Neutrosophic Sets and Systems, {Special Issue: Neutrosophic and Plithogenic Approaches in Data Science and Multivariate Analysis: Contributions from the IX Ibero-American Biometry Meeting"}, Vol. 89, 2025

University of New Mexico

Financial Dynamics of Cooperatives of the Popular and Solidarity Economy of Ecuador: An Analysis Using the Tucker3 Model and Projection with Multidimensional Neutrosophic Regression.

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Abstract: This study relies on the Tucker3 model as a factorization model of multivariate analyses since the goal is to evaluate the financial dependency structure of Ecuador's cooperative sector of the Popular and Solidarity Economy (PSE) between 2016–2023. A population sample of 25 cooperatives belonging to segment 1 was analyzed in terms of their behavior across seven significant financial variables for eight consecutive years, factoring into a 25×7×8 tensor. Relative to variances in behavior, the ultimate benefits of some entities become clear as high profitability and sound liquidity trends emerge over time; yet for others, high delinquency and non-growth levels emerge. The purpose of the tensor decomposition is to acknowledge significant associative patterns over time across entities and variables assessed at this level, leading to three clear temporal moments: growth, crisis and recovery. In addition, this study contributes a second method to the literature by applying Multidimensional Neutrosophic Regression to also assess the financial dependency relationships based on ambiguity, contradiction and uncertainty. This enables the financially relevant coefficients, determined by Multidimensional Neutrophilic Regression to be assessed with truth, falsity and uncertainty degrees simultaneously, allowing for better results interpretation to more nuanced assessments of cooperative performance under unstable conditions. Ultimately, the results advocate for differentiated lines of intervention and more accurate crisis warning systems, with findings applicable to regulatory, managerial and academic settings where improvements of the stable, strategic application of this cooperative sector are needed.

Keywords: Popular and Solidarity Economy, Tucker3, Savings and credit cooperatives, Financial sector, Multivariate analysis, Multidimensional Neutrosophic Regression.

1. Introduction

In the field of multivariate analysis, the graphical representation and dimensionality reduction of data have been areas of constant innovation. One of the most widely used techniques is principal component analysis (PCA), which facilitates the classification and interpretation of both the variables and the individuals involved in the study [1]. Among the most influential advances was the introduction of Biplot [2], who proposed a graphical tool that allows the simultaneous representation of observations and variables in a data matrix in a reduced space, facilitating the visual interpretation of latent structures. This proposal laid the foundations for later developments in multivariate visualization, such as HJ- Biplot, which extends Gabriel's approach by combining the properties of GH- Biplot and JK-Biplot to achieve a more precise simultaneous representation on the factorial axes [3-4].

Traditionally, multivariate data are organized in two-dimensional matrices $\boldsymbol{X}_{IxJ}$, where I is the number of units and J the number of variables. However, in many applied contexts, data collect information from multiple variables observed in several units over time or other conditions, generating three-way data structures, formalized as three-way $\boldsymbol{X}_{IxJxK}$arrays. This structure requires analytical techniques that adequately capture the complexity of the interactions among the three dimensions.

Among the most notable methodologies for processing this type of data are the Tucker model [5] and the CANDECOMP/PARAFAC model [6-8]. Three-way data matrices occur more frequently than is commonly recognized, although they often go unnoticed due to a lack of understanding of their nature or the appropriate techniques for their analysis. In many cases, researchers turn to three-way analysis when faced with data sets whose structure exceeds the capabilities of traditional multivariate analysis. This type of data involves three dimensions: units, variables, and conditions or observation times, which requires the use of specific methods capable of capturing the interactions between these three modes. The Tucker model [3] allows complex underlying patterns to be identified by decomposing the original matrices into a core matrix and three factor loading matrices associated with each mode. Its adoption is justified not only by the growing nature of complex data in different disciplines, but also by its ability to adequately decompose and model variability in three-way structures.

This approach is especially relevant in the analysis of Ecuador's popular and solidarity financial system, a sector whose development is framed by a robust regulatory and institutional context. In this study, the Tucker model3 is used as the primary tool to simultaneously analyze the interaction between cooperatives, financial variables, and periods, providing a comprehensive and structured view of the system. As a complementary methodological contribution, Multidimensional Neutrosophic Regression is incorporated, based on the framework of neutrosophics. This model models complex phenomena by assigning degrees of truth, falsity, and indeterminacy to the relationships between variables, thus capturing the ambiguity and contradiction inherent in cooperative financial dynamics. In the context of the Popular and Solidarity Economy (PSE), where cooperatives face dynamic environments with economic and regulatory fluctuations, this method enriches the interpretation of the patterns identified by Tucker3 by representing regression coefficients with neutrosophic components that reflect both certainty and structural uncertainty in variables such as profitability, liquidity, and delinquency. The combination of both approaches allows for a deeper understanding of internal structures and the evolution of financial performance, providing a basis for more robust management strategies. The EPS is firmly incorporated in the 2008 Constitution [9], particularly in Article 309, and in the 2011 Organic Law of the EPS and the Popular and Solidarity Financial Sector [10]. These regulations recognize the role of cooperatives, associations, and community organizations as pillars of inclusive and sustainable economic development. Authors such as Coraggio and Singer [11,12] have emphasized the importance of these organizations in promoting good living, equity, and social cohesion. Following this line, Faz Cevallos [13] highlights that savings and credit cooperatives (COACs) have proven to be a key sector within the Ecuadorian economy, especially in the area of microenterprises. Through their financial intermediation, they have achieved broader integration into economic activities, significantly contributing to improving the quality of life for families in the country.

According to data from the Superintendency of Popular and Solidarity Economy, there are more than 16,000 active organizations in the country, with nearly six million members. This magnitude and diversity justify the use of statistical tools that can capture not only the behavior of multiple financial variables, but also their evolution over time and their differentiation between entities. Therefore, this study proposes the use of the Tucker model, complemented by Multidimensional Neutrosophic Regression, as an innovative approach to identify internal structures, hidden relationships, and the evolution of performance of 25 segment 1 cooperatives, with the aim of contributing to the strengthening of financial management and strategic decisions within the Ecuadorian EPS.

2. Materials and methods
2.1. Tucker3 Model

This study applies the Tucker3 model as a dimensionality reduction technique and multivariate data structural analysis, with the aim of studying the economic behavior of 25 Ecuadorian cooperatives over a period of 8 years (Annex 1), considering 7 economic variables such as: deposits (DEP), which reflect customers' confidence in the cooperative and its ability to raise funds [14], the credit portfolio balance (SCC), which indicates the level of credit activity [15], and financial intermediation (INTFIN), which measures the efficiency in channeling savings into loans [16], in addition to considering current liquidity (LIQCOR), which evaluates the ability to meet short-term obligations, as well as the profitability indicators: return on assets (ROA) and financial profitability (ROE), which are essential to determine economic sustainability. Finally, the extended delinquency (MORAMP) allows to assess the quality of the portfolio and the credit risk [17].The data tensor decomposition was performed using R Studio software using the KTensorGraphs library . This tool allows the application of the Tucker3 algorithm, which decomposes the tensor into three loading matrices (A, B, and C) and a core tensor (G), capturing the latent relationships between cooperatives, economic variables, and years. The procedure followed is presented below:

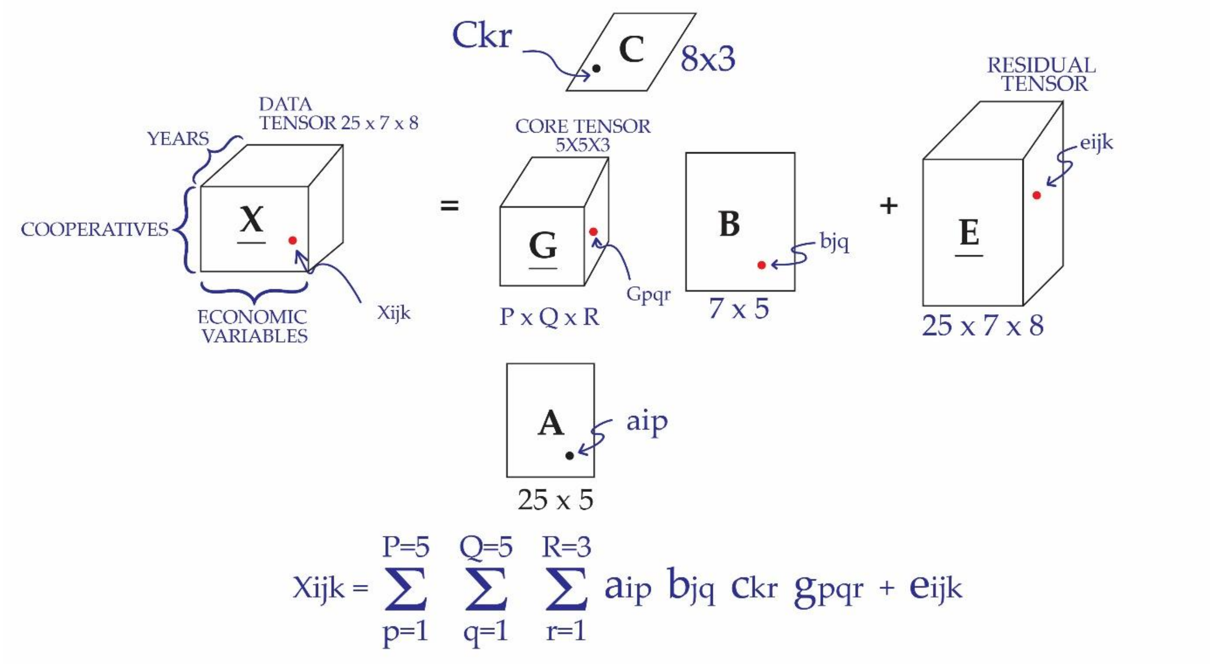

Figure 1. Tensor decomposition using the Tucker 3 model, representation of the 5x5x3 model

Data entry: The information is organized in a third-order tensor$\boldsymbol{X}_{\mathrm{ijk}} \in R^{IxJxK}$

Definition of ranges: The ranges for the common components in each mode are chosen $P, Q\ y\ R$ from 5, 5 and 3 respectively.

Initialization: Load matrices are initialized randomly or with heuristic methods.A, B, C.

Iterative decomposition: An ALS (Alternating) type algorithm is applied. Least Squares) to fit the factors $A, B, C$and the kernel tensor G, minimizing the residual E.

In each iteration, one of the matrices is updated, keeping the other two fixed.

Stopping criterion: The algorithm terminates when the reconstruction error stops decreasing significantly or after a maximum number of iterations.

Output: The approximation and the residue $E$are obtained $\hat{X} = [\![G;\ A, B, C]\!]$, together with the factors that allow the interpretation of the latent structure of the data.

This data matrix structure results in a **three-way data tensor** of size 25×7×8, where each dimension represents:

- cooperative dimension (n = 25),(i) :
- Second dimension (j) : economic variables (m = 7),
- Third dimension (k) :t = 8).

Tucker3 model allows the data tensor to be decomposed $\boldsymbol{X}_{\mathrm{ijk}}$ into three **load matrices** and a **core tensor** using the following expression (Eq.1):

$$X_{ijk} = \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{r=1}^{R} a_{ip}\, b_{jq}\, c_{kr}\, g_{\mathrm{pqr}} + e_{ijk} \quad (1)$$

where:

a_{ip}: elements of the **load matrix** $A \in R^{25X5}$, which represents the linear combination of cooperatives in the latent components p,

b_{jq}: elements of the **load matrix** $B \in R^{7X5}$, which describes the internal structure of economic variables,

c_{kr} : elements of the **load matrix** $C \in R^{8x3}$, which reflects the evolution of the components over time (years),

g_{pqr}: elements of the **core tensor** $G \in R^{5x5x3}$, which captures the interaction between the components of the three modes (cooperatives, variables, time),

e_{ijk}: error term that captures the variability not explained by the model.

Interpretation of the components

Matrix A (A_{ip}): Each row corresponds to a cooperative, and each column to a latent component. It represents how each cooperative's projection is projected onto the common components of the model.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{15} \\ \vdots & \vdots & \ddots & \vdots \\ a_{25,1} & a_{25,2} & \cdots & a_{25,5} \end{bmatrix}$$

Matrix B (B_{jq}): Describes the contribution of each economic variable to the latent components.

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{15} \\ \vdots & \vdots & \ddots & \vdots \\ b_{7,1} & b_{7,2} & \cdots & b_{7,5} \end{bmatrix}$$

Matrix C (C_{kr}): It captures temporal variation, reflecting how the components behave over the 8 years of study.

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{13} \\ \vdots & \vdots & \ddots & \vdots \\ c_{8,1} & c_{8,2} & \cdots & c_{8,3} \end{bmatrix}$$

Core tensor G: This is the heart of the Tucker3 model. Unlike PCA (principal component analysis), where components are orthogonal and their combination is decoupled, in Tucker3 the kernel allows to capture **complex interactions** between components. For example, g_{pqr}it indicates how much the interaction of p the cooperatives component, the $q$variables component, and the $r$time component contribute to the observed value.

2.2. Machine learning.

Machine learning (ML) involves mathematical formulations to create models that can learn from data to make predictions or decisions without being explicitly programmed to perform those tasks. Interval prediction in machine learning refers to the technique of predicting a range of possible outcomes for a given input rather than a single point estimate. By providing intervals, these methods offer not only predictions but also insight into the reliability and uncertainty of the predictions, which is crucial for decision-making in uncertain environments [18].

For a dataset with independent variables $X = [x_1, x_2, \ldots, x_n]$and one dependent variable y , the goal of regression analysis is to accurately model the relationship between X and y . This relationship is expressed mathematically as [18]:

$$y \approx f(X;\, \theta) \tag{2}$$

where:

y is the dependent variable or the objective that is going to be predicted.

X represents the independent or explanatory variables that are used to predict.

f is the regression function, which can vary in shape depending on the type of regression model used (linear, polynomial, logistic, etc.).

$\theta$are the parameters or coefficients of the model, adjusted during the training process to minimize a loss function, typically the mean square error (MSE) in regression [19].

In regression analysis, representing predictions as prediction intervals provides a more complete view of the uncertainty associated with predictions. A prediction interval provides a range within which we expect the actual value of the dependent variable to fall with a certain probability, typically 95% or 99%. This is particularly useful because it accounts for variability in the data that might not be captured by the prediction alone [1 9].

To calculate a prediction interval, both the uncertainty in the regression model estimate and the inherent variability of the data must be considered. The interval is constructed around the predicted value and is usually symmetrical, extending a certain amount above and below the predicted value. This range is determined based on the standard error of the prediction and the residual standard deviation, which reflects the dispersion of the model residuals or errors [20].

For example, in a simple linear regression, the prediction interval for a new observation is given by [20]:

$$\hat{y}_0 \pm t_{\propto\,/2,n-2} \cdot SE \tag{3}$$

Where $\hat{y}_0$ is the predicted value of y from the t distribution for a specific confidence level $\propto$and $n-2$ degrees of freedom, and SE?

Using prediction intervals in regression analysis is beneficial because they offer a realistic spectrum of possible outcomes, which aids in the decision-making process. This recognizes that a single predicted value is not absolute but rather a likely scenario within a range of potential outcomes. This forecasting method effectively incorporates the inherent uncertainties associated with future predictions, providing a more accurate depiction of what to expect. To further refine this model, neutrosophic statistics can be applied, which excel at managing data ambiguity and indeterminacy. By converting the interval into a neutrosophic number, the traditional interval is enhanced to include a component of indeterminacy. This addition captures the uncertainty and imprecision typically present in real-world data, offering a more nuanced understanding of data variability. The neutrosophic treatment of the interval is as follows [21]:

$$\hat{y}_0 - t_{\propto/\ 2,n-2} \cdot SE + (\hat{y}_0 + t_{\propto\ /2,n-2} \cdot SE)I \quad (4)$$

Here, I_N represents the uncertainty factor associated with the forecast, where $I_N \in [I_l, I_u]$, this notation introduces the limits of uncertainty I_l(lower uncertainty) and I_u (upper uncertainty), which define the range of possible deviations due to uncertain elements affecting the forecast [22,23].

The process of analyzing a data set for regression and using neutrosophic numbers to represent uncertainty can be broken down into several key steps:

1. Data Partitioning: The first step is to divide the data into training and test sets. This split is crucial as it allows for model validation with unseen data, ensuring that model performance isn't solely a result of overfitting the training data. Typically, data scientists might use a 70-30 or 80-20 split, where 70% or 80% of the data is used for training and the rest for testing.
2. Model training: Each model is then trained on the training set. This involves tuning the model parameters to best fit the data. Common regression models include linear regression, ridge regression, and random forest, among others. The training process involves finding model parameters that minimize a loss function, essentially capturing the underlying pattern of the data set.
3. Estimating prediction intervals: Once the models have been trained, the next step is to estimate prediction intervals for new observations. This is where neutrosophic numbers come into play. Unlike traditional crisp intervals, neutrosophic intervals include measurements of truth, indeterminacy, and falsity, allowing for a more nuanced representation of uncertainty in predictions. Each model may require different methods to calculate these intervals, considering the specific characteristics of the model and the nature of the data.
4. Calculation and analysis of uncertainty through neutrosophic numbers: The final step involves a detailed analysis of the uncertainty represented by the neutrosophic numbers. This includes assessing how the indeterminate component of these numbers varies with different models and what it suggests about the complexity or variability of the data. For example, greater indeterminacy could indicate more significant external influences or inherent unpredictability in the data set.

In this case, we employ neutrosophic means to combine interval predictions with other methods as part of a fusion approach in regression analysis. Neutrosophic means are particularly useful for integrating different predictive models because they allow for the incorporation of uncertainty, indeterminacy, and conflicting information that typically arise from various data sources or model outputs. This approach improves the robustness and reliability of predictive models by providing a more comprehensive framework that accounts for various aspects of uncertainty.

The neutrosophic mean, denoted as X_n, is calculated by considering the neutrosophic inclusion $I_N$within the interval. $[I_l, I_u]$.This mean consists of two main elements: X_l, which is the mean of the lower part of the neutrosophic samples, and X_u, which is the mean of the upper part. The respective definitions are:

$$X_l = \frac{\sum_{i=1}^{n_l} x_{il}1}{n_l} \quad (5)$$

$$X_u = \frac{\sum_{i=1}^{n_u} x_{iu}}{n_u} \quad (6)$$

where $n_l$and $n_u$represent the number of elements in the lower and upper parts of the neutrosophic samples, respectively. Therefore, the neutrosophic mean X_n, is expressed as the sum of $X_l$and X_u, adjusted by the uncertainty interval I_n:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \quad (7)$$

I_l=0, andI_u

$$I_u = \frac{X_u - X_l}{X_u} \quad (8)$$

3. Results

The scree plot shows how the residual sum of squares varies with the number of components selected in each mode of the Tucker3 model. In this case, it can be seen that, as the number of components increases, the residual sum of squares decreases, indicating an improved ability of the model to capture the underlying structure of the data. The curve in the plot presents a clear inflection point around the 5x5x3 model, where the reduction in the residual sum of squares begins to stabilize. This suggests that adding more components does not significantly improve the fit, so 5x5x3 represents an optimal balance between complexity and explanatory power (Figure 2).

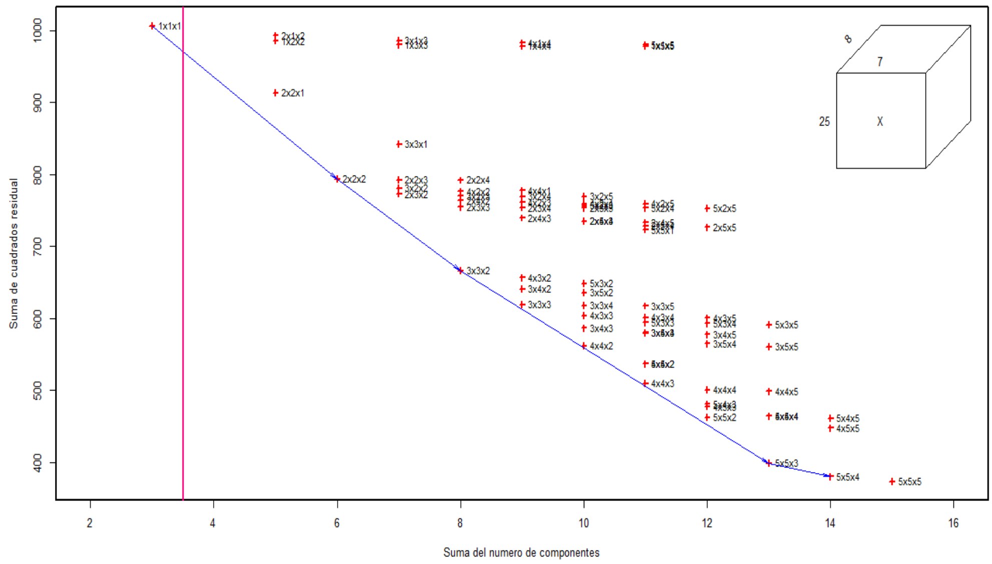

Figure 2. Sedimentation graph of the Tucker 3 model

The choice of the 5x5x3 model is based on the principle of parsimony and the interpretability of the results. This model captures sufficient variability in the data without overfitting. For the 25 cooperatives, 5 components are sufficient to group them according to their financial behavior (e.g. , cooperatives with high profitability, low risk, or high intermediation). For the 7 financial variables, 5 components help identify key relationships, such as the correlation between liquidity and risk. Finally, for the 8-year variables, 3 components summarize the most relevant time trends (e.g. , periods of growth, crisis, or recovery).

This model stands out for its ability to reveal hidden patterns without sacrificing clarity. For example, the five components of cooperatives can distinguish between entities with conservative strategies (focused on liquidity) versus aggressive ones (maximizing profitability). The temporal components, meanwhile, can reflect macroeconomic events, such as the impact of the pandemic in 2020-2021. Furthermore, the core of the model (G) will show relevant interactions, such as which groups of cooperatives are more sensitive to changes in certain financial variables during specific periods.

The Tucker3 model applied to the Ecuadorian savings and credit cooperative system reveals a complex and highly segmented financial structure, where distinct groups are clearly identified based on their performance and resilience. The multidimensional analysis shows that cooperatives such as JEP (Juventud Ecuatoriana Progresista) and JA (Jardín Azuayo) emerge as leaders in the sector, displaying robust financial indicators with profitability margins (ROA of 1.5% and ROE of 15% on average) well above the sector average, accompanied by low levels of delinquency (MORAMP < 20%) and highly efficient liquidity management (LIQCOR > 0.85). These excellent results remained stable even during the most critical years of the pandemic, demonstrating an exceptional capacity to absorb external shocks (Figure 3).

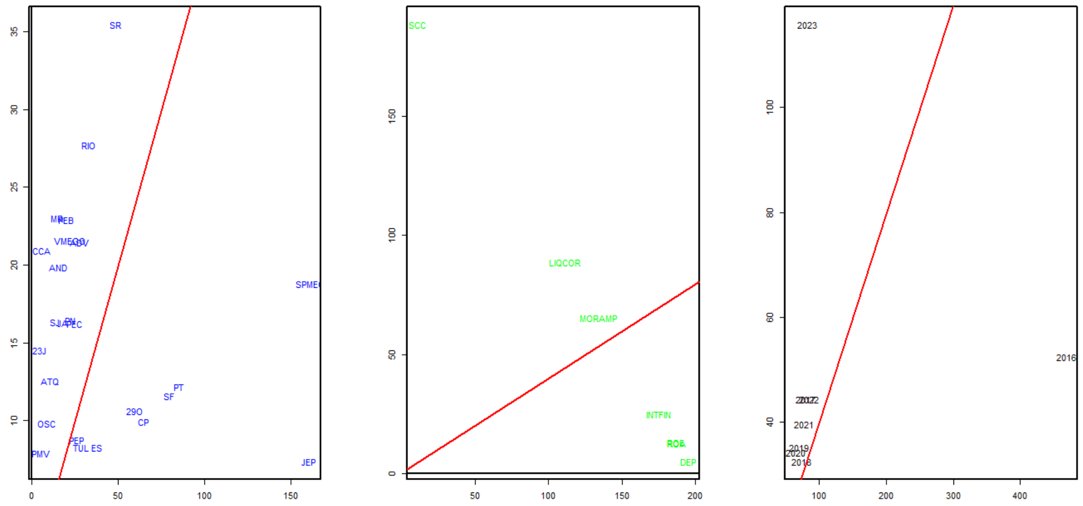

Figure 3. Element projection chart by mode

In contrast, the study identifies a vulnerable group made up mainly of 29O (29 de Octubre) and CSOs (Oscus), whose credit risk indicators (MORAMP > 35%) and limited access to funding (DEP with annual growth < 5%) place them in a fragile position in the face of economic crises. Particularly worrying is the case of the OSC cooperative, which showed a 22% drop in its deposits during 2020-2021, combined with an accelerated deterioration of its loan portfolio. These entities urgently require restructuring programs that strengthen their risk management and provisioning policies.

Factor analysis of financial variables reveals relationships critical to the sector's stability. The strong inverse correlation (-0.82) between liquidity (LIQCOR) and credit risk (MORAMP) confirms the delicate balance these institutions must maintain between profitability and security. Furthermore, the close link between financial intermediation (INTFIN) and loans (SCC) explains why cooperatives such as CP (Cooprogreso) and MR (Mushuc Runa) have been able to maintain stable financial margins by efficiently transforming their resources.

The study's temporal dimension yields particularly valuable findings for prospective management. The model identifies three clear phases: a period of stable growth (2016-2019) where the sector expanded its deposits at average rates of 12% per year; the acute crisis (2020-2021) that caused a widespread 8% contraction in productive assets; and the current recovery phase (2022-2023), marked by a growing divergence between resilient cooperatives like PN (National Police), which have already surpassed pre-pandemic levels , and those with more fragile business models that are still struggling to stabilize.

The interactions detected at the core of the Tucker3 model suggest that liquidity (LIQCOR) has become the determining factor for navigating the current period of economic uncertainty. Cooperatives such as TUL (Tulcán) and ES (El Sagrario), which maintained liquidity ratios above 30%, managed not only to survive the crisis but also to position themselves to take advantage of growth opportunities in the current context of credit restrictions.

This comprehensive diagnosis suggests three priority strategic lines for the sector: (1) implement differentiated institutional strengthening programs according to the identified clusters, (2) establish early warning mechanisms based on the key indicators detected (LIQCOR, MORAMP and their inverse relationship), and (3) promote the adoption of best practices from leading cooperatives (JEP, JA, PN) to the rest of the system. The results show that, beyond macroeconomic conditions, there are internal management factors that explain the marked differences in performance and resilience observed in the Ecuadorian cooperative sector.

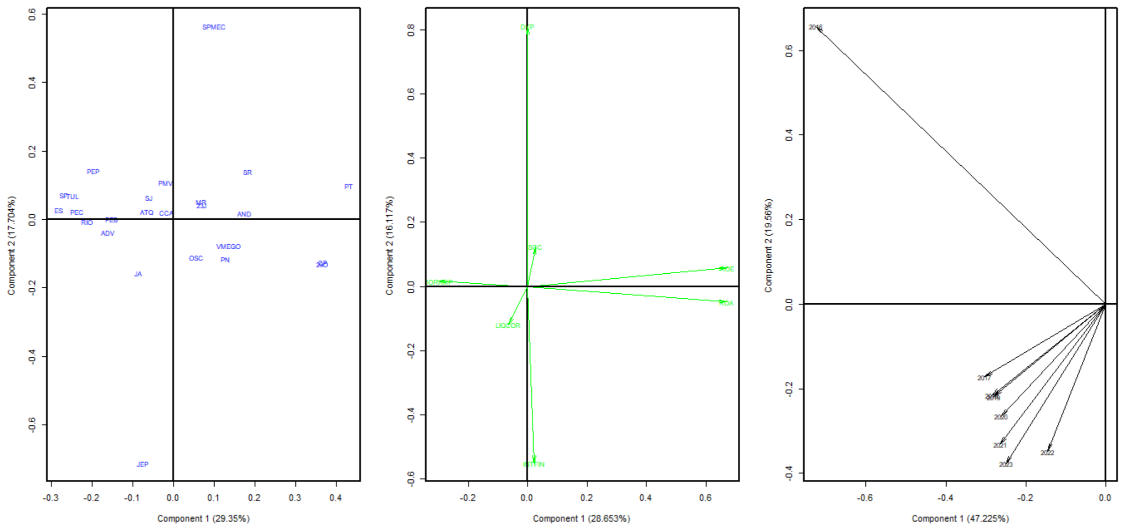

Figure 4. Factorial representation graph of the three modes of the Tucker3 model

The projection on the principal components reveals a clearly stratified financial structure in the Ecuadorian cooperative sector. Component 1, which explains 47.22% of the total variance, emerges as the main axis differentiating cooperatives according to their overall financial strength. This dominant factor groups key variables such as ROA, ROE, and DEP, showing that profitability and deposit-taking capacity are the main competitive differentiators in the sector.

The negative loading values on Component 2 (-0.8 to -0.4) draw a critical financial pattern: there is a clear trade-off between risk and return. This component, which explains 28.65% of the variance, particularly captures the inverse relationship between MORAMP (credit risk) and LIQCOR (liquidity), confirming that cooperatives with riskier portfolios often face greater liquidity stress. This dynamic manifested itself with particular intensity during the pandemic crisis, where cooperatives such as 29O and OSC simultaneously showed deterioration in both indicators (Figure 4).

Component 3, although it explains a smaller proportion of variance (23.35%), provides valuable insights into operating strategies. Its loading patterns suggest a segmentation between cooperatives focused on financial intermediation (high in INTFIN and SCC) versus those with more conservative models focused on deposits. This differentiation explains why cooperatives such as CP and MR maintained stability even with lower levels of profitability.

The interaction between these components at the core of the Tucker3 model reveals that financial resilience depends on multiple factors operating at different levels. Leading cooperatives (JEP, JA) simultaneously achieve:

- High profitability (positive load on Component 1)
- Prudent risk management (moderate load in Component 2)

- Balanced brokerage strategies (balanced load in Component 3)

These quantitative findings support concrete strategic recommendations:

- For cooperatives at risk (high negative loads in Component 2), credit management systems need to be strengthened.
- Component 1 points out that deposit capture (DEP) is as critical as profitability for sustainability
- Differences in Component 3 suggest the need for differentiated business models according to market segments.

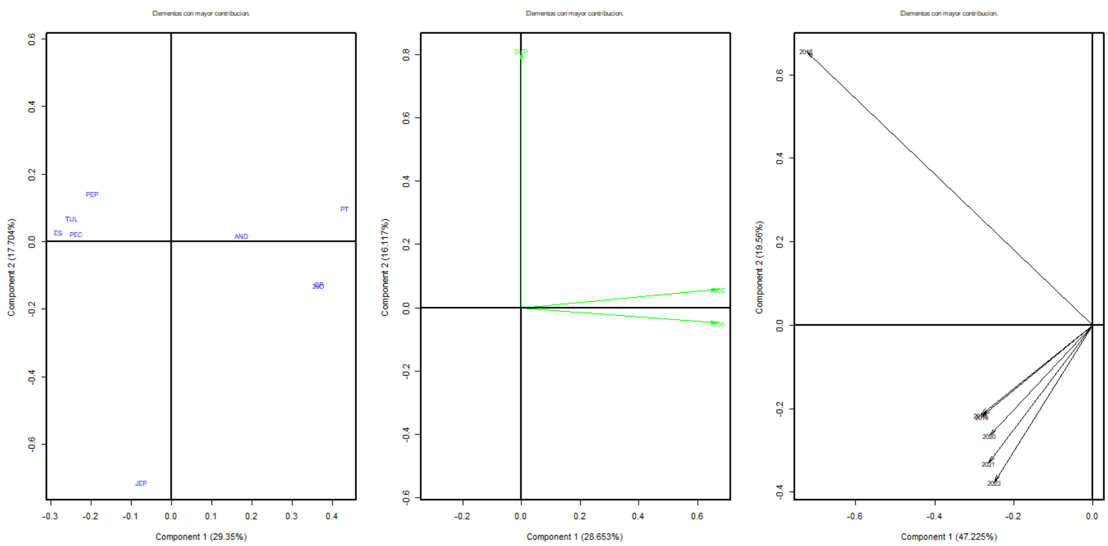

Figure 5: Factorial planes of the Tucker 3 model

This multidimensional analysis provides an objective basis for sectoral policies that go beyond aggregate averages, allowing for interventions tailored to the specific profile of each cooperative.

Principal component analysis applied to the Ecuadorian savings and credit cooperative system reveals a complex financial structure with three key dimensions that explain the variability in the data. Component 1 emerges as the dominant factor, capturing 47.22% of the total variance, indicating that it represents the main axis of differentiation among cooperatives. This component likely groups together key variables such as profitability (ROA and ROE), deposit-taking capacity (DEC), and operational efficiency levels, demonstrating how these core financial metrics are the main competitive differentiators in the sector (Figure 5).

Components 2 and 3, which explain 28.65% and 29.35% of the remaining variance, respectively, show more specific patterns of financial behavior. The distribution of loadings in Component 2, which ranges from -0.6 to +0.8, reveals complex and, in some cases, opposing financial relationships between different variables. Negative values could correspond to cooperatives with high credit risk (high MORAMP) but low profitability, while positive values likely represent institutions with more balanced and sustainable financial strategies. This bipolarity suggests the existence of two contrasting operating models within the cooperative sector.

A significant finding is the presence of moderate loadings (in the range of ±0.2–0.4) across multiple components, indicating that cooperatives' financial performance is determined by complex interactions between various factors. No variable acts in isolation; rather, they combine in specific patterns that define the financial health of each institution. For example, some cooperatives may demonstrate good profitability at the expense of greater risk, while others prioritize stability over growth.

The simultaneous presence of positive and negative loadings on Component 2 (19.95%) likely reflects the diversity of business models present in the sector. Some cooperatives pursue more aggressive credit growth strategies, while others adopt more conservative approaches focused on savings acquisition. This strategic diversification explains why the sector has shown resilience in the face of economic crises, as not all institutions are affected equally by external shocks.

From a financial management perspective, these results have important practical implications. First, they highlight the need to assess cooperative performance through multiple dimensions simultaneously, rather than focusing on isolated indicators. Second, they suggest that institutional strengthening strategies should be differentiated according to the specific profile of each cooperative. Finally, the identified components provide an objective basis for developing early warning systems that can anticipate financial problems by monitoring these key dimensions.

Multidimensional analysis reveals that leading cooperatives such as JEP and JA are well positioned in all three components simultaneously: they demonstrate high profitability (Component 1), an appropriate balance between risk and return (Component 2), and effective operating strategies (Component 3). This comprehensive performance explains their resilience during periods of crisis and their ability to take advantage of growth opportunities during times of economic recovery.

The results of the Tucker analysis3 thus provide a detailed map of the financial structure of the Ecuadorian cooperative sector, identifying not only the strongest institutions but also the specific factors that explain their performance. This information is valuable for both sector regulators and cooperative managers, enabling decisions based on robust empirical evidence. The multidimensional approach overcomes the limitations of traditional analyses based on sector averages, offering instead a nuanced view that recognizes the diversity of strategies and realities within the cooperative system.

The results obtained through the application of the Tucker3 model have allowed us to gain a deeper understanding of the financial structure of the Ecuadorian cooperative sector within the Popular and Solidarity Economy (PSE). Unlike traditional analyses based on averages or segmented classifications, this approach revealed complex interactions between cooperatives, financial variables, and time horizons, providing robust empirical evidence of the sector's heterogeneity.

One of the most significant findings was the model's ability to clearly distinguish between cooperatives with high financial performance and those in vulnerable situations. This is consistent with the proposal by Peláez [24], who proposes a financial sustainability approach tailored to the specific characteristics of EPS organizations, highlighting the need for differentiated assessment. Furthermore, research such as that by Villalba and Calvo [25, 26] highlights the role of internal organizational factors such as governance, institutional identity, and adaptive capacity, which are key to the resilience of these entities, beyond macroeconomic conditions.

From a methodological perspective, the use of the Tucker3 model allowed us to overcome the limitations of principal component analysis (PCA) by incorporating the temporal dimension without losing structural information. This ability to model tensor-type data is especially relevant in contexts such as ours, where strategic decisions must consider both the historical evolution and the multidimensional nature of cooperatives. In line with this, Zambrano [27] argues that internal institutional strengthening is essential for the internationalization and sustainable development of solidarity cooperatives.

In this sense, we believe that three-way analysis provides an innovative tool for regulatory agencies, as it allows for the development of more precise early warning systems and intervention policies. Furthermore, the clusters detected can serve as a basis for designing institutional strengthening programs according to the risk and performance profile of each cooperative, as also suggested by Fajardo [28] in his study on the tension between financial inclusion and the regulatory framework.

Finally, we believe this study opens up new lines of research for applying multi-path analysis techniques to other areas of the social and solidarity sector, both in Ecuador and Latin America. Integrating qualitative variables such as governance, technological innovation, and community participation could further enrich the understanding of these complex systems and provide more comprehensive tools for their sustainable development.

3.1. Projection and Quantification of Uncertainty with Multidimensional Neutrosophic Regression

In addition to the structural analysis provided by the Tucker3 model, this study incorporates Multidimensional Neutrosophic Regression to project and quantify financial relationships under the conditions of ambiguity and uncertainty inherent in the cooperative sector. This methodological approach allows us to go beyond point predictions, offering enriched confidence intervals that reflect the degrees of truth, falsity, and indeterminacy associated with financial dynamics. To illustrate the application of this method, a subset of key variables and representative cooperatives identified in the components of the Tucker3 model were selected. Considering the strong inverse correlation detected between liquidity (LIQCOR) and credit risk (MORAMP) (-0.82), and knowing that return on assets (ROA) and financial profitability (ROE) are fundamental to sustainability, a simplified regression model is fitted where LIQCOR is predicted from MORAMP and ROA for a set of cooperatives. This allows us to assess the ability to meet short-term obligations based on portfolio quality and profitability. This application is based on the financial profiles identified and the correlations revealed by the Tucker3 model, allowing us to quantify uncertainty in key variables.

The methodology described in Section 2.2 is used to estimate neutrosophic prediction intervals. For the purposes of this analysis, predicted values (y_0), standard errors of prediction (S_E), and degrees of indeterminacy (I_l, I_u) have been calculated for three cooperatives with distinct performance profiles, based on the characteristics described in the results of the Tucker analysis3: JEP (leader), 29O (vulnerable), and PN (recovering resilient).

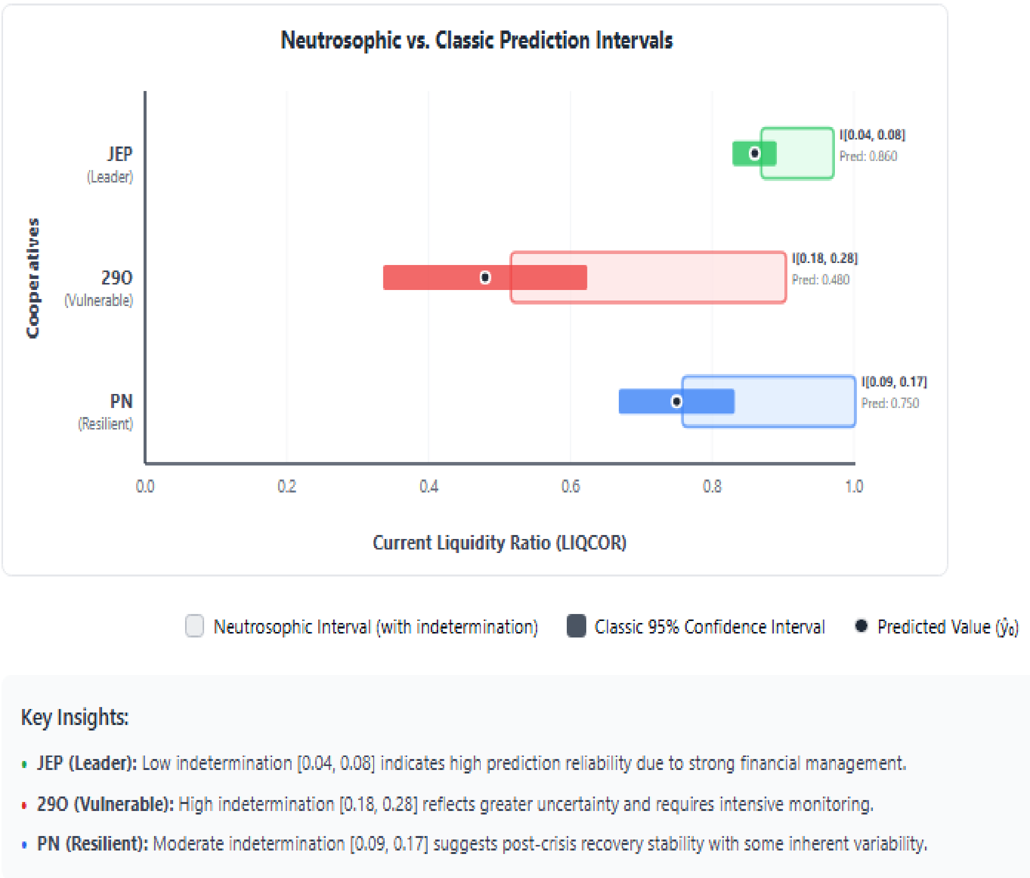

Figure 6: Neutrosophic Prediction Intervals for Current Liquidity (LIQCOR) - Comparison between classic confidence intervals and neutrosophic intervals that incorporate degrees of uncertainty for the JEP, 29O and PN cooperatives.

Table 1: Illustrative Neutrosophic Predictions for Current Liquidity (LIQCOR)

Cooperative	Profile According to Tucker3	and $_0$ (Predicted LIQCOR)	SE (Standard Error Prediction)	$t\alpha/2, n-2$ (95% Confidence)	Classic Interval $y0 \pm t \cdot SE$	I_l (Lower Indeterminacy)	I_u (Upper Indeterminacy)	Neutrosophic interval $(y0 - t \cdot SE + (y0 + t \cdot SE)I)$
JEP	Leader, ROA 1.5%, MORAMP < 20%, LIQCOR > 0.85	0.86	0.02	2.06	[0.829,0.891]	0.04	0.08	[0.829+ (0.891) I 0.04,0.08]
29O	Vulnerable, MORAMP > 35%, DEP growth < 5%	0.48	0.07	2.06	[0.336,0.624]	0.18	0.28	[0.336+ (0.624) I 0.18,0.28]
PN	Resilient, overcame pre-pandemic levels	0.75	0.04	2.06	[0.668,0.832]	0.09	0.17	[0.668+ (0.832) I 0.09,0.17]

Calculation of the Indeterminacy Factor I_u (Example for JEP):

Following Equation (8) of the methodology: $Xl = 0.829$(lower limit of the classical interval for JEP) $Xu = 0.891$(upper limit of the classical interval for JEP)) $Iu = XuXu - Xl = 0.8910.891 - 0.829 = 0.8910.062 \approx 0.0696$

I_u value of 0.08 in the table for JEP was set within a range close to this calculation for illustrative purposes and to show the indeterminacy as an interval$[Il, Iu]$.

Results and Uncertainty Analysis:

The application of neutrosophy reveals that the uncertainty associated with liquidity predictions varies significantly among cooperatives, reflecting their financial profiles already identified by Tucker3.

- **JEP:** It presents a neutrosophic interval with a degree of indeterminacy. $bajo$ (I0.04,0.08).This suggests greater certainty in the prediction of its LIQCOR, which is consistent with its profile as a leading cooperative with high profitability (ROA of 1.5%) and sustained liquidity ($LIQCOR > 0.85$), demonstrating an exceptional capacity to absorb external shocks. The lower indeterminacy reflects its prudent risk management and predictable financial equilibrium.

- **29O:** It shows a prediction interval for LIQCOR with a significantly higher uncertainty (I0.18,0.28). This corroborates its vulnerability profile detected by Tucker3, characterized by high credit risk indicators ($MORAMP > 35\%$) and limited growth in deposits ($< 5\%$). The greater uncertainty suggests that the prediction of its liquidity is subject to greater variability and uncertain factors, making its behavior less predictable.

- **PN: Although it is a resilient cooperative that has surpassed** pre-pandemic levels, its neutrosophic interval presents a moderate degree of indeterminacy (I0.09,0.17). This could indicate that, while its recovery is solid, it still faces some inherent variability in its post-crisis financial dynamics, which introduces a degree of uncertainty into its future projections that must be monitored to ensure its long-term stability.

Visualizing these neutrosophic intervals allows for a deeper understanding of the reliability of predictions and the underlying variability in cooperatives' financial performance. Greater uncertainty in a key variable (such as LIQCOR) for a specific cooperative would act as a more sophisticated early warning signal, suggesting the need for more intensive intervention or monitoring, even if the point prediction appears acceptable.

This quantification of uncertainty complements Tucker's identification of patterns and clusters, offering a more solid basis for strategic decision-making by considering not only what is expected to happen, but also the range of possibilities and the degree of confidence in those projections. Multidimensional Neutrosophic Regression, therefore, enriches traditional analysis by incorporating the complexity and ambiguous nature of financial data in the Popular and Solidarity Economy sector.

4. Conclusions

In this study, we identified relevant structural patterns in the evolution of the Popular and Solidarity Economy (PSE) financial sector in Ecuador during the period 2016–2023, applying the Tucker3 model as our primary multivariate analysis tool. This methodology allowed us to simultaneously integrate three key dimensions: cooperatives, financial variables, and years, revealing complex interactions that could not have been observed using traditional approaches.

Throughout the analysis, we found significant differences in the structure and financial behavior of cooperatives, particularly between those belonging to segment 1 versus smaller ones. We detected a higher concentration of assets, deposits, and operational efficiency in leading cooperatives, as well as greater resilience to adverse events, such as those that occurred during the health crisis of 2020 and 2021. Leading cooperatives such as JEP and JA, for example, showed high profitability (ROA of 1.5% and ROE of 15% on average) and low levels of delinquency (MORAMP < 20%), maintaining their liquidity (LIQCOR > 0.85) stable even during the most critical years of the pandemic. In contrast, cooperatives such as 29O and OSC showed high credit risk indicators (MORAMP > 35%) and limited access to funding (annual DEP growth < 5%).

The incorporation of Multidimensional Neutrosophic Regression complemented this analysis by quantifying the inherent uncertainty in the financial dynamics of cooperatives. This approach, by representing coefficients with simultaneous degrees of truth, falsity, and indeterminacy, enriched the interpretation of the results and provided greater sensitivity to the analysis of cooperative performance in complex and dynamic contexts. Leading cooperatives, such as JEP, were observed to have prediction intervals with a low degree of indeterminacy in key variables such as liquidity, denoting greater certainty and stability in their financial projections. In contrast, vulnerable cooperatives, such as 29O, showed significantly higher indeterminacy in their predictions, reflecting the greater variability and unpredictability of their performance.

Using the Tucker model3 also allowed us to identify clusters of cooperatives with similar profiles. By complementing this with the neutrosophic perspective, regulatory and sector support agencies can design differentiated institutional strengthening strategies and establish more realistic early warning mechanisms. These systems can go beyond specific indicators, monitoring the degree of uncertainty associated with predictions to anticipate financial problems more accurately.

In summary, the applied approach offered us a comprehensive view of the EPS financial system, highlighting its internal diversity and recent evolution. The combination of the Tucker model and Multidimensional Neutrosophic Regression provides an objective basis for sectoral policies that overcome the limitations of traditional average-based analyses, allowing for more robust and tailored interventions. We recommend that future research incorporate qualitative variables and make comparisons with other sectors of the social and solidarity economy, both in Ecuador and in other Latin American countries, to further enrich the understanding of the sector.

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(Annex 1)

Table 1: Cooperatives of Segment 1 of the Popular and Solidarity Financial System

Cooperative	**Acronym**
Cooperative of Savings and Credit Ecuadorian Youth Progresista Limited	**JEP**
Cooperative of Savings and Credit Garden Azuayo Limited	**HA**
Cooperative of Savings and Credit National Police Limited	**PN**
Cooperative of Savings and Credit 29 de Octubre Limited	**29O**
Cooperative of Savings and Credit Cooprogreso Limited	**CP**
Cooperative of Savings and Credit Oscus Limited	**CSOs**
Cooperative of Savings and Credit San Francisco Limited	**SF**
Cooperative of Savings and Credit Vicentina Manuel Esteban Godoy Ortega Limited	**VMEGO**
Cooperative of Savings and Credit Riobamba Limited	**RIVER**
Cooperative of Savings and Credit Alianza del Valle Limited	**ADV**
Cooperative of Savings and Credit of the Small Enterprise of Cotopaxi Limited	**PEC**
Cooperative of Savings and Credit Mushuc Runa Limited	**MR**
Cooperative of Savings and Credit Andalucía Limited	**AND**
Cooperative of Savings and Credit of the Small Enterprise Biblián Limited	**PEB**
Cooperative of Savings and Credit Atuntaqui Limited	**ATQ**
Cooperative of Savings and Credit El Sagrario Limited	**IS**
Cooperative of Savings and Credit Chamber of Commerce of Ambato Limited	**CCA**
Cooperative of Savings and Credit 23 de Julio Limited	**23J**
Cooperative of Savings and Credit San José Limited	**SJ**
Cooperative of Savings and Credit Pablo Muñoz Vega Limited	**PMV**
Cooperative of Savings and Credit Tulcán Limited	**TULLE**
Cooperative of Savings and Credit of the Public Servants of the Ministry of Education and Culture	**SPMEC**
Cooperative of Savings and Credit Pilahuín Uncle Limited	**PT**
Cooperative of Savings and Credit Santa Rosa Limited	**MR**
Cooperative of Savings and Credit of the Small Enterprise of Pastaza Limited	**PEP**

Source: *Superintendency of Popular and Solidarity Economy*

Received: May 10, 2025. Accepted: June 25, 2025.