} i_{a+2} \gamma}\right), \ldots, \left(\Delta p_{i_{b-1} i_b \alpha}, \Delta p_{i_{b-1} i_b \beta}, \Delta p_{i_{b-1} i_b \gamma}\right) > (0,0,1)$$

$\Rightarrow \left(\Delta p_{i_b i_{b-1}\alpha}, \Delta p_{i_b i_{b-1}\beta}, \Delta p_{i_b i_{b-1}\gamma}\right) > (0,0,1)$

Using the s-TNFM of the NFM ΔP, we get $\Delta p_{i_a i_a} = \Delta p_{i_a i_b} > \Delta p_{i_b i_a} = \Delta p_{i_a i_a}$ which is impossible.

Therefore, our assumption is incorrect and hence ΔP is nilpotent NFM.

Similar to the canonical form of TNFMs, a corresponding result also holds for s-TNFMs. The related theorem is stated as follows.

Theorem 5.3. For P is s-TNFM P, there exists an IFPM A, such that, $C = \left[\left(c_{ij}^{\alpha}, c_{ij}^{\beta}, c_{ij}^{\gamma}\right)\right] = A \times P \times A^T$ satisfies $c_{ij} \geq c_{ji}$ for all $i > j$.

Proof. Subsequently P is a TNFM, P can be expressed as, $P = (\Delta P) \vee (\nabla P)$

(By Theorem 5.2) ΔP is NNFM and ∇P is a SNFM (by Lemma4.1).

Then for a NFPM $\left(A \times \Delta P \times A^T\right)$ is strictly lower triangular and $\left(A \times \nabla P \times A^T\right)$ is SNFM.

Hence by Theorem 4.1, there exists a NFPM A implies that

$$C = \left(A \times \Delta P \times A^T\right) \vee \left(A \times \nabla P \times A^T\right)$$

$$= A \times (\Delta P \vee \nabla P) \times A^T$$

$= A \times P \times A^T$ satisfies $c_{ij} \geq c_{ji}$ for entirely $i > j$.

Example 5.1 Let us assume that NFM

$$P = \begin{bmatrix} <1,1,0> & <0.6,0.5,0.2> & <0.8,0.5,0.1> \\ <0.4,0.5,0.5> & <0,0,1> & <0.5,0.5,0.3> \\ <0.1,0.5,0.5> & <0.9,0.5,0.1> & <0,0,1> \end{bmatrix}$$

Then the transpose of P is $P^T = \begin{bmatrix} <1,1,0> & <0.4,0.5,0.5> & <0.1,0.5,0.5> \\ <0.6,0.5,0.2> & <0,0,1> & <0.9,0.5,0.1> \\ <0.8,0.5,0.1> & <0.5,0.5,0.3> & <0,0,1> \end{bmatrix}$

Also, $\Delta P = \begin{bmatrix} <0,0,1> & <0.6,0.5,0.2> & <0.8,0.5,0.1> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0.9,0.5,0.1> & <0,0,1> \end{bmatrix}$ and

$$(\Delta P)^2 = \begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \end{bmatrix}$$

Here it is observed that $(\Delta P)^2 \prec \Delta P$. ThusbytheTheorem5.2, the NFM P is s-transitive.

Again, $(\Delta P)^3 = \begin{bmatrix} <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0,0,1> & <0,0,1> & <0,0,1> \end{bmatrix} = 0$

ΔP is nilpotent.

Also, $\nabla P = \begin{bmatrix} <0,0,1> & <0.4,0.5,0.5> & <0.1,0.5,0.5> \\ <0.4,0.5,0.5> & <0,0,1> & <0.5,0.5,0.3> \\ <0.1,0.5,0.5> & <0.5,0.5,0.3> & <0,0,1> \end{bmatrix}$

Now, for the NFPM, $A = \begin{bmatrix} <0,0,1> & <1,1,0> & <0,0,1> \\ <0,0,1> & <0,0,1> & <1,1,0> \\ <1,1,0> & <0,0,1> & <0,0,1> \end{bmatrix}$

$$A(\Delta P)A^T = \begin{bmatrix} <0,0,1> & <1,1,0> & <0,0,1> \\ <0.9,0.5,0.1> & <0,0,1> & <1,1,0> \\ <0.6,0.5,0.2> & <0.8,0.5,0.1> & <0,0,1> \end{bmatrix}$$

And $A(\nabla P)A^T = \begin{bmatrix} <0,0,1> & <0.5,0.5,0.3> & <0.4,0.5,0.5> \\ <0.5,0.5,0.3> & <0,0,1> & <0.1,0.5,0.5> \\ <0.4,0.5,0.5> & <0.1,0.5,0.5> & <1,1,0> \end{bmatrix}$

So, $C = A \times P \times A^T = \begin{bmatrix} <0,0,1> & <0.5,0.5,0.3> & <0.4,0.5,0.5> \\ <0.9,0.5,0.1> & <0,0,1> & <0.1,0.5,0.5> \\ <0.6,0.5,0.2> & <0.8,0.5,0.1> & <1,1,0> \end{bmatrix}$ which satisfy the required condition.

6.Conclusions

Transitive Neutrosophic fuzzy relations, which correspond to TNFM, hold significant importance in both the theoretical framework of Neutrosophic fuzzy relations and their applications across various research domains. This paper explores fundamental properties of transitive NFMs. Subsequently, it presents a decomposition of transitive and STNFMs into the sum of a nilpotent NFM and an asymmetric NFM. Additionally, some intriguing results are discussed concerning nilpotent, transitive, and strongly transitive NFMs. A compelling question arises: can any NFM be represented in its canonical form this intriguing aspect will be addressed in a future study.

References

[1]. Zadeh, L.A. (1965). Fuzzy Sets, Journal of Information and Control, Vol. 8, No. 3, pp. 338-353.

[2]. K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 8796.

[3]. M. Bhowmik and M. Pal, Generalized intuitionistic fuzzy matrices, Far-East Journal of Math ematical Sciences 29 (3) (2008) 533554.

[4]. J. J. Buckley, Note on convergence of powers of a fuzzy matrix, Fuzzy Sets and Systems 121 (2001) 363364.

[5]. Z.-T. Fan and D.-F. Liu, Convergence of the power sequence of a nearly monotone increasing fuzzy matrix, Fuzzy Sets and Systems 88 (1997) 363372.

[6]. Z.-T. Fan and D.-F. Liu, On the oscillating power sequence of a fuzzy matrix, Fuzzy Sets and Systems 93 (1998) 7585.

[7]. Z.-T. Fan and D.-F. Liu, On the power sequence of a fuzzy matrix (III), A detailed study on the power sequence of matrices of commonly used types, Fuzzy Sets and Systems 99 (1998) 197203.

[8]. Z.-T. Fan, A note on the power sequence of a fuzzy matrix, Fuzzy Sets and Systems, 102 (1999) 281-286.

[9]. S.-M. Guu, H.-H. Chen and C.-T. Pang, Convergence of products of fuzzy matrices, Fuzzy Sets and Systems 121 (2001) 203207.

[10]. S.-M. Guu, Y.-Y. Lur and C.-T. Pang, On innite products of fuzzy matrices, SIAM J. Matrix Anal. Appl. 22 (4) (2001) 11901203.

[11]. S.-M. Guu and C.-T. Pang, On the convergence to zero of innite products of interval matrices, SIAM J. Matrix Anal. Appl. 25 (2004) 739751.

[12]. H. Hashimoto, Convergence of powers of a fuzzy transitive matrix, Fuzzy Sets and Systems 9 (1983) 153160.

[13]. H. Hashimoto, Canonical form of a transitive fuzzy matrix, Fuzzy Sets and Systems, 11 (1983) 157-162.

[14]. H. Hashimoto, Transitivity of generalized fuzzy matrices, Fuzzy Sets and Systems 17 (1985) 8390.

[15]. K. H. Kim, Boolean Matrix Theory and Applications, Marcel Dekker, NewYork 1982.

[16]. K. H. Kim and F. W. Roush, Fuzzy matrix theory, in: J.C. Bezdek (Ed.), Analysis of Fuzzy Information, vol. 1, CRC Press, Boca Raton, FL (1987) 107 129.

[17]. S. Mondal and M. Pal, Soft matrices, Journal of Uncertain Systems 7 (4) (2013) 254264.

[18]. S. Mondal and M. Pal, Intuitionistic fuzzy incline matrix and determinant, Ann. Fuzzy Math. Inform., 8(1) (2014) 19-32.

[19]. M. Pal, S. K. Khan and A. K. Shyamal, Intuitionistic fuzzy matrices, Notes on Intuitionistic Fuzzy Sets 8 (2) (2002) 5162.

[20]. M. Pal, Intuitionistic fuzzy determinant, J. Phys. Sci. 7 (2001) 8793.

[21]. R. Pradhan and M. Pal, Convergence of maxgeneralized mean-mingeneralized mean powers of intuitionistic fuzzy matrices, J. Fuzzy Math. 22 (2) (2014) 477492.

[22]. R. Pradhan and M. Pal, Convergence of maxarithmetic mean-minarithmetic mean powers of intuitionistic fuzzy matrices, International Journal of Fuzzy Mathematical Archive 2 (2013) 3869.

[23]. R. Pradhan and M. Pal, Intuitionistic fuzzy linear transformations, Annals of Pure and Applied Mathematics 1 (1) (2012) 5768.

[24]. R. Pradhan and M. Pal, Generalized inverse of block intuitionistic fuzzy matrices, International Journal of Applications of Fuzzy Sets and Arti cial Intelligence 3 (2013) 2338.

[25]. A. K. Shyamal and M. Pal, Interval-valued fuzzy matrices, J. Fuzzy Math. 14 (3) (2006) 583604.

[26]. A. K. Shyamal and M. Pal, Distance between intuitionistic fuzzy matrices and its applicatins, Natural and Physical Science 19 (1) (2005) 3958.

[27]. Rajkumar Pradhan, Madhumangal Pal, Transitive and strongly transitive intuitionistic fuzzy matrices, Annals of Fuzzy Mathematics and Informatics Volume 13, No. 4, (April 2017), pp. 485-498.

[28]. E. G. Emam, On Consistent and Weak Transitive Intuitionistic Fuzzy Matrices, Fuzzy Information and Engineering, Volume 14, 2022 - Issue 1, Pages 16-25.

[29]. M. Anandhkumar , T. Harikrishnan , S. M. Chithra , V. Kamalakannan , B. Kanimozhi, Partial orderings, Characterizations and Generalization of k-idempotent Neutrosophic fuzzy matrices, International Journal of Neutrosophic Science, Volume 23 , Issue 2 , PP: 286-295, 2024.

[30]. R.Uma, P. Murugadas and S. Sriram, Fuzzy Neutrosophic Soft Matrices of Type I and Type II, Volume 13, Pages 211-222, 2021.

[31]. M. Anandhkumar , V. Kamalakannan , S. M. Chithra , Broumi Said, Pseudo Similarity of Neutrosophic Fuzzy matrices, International Journal of Neutrosophic Science, , Issue 2 ,Volume 20 , PP: 191-196, 2023.

[32]. Smarandache,F, Neutrosophic set, a generalization of the intuitionistic fuzzy set. Int J Pure Appl Math.; .,(2005),.24(3):287–297.

[33]. M. A. Mishref and E. G. Emam, "Transitivity and subinverses in fuzzy matrices," Fuzzy Sets and Systems, vol. 52, no. 3, pp. 337-343, 1992.

[34]. Ye, J., Yong, R., & Du, W. MAGDM Model Using Single-Valued Neutrosophic Credibility Matrix Energy and Its Decision-Making Application. Neutrosophic Systems with Applications, 17, 1-20. 2024.

[35]. J, J., & S, R. Some Operations on Neutrosophic Hypersoft Matrices and Their Applications. Neutrosophic Systems with Applications, 21, 46-62. 2024.

[36]. J, J., & S, R. Some Operations on Neutrosophic Hypersoft Matrices and Their Applications. Neutrosophic Systems with Applications, 21, 46-62. 2024.

[37]. M.Anandhkumar , S. Prathap , R. Ambrose Prabhu , P.Tharaniya , K. Thirumalai , B. Kanimozhi, Determinant Theory of Quadri-Partitioned Neutrosophic Fuzzy Matrices and its Application to Multi-Criteria Decision-Making Problems, Neutrosophic Sets and Systems, Vol. 79, 2025.

[38]. K. Radhika, S. Senthil , N. Kavitha , R.Jegan , M.Anandhkumar, A. Bobin, Interval Valued Secondary k-Range Symmetric Quadri Partitioned Neutrosophic Fuzzy Matrices with Decision Making, Neutrosophic Sets and Systems, Vol. 78, 2025.

[39]. Anandhkumar, M., Kanimozhi, B., Chithra, S.M., Kamalakannan, V., Said, B., "On various Inverse of Neutrosophic Fuzzy Matrices", International Journal of Neutrosophic Science, Vol. 21, No. 02, PP. 20-31, 2023.

[40]. C. Devi Shyamala Mary, C. Kayelvizhi, Shriram kalathian, P. Tharaniya, M. Anandhkumar, & S. M. Chithra. (2025). Decomposition of Neutrosophic Fuzzy Matrices Using Some Alpha – Cuts. Neutrosophic Sets and Systems, 86, 126-158.

[41]. K. Radhika, T. Harikrishnan, R. Ambrose Prabhu, , P.Tharaniya , M.John peter, M.Anandhkumar, On Schur Complement in k-Kernel Symmetric Block Quadri Partitioned Neutrosophic Fuzzy Matrices, Neutrosophic Sets and Systems, Vol. 78, 2025.

[42]. H. Prathab, N. Ramalingam, E. Janaki, A. Bobin, V. Kamalakannan and M. Anandhkumar, Interval Valued Secondary k-Range Symmetric Fuzzy Matrices with Generalized Inverses, IAENG International Journal of Computer Science, Volume 51, Issue 12, December 2024, Pages 2051-2066.

[43]. M.Anandhkumar, B.Kanimozhi, S.M. Chithra, V.Kamalakannan, Reverse Tilde (T) and Minus Partial Ordering on Intuitionistic fuzzy matrices, Mathematical Modelling of Engineering Problems, 2023, 10(4), pp. 1427–1432.

[44]. G. Punithavalli1, M. Anandhkumar, Reverse Sharp and Left-T Right-T Partial Ordering on Intuitionistic Fuzzy Matrices, TWMS J. App. and Eng. Math. V.14, N.4, 2024, pp. 1772-1783.

[45]. M. Anandhkumar, H. Prathab, S. M. Chithra, A. S. Prakaash, A. Bobin, Secondary K-Range Symmetric Neutrosophic Fuzzy Matrices, International Journal of Neutrosophic Science, vol. 23, no. 4, 2024, pp. 23-28.

[46]. T. Harikrishnan, M. Anandhkumar, S. Prathap, S. Subramanian, D. Ramesh, & M.Raji. Min(Max) – Min(Max) – Max(Min) (∗): Compositions of Neutrosophic Fuzzy Matrices and its Application in Medical Diagnosis . Neutrosophic Sets and Systems, 86, 212-244 (2025).

[47]. G. Punithavalli, M. Anandhkumar, Kernel and K-Kernel Symmetric Intuitionistic Fuzzy Matrices,TWMS J. App. and Eng. Math. V.14, N.3, 2024, pp. 1231-1240.

[48]. Anandhkumar, M., Harikrishnan, T., Chithra, S.M., ...Kanimozhi, B., Said, B. "Reverse Sharp and Left-T Right-T Partial Ordering on Neutrosophic Fuzzy Matrices" International Journal of Neutrosophic Science, 2023, 21(4), pp. 135–145.

[49]. Anandhkumar, M.; G. Punithavalli; and E. Janaki. "Secondary k-column symmetric Neutrosophic Fuzzy Matrices." Neutrosophic Sets and Systems 64, 1 (2024).

[50]. Anandhkumar, M.; G. Punithavalli; T. Soupramanien; and Said Broumi. "Generalized Symmetric Neutrosophic Fuzzy Matrices." Neutrosophic Sets and Systems 57, 1 (2023).

[51]. Anandhkumar, M.; G. Punithavalli; R. Jegan; and Said Broumi. "Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices." Neutrosophic Sets and Systems 61, 1 (2023).

Received: Dec. 21, 2024. Accepted: July 7, 2025