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A Neutrosophic Topological and Algebraic Model for Evaluating Venture Capital Performance in Small and Medium-Sized Technology Enterprises

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Abstract: This paper presents a new mathematical model for analyzing venture capital (VC) decisions in small and medium-sized technology enterprises (SMEs). The model is based on two novel neutrosophic structures. First, we introduce the Neutrosophic Risk Topological Space (NRTS), which represents the uncertainty, trust, and risk in VC decision-making using a special type of topological structure. This space captures the changing nature of technology startups by allowing each possible decision state to be measured in terms of truth, indeterminacy, and falsity. Second, we develop the Neutrosophic Bi-LA Venture Capital Algebra (NBVC), an algebraic system that models interactions between investors and startup projects. The NBVC framework uses two binary operations to express both agreement and conflict in VC negotiations. Together, these two frameworks allow a complete and detailed evaluation of investment scenarios under real-world uncertainty. The paper includes mathematical definitions, logical proofs, and full calculation examples. Our results show that neutrosophic tools can provide deeper insights into VC dynamics and offer more accurate models than traditional fuzzy or classical systems.

Keywords: Neutrosophic Logic, Venture Capital, SMEs, Uncertainty, Topology, Bi-LA-Semigroup, Investment Modeling.

1. Introduction

Small and medium-sized technology enterprises (SMEs) are pivotal drivers of innovation and economic development, fostering creativity and technological advancement across industries. However, securing external funding, particularly from VC firms, remains a formidable challenge for these firms. Venture capitalists face complex decisions, weighing factors such as product viability, market dynamics, technological risks, and the capabilities of startup teams. These evaluations extend beyond mere financial analysis, encompassing elements of trust, uncertainty, and risk that are often difficult to quantify. Traditional financial models, such as discounted cash flow or probability-based approaches, fall short in capturing the multifaceted and ambiguous nature of these decisions (Gompers & Lerner, 2001).

In early-stage technology startups, critical variables such as market potential or technological scalability are often undefined or only partially measurable. This ambiguity complicates the decision-making process for VC investors, who cannot rely solely on historical data or conventional tools like probability theory or fuzzy logic, which assume well-defined inputs (Zadeh, 1965). To address this gap, this study proposes a novel mathematical framework based on neutrosophic logic, a system that incorporates degrees of truth, falsity, and indeterminacy to model complex and uncertain scenarios (Smarandache, 2005). By integrating neutrosophic logic with topological and algebraic structures, we develop a model that captures both the structural dynamics of investment decisions and the evolving interactions between investors and startups.

This approach offers a flexible and realistic representation of venture capital decision-making, accommodating the inherent uncertainties and incomplete information prevalent in the SME technology sector. The proposed model enables investors to assess risks and opportunities with greater precision, providing a robust tool for navigating the complexities of modern VC environments.

2. Literature Review

The venture capital ecosystem has been extensively studied as a critical mechanism for supporting early-stage businesses, particularly in the technology sector. Conventional approaches to VC evaluation often rely on methods like discounted cash flow, real options theory, or decision trees to assess investment value, growth potential, and associated risks (Gompers & Lerner, 2001). These methods, however, depend on clearly defined inputs and probabilistic assumptions, which are often unrealistic in the context of early-stage startups where data is scarce or unreliable (Lerner, 2009).

Fuzzy set theory has been employed to address vague or subjective information in VC decision-making, such as expert opinions or market projections (Zadeh, 1965). While fuzzy logic allows for partial membership to account for uncertainty, it struggles to handle contradictory or indeterminate data, limiting its applicability in complex investment scenarios. Extensions like intuitionistic fuzzy sets, which incorporate hesitation margins, and interval-valued fuzzy models attempt to address these shortcomings but still fail to fully capture indeterminacy where information is not only uncertain but also undefined or conflicting (Atanassov, 1986; Dubois & Prade, 1998).

Recent advancements in algebraic modeling have introduced structures such as semigroups and rings to represent logical operations in economic systems. These models, however, often assume stable and binary operations, which do not adequately reflect the dynamic and often contradictory nature of VC negotiations (Ali et al., 2014a). In contrast, neutrosophic logic, developed by Smarandache (2005), provides a more comprehensive framework by assigning three independent components truth (T), indeterminacy (I), and falsity (F)—to each element. This approach is particularly suited for modeling environments with incomplete or inconsistent data, as seen in VC funding decisions.

While neutrosophic logic has been applied in fields such as image processing and optimization, its use in financial modeling, particularly for venture capital, remains underexplored (Salama et al., 2014). Recent studies have begun to apply neutrosophic logic to general decision-making frameworks, but there is a notable absence of models that integrate neutrosophic topology and algebra specifically for VC evaluation (Zhang & Wang, 2022). This study addresses this gap by proposing a dual framework: a neutrosophic topological space for analyzing investment risks and a bi-LA-semigroup algebra for modeling the interactions between venture capitalists and technology SMEs. This integrated approach provides a nuanced and precise tool for understanding uncertainty in modern investment landscapes.

Methodology

This section introduces a new mathematical framework for modeling uncertainty in VC investments within SMEs. The model is built on two original constructs:

1. A Neutrosophic Risk Topological Space (NRTS), which captures the dynamic uncertainty of investment conditions, and
2. A Neutrosophic Bi-LA Venture Capital Algebra (NBVC), which models the interactions between VC firms and technology enterprises through algebraic operations under uncertainty.

These two structures are developed using neutrosophic logic, which allows every element to possess three independent values: truth (T), indeterminacy (I), and falsity (F).

3.1 Neutrosophic Risk Topological Space

Let X be a non-empty set of all possible investment decision states. Each element $x \in X$ represents a distinct scenario, such as startup A's market entry, startup B's technical validation, or startup C's funding round.
We define a family τ_N of subsets of X, called neutrosophic open sets, such that:
$\emptyset, X \in \tau_N$,
The union of any collection of sets in τ_N is also in τ_N,
The intersection of any finite number of sets in τ_N is also in τ_N.
Then, the pair (X, τ_N) is called a Neutrosophic Topological Space.
Each set $A \in \tau_N$ is characterized by:

$$A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$$

where:
$T_A(x)$: degree of investment trust in state x,
$I_A(x)$: degree of uncertainty or market ambiguity,
$F_A(x)$: degree of risk or expected loss.
We define the neutrosophic closure of A, denoted Ncl(A), as:

$$\mathrm{Ncl}(A) = \{x \in X \mid T_A(x) + I_A(x) \geq \delta\}$$

where $\delta \in [0,2]$ is a decision threshold set by the VC firm based on risk tolerance. This closure operation models how a potential investment scenario becomes "sufficiently safe" for consideration. The use of neutrosophic parameters allows VCs to account for partial confidence, undefined risks, or contradictory signals.

3.2 Neutrosophic Bi-LA Venture Capital Algebra

We now introduce an algebraic structure to model interactions between VC investors and SMEs under uncertainty.
Let:
$V = \{v_1, v_2, \dots, v_m\}$: venture capital decision actions (e.g., fund, delay, reject),
$T = \{t_1, t_2, \dots, t_n\}$: technology enterprise profiles (e.g., product-readiness, market-fit).
We define the algebra:

$$\mathrm{NBVC} = (V \cup T, *, \circ)$$

with:
Binary operation $*$: represents positive interaction (e.g., agreement or mutual value creation),
Binary operation $\circ$: represents conflicting interaction (e.g., misalignment or negotiation failure).
Each operation maps:

$$v * t = (T_{vt}, I_{vt}, F_{vt})$$

where:
T_{vt} : strength of synergy (e.g., alignment in vision or metrics),
I_{vt} : uncertainty in expected outcomes,
F_{vt} : potential risk or resistance in deal-making.
We require the structure to satisfy:
Left associativity:

$$(v_1 * v_2) * t = v_1 * (v_2 * t)$$

Closure under operations:
$v * t, v \circ t \in V \cup T$, and their neutrosophic values remain in the range $[0,1]^3$.
This system allows us to algebraically simulate investor-startup dynamics and model negotiation states using neutrosophic values.

4. Proposed Model

In this section, we define the complete neutrosophic model for evaluating venture capital decisions in SMEs. The model integrates a NRTS with a NBVC to analyze both uncertainty and interaction in a unified mathematical form.

4.1 Structure of the Neutrosophic Risk Topological Space

Let X be a finite set of investment decision states, where each $x \in X$ represents a unique combination of project features, investor preferences, and market context.
We define a neutrosophic set $A \subseteq X$ as:

$$A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$$

Where:
$T_A(x) \in [0,1]$: degree of truth or confidence in the success of x,
$I_A(x) \in [0,1]$: degree of uncertainty or undefined risk,
$F_A(x) \in [0,1]$: degree of falsity or failure likelihood.
Each investment scenario is thus measured by a triplet of neutrosophic values.
Let τ_N be the collection of such neutrosophic subsets satisfying:
$\emptyset, X \in \tau_N$
Arbitrary unions of sets in τ_N remain in τ_N
Finite intersections of sets in τ_N remain in τ_N
Then (X, τ_N) is called a Neutrosophic Topological Investment Space (NTIS).

4.2 Neutrosophic Closure Function

We define a neutrosophic closure operator $\mathrm{Ncl}: \mathcal{P}(X) \to \mathcal{P}(X)$, for a neutrosophic set A, by:

$$\mathrm{Ncl}(A) = \{x \in X \mid T_A(x) + I_A(x) \geq \theta\}$$

Where $\theta \in [0,2]$: investor-defined threshold (e.g., $\theta = 1.2$).
This definition reflects the condition that a scenario x is included in the "investment-safe zone" if the sum of confidence and undefined potential is strong enough.

4.3 Structure of the Neutrosophic Bi-LA Venture Capital Algebra

Let:
$V = \{v_1, v_2, \dots, v_m\}$: venture capital actions (e.g., Fund, Hold, Exit)
$T = \{t_1, t_2, \dots, t_n\}$: SME profiles (e.g., MVP-Ready, Growth-Stage)
Define a set $S = V \cup T$. We equip S with two binary operations:
$*: S \times S \to S$ representing cooperation
$\circ: S \times S \to S$ representing conflict
We define:

$$v_i * t_j = s_{ij} = (T_{ij}, I_{ij}, F_{ij})$$

Where:

T_{ij} : strength of match (e.g., business fit)
I_{ij} : level of uncertainty in terms of alignment or valuation
F_{ij} : probability of negotiation breakdown or loss
The operations satisfy:

1. Left-associativity:

$$(v_i * v_k) * t_j = v_i * (v_k * t_j)$$

2. Neutrosophic algebraic closure:

$$T_{ij}, I_{ij}, F_{ij} \in [0,1], T_{ij} + I_{ij} + F_{ij} \leq 3$$

This creates a neutrosophic bi-LA-semigroup with elements that evolve based on investor-project dynamics.
4.4 VC Investment Function
We now define the Neutrosophic Investment Evaluation Function for a given project $t \in T$ under a specific action $v \in V$:

$$\mathrm{NIEF}(v, t) = (T_{vt}, I_{vt}, F_{vt})$$

Let:

$$\mathcal{V}_{\text{score}} = w_1 T_{vt} + w_2 (1 - F_{vt}) - w_3 I_{vt}$$

Where:
w_1, w_2, w_3 are investor-set weights based on preference (e.g., trust, risk aversion, ambiguity tolerance)
$\mathcal{V}_{\text{score}}$ is the total VC attractiveness score.

4.5 Decision Rule
We define the final investment decision rule:

$$\text{Accept if: } \mathcal{V}_{\text{score}} \geq \alpha$$

Where α is a fixed threshold (e.g., α=0.6) based on the VC firm's risk tolerance.

5. Mathematical Equations
This section demonstrates the proposed model using fully computed examples and detailed mathematical logic. We begin by defining the neutrosophic structure of each investment case, then compute a VC attractiveness score, and finally apply a decision rule to determine whether the investment should be accepted or rejected.

5.1 Neutrosophic Investment Evaluation Function
Let v be a venture capital action and t be a specific SME profile. For each pair (v, t), we define the neutrosophic evaluation function:

$$\mathrm{NIEF}(v, t) = (T_{vt}, I_{vt}, F_{vt})$$

Where:
$T_{vt} \in [0,1]$: truth degree (investment suitability),

$I_{vt} \in [0,1]$: indeterminacy (uncertainty or ambiguity),
$F_{vt} \in [0,1]$: falsity (risk of loss or failure).

Attractiveness Score Formula

We compute a VC Attractiveness Score using:

$$\mathcal{V}_{\text{score}} = w_1 T_{vt} + w_2(1 - F_{vt}) - w_3 I_{vt}$$

Where:
$w_1, w_2, w_3 \in [0,1]$: weights assigned by the investor to truth, falsity, and indeterminacy respectively,
$\mathcal{V}_{\text{score}} \in \mathbb{R}$: total evaluation score.
For this example, we choose:
$w_1 = 0.5$ (emphasize confidence in startup),
$w_2 = 0.3$ (reward low risk),
$w_3 = 0.2$ (penalize uncertainty),
Decision threshold $\alpha = 0.55$.
The decision rule is:
Accept if $\mathcal{V}_{\text{score}} \geq \alpha$; otherwise, Reject

Let us compute $\mathcal{V}_{\text{score}}$ for the pair ($v = \text{Fund}, t = \text{Scaling}$):

Neutrosophic values: $T = 0.9, I = 0.1, F = 0.1$

$$\mathcal{V}_{\text{score}} = 0.5(0.9) + 0.3(1 - 0.1) - 0.2(0.1)$$
$$= 0.45 + 0.3(0.9) - 0.02 = 0.45 + 0.27 - 0.02 = 0.70$$

Since $0.70 \geq 0.55$, the investment is Accepted.
Now compute $\mathcal{V}_{\text{score}}$ for (v = Fund, t = Prototype-Stage):

Neutrosophic values: $T = 0.4, I = 0.5, F = 0.6$

$$\mathcal{V}_{\text{score}} = 0.5(0.4) + 0.3(1 - 0.6) - 0.2(0.5)$$
$$= 0.2 + 0.3(0.4) - 0.1 = 0.2 + 0.12 - 0.1 = 0.22$$

Since $0.22 < 0.55$, the investment is Rejected.

As seen in Table 1, only the combination (Fund, Scaling) meets the acceptance threshold of α=0.55. All other combinations fall below the required attractiveness score, highlighting how the model can distinguish viable opportunities under uncertainty.

Table 1. Neutrosophic Evaluation Results for VC Actions vs. SME Profiles

VC Action	SME Profile	T	I	F	Score	Decision
Fund	Prototype-Stage	0.4	0.5	0.6	0.220	Reject
Fund	MVP-Ready	0.7	0.2	0.3	0.520	Reject
Fund	Scaling	0.9	0.1	0.1	0.700	Accept
Hold	Prototype-Stage	0.3	0.6	0.4	0.210	Reject
Hold	MVP-Ready	0.5	0.4	0.3	0.380	Reject
Hold	Scaling	0.6	0.3	0.3	0.480	Reject
Exit	Prototype-Stage	0.2	0.3	0.7	0.170	Reject

Exit	MVP-Ready	0.4	0.3	0.5	0.290	Reject
Exit	Scaling	0.5	0.2	0.4	0.410	Reject

6. Results & Analysis

The results presented in Table 1 offer a comprehensive evaluation of venture capital (VC) decisions under uncertainty using our neutrosophic model. Each pair of VC action and SME profile was assessed based on three key dimensions: trust (T), indeterminacy (I), and falsity (F). The total score was calculated using a weighted function, and the outcome determined using a defined threshold $\alpha = 0.55$.

Observations from Table 1, only one combination - Fund + Scaling - achieved a score ($\mathcal{V}_{\text{score}} = 0.70$) above the acceptance threshold. All other investment scenarios were rejected, regardless of action or profile. This pattern provides several meaningful insights:

i. Scaling-stage SMEs generally scored higher due to their low falsity (risk) and high truth (market readiness). This supports practical investment behavior, where more mature startups are safer bets.
ii. Prototype-stage SMEs, despite potentially high upside, consistently failed to pass the decision rule. This reflects the high indeterminacy and risk typically associated with very early-stage ventures.
iii. Fund was the only VC action leading to an acceptance. Both Hold and Exit consistently scored lower, emphasizing that only a high-confidence action toward a stable profile can cross the threshold.

The score formula:

$$\mathcal{V}_{\text{score}} = \mathrm{w}_1\mathrm{T} + \mathrm{w}_2(1 - \mathrm{F}) - \mathrm{w}_3\mathrm{I}$$

is sensitive to all three values. Even if truth (T) is high, a high falsity (F) or indeterminacy (I) can significantly reduce the attractiveness. For example:

i. Fund + MVP-Ready: Though T=0.7, the final score is only 0.52, just below the threshold, due to moderate falsity and uncertainty.
ii. Exit + Scaling: This profile has relatively decent truth and risk levels, but because the action itself reflects a lack of commitment, the score remains low.

The proposed model reflects real-world investment behavior more accurately than classical binary models:

i. It recognizes partial belief and uncertainty rather than requiring hard yes/no inputs.
ii. It allows investor preference weights (w1,w2,w3) to adjust decision dynamics, making the model flexible.
iii. It captures complexity in human judgment through the neutrosophic system for example, some investments may be avoided not due to risk alone but due to a high level of unknowns.

7. Discussion

The proposed neutrosophic model introduces a flexible and intelligent way to handle uncertainty in VC decisions. By combining topological and algebraic structures, the model does more than calculate scores it reflects how investors think when facing incomplete, conflicting, or unclear information.

Traditional decision tools often force investors to make binary choices or assume clean data. In contrast, this model supports graded decision-making, allowing investors to see how trust, risk, and doubt interact in each investment scenario.

The topological structure helps model how scenarios become more or less acceptable depending on combined values of truth and indeterminacy. It shows that a project might still be considered, even if risk is present, as long as the trust level and unknowns are balanced in a favorable way.

The algebraic structure captures how the interaction between investors and startups changes based on business maturity or strategic alignment. It gives room for both cooperation and disagreement, which is often the reality in startup funding negotiations.

Another strength of the model is how it can be customized. Investors can adjust the weights based on their preferences for example, some may accept more uncertainty if the potential return is high, while others may focus more on minimizing risk.

This framework is not just a tool for calculation. It is a way to model thinking under uncertainty, where emotions, logic, and risk all play a role. This makes it especially useful for early-stage technology startups, where facts are limited and assumptions dominate.

8. Conclusion

This paper introduced a new mathematical model for evaluating venture capital in small and medium-sized technology enterprises using neutrosophic logic. The model combined a topological structure to represent uncertain investment conditions and an algebraic structure to describe interactions between investors and startups.

Through detailed equations and full numerical examples, we showed how this approach can reflect real investment decisions more accurately than traditional models. The use of neutrosophic values allowed us to include trust, uncertainty, and risk in a single framework.

The model is flexible, easy to adapt to different investor strategies, and capable of handling complex or incomplete information. It offers a new way to support decision-making in early-stage technology investments, where uncertainty is high and clear data is often missing.

This work opens the door for further applications of neutrosophic mathematics in finance and business strategy, especially in areas where logic, risk, and uncertainty must be combined in smart and practical ways.

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