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On the Lattice Structure of Neutrosophic Open Sets for Risk Evaluation of Supply Chain Finance in International Cross-Border E-Commerce

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Abstract: In complex financial ecosystems, firms interact through asymmetric and uncertain relationships that classical models fail to capture adequately. With the increase of complexity of these ecosystems, standard topological and fuzzy approaches have become no longer able to expressively model indeterminacy intrinsic in financial interactions. To address this limitation, we propose a novel neutrosophic topological framework for corporate financial management, which introduces neutrosophic financial relations theories to enable a granular representation of membership influence, indeterminacy, and resistance in inter-firm dynamics. Our framework, then, drives neutrosophic open sets using upper and lower contour mappings and organizes them into a complete lattice structure, with support of formal set of operations. Furthermore, we propose Contour Index as a new centrality metric to quantify financial importance the neutrosophic topology. With the introduction of realistic case study from financial corporations, we investigate the applicability of our framework, and the results demonstrate its ability to identify influential firms, supporting ranks under uncertainty, and enhances strategic financial planning. Extensive analysis proves the ability to offer a powerful, uncertainty-aware foundation for financial analytics as well as decision-making in dynamic corporate networks.

Keywords: Neutrosophic Set, Neutrosophic Relation, Neutrosophic Topology, Neutrosophic Open Set, Corporate Financial Management, Lattice Theory, Financial Topological Structures.

1. Introduction

Uncertainty is an intrinsic feature of corporate financial systems due to fluctuating markets, incomplete information, complex interdependencies, and the coexistence of risk, ambiguity, and vagueness [1], [2]. Common mathematical models [3]—like crisp or probabilistic techniques—are no longer able to capture the multidimensional nature of uncertainty that had been arise in financial decision-making, particularly when data is imprecise, conflicting, or partially known [4].

In responding to these challenges, neutrosophic set theory, proposed with Smarandache [5], [6], provides a powerful extension of classical [7], fuzzy [8], [9], and intuitionistic fuzzy sets [10], [11] by incorporatine truth, indeterminacy, and falsity components in and Independent manner. This triadic enabled flexible modeling of incomplete and inconsistent information, made it a capable candidate for application in financial systems where such conditions are dominant [12].

Building upon these foundations, neutrosophic topologies constructed from neutrosophic relations have developed as a method for structural modeling information flow, interaction patterns, and decision boundaries in uncertain environments. Apart from classical topological constructs, neutrosophic topologies enabled for the integration of multiple layers of uncertainty into the spatial and relational representation of entities, which made them particularly appropriate for financial analytics involved dynamic corporate relations, like credit networks, investment dependencies, as well as ownership hierarchies.

Despite theoretical richness of neutrosophic topologies, their application to corporate financial management remain in early phases [13], [14]. Current literature lacks a formal framework that bridges neutrosophic relational structures and topological modeling with financial systems, especially in terms of how neutrosophic-open-sets [15] can represent meaningful financial paradigms such as risk clusters, stability regions, or influence zones within corporate networks. Moreover, the algebraic and lattice-theoretic properties of these structures have not been broadly investigated in financial contexts, limiting their integration into decision-support tools.

This seeks to fulfill existing research gaps through introducing a formal study of neutrosophic open sets in a neutrosophic topology generated via a neutrosophic relation and exploring their applicability to corporate financial management. Specifically, we:

1. We introduce a novel neutrosophic relational framework to represent financial influence between corporate entities, by explicitly modeling triadic dimensions (membership, non-membership, and indeterminacy) in corporate financial management.
2. Our framework develops a formal method to generate lower and upper contour neutrosophic open sets from proposed financial relations, which provides a topological view on the way the financial firms are influenced by or exert influence over others.
3. Then, the generated neutrosophic open sets are intelligently mapped to complete and distributive lattice, in which a group neutrosophic operations, like union, intersection, and containment, which enabled rigorous topological manipulation and algebraic reasoning over financial influence patterns.

4. Our framework introduces a novel metric called the Contour Index, derived from upper contour sets, to enumerate financial influence centrality of financial firms. Contour Index provides novel integration of both trust as well as resistance allowing a robust ranking of firms within uncertain environments.
5. We apply the framework to a realistic case involving a five-entity supply chain (Supplier, Manufacturer, Logistics, Wholesaler, Retailer). We demonstrate how the model captures asymmetric influence, ranks financial importance, and identifies the central players under indeterminacy.

The remainder of this paper is organized as follows: Section 2 presents the necessary preliminaries and notation. Section 3 introduces neutrosophic topologies. Section 4 explores the neutrosophic financial topology framework. Section 5 applies the proposed framework to corporate financial systems, while Section 6 concludes the research.

2. Preliminaries

In this section, we recap the fundamentals of neutrosophic sets and numerous related concepts that henceforward required through remaining of this study.

Definition 1 ([1]) -- Neutrosophic Set (NS) is a further generalization of the intuitionistic fuzzy set, introduced by Smarandache. Let E be a crisp set. A neutrosophic set Ω on E is defined as:

$$\Omega = \{ \langle \varsigma, ר_{\Omega}(\varsigma), ત_{\Omega}(\varsigma), \mathrm{F}_{\Omega}(\varsigma) \rangle \mid \varsigma \in \mathcal{E} \} \quad (1)$$

where $ר_{\Omega}: E \to \mathcal{J}$ represent the truth membership, $ત_{\Omega}: E \to \mathcal{J}$ represent the indeterminacy membership, and $\mathrm{F}_{\Omega}: E \to \mathcal{J}$. Here $\mathcal{J}: =]^{-}0, 1^{+}[$ is a non-standard interval extending beyond the classical unit interval to allow for over- and under-estimation. The values satisfy:

$$0 \leqslant \mu_{\Omega}(\varsigma) + \eta_{\Omega}(\varsigma) + \nu_{\Omega}(\varsigma) \leqslant 3^{+}, for\ all \varsigma \in E. \quad (2)$$

Definition 2 – Single-Valued Neutrosophic Set (SVNS) [16]: To facilitate the practical application of neutrosophic sets in real-world scenarios, a simplified subclass known as the *Single-Valued Neutrosophic Set (SVNS)* was proposed. Let E be a classical (crisp) set. A single-valued neutrosophic set Ω on E is defined as:

$$\Omega = \{ \langle \varsigma, \mu_{\Omega}(\varsigma), \eta_{\Omega}(\varsigma), \nu_{\Omega}(\varsigma) \rangle \mid \varsigma \in \mathcal{E} \} \quad (3)$$

where $ר_{\Omega}: E \to [0,1]$ represent the truth membership, $ત_{\Omega}: E \to [0,1]$ represent the indeterminacy membership, and $\mathrm{F}_{\Omega}: E \to [0,1]$.

Definition 3 ([5]) – Neutrosophic Set Operations: Let Ω and Δ be two neutrosophic sets defined over a crisp universe E. Various set-theoretic operations have been defined in the literature. The operations relevant to the present work are formally defined as follows:

Inclusion:

$$\Omega \subseteq \Delta \Leftrightarrow \mu_{\Omega}(\varsigma) \leq \mu_{\Delta}(\varsigma), \eta_{\Omega}(\varsigma) \leq \eta_{\Delta}(\varsigma), \nu_{\Omega}(\varsigma) \geq \nu_{\Delta}(\varsigma), \forall \varsigma \in E (4)$$

Equality:

$$\Omega = \Delta \Leftrightarrow \mu_{\Omega}(\varsigma) = \mu_{\Delta}(\varsigma), \eta_{\Omega}(\varsigma) = \eta_{\Delta}(\varsigma), \nu_{\Omega}(\varsigma) = \nu_{\Delta}(\varsigma), \forall \varsigma \in E \quad (5)$$

Intersection:

$$\Omega \cap \Delta = \{\langle \varsigma, \mu_{\Omega}(\varsigma) \wedge \mu_{\Delta}(\varsigma), \eta_{\Omega}(\varsigma) \wedge \eta_{\Delta}(\varsigma), \nu_{\Omega}(\varsigma) \vee \nu_{\Delta}(\varsigma) \rangle \mid \varsigma \in E\} \quad (6)$$

where $\wedge$ and $\vee$ denote the minimum and maximum operators, respectively.

Union:

$$\Omega \cup \Delta = \{\langle \varsigma, \mu_{\Omega}(\varsigma) \vee \mu_{\Delta}(\varsigma), \eta_{\Omega}(\varsigma) \vee \eta_{\Delta}(\varsigma), \nu_{\Omega}(\varsigma) \wedge \nu_{\Delta}(\varsigma) \rangle \mid \varsigma \in E\} \quad (7)$$

Complement:

$$\bar{\Omega} = \{\langle \varsigma, \nu_{\Omega}(\varsigma), \eta_{\Omega}(\varsigma), \mu_{\Omega}(\varsigma) \rangle \mid \varsigma \in E\} \quad (8)$$

Truth-Dominant Transformation:

$$[\Omega] = \{\langle \varsigma, \mu_{\Omega}(\varsigma), \eta_{\Omega}(\varsigma), 1 - \mu_{\Omega}(\varsigma) \rangle \mid \varsigma \in E\} \quad (9)$$

Falsity-Dominant Transformation:

$$\langle \Omega \rangle = \{\langle \varsigma, 1 - \nu_{\Omega}(\varsigma), \eta_{\Omega}(\varsigma), \nu_{\Omega}(\varsigma) \rangle \mid \varsigma \in E\} \quad (10)$$

Definition 5 - Support of a Neutrosophic Set: Let Ω be a neutrosophic set on the universe E. The support of Ω, denoted $\mathcal{S}(\Omega)$, is defined as the classical subset of E containing all elements with non-zero degrees in all three neutrosophic components:

$$\mathcal{S}(\Omega) = \{\varsigma \in E \mid \mu_{\Omega}(\varsigma) \neq 0, \eta_{\Omega}(\varsigma) \neq 0, \nu_{\Omega}(\varsigma) \neq 0\} \quad (11)$$

This definition identifies the region where the neutrosophic set has meaningful (non-null) information.

Definition 6 - (α, β, γ)-Cut of a Neutrosophic Set (Level Set) [17]: Let Ω be a neutrosophic set defined over a universe E. For given thresholds $\alpha, \beta, \gamma \in (0,1]$, the (α, β, γ) cut (also referred to as a level set) of Ω is defined as the classical (crisp) subset of E given by:

$$\Omega_{(\alpha,\beta,\gamma)} = \{\varsigma \in E \mid \mu_{\Omega}(\varsigma) \geq \alpha, \eta_{\Omega}(\varsigma) \geq \beta, \nu_{\Omega}(\varsigma) \leq \gamma\} \quad (12)$$

This subset consists of all elements in E whose membership degree is at least α, indeterminacy degree is at least β, and non-membership degree is at most γ.

Definition 7 - Neutrosophic Binary Relation [18], [19]: Let E and Z be two crisp (classical) sets. A neutrosophic binary relation N from E to Z is defined as a neutrosophic subset of the Cartesian product $E \times Z$. Formally, it is expressed as:

$$N = \{\langle(\varsigma,\sigma), \mu_N(\varsigma,\sigma), \eta_N(\varsigma,\sigma), \nu_N(\varsigma,\sigma)\rangle \mid (\varsigma,\sigma) \in E \times Z\} \quad (13)$$

where $\mu_N: E \times Z \to [0,1]$ denotes the truth-membership function, $\eta_N: E \times Z \to [0,1]$ denotes the indeterminacy-membership function, and $\nu_N: E \times Z \to [0,1]$ denotes the falsity-membership (non-membership) function.

Example 1. Given $E = \{a, b, c\}$ as the domain, and $Z = \{x, y\}$ represent the codomain. We define a neutrosophic relation $N \subseteq E \times Z$, represented by three component matrices: $\mu_N(\cdot,\cdot), \eta_N(\cdot,\cdot), \nu_N(\cdot,\cdot)$; which are given in Figure 1. These visualizations provide an intuitive and comparative understanding of how the relation behaves across different element pairs.

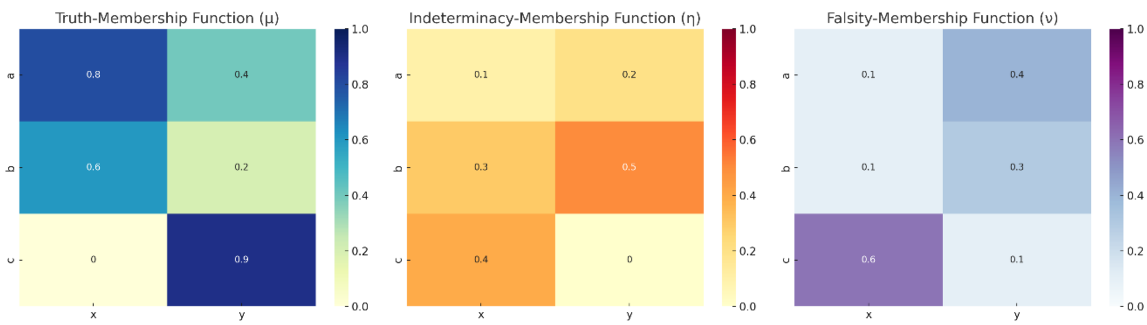

Figure 1. visualization of the neutrosophic relation between sets $E = \{a, b, c\}$ and $Z = \{x, y\}$.

Definition 8 - Transpose of a Neutrosophic Relation [20]. Let N and M be two neutrosophic relations defined from a universe E to a universe Z. The transpose (also referred to as the inverse) of a neutrosophic relation N, denoted N^t, is a neutrosophic relation from Z to E, and is defined by:

$$N^t = \{\langle(\varsigma,\sigma), \mu_{N^t}(\varsigma,\sigma), \eta_{N^t}(\varsigma,\sigma), \nu_{N^t}(\varsigma,\sigma)\rangle \mid (\varsigma,\sigma) \in Z \times E\} \quad (14)$$

with the component functions given by:

$$\mu_{N^t}(\varsigma,\sigma) = \mu_N(\sigma,\varsigma), \eta_{N^t}(\varsigma,\sigma) = \eta_N(\sigma,\varsigma), \nu_{N^t}(\varsigma,\sigma) = \nu_N(\sigma,\varsigma) \quad (15)$$

for all $(\varsigma,\sigma) \in Z \times E$.

Definition 9 - Containment of Neutrosophic Relations [20]. The relation N is said to be contained in M, denoted $N \subseteq M$, if for all $(\varsigma,\sigma) \in E \times Z$, the following inequalities hold:

$$\mu_N(\varsigma,\sigma) \leq \mu_M(\varsigma,\sigma), \eta_N(\varsigma,\sigma) \leq \eta_M(\varsigma,\sigma), \nu_N(\varsigma,\sigma) \geq \nu_M(\varsigma,\sigma) \quad (16)$$

Definition 10 - Intersection of Neutrosophic Relations: The intersection of two neutrosophic relations N and M, denoted $N \cap M$, is defined as:

$$N \cap M = \left\{ \left\langle \begin{matrix} (\varsigma, \sigma), \min(\mu_N(\varsigma, \sigma), \mu_M(\varsigma, \sigma)), \\ \min(\eta_N(\varsigma, \sigma), \eta_M(\varsigma, \sigma)), \\ \max(\nu_N(\varsigma, \sigma), \nu_M(\varsigma, \sigma)) \end{matrix} \right\rangle \mid (\varsigma, \sigma) \in E \times Z \right\} \quad (17)$$

Definition 11 - Union of Neutrosophic Relations: The union of two neutrosophic relations N and M, denoted $N \cup M$, is defined as:

$$N \cup M = \left\{ \left\langle \begin{matrix} (\varsigma, \sigma), \max(\mu_N(\varsigma, \sigma), \mu_M(\varsigma, \sigma)), \\ \max(\eta_N(\varsigma, \sigma), \eta_M(\varsigma, \sigma)), \\ \min(\nu_N(\varsigma, \sigma), \nu_M(\varsigma, \sigma)) \end{matrix} \right\rangle \mid (\varsigma, \sigma) \in E \times Z \right\} \quad (18)$$

Definition 12- Properties of Neutrosophic Relations on a Set. Let N be a neutrosophic relation defined on a universe E, i.e., $N \subseteq E \times E$. Then N is said to satisfy the following classical relation properties extended to the neutrosophic context:

Reflexivity

The neutrosophic relation N is reflexive if for all $\varsigma \in E$, the following conditions hold:

$$\mu_N(\varsigma, \varsigma) = 1, \eta_N(\varsigma, \varsigma) = 1, \nu_N(\varsigma, \varsigma) = 0 \quad (19)$$

That is, each element is fully related to itself with complete certainty and no falsity.

Symmetry

The neutrosophic relation N is symmetric if for all $\varsigma, \sigma \in E$:

$$\mu_N(\varsigma, \sigma) = \mu_N(\sigma, \varsigma), \eta_N(\varsigma, \sigma) = \eta_N(\sigma, \varsigma), \nu_N(\varsigma, \sigma) = \nu_N(\sigma, \varsigma) \quad (20)$$

This ensures that the relation holds in both directions with equal membership, indeterminacy, and nonmembership values.

Antisymmetry

The neutrosophic relation N is antisymmetric if for all distinct $\varsigma, \sigma \in E$ (i.e., $\varsigma \neq \sigma$):

$$\mu_N(\varsigma, \sigma) \neq \mu_N(\sigma, \varsigma), \eta_N(\varsigma, \sigma) \neq \eta_N(\sigma, \varsigma), \nu_N(\varsigma, \sigma) \neq \nu_N(\sigma, \varsigma) \quad (21)$$

This condition reflects that mutual relationships between distinct elements must differ in at least one of the three components.

Transitivity

The neutrosophic relation N is transitive if the composition of N with itself is contained in N, i.e.,

$$N \circ N \subseteq N \quad (22)$$

Here, the composition $N \circ N$ is computed using an appropriate aggregation of membership, indeterminacy, and non-membership values across intermediate elements in E.

Definition 13 - Lower and Upper Contours of a Neutrosophic Relation: Let E be a universe and let $N = \{\langle(\varsigma,\sigma), \mu_N(\varsigma,\sigma), \eta_N(\varsigma,\sigma), \nu_N(\varsigma,\sigma)\rangle \mid \varsigma,\sigma \in E\}$ be a neutrosophic relation on E. For each $\varsigma \in E$, define the following neutrosophic sets:

The lower contour of ς, denoted L_ς, is given by:

$$\mu_{L_\varsigma}(\sigma) = \mu_N(\sigma,\varsigma), \eta_{L_\varsigma}(\sigma) = \eta_N(\sigma,\varsigma), \nu_{L_\varsigma}(\sigma) = \nu_N(\sigma,\varsigma), \forall\sigma \in E \quad (23)$$

The upper contour of ς, denoted R_ς, is given by:

$$\mu_{R_\varsigma}(\sigma) = \mu_N(\varsigma,\sigma), \eta_{R_\varsigma}(\sigma) = \eta_N(\varsigma,\sigma), \nu_{R_\varsigma}(\sigma) = \nu_N(\varsigma,\sigma), \forall\sigma \in E \quad (24)$$

The families of all lower and upper contours generate the following neutrosophic topologies, where τ_1 is generated by lower contours $\{L_\varsigma \mid \varsigma \in E\}$, τ_2 is generated by upper contours $\{R_\varsigma \mid \varsigma \in E\}$, and τ_N represent neutrosophic topology generated by the union of lower and upper contours. Since the sets L_S and R_S are derived from the neutrosophic relation N, they satisfy the condition $0 \le \mu + \eta + \nu \le 3$, where $0 \le \mu_{L_c}(\sigma) + \eta_{L_s}(\sigma) + \nu_{L_c}(\sigma) \le 3$, and $0 \le \mu_{R_\varsigma}(\sigma) + \eta_{R_\varsigma}(\sigma) + \nu_{R_\varsigma}(\sigma) \le 3$, for all $\varsigma, \sigma \in E$.

Definition 14 - Neutrosophic Intersection of Open Sets: Let τ_N be the neutrosophic topology on the universe E generated by the neutrosophic relation N. Let W_1 and W_2 be two neutrosophic open sets in τ_N. Their intersection, denoted $W_1 \sqcap W_2$, is the neutrosophic open set $V \subseteq E$ defined by:

$$\begin{aligned} \mu_V(\varsigma) &= \min\left(\mu_{W_1}(\varsigma), \mu_{W_2}(\varsigma)\right) \\ \eta_V(\varsigma) &= \min\left(\eta_{W_1}(\varsigma), \eta_{W_2}(\varsigma)\right) \\ \nu_V(\varsigma) &= \max\left(\nu_{W_1}(\varsigma), \nu_{W_2}(\varsigma)\right) \end{aligned} \quad (26)$$

for all $\varsigma \in E$. Moreover, for an indexed family $\{W_i\}_{i\subset I} \subseteq \tau_N$, the general intersection is defined as: $\sqcup_{i\in I} W_i \in \tau_N$.

Definition 12 - Neutrosophic Union of Open Sets: Let τ_N be the neutrosophic topology on the universe E generated by the neutrosophic relation N. Let W_1 and W_2 be two neutrosophic open sets in τ_N. Their union, denoted $W_1 \sqcup W_2$, is the neutrosophic open set $V \subseteq E$ defined by:

$$\begin{aligned} \mu_V(\varsigma) &= \max\left(\mu_{W_1}(\varsigma), \mu_{W_2}(\varsigma)\right) \\ \eta_V(\varsigma) &= \max\left(\eta_{W_1}(\varsigma), \eta_{W_2}(\varsigma)\right) \\ \nu_V(\varsigma) &= \min\left(\nu_{W_1}(\varsigma), \nu_{W_2}(\varsigma)\right) \end{aligned} \quad (27)$$

for all $\varsigma \in E$. Similarly, the general union of a family $\{W_i\}_{i\subset I} \subseteq \tau_N$ is:

$$\sqcup_{i\in I} W_i \in \tau_N \quad (28)$$

In Figure 2, we provide visualization of example on neutrosophic open sets and their operations over the universe $\mathrm{E} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$.

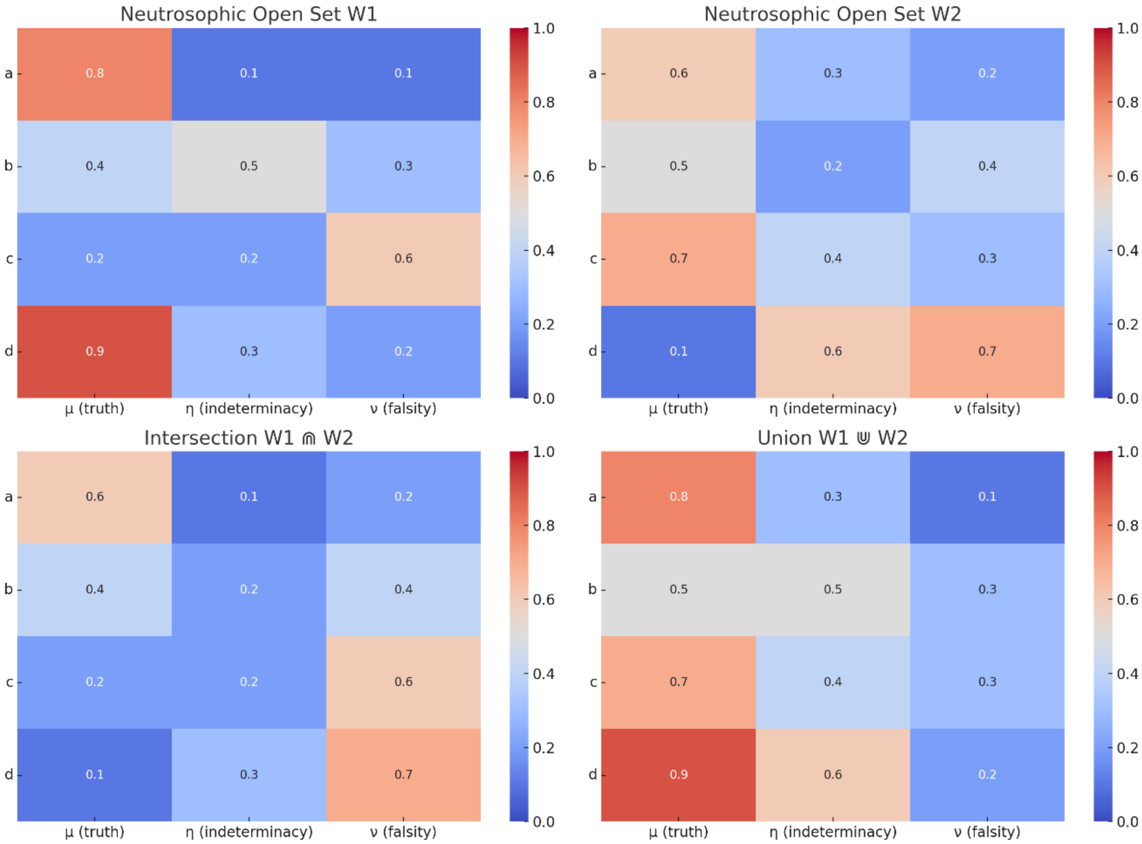

Figure 2. visualization of neutrosophic open sets and their operations

3. Neutrosophic Financial Topology Framework

This section introduces a novel multi-step framework for modeling financial relationships using neutrosophic logic. Rather than directly mapping numeric financial metrics into neutrosophic values, we propose a layered modeling approach that captures financial strength, uncertainty, and resistance through dedicated transformation stages.

In step 1, we construct a financial Indicator Vector that enables semantic decomposition of financial behavior. Let the financial universe be defined as:

$$E = \{\rho_1, \rho_2, \dots, \rho_n\} \qquad (29)$$

For each entity $\rho_i \in E$, we construct a normalized Financial Indicator Vector (FIV):

$$f_i = \left[f_i^{(L)}, f_i^{(P)}, f_i^{(R)}\right] \qquad (30)$$

where $f_i^{(L)}$ is a liquidity index, $f_i^{(P)}$ is a profitability index, and $f_i^{(R)}$ is a risk index. Each component is normalized into [0,1] and computed as a nonlinear aggregation of real-world financial ratios.

In step 2, we compute context-aware influence scoring, which acts as multi-angle influence metric using contextual kernels. For each pair $(\rho_i, \rho_j) \in E \times E$, we compute context-aware influence using a multi-kernel model:

$$\phi_{ij} = \lambda_L \cdot \kappa_L \left(f_i^{(L)}, f_j^{(L)}\right) + \lambda_P \cdot \kappa_P \left(f_i^{(P)}, f_j^{(P)}\right) - \lambda_R \cdot \kappa_R \left(f_j^{(R)}\right) - \lambda_V \cdot \kappa_V \left(f_j^{(V)}\right) \quad (31)$$

where: κ_* are similarity kernels (i.e., $\kappa_P \left(f_i^{(L)}, f_j^{(L)}\right) = 1 - \left|f_i^{(L)} - f_j^{(L)}\right|$), λ_* are tunable weights (financial scenario-dependent), the negative weight for risk expresses inverse influence due to financial stress. This score $\phi_{ij} \in [-1,1]$ model's directional financial influence.

In step 3, we perform structured mapping from influence score to neutrosophic triple. Let us now propose a new mapping where each component is separately computed from different semantics of ϕ_{ij} . Let high ϕ_{ij} imply strong positive support:

$$\mu_{ij} = \text{sigmoid}(\alpha \cdot \phi_{ij}) = \frac{1}{1 + e^{-\alpha \cdot \phi_{ij}}}$$

$$\nu_{ij} = \text{sigmoid}(-\beta \cdot \phi_{ij}) \quad (32)$$

$$\eta_{ij} = \exp\left(-\gamma \cdot \phi_{ij}^2\right)$$

With $\alpha \in [2,6]$, e.g., $\alpha = 4$, controlling sensitivity. $\beta \approx 4$, or use a different slope if needed. $\gamma > 0$, making uncertainty drop off quickly. This ensures that very positive influence imply μ domination, very high negative influence imply that ν dominates, while ambiguous influence imply that dominates. To incorporate external uncertainty factors (e.g., economic policy shifts, global risks), we introduce an adaptive amplification factor $\delta_{ij} \in [0,1]$ that scales η:

$$\eta'_N(\rho_i, \rho_j) = \delta_{ij} \cdot \eta_N(\rho_i, \rho_j) \quad (33)$$

The triple is now:

$$\Omega_{ij} = \langle \mu_N(\rho_i, \rho_j), \eta'_N(\rho_i, \rho_j), \nu_N(\rho_i, \rho_j) \rangle \quad (34)$$

In step 5, we perform constraining normalization and filtering. Here, we bounded total belief constraint with risk-pruning to ensure compliance with neutrosophic set theory, we normalize:

$$T_{ij} = \mu_N + \eta'_N + \nu_N \quad (35)$$

If $T_{ij} > 1$, apply scaling:

$$(\mu, \eta, \nu) \leftarrow \left(\frac{\mu}{T}, \frac{\eta}{T}, \frac{\nu}{T}\right) \quad (36)$$

We also define a risk threshold $\theta \in [0,1]$. If $\nu_N(\rho_i, \rho_j) > \theta$, we flag the relation as financially adverse, enabling the pruning of destabilizing links in the generated topology.
In step 6, we generate matrix representation for topological construction. In particular, we represent the neutrosophic relation N as three $n \times n$ matrices:

$$M_\mu = [\mu_N(\rho_i, \rho_j)],\ M_\eta = [\eta'_N(\rho_i, \rho_j)],\ M_\nu = [\nu_N(\rho_i, \rho_j)] \quad (37)$$

Based on the above, we build upon the neutrosophic relation N, constructed in Section 3.1, to generate a neutrosophic topology over the financial universe E. This is achieved through the introduction of lower and upper neutrosophic contours, followed by the derivation of neutrosophic open sets that form the topological space. The process is presented as a multi-step construction, with each step offering an original contribution.

In step 7, we drive lower and Upper Neutrosophic Contours. Given the neutrosophic relation:

$$N = \{\langle(\rho_i, \rho_j), \mu_N(\rho_i, \rho_j), \eta_N(\rho_i, \rho_j), \nu_N(\rho_i, \rho_j)\rangle \mid \rho_i, \rho_j \in E\} \quad (38)$$

We define the lower contour of a financial entity ρ_i as:

$$L_{\rho_i} = \{\langle\rho_j, \mu_L(\rho_j), \eta_L(\rho_j), \nu_L(\rho_j)\rangle \mid \rho_j \in E\}$$
$$\mu_L(\rho_j) = \mu_N(\rho_j, \rho_i), \eta_L(\rho_j) = \eta_N(\rho_j, \rho_i), \nu_L(\rho_j) = \nu_N(\rho_j, \rho_i) \quad (39)$$

Analogously, the upper contour is:

$$R_{\rho_i} = \{\langle\rho_j, \mu_R(\rho_j), \eta_R(\rho_j), \nu_R(\rho_j)\rangle \mid \rho_j \in E\}$$
$$\mu_R(\rho_j) = \mu_N(\rho_i, \rho_j), \eta_R(\rho_j) = \eta_N(\rho_i, \rho_j), \nu_R(\rho_j) = \nu_N(\rho_i, \rho_j) \quad (40)$$

where L_{ρ_i} denote backward-looking financial influences, and R_{ρ_i} denote forward-looking exposures.

In step 8, we can compute an aggregated temporal contour that capture both incoming and outgoing influence. We define the temporal composite contour as a convex combination of lower and upper contours:

$$T_{\rho_i}(\rho_j) = \begin{pmatrix} \alpha \cdot \mu_L(\rho_j) + (1-\alpha) \cdot \mu_R(\rho_j), \\ \alpha \cdot \eta_L(\rho_j) + (1-\alpha) \cdot \eta_R(\rho_j), \\ \alpha \cdot \nu_L(\rho_j) + (1-\alpha) \cdot \nu_R(\rho_j) \end{pmatrix} \quad (41)$$

Where $\alpha \in [0,1]$ controls temporal directionality: $\alpha = 1$: purely backward-looking (risk absorption), $\alpha = 0$: purely forward-looking (risk emission), $\alpha = 0.5$: balanced systemic view.

In step 8, we define Cut-based financial state filtering using triple thresholding. We define the (α, β, γ)-cut open set for a contour C_{ρ_1} (e.g., lower, upper, or composite) as:

$$\mathcal{O}_{\rho_i}^{(\alpha,\beta,\gamma)} = \{\rho_j \in E \mid \mu_C(\rho_j) \geq \alpha, \eta_C(\rho_j) \geq \beta, \nu_C(\rho_j) \leq \gamma\} \tag{42}$$

where α denote minimum strength required, β denote minimum uncertainty tolerated, γ represent maximum risk tolerated.

In step 9, we proceed to topology generation from both symmetric and asymmetric relations. Let:

$$\mathcal{B}_1 = \{L_{\rho_i} \mid \rho_i \in E\}, \mathcal{B}_2 = \{R_{\rho_i} \mid \rho_i \in E\} \tag{43}$$

We define the neutrosophic topology as τ_1 : generated by lower contours $\mathcal{B}_1$, τ_2 : generated by upper contours $\mathcal{B}_2$, and $\tau_N = \tau_1 \cup \tau_2$: bidirectional neutrosophic topology.

If N is symmetric, then $\tau_1 = \tau_2$, and $\tau_N = \tau_1$.

In step 10, open set operations for Financial Semantics. For two neutrosophic open sets $W_1, W_2 \subseteq \tau_N$, we define:
Intersection (Joint Conformity)

$$\mu_V = \min(\mu_{W_1}, \mu_{W_2}), \eta_V = \min(\eta_{W_1}, \eta_{W_2}), \nu_V = \max(\nu_{W_1}, \nu_{W_2}) \tag{44}$$

Union (Alternative Acceptability)

$$\mu_V = \max(\mu_{W_1}, \mu_{W_2}), \eta_V = \max(\eta_{W_1}, \eta_{W_2}), \nu_V = \min(\nu_{W_1}, \nu_{W_2}) \tag{45}$$

In step 11, we introduce contour index mapping function to map of open set centrality for financial importance ranking. This can be defining the contour index of a financial entity ρ_i as:

$$\kappa(\rho_i) = \frac{1}{|E|} \sum_{\rho_j \in E} \mu_{T_{\rho_i}}(\rho_j) - \nu_{T_{\rho_i}}(\rho_j)$$

4. Application for Financial Management

In this section, we demonstrate the practical applicability of the proposed neutrosophic topological framework using a realistic corporate financial management scenario. The objective is to assess inter-company financial relationships and identify risk zones, dominant entities, and clusters of financial stability or volatility using neutrosophic open sets and their lattice structure.

We consider a hypothetical but realistic ecosystem consisting of five interconnected firms operating in a supply-chain network: a raw material supplier, a manufacturer, a logistics firm, a wholesaler, and a retailer. These firms are denoted by $E = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5\}$, respectively. The financial health and inter-dependence of these firms are influenced by several factors:

- Liquidity: ability to meet short-term obligations,
- Profitability: return on assets and equity,
- Leverage (Risk): reliance on debt financing,
- Volatility: historical fluctuation in performance.

The data shown in Table 1 are derived from realistic financial ratios for one fiscal year. Each firm's indicators are normalized to [0,1] to allow integration into the neutrosophic model.

Table 1. realistic financial ratios for one fiscal year

Firm ($\boldsymbol{\rho}$)	Liquidity $(\boldsymbol{f}^{(L)})$	Profitability $\boldsymbol{f}_i^{(P)}$	Leverage $\boldsymbol{f}_i^{(R)}$	Volatility $(\boldsymbol{f}^{(V)})$
$\boldsymbol{\rho_1}$	0.70	0.40	0.30	0.20
$\boldsymbol{\rho_2}$	0.60	0.60	0.40	0.40
$\boldsymbol{\rho_3}$	0.40	0.50	0.80	0.90
$\boldsymbol{\rho_4}$	0.90	0.30	0.20	0.30
$\boldsymbol{\rho_5}$	0.50	0.70	0.60	0.70

In Table 2, we show the influence score matrix ϕ_{ij}, capturing the directional financial influence from firm ρ_i to firm ρ_j in the corporate ecosystem. These scores integrate liquidity, profitability, leverage, and volatility, applying your selected weight scheme and similarity-based kernel.

Table 2. Influence Score Matrix derived from our case study.

	ϱ_1 (Supplier)	ϱ_2 (Manufacturer)	ϱ_3 (Logistics)	ϱ_4 (Wholesaler)	ϱ_5 (Retailer)
ϱ_1 (Supplier)	1.00	0.52	0.30	0.58	0.34
ϱ_2 (Manufacturer)	0.58	1.00	0.34	0.46	0.46
ϱ_3 (Logistics)	0.54	0.52	1.00	0.42	0.42
ϱ_4 (Wholesaler)	0.58	0.40	0.18	1.00	0.22
ϱ_5 (Retailer)	0.50	0.56	0.34	0.38	1.00

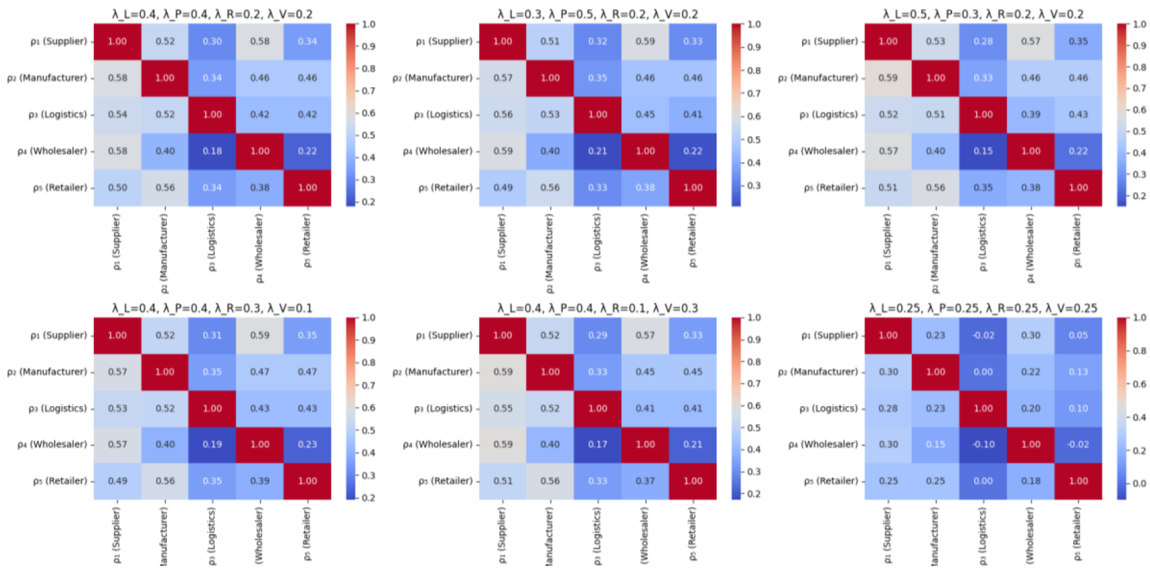

Figure 3. Heatmap visualization of influence score matrices ϕ_{ij} under six different λ-weight configurations.

The heatmaps in Figure 3 demonstrate the sensitivity of the influence score matrix ϕ_{ij} to variations in the λ-weight configuration. When emphasis is placed on liquidity and profitability (e.g., in Configs 1–3), firms like ρ_1 and ρ_5 (Retailer) show relatively strong positive influence across the network. Conversely, configurations with higher weights on leverage and volatility (Configs 4–6) amplify the negative effects from financially volatile firms such as ρ_3 (Logistics), whose influence scores tend to decrease or become negative. This highlights the ability of the proposed model to adaptively capture financial propagation dynamics, aiding in the identification of financially stabilizing vs. destabilizing agents within the ecosystem.

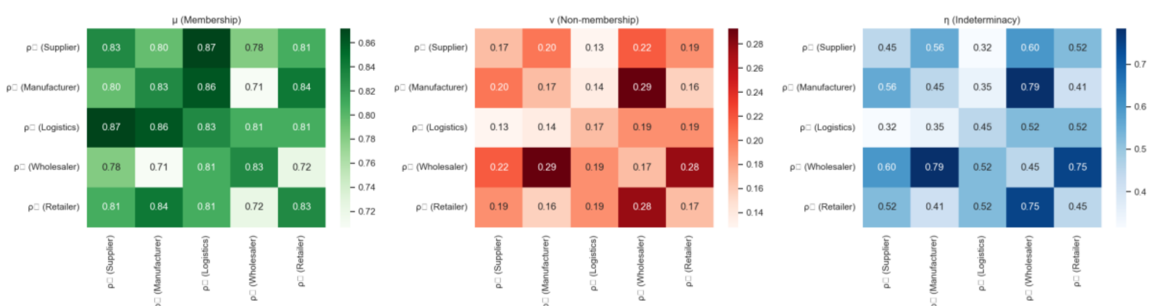

Figure 4. Heatmaps of the neutrosophic influence matrices among corporate entities

The heatmaps in Figure 4 provide a comprehensive visualization of the neutrosophic relations derived from corporate financial indicators. The μ reveals strong directional influences between certain firm pairs notably, the wholesaler (ϱ_4) exerts consistent supportive influence across others. Conversely, the ν highlights asymmetric resistance patterns, especially where financial leverage or volatility diverge significantly. The η matrix is highest where firms show ambiguous interaction suggesting uncertain or unstable financial ties.

Table 3. Lower Contour Neutrosophic Open Sets

	ϱ_1 (Supplier)	ϱ_2 (Manufacturer)	ϱ_3 (Logistics)	ϱ_4 (Wholesaler)	ϱ_5 (Retailer)
ϱ_1 (Supplier)	<0.98, 0.02, 0.01>	<0.89, 0.11, 0.26>	<0.77, 0.23, 0.64>	<0.91, 0.09, 0.19>	<0.80, 0.20, 0.56>
ϱ_2 (Manufacturer)	<0.91, 0.09, 0.19>	<0.98, 0.02, 0.01>	<0.80, 0.20, 0.56>	<0.86, 0.14, 0.35>	<0.86, 0.14, 0.35>
ϱ_3 (Logistics)	<0.90, 0.10, 0.23>	<0.89, 0.11, 0.26>	<0.98, 0.02, 0.01>	<0.84, 0.16, 0.41>	<0.84, 0.16, 0.41>
ϱ_4 (Wholesaler)	<0.91, 0.09, 0.19>	<0.83, 0.17, 0.45>	<0.67, 0.33, 0.85>	<0.98, 0.02, 0.01>	<0.71, 0.29, 0.79>
ϱ_5 (Retailer)	<0.88, 0.12, 0.29>	<0.90, 0.10, 0.21>	<0.80, 0.20, 0.56>	<0.82, 0.18, 0.49>	<0.98, 0.02, 0.01>

Table 4. Upper Contour Neutrosophic Open Sets

	ϱ_1 (Supplier)	ϱ_2 (Manufacturer)	ϱ_3 (Logistics)	ϱ_4 (Wholesaler)	ϱ_5 (Retailer)
ϱ_1 (Supplier)	<0.98, 0.02, 0.01>	<0.91, 0.09, 0.19>	<0.90, 0.10, 0.23>	<0.91, 0.09, 0.19>	<0.88, 0.12, 0.29>
ϱ_2 (Manufacturer)	<0.89, 0.11, 0.26>	<0.98, 0.02, 0.01>	<0.89, 0.11, 0.26>	<0.83, 0.17, 0.45>	<0.90, 0.10, 0.21>
ϱ_3 (Logistics)	<0.77, 0.23, 0.64>	<0.80, 0.20, 0.56>	<0.98, 0.02, 0.01>	<0.67, 0.33, 0.85>	<0.80, 0.20, 0.56>
ϱ_4 (Wholesaler)	<0.91, 0.09, 0.19>	<0.86, 0.14, 0.35>	<0.84, 0.16, 0.41>	<0.98, 0.02, 0.01>	<0.82, 0.18, 0.49>
ϱ_5 (Retailer)	<0.80, 0.20, 0.56>	<0.86, 0.14, 0.35>	<0.84, 0.16, 0.41>	<0.71, 0.29, 0.79>	<0.98, 0.02, 0.01>

In Table 3 and Table 4, we present the lower and upper contour neutrosophic open sets provide insights into the directional dynamics of corporate influence within the financial network. The lower contours capture how each firm is impacted by others, offering a view of their susceptibility or dependency on peers. In contrast, the upper contours reflect each firm's outward influence across the network, quantifying how much it shapes the financial environment of others. Notably, these sets are not symmetric because they are derived from an asymmetric influence model, which penalizes firms with higher financial risk during outgoing influence computation.

The lattice operations have been performed for all unique pairs of firms, and the corresponding results are displayed in Tables 5-14, identifying collaboration potential between firms with complementary influence zones.

Table 5. Lattice Operations between ϱ_1 (Supplier) and ϱ_2 (Manufacturer)

	Intersection	Union
ϱ_1 (Supplier)	<0.91, 0.02, 0.19>	<0.98, 0.09, 0.01>
ϱ_2 (Manufacturer)	<0.89, 0.02, 0.26>	<0.98, 0.11, 0.01>
ϱ_3 (Logistics)	<0.77, 0.20, 0.64>	<0.80, 0.23, 0.56>
ϱ_4 (Wholesaler)	<0.86, 0.09, 0.35>	<0.91, 0.14, 0.19>
ϱ_5 (Retailer)	<0.80, 0.14, 0.56>	<0.86, 0.20, 0.35>

Table 6. Lattice Operations between ϱ_1 (Supplier) and ϱ_3 (Logistics)

	Intersection	Union
ϱ_1 (Supplier)	<0.90, 0.02, 0.23>	<0.98, 0.10, 0.01>
ϱ_2 (Manufacturer)	<0.89, 0.11, 0.26>	<0.89, 0.11, 0.26>

ϱ$_3$ (Logistics)	<0.77, 0.02, 0.64>	<0.98, 0.23, 0.01>
ϱ$_4$ (Wholesaler)	<0.84, 0.09, 0.41>	<0.91, 0.16, 0.19>
ϱ$_5$ (Retailer)	<0.80, 0.16, 0.56>	<0.84, 0.20, 0.41>

Table 7. Lattice Operations between ϱ$_1$ (Supplier) and ϱ$_4$ (Wholesaler)

	Intersection	Union
ϱ$_1$ (Supplier)	<0.91, 0.02, 0.19>	<0.98, 0.09, 0.01>
ϱ$_2$ (Manufacturer)	<0.83, 0.11, 0.45>	<0.89, 0.17, 0.26>
ϱ$_3$ (Logistics)	<0.67, 0.23, 0.85>	<0.77, 0.33, 0.64>
ϱ$_4$ (Wholesaler)	<0.91, 0.02, 0.19>	<0.98, 0.09, 0.01>
ϱ$_5$ (Retailer)	<0.71, 0.20, 0.79>	<0.80, 0.29, 0.56>

Table 8. Lattice Operations between ϱ$_1$ (Supplier) and ϱ$_5$ (Retailer)

	Intersection	Union
ϱ$_1$ (Supplier)	<0.88, 0.02, 0.29>	<0.98, 0.12, 0.01>
ϱ$_2$ (Manufacturer)	<0.89, 0.10, 0.26>	<0.90, 0.11, 0.21>
ϱ$_3$ (Logistics)	<0.77, 0.20, 0.64>	<0.80, 0.23, 0.56>
ϱ$_4$ (Wholesaler)	<0.82, 0.09, 0.49>	<0.91, 0.18, 0.19>
ϱ$_5$ (Retailer)	<0.80, 0.02, 0.56>	<0.98, 0.20, 0.01>

Table 9. Lattice Operations between ϱ$_2$ (Manufacturer) and ϱ$_3$ (Logistics)

	Intersection	Union
ϱ$_1$ (Supplier)	<0.90, 0.09, 0.23>	<0.91, 0.10, 0.19>
ϱ$_2$ (Manufacturer)	<0.89, 0.02, 0.26>	<0.98, 0.11, 0.01>
ϱ$_3$ (Logistics)	<0.80, 0.02, 0.56>	<0.98, 0.20, 0.01>
ϱ$_4$ (Wholesaler)	<0.84, 0.14, 0.41>	<0.86, 0.16, 0.35>
ϱ$_5$ (Retailer)	<0.84, 0.14, 0.41>	<0.86, 0.16, 0.35>

Table 10. Lattice Operations between ϱ$_2$ (Manufacturer) and ϱ$_4$ (Wholesaler)

	Intersection	Union
ϱ$_1$ (Supplier)	<0.91, 0.09, 0.19>	<0.91, 0.09, 0.19>
ϱ$_2$ (Manufacturer)	<0.83, 0.02, 0.45>	<0.98, 0.17, 0.01>
ϱ$_3$ (Logistics)	<0.67, 0.20, 0.85>	<0.80, 0.33, 0.56>
ϱ$_4$ (Wholesaler)	<0.86, 0.02, 0.35>	<0.98, 0.14, 0.01>
ϱ$_5$ (Retailer)	<0.71, 0.14, 0.79>	<0.86, 0.29, 0.35>

Table 11. Lattice Operations between ϱ$_2$ (Manufacturer) and ϱ$_5$ (Retailer)

	Intersection	Union
ϱ$_1$ (Supplier)	<0.88, 0.09, 0.29>	<0.91, 0.12, 0.19>
ϱ$_2$ (Manufacturer)	<0.90, 0.02, 0.21>	<0.98, 0.10, 0.01>
ϱ$_3$ (Logistics)	<0.80, 0.20, 0.56>	<0.80, 0.20, 0.56>
ϱ$_4$ (Wholesaler)	<0.82, 0.14, 0.49>	<0.86, 0.18, 0.35>
ϱ$_5$ (Retailer)	<0.86, 0.02, 0.35>	<0.98, 0.14, 0.01>

Table 12. Lattice Operations between ϱ_3 (Logistics) and ϱ_4 (Wholesaler)

	Intersection	Union
ϱ_1 (Supplier)	<0.90, 0.09, 0.23>	<0.91, 0.10, 0.19>
ϱ_2 (Manufacturer)	<0.83, 0.11, 0.45>	<0.89, 0.17, 0.26>
ϱ_3 (Logistics)	<0.67, 0.02, 0.85>	<0.98, 0.33, 0.01>
ϱ_4 (Wholesaler)	<0.84, 0.02, 0.41>	<0.98, 0.16, 0.01>
ϱ_5 (Retailer)	<0.71, 0.16, 0.79>	<0.84, 0.29, 0.41>

Table 13. Lattice Operations between ϱ_3 (Logistics) and ϱ_5 (Retailer)

	Intersection	Union
ϱ_1 (Supplier)	<0.88, 0.10, 0.29>	<0.90, 0.12, 0.23>
ϱ_2 (Manufacturer)	<0.89, 0.10, 0.26>	<0.90, 0.11, 0.21>
ϱ_3 (Logistics)	<0.80, 0.02, 0.56>	<0.98, 0.20, 0.01>
ϱ_4 (Wholesaler)	<0.82, 0.16, 0.49>	<0.84, 0.18, 0.41>
ϱ_5 (Retailer)	<0.84, 0.02, 0.41>	<0.98, 0.16, 0.01>

Table 14. Lattice Operations between ϱ_4 (Wholesaler) and ϱ_5 (Retailer)

	Intersection	Union
ϱ_1 (Supplier)	<0.88, 0.09, 0.29>	<0.91, 0.12, 0.19>
ϱ_2 (Manufacturer)	<0.83, 0.10, 0.45>	<0.90, 0.17, 0.21>
ϱ_3 (Logistics)	<0.67, 0.20, 0.85>	<0.80, 0.33, 0.56>
ϱ_4 (Wholesaler)	<0.82, 0.02, 0.49>	<0.98, 0.18, 0.01>
ϱ_5 (Retailer)	<0.71, 0.02, 0.79>	<0.98, 0.29, 0.01>

The influence network graph illustrates the directional financial relationships among supply chain entities using neutrosophic membership values (μ). An edge from firm ρ_i to firm ρ_j signifies that ρ_i exerts notable influence on ρ_j, with influence strength proportional to the μ value. Thicker edges denote stronger confidence in this influence, and only links with $\mu > 0.7$ are visualized for clarity. The diagram highlights key influencers and recipients in the network. For instance, firms with multiple outgoing arrows act as financial anchors or decision leaders, while those with numerous incoming edges are more sensitive to external financial shifts.

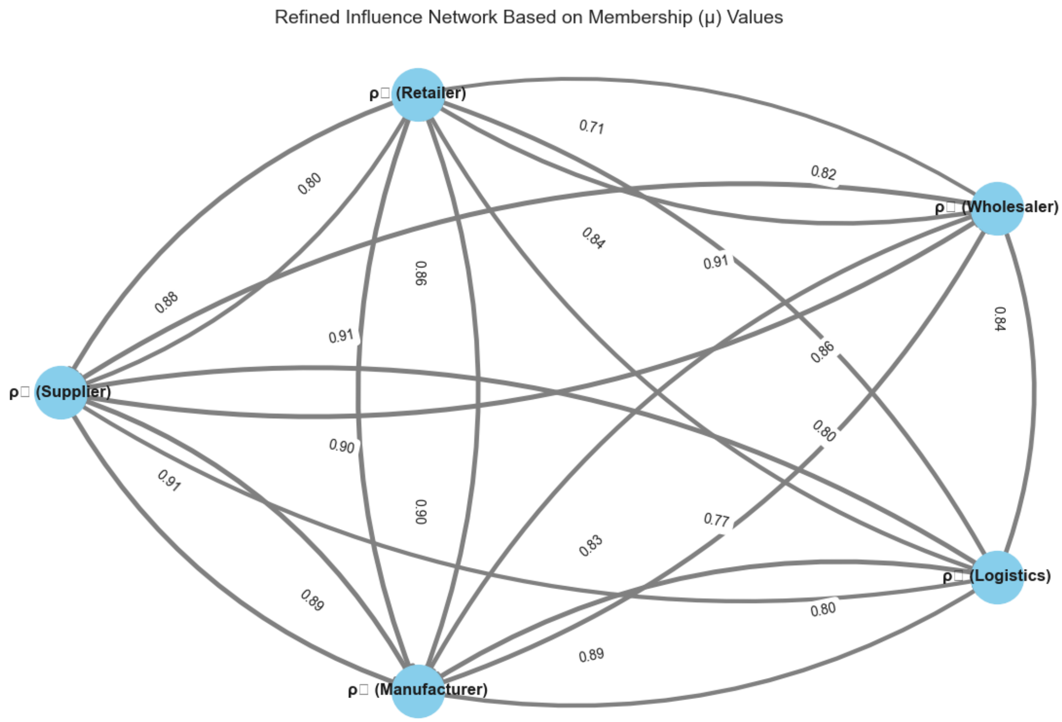

Figure 5. Influence Network Among Firms Based on Neutrosophic Membership

The graph in Figure 5 presents asymmetric influence dynamics between five firms in the supply chain, where edge direction represents the flow of influence and edge thickness denotes its strength. The network is constructed using the μ, with only connections exceeding a threshold of 0.7 visualized to emphasize meaningful financial interactions. Firms such as ϱ_1 (Supplier) and ϱ_2 (Manufacturer) appear as major influencers, while exercising noteworthy financial effect on others, as shown by their manifold outgoing edges.

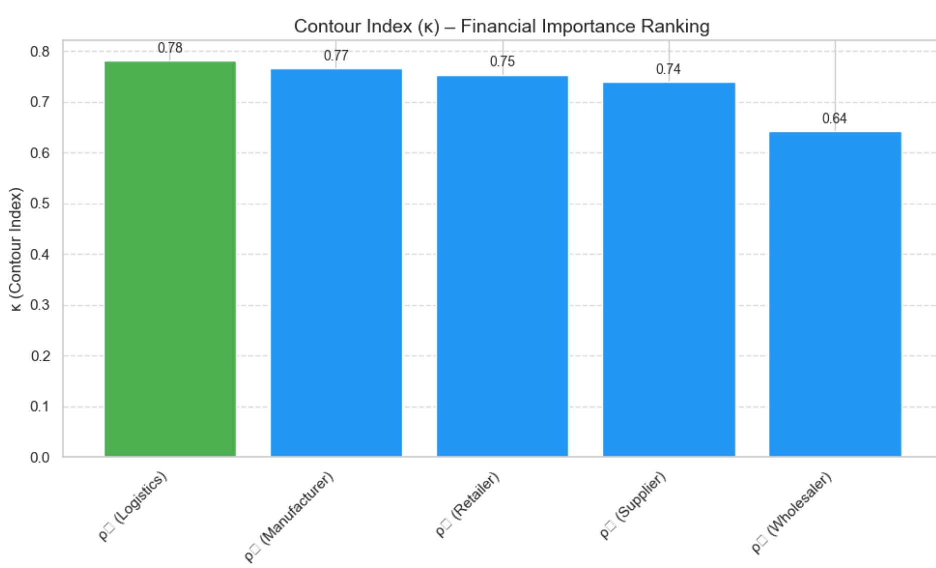

Figure 6. Visualization of ranks of firms based on their Contour Index (κ).

In Figure 6, we highlight ϱ_3 (Logistics) as the most financially influential entity, which achieve highest contour index. This can be explained like ϱ_3 consistently exhibits trust and low resistance levels within upper contour communications, which position it as a planned hub in the financial relationship network. In contrast, ϱ_4 (Wholesaler) score the lowest, indicating weaker or less convinced financial influence across the supply structure. With this ranking in mind, stakeholders have valuable insight into financial management, investment prioritization, and supply chain resilience planning.

5. Conclusion

In this study, we introduce a simple but effective neutrosophic topological framework to modeling and analyzing financial influence among corporate entities. With the use of neutrosophic relations and connaturalized open sets, our framework can quantify both symmetric as well as asymmetric financial interactions thereby can construct comprehensive lattice of influence. Moreover, our framework introduces the proposed Contour Index, which enables ranking firms according to their financial centrality, which offer critical insights into decision-making in corporate financial management. The outcomes of our framework bargain for a powerful, uncertainty-aware tool for identifying influential players, managing risk, and guiding strategic interferences in composite financial ecosystems.

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