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A Trilevel Transportation Problem for Traffic Management in Neutrosophic Environment

Prasanta Kumar Raut[1], R. Rajalakshmi[2], Ananya Mishra[3], Surapati Pramanik[4], Mana Donganont[5*] ,Siva Prasad Behera[3]

[1]Department of Mathematics, Trident Academy of Technology, Bhubaneswar, Odisha, India, Email: prasantaraut95@gmail.com

[2]Assistant Professor, Department of Mathematics, Panimalar Engineering College, Chennai-600123.Email: rajimat2020@gmail.com

[3]Department of Mathematics, C.V. Raman Global University, Bhubaneswar-752054, Odisha, India, Email: ananyamishra4378@gmail.com , sivaiitkgp12@gmail.com

[4]Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, Narayanpur, Dist-North 24 Parganas, West Bengal, India, PIN-743126, Email: surapati.math@gmail.com

[5]Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand, Email: mana.do@up.ac.th

Corresponding Author: mana.do@up.ac.th

ABSTRACT:

Traffic congestion in urban areas presents a complex challenge due to the multi-level nature of transportation networks and the presence of uncertain, imprecise data. Existing models often fail to comprehensively address hierarchical decision-making under uncertainty. This paper fills this gap by proposing a novel interval neutrosophic goal programming model for the Trilevel Transportation Problem (TTP), which considers decision-making across three interconnected levels: highways, traffic hubs, and city streets. The proposed framework incorporates interval neutrosophic numbers to effectively represent uncertainty and indeterminacy in traffic data. We develop a structured goal programming approach to optimize traffic flow, minimize congestion, and improve travel efficiency. A detailed numerical example illustrates the applicability and superiority of the model compared to conventional methods. The results demonstrate that our model provides a more flexible and accurate solution for traffic planning under uncertain environments, offering valuable insights for urban transportation management and policy formulation.

Keywords: Interval neutrosophic numbers, Trilevel Transportation Problem, Fractional Transportation Problem.

1. Introduction

In the modern era, transportation systems play a crucial role in ensuring the efficient movement of people and goods, especially in densely populated and industrially developed regions. However, real-world transportation networks are inherently complex and are often influenced by numerous uncertain factors such as traffic congestion, infrastructure limitations, varying road conditions, and unpredictable human behaviour. These uncertainties pose significant challenges to the design and implementation of optimal transportation strategies. While classical mathematical models are effective in structured and deterministic environments, they often fall short in addressing the vagueness and indeterminacy prevalent in real-world traffic systems.

To tackle these challenges, the concept of neutrosophic sets, introduced by Smarandache [1], has emerged as a powerful mathematical framework capable of handling indeterminate, inconsistent, and incomplete information. Neutrosophic logic generalizes classical and fuzzy logic by incorporating independent degrees of truth, indeterminacy, and falsity, offering a more flexible and comprehensive structure for modelling real-life problems characterized by ambiguity and incomplete knowledge.

In recent years, neutrosophic theories have been successfully applied in various domains, including decision-making [2-16], disease prediction [17], and network optimization [18-20]. Particularly in graph theory, neutrosophic graphs have been effectively utilized to model uncertain relationships among entities [21, 22], leading to improved algorithms for solving problems such as shortest path, minimum spanning tree, and other critical network-related computations [23-29], medical diagnosis [30-32], hierarchical problems [33-36] .

The integration of neutrosophic environments [37, 38] into transportation problem modelling [39-45] is a relatively recent but rapidly developing research area. Traditionally, transportation problems have been addressed using single-level or bilevel optimization frameworks. However, the increasing complexity of urban traffic systems necessitates more realistic and hierarchical models. This demand has led to the development of bilevel transportation problem [46], trilevel transportation models [47], which consider the interactions among various decision-makers, such as urban planners, transportation authorities, and commuters.

The model proposed in this study incorporates neutrosophic theory into a hierarchical decision-making framework to optimize traffic flow under uncertain conditions more effectively.

A Trilevel Programming Problem (TLPP) [48] consists of three hierarchical decision levels: the upper, middle, and lower levels. Each level involves a distinct decision-maker focused on optimizing their own objectives. In this structure, the leader at the top level makes the initial decisions, followed by the intermediary at the middle level who optimizes their response accordingly, and finally, the follower at the bottom level adapts their choices based on the decisions made above.

The concept of TLPP originates from game theory, particularly from Stackelberg's model [49]), which focuses on hierarchical decision-making structures. While Bilevel Programming Problems (BLPPs) [50, 51] have been widely studied, the multilevel extensions [52-57] have also garnered considerable attention. Lai [58] at first proposed hierarchical optimization method and obtained a satisfactory solution using the concept of tolerance membership functions based on fuzzy set theory in 1996. In fuzzy environments several studies for BLPPs [59-67] and multi-level programming problems [68-72] have been studied. Candler and Townsley [73] introduced an enumeration method for solving linear BLPPs. Bialas and Karwan [74] extended the Kth-best method to handle more complex BLPPs.

In the context of transportation, trilevel models have shown promise in addressing multifaceted decision-making challenges. Khandelwal and Puri [75] examined time minimization and capacitated fixed-charge transportation issues in a three-level framework. More recently, Kaushal et al. [76] proposed a TLPP in which the upper level involves a fractional transportation objective, the middle level handles a fixed-charge problem, and the lower level addresses a separate transportation-related goal such as cost minimization.

The transition from two-level to three-level transportation problems enables a more comprehensive analysis of complex decision-making systems, particularly in areas such as supply chain management, logistics, and multi-echelon transportation networks. In this paper, we propose a novel Trilevel Transportation Problem (TTP) formulated within a neutrosophic environment to effectively capture the layered decision-making process involved in traffic management under uncertainty.

2. Motivation and Research Gaps

In this context, the motivation behind the present study stems from the necessity to develop a traffic management approach that can simultaneously address (i) the multi-level decision-making hierarchy involved in real transportation systems, and (ii) the uncertainty and vagueness present in traffic data. The adoption of neutrosophic sets—which allow for the representation of truth, indeterminacy, and falsity degrees independently—offers a promising direction to overcome these challenges.

The novelty of this work lies in the formulation of a Trilevel Transportation Problem (TTP) under a neutrosophic environment, where three decision-makers operate at different levels: highways (upper), traffic hubs (middle), and city roads (lower). Unlike prior works that rely on crisp or fuzzy values, the proposed model incorporates interval neutrosophic numbers within a goal programming framework, enabling a more realistic representation of uncertain parameters in traffic planning.

This study addresses a significant research gap by integrating hierarchical decision-making with neutrosophic logic—a combination not adequately explored in existing literature. It contributes a new methodology that enhances the flexibility, adaptability, and accuracy of traffic management models under uncertain conditions. Through a comprehensive numerical illustration, the effectiveness of the proposed approach is demonstrated, offering practical relevance to urban traffic planners and decision-makers.

3. Preliminaries

Definition 3.1: Let c be the generic element of C, and let C be a space of points. The definition of a neutrosophic set [1] is

$$U = \{\langle z\colon \alpha_{1_U}(c), \alpha_{2_U}(c), \alpha_{3_U}(c)\rangle c \in C\}$$

Where,

α_{1_U} = membership function of truth value

α_{2_U} = membership function of indeterminacy value

α_{3_U} = membership function of falsity

$\alpha_{1_U}, \alpha_{2_U}, \alpha_{3_U}\colon C \to]-0,1+[$ satisfy the condition,

$$-0 \le \alpha_{1_U}(c) + \alpha_{2_U}(c) + \alpha_{3_U}(c) \le 3^+$$

Definition 3.2: A notion in neutrosophic statistics, a neutrosophic number [77]is a number that has both a determinate and an indeterminate portion. It is written as

N = p + qJ.

where p= the determinate part

qJ = the indeterminate part.

4. Notations

A Neutrosophic number [77]:$b_n = b + \tilde{b}J$ where, J∈ $[J', J'']$

$P_u = \{1,2,\dots,p_1\}$ = Number of sources, leader's problem

$P_v = \{p_1+1,\dots,p_o\}$ = Number of sources, follower 1's problem

$P_w = \{p_o+1,\dots,p\}$ = Number of sources, follower 2's problem

$Q_u = \{1,2,\dots,q_1\}$ = Number of destinations, leader's problem

$Q_v = \{q_1+1,\dots,q_o\}$ = Number of destinations, follower 1's problem

$Q_w = \{q_o+1,\dots,q\}$ = Number of destinations, follower 2's problem

P = $P_u + P_v + P_w$: Total number of sources

Q = $Q_u + Q_v + Q_w$: Total number of destinations

$A_1 = a_{ef}\colon e \in P_u, f \in Q_u$ (variables controlled by the leader)

$A_2 = a_{ef}\colon e \in P_v, f \in Q_v$ (variables controlled by the follower 1)

$A_3 = a_{ef}\colon e \in P_w, f \in Q_w$ (variables controlled by the follower 2)

5. Formulation of a three-level linear fractional transportation problem incorporating neutrosophic numbers.

Mathematically, Trilevel linear fractional transportation problem with interval neutrosophic number (TLFTP-NN) are comprised of the following steps

- Step1
- Step-2
- Step-3

Step1(leader)

(highways to traffic hubs**)**

$$(R_1)\ \min_{A_1} z_1(A_1, A_2, A_3) = \frac{(d_{1}{}_{m}^{S})A_1+(d_{2}{}_{m}^{S})A_2+(d_{3}{}_{m}^{S})a_3}{(x_{1}{}_{m}^{S})A_1+(x_{2}{}_{m}^{S})A_2+(x_3)_{m}^{S})a_3} \tag{1}$$

Subject to

$$\left.\begin{aligned} \sum_{f\in Q_U} a_{ef} &\le (B_m)_e^U, \quad \forall\, e \in P_U \\ \sum_{e\in P_U} a_{ef} &\le (D_m)_f^U, \quad \forall\, f \in Q_U \end{aligned}\right\} \tag{2}$$

Step2(follower-1)

(traffic hubs to city regions**)**

$$(R_2)\ \min_{A_2} z_2(A_1, A_2, A_3) = \frac{(g_{1}{}_{m}^{S})A_1+(g_{2}{}_{m}^{S})A_2+(g_{3}{}_{m}^{S})a_3}{(y_{1}{}_{m}^{S})A_1+(y_{2}{}_{m}^{S})A_2+(y_3)_{m}^{S})a_3} \tag{3}$$

Subject to

$$\left.\begin{aligned} \sum_{f\in Q_V} a_{ef} &\le (B_m)_e^V, \quad \forall\, e \in P_V \\ \sum_{e\in P_V} a_{ef} &\le (D_m)_f^V, \quad \forall\, f \in Q_V \end{aligned}\right\} \tag{4}$$

Step-3(follower-2)

(street roads to city regions**)**

$$(R_3)\ \min_{A_3} z_3(A_1, A_2, A_3) = \frac{(h_{1}{}_{m}^{S})A_1+(h_{2}{}_{m}^{S})A_2+(h_{3}{}_{m}^{S})a_3}{(o_{1}{}_{m}^{S})A_1+(o_2)_{m}^{S})A_2+(o_3)_{m}^{S})a_3} \tag{5}$$

Subject to

$$\left.\begin{aligned} \sum_{f\in Q_W} a_{ef} &\le (B_m)_e^W, \quad \forall\, e \in P_W \\ \sum_{e\in P_W} a_{ef} &\le (D_m)_f^W, \quad \forall\, f \in Q_W \end{aligned}\right\} \tag{6}$$

Where,

$$\left.\begin{aligned} (d_1)_m &= \left[(d_m)_{ef}^U\right] \quad e \in P_U, f \in Q_U \\ (g_1)_m &= \left[(g_m)_{ef}^U\right] \quad e \in P_U, f \in Q_U \\ (h_1)_m &= \left[(h_m)_{ef}^U\right] \quad e \in P_U, f \in Q_U \end{aligned}\right\} \text{neutrosophic cost parameters of leader's problem}$$

$(d_2)_m = \left[(d_m)_{ef}^{V}\right] \quad e \in P_V, f \in Q_V$
$(g_2)_m = \left[(d_m)_{ef}^{V}\right] \quad e \in P_V, f \in Q_V$
$(h_2)_m = \left[(d_m)_{ef}^{V}\right] \quad e \in P_V, f \in Q_V$

neutrosophic cost parameters of follower-1's problem

$(d_3)_m = \left[(d_m)_{ef}^{W}\right] \quad e \in P_W, f \in Q_W$
$(g_3)_m = \left[(d_m)_{ef}^{W}\right] \quad e \in P_W, f \in Q_W$
$(h_3)_m = \left[(d_m)_{ef}^{W}\right] \quad e \in P_W, f \in Q_W$

neutrosophic cost parameters of follower-2's problem

$(x_1)_m = \left[(x_m)_{ef}^{U}\right] \quad e \in P_U, f \in Q_U$
$(y_1)_m = \left[(y_m)_{ef}^{U}\right] \quad e \in P_U, f \in Q_U$
$(o_1)_m = \left[(o_m)_{ef}^{U}\right] \quad e \in P_U, f \in Q_U$

Number of neutrosophic parameters of leader's problem

$(x_2)_m = \left[(x_m)_{ef}^{V}\right] \quad e \in P_V, f \in Q_V$
$(y_2)_m = \left[(y_m)_{ef}^{V}\right] \quad e \in P_V, f \in Q_V$
$(o_2)_m = \left[(o_m)_{ef}^{V}\right] \quad e \in P_V, f \in Q_V$

Number of neutrosophic parameters of follower-1's problem

$(x_3)_m = \left[(x_m)_{ef}^{W}\right] \quad e \in P_U, f \in Q_U$
$(y_3)_m = \left[(y_m)_{ef}^{W}\right] \quad e \in P_U, f \in Q_U$
$(o_3)_m = \left[(o_m)_{ef}^{U}\right] \quad e \in P_U, f \in Q_U$

Number of neutrosophic parameters of follower-2's problem

And $A_{ef} \geq 0 \ \forall (e, f) \ \in P \times Q \ where,$
P= $P_U \cup P_V \cup P_W, \ \ Q = Q_U \cup Q_V \cup Q_W,$
$P_u = \{1,2, \dots, p_1\}$, $P_v = \{p_1 + 1, \dots, p_o\}$, $P_w = \{p_o + 1, \dots, p\}$
$Q_u = \{1,2, \dots, q_1\}, \mathrm{Q_v} = \{\mathrm{q_1} + 1, \dots, \mathrm{q_o}\}$, $Q_w = \{q_o + 1, \dots, q\}$
Also, we have, $(A_1, A_2, A_3) \in D_1$
D= $\{(A_1, A_2, A_3) \in \alpha, A_2, A_3 \in \beta(A_1), (x_1)_m^S A_1 + (x_2)_m^S A_2) + (x_3)_m^S A_3 > 0, (y_1)_m^S A_1 + (y_2)_m^S A_2) + (y_3)_m^S A_3 > 0, \ (o_1)_m^S A_1 + (o_2)_m^S A_2) + (o_3)_m^S A_3 \ \}$
$\beta = \{(A_1, A_{2,} A_3) : (A_1, A_2, A_3) : (A_1, A_{2,} A_3)\}$ satisfy constraints in equation 2 and 4 and 6

5.1 Solution methodology for (TLFTP-NN) using goal programming.

$$(b_1)_m^S A_1 + (b_2)_m^S A_2 + (b_3)_m^S A_3 = \sum_{e=1}^{P}\sum_{f=1}^{Q}[(b_m)_{ef}]A_{ef}$$
$$= \sum_{e=1}^{P}\sum_{f=1}^{Q}[b_{ef} + \tilde{b}_{ef}J]A_{ef} \qquad where\ J \in [j', J''] \tag{7}$$

Now the problem (TLFTP-NN) simplifies to

$(R_1)\ \lim_{A_1} z_1(A_1, A_2, A_3) = \frac{\sum_{e=1}^{P}\sum_{f=1}^{Q}[d_{ef}+\tilde{d}_{ef}J]a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q}[x_{ef}+\tilde{x}_{ef}J]a_{ef}}$

Subject to

$\sum_{f\in Q_U} a_{ef} \le [B_e^U + \tilde{B}_e^U J]\ \forall e \in P_U$,

$\sum_{e\in P_U} a_{ef} \le [D_e^U + \tilde{D}_e^U J]\ \forall f \in Q_U$

Where A_2 solves

$(R_2)\ \lim_{A_2} z_2(A_1, A_2, A_3) = \frac{\sum_{e=1}^{P}\sum_{f=1}^{Q}[g_{ef}+\tilde{g}_{ef}J]a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q}[y_{ef}+\tilde{y}_{ef}J]a_{ef}}$

Subject to

$\sum_{f\in Q_V} a_{ef} \le [B_e^V + \tilde{B}_e^V J]\ \forall e \in P_V$,

$\sum_{e\in P_V} a_{ef} \le [D_e^V + \tilde{D}_e^V J]\ \forall f \in Q_V$

A_3 Solves

$(R_3)\ \lim_{A_3} z_3(A_1, A_2, A_3) = \frac{\sum_{e=1}^{P}\sum_{f=1}^{Q}[h_{ef}+\tilde{h}_{ef}J]a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q}[o_{ef}+\tilde{o}_{ef}J]a_{ef}}$

Subject to

$\sum_{f\in Q_W} a_{ef} \le [B_e^W + \tilde{B}_e^W J]\ \forall e \in P_W$,

$\sum_{e\in P_W} a_{ef} \le [D_e^W + \tilde{D}_e^W J]\ \forall f \in Q_W$

Also, $a_{ef} \ge 0\ \ \forall (e, f) \in P \times Q$. Let J$\in [J', J'']$ and using Sect. 3, we convert each NN to an interval number. So, we get

$(R_1)\ \lim_{A_1} z_1(A_1, A_2, A_3) = \frac{\sum_{e=1}^{P}\sum_{f=1}^{Q}[d'_{ef}, d''_{ef}]a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q}[x'_{ef}, x''_{ef}]a_{ef}}$

Subject to

$\sum_{f\in Q_U} a_{ef} \le [B'^U_e, B''^U_e]\ \forall e \in P_U$,

$\sum_{e\in P_U} a_{ef} \le [D''^U_e, D''^U_e]\ \forall f \in Q_U$

And A_2 solves (8)

$(R_2)\ \lim_{A_2} z_2(A_1, A_2, A_3) = \frac{\sum_{e=1}^{P}\sum_{f=1}^{Q}[g'_{ef}, g''_{ef}]a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q}[y'_{ef}, y''_{ef}]a_{ef}}$

Subject to

$\sum_{f\in Q_V} a_{ef} \le [B'^V_e, B''^V_e]\ \forall e \in P_V$,

$\sum_{e\in P_V} a_{ef} \le [D''^V_e, D''^V_e]\ \forall f \in Q_V$ (9)

And A_3 solves

$$(R_3)\ \lim_{A_3} z_3(A_1, A_2, A_3) = \frac{\sum_{e=1}^{P}\sum_{f=1}^{Q}[h'_{ef}, h''_{ef}]a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q}[o'_{ef}, o''_{ef}]a_{ef}}$$

$$\left.\begin{array}{l}\text{Subject to} \\ \sum_{f\in Q_W} a_{ef} \le \left[B'^{W}_{e}, B''^{W}_{e}\right]\ \forall e \in P_W\ , \\ \sum_{e\in P_W} a_{ef} \le \left[D''^{W}_{e}, D''^{W}_{e}\right]\ \forall f \in Q_W \\ \text{Where } a_{ef} \ge 0\ \ \forall (e,f) \in P \times Q. \end{array}\right\} \qquad (10)$$

To find the optimal solution range, we adjust coefficients within their intervals in the objective functions and constraints. This approach helps determine the best and worst optimal solutions, considering system limitations

Theorem 1.

Suppose $\sum_{f=1}^{m}\left[G_1^f, G_2^f\right]Z_f \ge [h_1, h_2]$ *then* $\sum_{f=1}^{m}\left[G_2^f\right]Z_f \ge [h_1]$ *and* $\sum_{f=1}^{m}\left[G_1^f\right]Z_f \ge [h_2]$ *represent the upper and lower limits, respectively, of the range of values that satisfy the constraint condition.*

Applying the proposition to constraints (Eqs. 6-7) with $[G_1^f, G_2^f]$ = 1 yields the maximum and minimum feasible regions for both levels, representing the best and worst optimal solutions (Shaocheng, 1994; Chinneck & Ramadan, 2000). To minimize At each level, the coefficients in the interval fractional objective function are modified accordingly. Optimization problems for the best solutions are summarized at table 1 and the worst solutions summarized in Tables 2.

Table 1. Best solution

Leader's Problem	$\lim_{a\in A} z_1(A_1,A_2,A_3)=\frac{\sum_{e=1}^{P}\sum_{f=1}^{Q} d'_{ef}a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q} x''_{ef}a_{ef}}$ s.t. $\sum_{f\in Q_U} a_{ef} \le B_e''^{U} \quad \forall e \in P_U$ $\sum_{e\in P_U} a_{ef} \ge D_f'^{U} \quad \forall f \in Q_U$
Follower-1's Problem	$\lim_{a\in A} z_2(A_1,A_2,A_3)=\frac{\sum_{e=1}^{P}\sum_{f=1}^{Q} g'_{ef}a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q} y''_{ef}a_{ef}}$ s.t. $\sum_{f\in Q_V} a_{ef} \le B_e''^{V} \quad \forall e \in P_V$ $\sum_{e\in P_V} a_{ef} \ge D_f'^{V} \quad \forall f \in Q_V$
Follower-2's Problem	$\lim_{a\in A} z_3(A_1,A_2,A_3)=\frac{\sum_{e=1}^{P}\sum_{f=1}^{Q} h'_{ef}a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q} o''_{ef}a_{ef}}$ s.t. $\sum_{f\in Q_W} a_{ef} \le B_e''^{W} \quad \forall e \in P_W$ $\sum_{e\in P_W} a_{ef} \ge D_f'^{W} \quad \forall f \in Q_W$

Table 2. Worst solution

Leader's Problem	$\lim_{a\in A} z_1(A_1,A_2,A_3)=\frac{\sum_{e=1}^{P}\sum_{f=1}^{Q} d''_{ef}a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q} x'_{ef}a_{ef}}$ s.t. $\sum_{f\in Q_U} a_{ef} \le B_e'^{U} \quad \forall e \in P_U$ $\sum_{e\in P_U} a_{ef} \ge D_f''^{U} \quad \forall f \in Q_U$

Follower-1's Problem	$$\lim_{a\in A} z_2(A_1,A_2,A_3)=\frac{\sum_{e=1}^{P}\sum_{f=1}^{Q} g''_{ef}a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q} y'_{ef}a_{ef}}$$ s.t. $$\sum_{f\in Q_V} a_{ef}\le B_e'^{V} \quad \forall e\in P_V$$ $$\sum_{e\in P_V} a_{ef}\ge D_f''^{V} \quad \forall f\in Q_V$$
Follower-2's Problem	$$\lim_{a\in A} z_3(A_1,A_2,A_3)=\frac{\sum_{e=1}^{P}\sum_{f=1}^{Q} h''_{ef}a_{ef}}{\sum_{e=1}^{P}\sum_{f=1}^{Q} o'_{ef}a_{ef}}$$ s.t. $$\sum_{f\in Q_W} a_{ef}\le B_e'^{W} \quad \forall e\in P_W$$ $$\sum_{e\in P_W} a_{ef}\ge D_f''^{W} \quad \forall f\in Q_W$$

For the mth level decision maker, m=1,2,3, let
Individual best solution is
$a_m^c=((a_m^c)_{11},(a_m^c)_{12},\dots(a_m^c)_{p_1q_1},(a_m^c)_{m_1+1,n_1+1},\dots\ (a_m^c)_{PQ})$
Individual worst solution is

$$a_m^l=((a_m^l)_{11},(a_m^l)_{12},\dots(a_m^l)_{p_1q_1},(a_m^l)_{m_1+1,n_1+1},\dots\ (a_m^l)_{PQ})$$

Let $z_m(a_m^c)$ represent the best possible outcome (individual optimal value) and $z_m(a_m^l)$ represent the worst possible outcome (individual worst value) for the objective function. Then the best possible outcomes for the m-th level decision-maker fall within the range of $[\bar{z}_m,\bar{\bar{z}}_m]$C, which is between $z_m(a_m^c)$ and $z_m(a_m^l)$.
To achieve an optimal compromise solution, a target range [Y_m^*, Y_m^{**}] is set for the m-th level decision-maker, representing a middle ground for negotiation.

$$\bar{\bar{z}}_m\ge Y_m^*$$
$$\bar{z}_m\le Y_m^{**}$$

Therefore, this goal achievement functions can be mathematically represented as:

$$-\bar{\bar{z}}_m+\bar{\bar{G}}_m=-Y_m^*, \qquad (11)$$
$$\bar{z}_m+\bar{G}_m=Y_m^{**} \qquad (12)$$

Let
$(a_1^D)_{ef}-(q_1)_{ef} and\ (a_1^D)_{ef}+(p_1)_{ef}, e\in P_U, f\in Q_U$ be the upper and lower bounds of the decision vector given by leader.
$(a_2^D)_{ef}-(q_2)_{ef} and\ (a_2^D)_{ef}+(p_2)_{ef}, e\in P_V, f\in Q_V$ be the upper and lower bounds of the decision vector given by follower-1.

$(a_3^D)_{ef} - (q_3)_{ef} \leq (a_3^D)_{ef} \leq (a_3^D)_{ef} + (p_3)_{ef}$, $e \in P_V, f \in Q_V$ be the lower and upper bounds of the decision vector given by follower-2.

Then we have

$$(a_1^D)_{ef} - (q_1)_{ef} \leq (a_1^D)_{ef} \leq (a_1^D)_{ef} + (p_1)_{ef}, e \in P_U, f \in Q_U \quad (14$$

$$(a_2^D)_{ef} - (q_2)_{ef} \leq (a_2^D)_{ef} \leq (a_2^D)_{ef} + (p_2)_{ef}, e \in P_V, f \in Q_V \quad (13)$$

$$(a_3^D)_{ef} - (q_3)_{ef} \leq (a_3^D)_{ef} \leq (a_3^D)_{ef} + (p_3)_{ef}, e \in P_W, f \in Q_W \quad (15$$

The following goal programming model is formulated to obtain a satisfactory solution:

$$Min \sum_{m=1}^{3} (\bar{G}_M + \bar{\bar{G}}_M)$$

Subject to

$$\sum_{f \in Q_U} a_{ef} \leq B_e''^U, \ \sum_{f \in Q_U} a_{ef} \leq B_e'^U \quad \forall e \in P_U,$$

$$\sum_{e \in P_U} a_{ef} \leq D_e''^U, \ \sum_{e \in P_U} a_{ef} \leq D_e'^U \quad \forall e \in Q_U,$$

$$\sum_{f \in Q_V} a_{ef} \leq B_e''^V, \ \sum_{f \in Q_V} a_{ef} \leq B_e'^V \quad \forall e \in P_V,$$

$$\sum_{e \in P_V} a_{ef} \leq D_e''^V, \ \sum_{e \in Q_V} a_{ef} \leq D_e'^V \quad \forall e \in Q_V,$$

$$\sum_{f \in Q_W} a_{ef} \leq B_e''^W, \ \sum_{f \in Q_W} a_{ef} \leq B_e'^W \quad \forall e \in P_U,$$

$$\sum_{e \in P_W} a_{ef} \leq D_e''^W, \ \sum_{e \in P_W} a_{ef} \leq D_e'^W \quad \forall e \in Q_W,$$

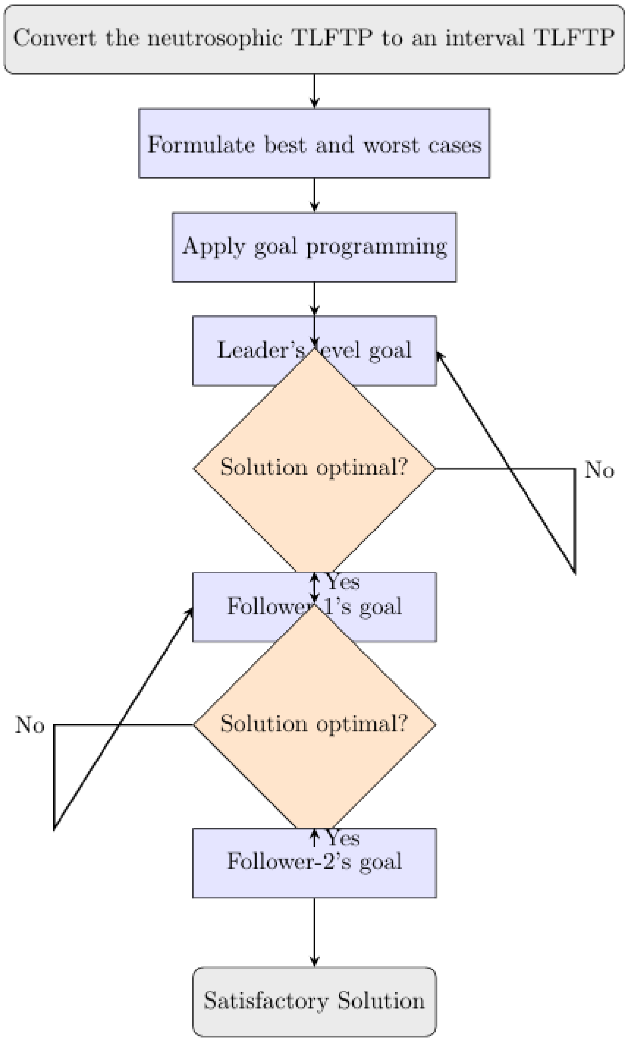
$$(a_1^D)_{ef} - (q_1)_{ef} \leq (a_1^D)_{ef} \leq (a_1^D)_{ef} + (p_1)_{ef}, e \in P_U, f \in Q_U$$

$$(a_2^D)_{ef} - (q_2)_{ef} \leq (a_2^D)_{ef} \leq (a_2^D)_{ef} + (p_2)_{ef}, e \in P_V, f \in Q_V$$

$$(a_3^D)_{ef} - (q_3)_{ef} \leq (a_3^D)_{ef} \leq (a_3^D)_{ef} + (p_3)_{ef}, e \in P_W, f \in Q_W$$

$$\bar{G}_m, \bar{\bar{G}}_m, A \geq 0, m = 1,2,3.$$

5.2 Flowchart of the Proposed Methodology



6. Numerical example of TLFTP-NN

6.1 Definition of Problem

A traffic management authority is responsible for optimizing the movement of vehicles across a city. The transportation network consists of:

Top Level (Leader - Government Authority): Controls the main highways connecting different cities and aims to minimize overall traffic congestion. Middle Level (Follower 1 - City Traffic Department): Regulates the flow of traffic from major highways to urban distribution centers while minimizing delays. Lower Level (Follower 2 - Local Authorities): Manages vehicle flow within urban areas and optimizes road usage for local traffic.

6.2 Solved numerical (TLFTP-NN)

In this scenario, three entities collaborate to optimize traffic flow:

Government Traffic Authority (Leader): Aims to minimize neutrosophic transportation costs while maximizing vehicle flow on highways connecting cities.

City Traffic Management Department (First Follower): Seeks to minimize neutrosophic transportation costs while ensuring optimal traffic distribution from highways to urban traffic hubs.

Local Traffic Control Authority (Second Follower): Focuses on minimizing traffic congestion in city regions by maximizing efficient vehicle distribution from urban hubs to different city regions (North, East, West, and South).

We consider a three- level transportation system for managing traffic in a metropolitan region;

At upper Level, (highway to traffic Hubs) (see table 3)

Four highway points serve as supply i.e.-

(A=6+5J), (B=6+4J), (C=6+7J), (D=6+3J)

Let four traffic hubs points serve as demand

(W=6+3J), (X=5+6J), (Y=7+5J), (Z=6+5J)

Table 3. Upper level neutrosophic transportation problem

Highway to traffic Hubs	W	X	Y	Z
A	2+3J 5+2J	5+6J 4+2J	7+J 2+4J	4+3J 7+3J
B	3+2J 4+2J	4+5J 2+5J	5+3J 3+4J	8+3J 4+5J
C	10+7J 6+3J	7+3J 3+5J	2+5J 5+6J	10+3J 3+6J
D	3+4J 3+7J	2+6J 4+5J	3+4J 4+3J	5+3J 6+2J

$[d_{ef} + \tilde{d}_{ef}J]$

$[x_{ef} + \tilde{x}_{ef}J]$

At follower-1 level (traffic Hubs to city regions) Each Hub must send vehicles to city regions while minimizing congestion.

So, at middle level (see Table 4) we get supply (W=6+3J), (X=5+6J), (Y=7+5J), (Z=6+5J)

Demand is (North=7+5J), (East=7+4J), (West=6+5J), (South=4+5J)

Table 4. Middle level neutrosophic transportation problem

traffic Hubs to city regions	NORTH	EAST	WEST	SOUTH
W	5+3J 6+2J	6+8J 4+2J	3+2J 5+3J	7+3J 11+2J
X	8+6J 3+3J	11+2J 4+5J	9+5J 7+4J	9+2J 2+6J
Y	7+10J 3+3J	5+5J 4+3J	7+4J 2+4J	10+3J 4+7J
Z	9+2J	5+4J	3+5J	6+5J

$[a_{ef} + \tilde{a}_{ef}J]$

$[y_{ef} + \tilde{y}_{ef}J]$

	6+2J	4+3J	7+3J	4+5J

And at lower level (City Reasons to Local Streets) (see Table 5) we obtain
Supply is (North=7+5J), (East=7+4J), (West=6+5J), (South=4+5J)
Demand is (Street -1=8+6J), (Street-2=6+3J), (Street-3=5+6J), (Street-4=5+4J)

Table 5. Lower level neutrosophic transportation problem

City Reasons to Local Streets	STREET-1	STREET-2	STREET-3	STREET-4
NORTH	2+5J 5+6J	3+2J 4+2J	3+2J 4+2J	3+5J 7+3J
EAST	3+2J 4+2J	3+2J 4+2J	9+5J 7+4J	5+4J 4+3J
WEST	4+3J 7+3J	5+6J 4+2J	5+6J 4+2J	7+3J 11+2J
SOUTH	7+3J 11+2J	7+10J 3+3J	3+4J 3+7J	3+4J 3+7J

$[h_{ef} + \tilde{h}_{ef}J$

$[o_{ef} + \tilde{o}_{ef}J$

At each level, the transportation problem is considered balanced, with total supply equalling total demand amounting to 24+19J.

Tables 4 and 5 display that neutrosophic transportation costs and neutrosophic parameter numbers for the upper, middle and lower levels, respectively. Each cell (e, f) in both tables contains two entries:

The source and demand constraints at upper level:

$$\sum_{f=1}^{4} a_{1f} \leq 6+5J, \sum_{f=1}^{4} a_{2f} \leq 6+4J, \sum_{f=1}^{4} a_{3f} \leq 6+7J, \sum_{f=1}^{4} a_{4f} \leq 6+3J,$$

$$\sum_{e=1}^{4} a_{e1} \leq 6+3J, \sum_{e=1}^{4} a_{e2} \leq 5+6J, \sum_{e=1}^{4} a_{e3} \leq 7+5J, \sum_{e=1}^{4} a_{e4} \leq 6+5J$$

The source and demand constraints at middle level:

$$\sum_{f=1}^{4} a_{1f} \leq 6+3J, \sum_{f=1}^{4} a_{2f} \leq 5+6J, \sum_{f=1}^{4} a_{3f} \leq 7+5J, \sum_{f=1}^{4} a_{4f} \leq 6+5J$$

$$\sum_{e=1}^{4} a_{e1} \leq 7+5J, \sum_{e=1}^{4} a_{e2} \leq 7+4J, \sum_{e=1}^{4} a_{e3} \leq 6+5J, \sum_{e=1}^{4} a_{e4} \leq 4+5J$$

The source and demand constraints at lower level:

$$\sum_{f=1}^{4} a_{1f} \leq 7 + 5J, \sum_{f=1}^{4} a_{2f} \leq 7 + 4J, \sum_{f=1}^{4} a_{3f} \leq 6 + 5J, \sum_{f=1}^{4} a_{4f} \leq 4 + 5J$$

$$\sum_{e=1}^{4} a_{e1} \leq 8 + 6J, \sum_{e=1}^{4} a_{e2} \leq 6 + 3J, \sum_{e=1}^{4} a_{e3} \leq 5 + 6J, \sum_{e=1}^{4} a_{e4} \leq 5 + 4J$$

By setting J ∈ [0,1], we transform the trilevel Transportation Problem with Neutrosophic Numbers (TLTP-NN) into a Trilevel Transportation Problem (TLTP) with interval numbers (see table 6 & Table 7). This conversion enables us to derive optimization problems, as shown in Tables 1 and 2, which help determine the best and worst upper-level solutions.

Next, we create goal achievement functions for both levels' objective functions using the obtained results. We then set preference bounds for the decision variables to guide the decision-making process.

Now, Let the target level of the leader's objective function be given by $[Y_1^*, Y_2^{**}] = [0.2,0.5]$

the target level of the follower-1's objective function be given by $[Y_2^*, Y_2^{**}] = [0.5,0.8]$

the target level of the follower-2's objective function be given by $[Y_3^*, Y_3^{**}] = [0.8,1.1]$

A goal programming model is formulated, incorporating preference bounds on the decision variables, and is represented as follows:

Table 6. Best upper-level optimization problem

Highway to traffic Hubs	W	X	Y	Z	supply
A	2 7	5 6	7 6	4 10	11
B	3 6	4 7	5 7	8 9	10
C	10 9	7 8	2 11	10 9	13
D	3 10	2 9	3 7	5 8	9
demand	6	5	7	6	

Table 7 Worst upper-level optimization problem

Highway to traffic Hubs	W	X	Y	Z	supply
A	5 5	11 4	8 2	7 7	6
B	5	9	8	11	6

	4	2	3	4	
C	17 6	10 3	7 5	13 3	6
D	7 3	8 4	7 4	8 6	6
demand	9	11	12	11	

By setting J ∈ [0,1] in this given problem, we convert the TLTP-NN into a TLTP with the interval numbers. Tables 8, and Table-9 represent that optimization problems derived from Table 1 and table 2, used to determine the best and worst upper-level solutions, respectively.

Table 8. Best middle level optimization problem

Traffic Hubs to city regions	NORTH	EAST	WEST	SOUTH	supply
W	5 8	6 6	3 8	7 13	9
X	8 6	11 9	9 11	9 8	11
Y	7 6	5 7	7 6	10 11	12
Z	9 8	5 7	3 10	6 9	11
demand	7	7	6	4	

Table 9. Worst middle level optimization problem

Traffic Hubs to city regions	NORTH	EAST	WEST	SOUTH	supply
W	8 6	14 4	5 5	10 11	6
X	14 3	13 4	14 7	11 2	5
Y	17 3	10 4	11 2	13 4	7
Z	11 6	9 4	8 7	11 4	6
demand	12	11	11	9	

By setting J ∈ [0,1] in this given problem, we convert the TLTP-NN into a TLTP with the interval numbers. Tables 10 and Table-11 represent that optimization problems derived from Table 1 and table 2, used to determine the best and worst upper-level solutions, respectively.

Table10. Best lower-level optimization problem

City Reasons to Local Streets	NORTH	EAST	WEST	SOUTH	supply
W	5 8	6 6	3 8	7 13	12
X	8 6	11 9	9 11	9 8	11
Y	7 6	5 7	7 6	10 11	11
Z	9 8	5 7	3 10	6 9	9
demand	8	6	5	5	

Table 11. Worst lower-level optimization problem

City Reasons to Local Streets	STREET-1	STREET-2	STREET-3	STREET-4	supply
NORTH	7 5	5 4	5 4	8 7	7
EAST	5 4	5 4	14 7	9 4	7
WEST	7 7	11 4	11 4	10 11	6
SOUTH	10 11	17 3	7 3	7 3	4
demand	14	9	11	9	

Dummy sources and destinations are added to balance the transportation problem and identify the optimal and least favourable solutions./ The compiled results are presented in Table 12.

Table 12. Best and worst solution

Solution	Best	Worst
Upper level	$\bar{z}_1 = 0.1063$ At $a_1^{best} =$ (0,0,0,6,2,0,0,0,0,0,7,0,4,5,0,0)	$\bar{z}_1 = 0.2063$ At $a_1^{worst} =$ (6,0,0,0,0,0,6,0,0,0,6,0,0,5,0,1)
solution	Best	worst
Middle level	$\bar{z}_1 = 0.1333$ At $a_1^{best} =$ (0,0,6,0,0,0,0,0,7,0,0,0,0,7,0,4)	$\bar{z}_1 = 0.333$ At $a_1^{worst} =$ (0,0,6,0,0,0,0,5,0,7,0,0,0,0,5,0)
solution	Best	worst
Lower level	$\bar{z}_1 = 0.36$ At $a_1^{best} =$ (8,4,0,0,0,2,5,4,0,0,0,11,0,0,0,0)	$\bar{z}_1 = 0.389$ At $a_1^{worst} =$ (7,0,0,0,7,0,0,0,0,1,6,0,7,7,5,0)

Minimize ($\overline{G_1} + \overline{G_2} + \overline{G_3} + \overline{\overline{G_1}} + \overline{\overline{G_1}} + \overline{\overline{G_3}}$

Subject to $\frac{2a_{11}+5a_{12}+7a_{13}+4a_{14}+3a_{21}+4a_{22}+5a_{23}+8a_{24}+10a_{31}+7a_{32}+2a_{33}+10a_{34}+3a_{41}+2a_{42}+3a_{43}+5a_{44}}{7a_{11}+6a_{12}+6a_{13}+10a_{14}+6a_{21}+7a_{22}+7a_{23}+9a_{24}+9a_{31}+8a_{32}+11a_{33}+9a_{34}+10a_{41}+9a_{42}+7a_{43}+8a_{44}}$

$\overline{G_1}$ =0.6201

$\frac{-5a_{11}-11a_{12}-8a_{13}-7a_{14}-5a_{21}-9a_{22}-8a_{23}-11a_{24}-17a_{31}-10a_{32}-7a_{33}-13a_{34}-7a_{41}-8a_{42}-7a_{43}-8a_{44}}{5a_{11}+4a_{12}+2a_{13}+7a_{14}+4a_{21}+2a_{22}+3a_{23}+4a_{24}+6a_{31}+3a_{32}+5a_{33}+3a_{34}+3a_{41}+4a_{42}+4a_{43}+6a_{44}}$ $\overline{\overline{G_2}} =$ -2.16

$\frac{5a_{11}+6a_{12}+3a_{13}+7a_{14}+8a_{21}+11a_{22}+9a_{23}+9a_{24}+7a_{31}+5a_{32}+7a_{33}+10a_{34}+9a_{41}+5a_{42}+3a_{43}+6a_{44}}{8a_{11}+6a_{12}+8a_{13}+13a_{14}+6a_{21}+9a_{22}+11a_{23}+8a_{24}+6a_{31}+7a_{32}+6a_{33}+11a_{34}+8a_{41}+7a_{42}+10a_{43}+9a_{44}}$

$\overline{G_2} = 0.827$

$\frac{-8a_{11}-14a_{12}-5a_{13}-10a_{14}-14a_{21}-13a_{22}-14a_{23}-11a_{24}-17a_{31}-10a_{32}-11a_{33}-13a_{34}-11a_{41}-9a_{42}-8a_{43}-11a_{44}}{5a_{11}+4a_{12}+2a_{13}+7a_{14}+4a_{21}+2a_{22}+3a_{23}+4a_{24}+6a_{31}+3a_{32}+5a_{33}+3a_{34}+3a_{41}+4a_{42}+4a_{43}+6a_{44}}$

$\overline{\overline{G_2}} = 2.35$

$\frac{5a_{11}+6a_{12}+3a_{13}+7a_{14}+8a_{21}+11a_{22}+9a_{23}+9a_{24}+7a_{31}+5a_{32}+7a_{33}+10a_{34}+9a_{41}+5a_{42}+3a_{43}+6a_{44}}{8a_{11}+6a_{12}+8a_{13}+13a_{14}+6a_{21}+9a_{22}+11a_{23}+8a_{24}+6a_{31}+7a_{32}+6a_{33}+11a_{34}+8a_{41}+7a_{42}+10a_{43}+9a_{44}}$

$\overline{G_3} = 0.83$

$\frac{-7a_{11}-5a_{12}-5a_{13}-8a_{14}-5a_{21}-5a_{22}-14a_{23}-9a_{24}-7a_{31}-11a_{32}-11a_{33}-10a_{34}-10a_{41}-17a_{42}-7a_{43}-7a_{44}}{5a_{11}+4a_{12}+4a_{13}+7a_{14}+4a_{21}+4a_{22}+7a_{23}+4a_{24}+7a_{31}+4a_{32}+4a_{33}+11a_{34}+11a_{41}+3a_{42}+3a_{43}+3a_{44}}$

$\overline{\overline{G_3}} = 1.62$

The results obtained from the proposed goal programming model are displayed in Table 13.

Table 13. Satisfactory solution of TLFTP-NN

Solution point	(1,0,0,0,6,5,0,0,0,0,5,1,0,1,1,5,)
Objective values of leader	(0.1063,0.2063)
Objective values of follower-1	(0.1333,0.333)
Objective values of follower-2	(0.36,0.389)

7. Comparative Study

To assess the performance of the proposed Trilevel Linear Fractional Transportation Problem under the neutrosophic framework (TLFTP-INN), a comparison has been made with a standard trilevel transportation model designed under a deterministic (crisp) setting, referred to here as TLFTP-C.
In both models, the structure of decision-making is the same and follows three levels:

- **Leader level**: Transportation from highways to traffic hubs.
- **First follower level**: Distribution from traffic hubs to urban city zones.
- **Second follower level**: Allocation from city zones to local streets.

7.1 Objective of the Comparison: The comparison aims to evaluate the two models based on their total transportation cost across the three hierarchical levels, ability to handle uncertain data, adaptability and robustness in decision-making, and effectiveness in responding to changing or imprecise conditions.

7.2 Classical TLFTP Model (Deterministic)

In this approach, all parameters related to cost, supply, and demand are treated as fixed numerical values. The uncertainty component J is assumed to be zero, which essentially eliminates the indeterminate nature of data. The optimization is carried out independently at each level using conventional linear programming techniques.

7.3 Neutrosophic TLFTP Model (Proposed)

In contrast, the proposed model considers all costs and capacities as interval neutrosophic numbers of the form $p + qJ$, where $J \in [0,1]$. This structure allows the model to incorporate both determinacy and indeterminacy. The optimization is handled using a goal programming strategy, which considers the best-case and worst-case performance scenarios.

7.4 Comparative Results

Comparison of crisp TLFTP with Neutrosophic TLFTP-INN is shown in Table 14.

Table 14. Comparison of Crisp TLFTP with Neutrosophic TLFTP-INN

Evaluation Criteria	Crisp TLFTP	Neutrosophic TLFTP-INN
Treatment of Uncertainty	Not Addressed	Incorporated via J
Flexibility of Decision Variables	Rigid (Single Values)	Adaptive (Interval-based)
Use of Goal Programming	Not Included	Fully Integrated
Resistance to Parameter Changes	Low	High
Availability of Best/Worst Scenarios	Absent	Present
Practical Suitability for Real Systems	Limited	Highly Suitable

8. Sensitive Analysis

The classical TLFTP model produces a fixed solution, which may not reflect the variability or indeterminacy present in real-world traffic conditions. On the other hand, the neutrosophic TLFTP-INN model offers a broader and more flexible solution set. It adapts better to uncertain environments, provides more informed decisions under ambiguity, and allows planners to evaluate outcomes under multiple scenarios. This makes the proposed model particularly effective for complex traffic networks subject to unpredictable behaviours.

Sensitivity analysis is a crucial tool to evaluate the stability and robustness of a mathematical model under variations in key parameters. In the proposed TLFTP-INN model, uncertainty is captured through interval neutrosophic numbers of the form $p + qJ$, where $J \in [0,1]$ represents the degree of indeterminacy. This analysis investigates how changes in the indeterminacy factor J affect the optimal solutions across the three hierarchical decision levels.

To perform the sensitivity analysis, the value of J is systematically varied from 0 to 1 in increments of 0.2 (i.e., $J = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$). At each level, we observe (see table 15) the changes in the objective function values corresponding to the leader, follower-1, and follower-2 under the given neutrosophic transportation cost and supply-demand structures.

Table 15. Observations

Value of J	Leader's Objective Value	Follower-1's Objective Value	Follower-2's Objective Value
0.0	0.83	1.14	0.36
0.2	1.04	1.35	0.38
0.4	1.27	1.56	0.40
0.6	1.62	1.79	0.43
0.8	2.03	2.03	0.45
1.0	2.35	2.28	0.48

- As the value of J increases, the objective function values at all three levels increase, reflecting the growing influence of uncertainty in cost estimation.
- The leader's problem shows a sharper increase in cost compared to the followers, indicating a higher sensitivity to uncertain conditions at the top decision level.
- The follower-2 level (local streets) demonstrates relatively stable behavior, suggesting more resilience to uncertainty at micro-level traffic zones.

The results of this sensitivity analysis confirm that the proposed neutrosophic model maintains a consistent and logical response to changes in the indeterminacy level J. This validates the robustness of the TLFTP-INN model in handling real-world uncertainty. The findings also highlight the importance of accounting for indeterminacy in high-level traffic planning, as decision errors at the top level can cascade through the system.

9. Conclusions

This study introduces a novel and efficient framework for addressing the Trilevel Transportation Problem (TLTP) in urban traffic systems by incorporating Interval Neutrosophic Numbers (INNs) into a Trilevel Linear Fractional Transportation Problem (TLFTP), solved through the Goal Programming (GP) method. The proposed approach effectively models the uncertainty and vagueness inherent in real-world transportation scenarios, while capturing the hierarchical nature of decision-making across different network levels—namely highways, traffic hubs, and urban roads.

The primary contribution of this work is the development of a neutrosophic-based trilevel decision-making model that optimizes traffic flow while considering the distinct yet interrelated objectives of various transportation authorities. The integration of neutrosophic logic improves the adaptability and realism of planning processes under uncertain and imprecise conditions. Looking ahead, this model offers several promising directions for future research. Potential extensions include the integration of dynamic traffic management systems using real-time data, expansion to multi-modal transport networks, incorporation of environmental and economic objectives, and alignment with smart city initiatives such as adaptive signal control, autonomous vehicle navigation, and variable toll pricing. Additionally, improving scalability and computational efficiency through the use of heuristic or metaheuristic algorithms will enhance the model's applicability to large and complex transportation networks.

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