University of New Mexico

Neutrosophic Inference Systems Using Takagi-Sugeno-Kang Model and Its Application

İrfan Deli[1], Vakkas Uluçay[2] and Zeynep Başer[3,*]
[1]Kilis 7 Aralik University, 79000 Kilis, Türkiye ; irfandeli@kilis.edu.tr
[2]Kilis 7 Aralik University, 79000 Kilis, Türkiye ; vakkas.ulucay@kilis.edu.tr
[3]Kilis 7 Aralik University, 79000 Kilis, Türkiye ; zeynep.baser@kilis.edu.tr
*Correspondence: zeynep.baser@kilis.edu.tr;

Abstract. In this article, a new Takagi-Sugeno-Kang (TSK) model was developed for neutrosophic environments based on existing the fuzzy TSK models. The developed model establishes inference relations based on past experiences or data and offers a new solution with IF-THEN rules for future events. In this direction, the neutrosophic TSK model was created using non-linear systems and made more effective by using t and s norms in its solutions. In addition, this model is defined for $n-$input systems and finally an example application is given in non-linear systems with relationship between energy consumption and heating time to demonstrate the applicability of the two-input model.

Keywords: Takagi-Sugeno-Kang; Neutrosophic sets; Neutrosophic inference system.

1. Introduction

After Zadeh's introduction of fuzzy set theory [1], there has been substantial development in fuzzy sets [2–10] and decision-making [11–20]. Decision-making processes involve the development of an inference system based on past experiences and existing data, then utilize to predict future occurrences. One of these inference methods, the concept of Mamdani Fuzzy Inference System, was first introduced by Mamdani [21] in 1974. The Mandani method, based on the max-min technique and the Zadeh method, which follows the $f(a, b) = min\{1, 1-a+b\}$ approach, are among the most commonly employed methods for deriving inference relations in fuzzy sets [22]. Given in [22] fuzzy inference systems generally consists of four main components. These are the fuzzification process, fuzzy rules based on IF-THEN statements, the inference mechanism and the defuzzification process. Among other fuzzy inference methods, the Larsen method [23], which utilizes mathematical functions the Adaptive neuro-fuzzy inference system [24] that enhances learning capabilities by integrating fuzzy logic with artificial neural networks, and fuzzy cognitive maps [25], which is particularly effective in modeling and simulating dynamic systems. Studies in this field have applied including solving traffic problems [26], predicting diseases [27], [28], measuring groundwater quality [29], forecasting wave parameters [30], predicting bridge degradation rates [31], controlling greenhouse climate [32], architectural space selection [33], disaster scenario prediction [34], [35], supplier selection [36] and forecasting tennis match outcomes [37]. This diversity highlights the flexibility and accuracy of fuzzy inference systems in providing solutions to problems involving uncertainty and complexity across various disciplines. Among the various types of fuzzy models, The TSK model [38] [39], fuzzy model has been shown to be an efficient and powerful approach for addressing analysis and synthesis problems in complex non-linear systems. The TSK method has been effectively applied in various areas such as hydrogen fuel usage [40], highway travel time prediction [41] and ozone concentration modeling [42]. Additionally, significant research has been conducted in fields such as healthcare and medical applications [43], data mining [44] and control systems [45] using the TSK method. Among the recent contributions to the field of neutrosophic inference systems, [46–48] stand out as representative examples. Recently, several studies such as [49–51] have focused on the Takagi-Sugeno-Kang (TSK) model, contributing to the growing body of literature in this area.

1.1. *Motivation and Research Gap*

In the above-mentioned studies, TSK fuzzy models are generally considered within the framework of fuzzy set theory and intuitionistic fuzzy set theory [52], and these models are generalized to express different types of uncertainties. However, in order to further improve the performance of TSK models, it is necessary to consider not only the degree of membership or non-membership but also the degree of indeterminacy. At this point, the neutrosophic set theory developed by Smarandache [53] offers an important alternative. Neutrosophic set theory can operate in the non-standard interval $]^{-}0, 1^{+}[$ in addition to the standard interval $[0, 1]$ through functions that independently define the degrees of truth, indeterminacy and falsity [62]. This flexible structure allows for more effective modeling of multidimensional and complex uncertainties that cannot be adequately represented in fuzzy systems.

1.2. *Contributions and Novelty*

The most commonly used fuzzy inference systems [54–59] face significant limitations when working with data containing uncertainty. These systems operate only with membership degrees of fuzzy set theory and degree of membership and non-membership of intuitionistic fuzzy set theory and do not take into account the value of indeterminacy. However, this approach may not fully reflect the differences in indeterminacies in real-life conditions. This limitation highlights the need for more flexible neutrosophic inference systems approaches that can effectively handle the complexity of indeterminacy and at the same time be suitable for situations of lack of confidence or hesitation in the information field. Thus, the reliability of decision-making processes can be increased.

The motivation and contributions of the study are considered as:

(1) This paper develop a new model by generalized fuzzy inference systems to Neutrosophic inference systems based on TSK method under Mehran [61].
(2) This model enables indeterminacy to be modeled more precisely and effectively by evaluating the degrees of accuracy, uncertainty and inaccuracy together, thus significantly improving the accuracy of the outputs.
(3) An example for application is presented to demonstrate the effectiveness and advantage of the proposed model.

This paper summarizes as follows: In section 2, we gave Mandani and Zadeh methods, which are inference methods on fuzzy sets thanks to IF-THEN rules, are given with the help of algorithms. In section 3, we introduced a new Takagi-Sugeno-Kang (TSK) model has been introduced for neutrosophic environments, building upon existing TSK models. The neutrosophic TSK model is formulated with non-linear systems and enhanced by incorporating t and s norms, improving its solution capabilities. In Section 4, the model defined for n-input systems is developed and its practical applicability is demonstrated through a model with two inputs, energy consumption and heating time, to reduce carbon emissions, which are directly related to energy efficiency. In section 5, we proposed a conclusion.

2. **Preliminary**

In this section, we gave some basic definitions of fuzzy set and neutrosophic sets and then Mandani and Zadeh methods, which are inference methods on fuzzy sets based on IF-THEN rules, are given with the help of algorithms.

Definition 2.1. *[1] Let X be a non-empty set, and for each $x \in X$, $T_A(x) \in [0,1]$ such that the function $T_A : X \to [0,1]$ defines a fuzzy set*

$$A = \{\langle x, T_A(x)\rangle : x \in X\}. \tag{1}$$

Definition 2.2. *[60] Consider a function t from $[0,1] \times [0,1]$ to $[0,1]$. If t satisfies the following properties with $a, b, c, d \in [0,1]$, then t is defined as t-norm.*

(*i*) $t(0,a) = t(a,0) = 0$ *and* $t(a,1) = t(1,a) = a$
(*ii*) *If* $a \leq c$ *and* $b \leq d$, *then* $t(a,b) \leq t(c,d)$

(iii) $t(a,b) = t(b,a)$

(iv) $t(a,t(b,c)) = t(t(a,b),c)$

Definition 2.3. *[60] Consider a function s from $[0,1]\times[0,1]$ to $[0,1]$. If s satisfies the following properties with $a,b,c,d\in[0,1]$, then s is defined as s-norm.*

(i) $s(0,a) = s(a,0) = a$ *and* $s(a,1) = s(1,a) = 1$

(ii) *If* $a \le c$ *and* $b \le d$, *then* $s(a,b) \le s(c,d)$

(iii) $s(a,b) = s(b,a)$

(iv) $s(a,s(b,c)) = s(s(a,b),c)$

Definition 2.4. *[62] Let E be a universe of discourse. A neutrosophic set $\bar{\mathcal{N}}$ in E is characterized by a truth-membership function $T_{\bar{\mathcal{N}}}$, a indeterminacy-membership function $I_{\bar{\mathcal{N}}}$ and a falsity-membership function $F_{\bar{\mathcal{N}}}$. $T_{\bar{\mathcal{N}}}(x)$, $I_{\bar{\mathcal{N}}}(x)$ and $F_{\bar{\mathcal{N}}}(x)$ are real standard elements of $[0,1]$. It can be written*

$$\bar{\mathcal{N}} = \{< x, (T_{\bar{\mathcal{N}}}(x), I_{\bar{\mathcal{N}}}(x), F_{\bar{\mathcal{N}}}(x)) >: x \in E, T_{\bar{\mathcal{N}}}(x), I_{\bar{\mathcal{N}}}(x), F_{\bar{\mathcal{N}}}(x) \in [0,1]\} \qquad (2)$$

Definition 2.5. *[62] Let E be a universe of discourse, $\bar{A} = \{< x, T_{\bar{A}}(x), I_{\bar{A}}(x), F_{\bar{A}}(x) >: x \in X\}$ and $\bar{B} = \{< x, T_{\bar{B}}(x), I_{\bar{B}}(x), F_{\bar{B}}(x) >: x \in X\}$ be the neutrosophic sets. Then for $\bar{A}$ and $\bar{B}$ set operations:*

(1) $\bar{A}\hat{\cup}\bar{B} = \{< x, \max\{T_{\bar{A}(x)}, T_{\bar{B}}(x)\}, \max\{I_{\bar{A}}(x), I_{\bar{B}}(x)\}, \min\{F_{\bar{A}}(x), F_{\bar{B}}(x)\} >: x \in X\}$

(2) $\bar{A}\hat{\cap}\bar{B} = \{< x, \min\{T_{\bar{A}}(x), T_{\bar{B}}(x)\}, \min\{I_{\bar{A}}(x), I_{\bar{B}}(x)\}, \max\{F_{\bar{A}}(x), F_{\bar{B}}(x)\} >: x \in X\}$

(3) $\bar{A}^{\hat{c}} = \{< x, F_{\bar{A}}(x), 1 - I_{\bar{A}}(x), T_{\bar{A}(x)} >: x \in X\}$

(4) $\bar{A}\hat{+}\bar{B} = \{< x, T_{\bar{A}}(x) + T_{\bar{B}}(x) - T_{\bar{A}}(x)\cdot T_{\bar{B}}(x), I_{\bar{A}}(x)\cdot I_{\bar{B}}(x), F_{\bar{A}}(x)\cdot F_{\bar{A}}(x) >: x \in X\}$

(5) $\bar{A}\hat{\cdot}\bar{B} = \{< x, T_{\bar{A}}(x)\cdot T_{\bar{B}}(x), I_{\bar{A}}(x) + I_{\bar{B}}(x) - I_{\bar{A}}(x)\cdot I_{\bar{B}}(x), F_{\bar{A}}(x) + F_{\bar{B}}(x) - F_{\bar{A}}(x)\cdot F_{\bar{B}}(x) >: x \in X\}$

(6) $\bar{A}^{\lambda} = \{< x, T_{\bar{A}^{\lambda}}(x), 1 - (1 - I_{\bar{A}}(x))^{\lambda}, 1 - (1 - F_{\bar{A}}(x))^{\lambda} >: x \in X\}$

(7) $\lambda\cdot\bar{A} = \{< x, 1 - (1 - T_{\bar{A}}(x))^{\lambda}, I_{\bar{A}^{\lambda}}(x), F_{\bar{A}^{\lambda}}(x) >: x \in X\}$

(8) $\bar{A}\hat{\subseteq}\bar{B} \Leftrightarrow \{< x, T_{\bar{A}}(x) \le T_{\bar{B}}(x), I_{\bar{A}}(x) \le I_{\bar{B}}(x), F_{\bar{A}}(x) \ge F_{\bar{B}}(x) >: x \in X\}$

(9) $\bar{A} = \bar{B} \Leftrightarrow \{\bar{A}\hat{\subseteq}\bar{B}, \bar{B}\hat{\subseteq}\bar{A}\}$

Definition 2.6. *[63] A single valued trapezoidal neutrosophic number $\bar{a} =< (a', b', c', d'); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}} >$ is a special neutrosophic set on the real number set R, whose truth−membership, indeterminacy−membership, and a falsity−membership are given as follows:*

$$T_{\bar{a}}(x) = \begin{cases} (x - a')w_{\bar{a}}/(b' - a'), & a' \le x \le b' \\ w_{\bar{a}}, & b' \le x \le c' \\ (d' - x)w_{\bar{a}}/(d' - c'), & c' \le x \le d' \\ 0, & otherwise \end{cases}$$

$$I_{\bar{a}}(x) = \begin{cases} b' - x + u_{\bar{a}}(x - a')/(b' - a'), & a' \leq x \leq b' \\ u_{\bar{a}}, & b' \leq x \leq c' \\ x - c' + u_{\bar{a}}(d' - x)/(d' - c'), & c' \leq x \leq d' \\ 1, & otherwise \end{cases} \tag{3}$$

$$F_{\bar{a}}(x) = \begin{cases} b' - x + y_{\bar{a}}(x - a')/(b' - a'), & a' \leq x \leq b' \\ y_{\bar{a}}, & b' \leq x \leq c' \\ x - c' + y_{\bar{a}}(d' - x)/(d' - c'), & c' \leq x \leq d' \\ 1, & otherwise \end{cases} \tag{4}$$

where $w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}} \in [0, 1]$.

Definition 2.7. *[65] The expectation score function of neutrosophic values is the application* $S^{\star} : L^{*} \to [0, 1]$ *defined by the expression:*

$$S^{\star}(\bar{x}) = \frac{2 + T_{\bar{A}}(x) - I_{\bar{A}}(x) - F_{\bar{A}}(x)}{3} \tag{5}$$

for all $\bar{x} =< x; T_{\bar{A}}(x); I_{\bar{A}}(x); F_{\bar{A}}(x) >\in L^{*}$.

Definition 2.8. *[64] The Mandani method for two-input single−output inference relations is based on the max−min approach. The number of rules can be increased or decreased as desired. The algorithm for two rules is as follows:*

For the classical sets $X = \{x_1, x_2, \ldots, x_i\}, Y = \{y_1, y_2, \ldots, y_j\}$ *and* $Z = \{z_1, z_2, \ldots, z_k\}$, *let neutrosophic sets* $\bar{A}_1, \bar{A}_2 \subseteq X, \bar{B}_1, \bar{B}_2 \subseteq Y$ *and* $\bar{C}_1, \bar{C}_2 \subseteq Z$ *be defined. Throughout this paper, we will use the notation* $\bar{\bar{x}}$ *instead of the notation* $< x; T_{\bar{A}}(x); I_{\bar{A}}(x); F_{\bar{A}}(x) >$ *for the neutrosophic element on* $x \in X$.

Algorithm

Step 1 *The following rules will be defined with a specific number of variables. (The number of rules can be increased or decreased.)*

Rule 1 *: IF* $\bar{\bar{x}} \in \bar{A}_1$ *AND* $\bar{\bar{y}} \in \bar{B}_1$ *THEN* $\bar{\bar{z}} \in \bar{C}_1$

Rule 2 *: IF* $\bar{\bar{x}} \in \bar{A}_2$ *AND* $\bar{\bar{y}} \in \bar{B}_2$ *THEN* $\bar{\bar{z}} \in \bar{C}_2$

Step 2 $\forall (x_i, y_j, z_k) \in X \times Y \times Z$, *calculate the result rules.*

Final Rule 1 *:*

$$\begin{aligned} &\langle T_{\bar{R}_1}(x_i, y_j, z_k), I_{\bar{R}_1}(x_i, y_j, z_k), F_{\bar{R}_1}(x_i, y_j, z_k)\rangle \\ &= \langle T_{\bar{A}_1}(x_i) \wedge T_{\bar{B}_1}(y_j) \wedge T_{\bar{C}_1}(z_k), I_{\bar{A}_1}(x_i) \vee I_{\bar{B}_1}(y_j) \vee I_{\bar{C}_1}(z_k), F_{\bar{A}_1}(x_i) \vee F_{\bar{B}_1}(y_j) \vee F_{\bar{C}_1}(z_k)\rangle \end{aligned}$$

calculate the relation $\bar{R}_1$.

Final Rule 2 *:*

$$\begin{aligned} &\langle T_{\bar{R}_2}(x_i, y_j, z_k), I_{\bar{R}_2}(x_i, y_j, z_k), F_{\bar{R}_2}(x_i, y_j, z_k)\rangle \\ &= \langle T_{\bar{A}_2}(x_i) \wedge T_{\bar{B}_2}(y_j) \wedge T_{\bar{C}_2}(z_k), I_{\bar{A_2 1}}(x_i) \vee I_{\bar{B}_2}(y_j) \vee I_{\bar{C}_2}(z_k), F_{\bar{A}_2}(x_i) \vee F_{\bar{B}_2}(y_j) \vee F_{\bar{C}_2}(z_k)\rangle \end{aligned}$$

calculate the relation $\bar{R}_2$*.*

Step 3 $\forall(x_i, y_j, z_k) \in X \times Y \times Z$*, combine the results of each result rule in Step 2 as follows:*

$$\langle T_{\bar{R}}(x_i, y_j, z_k), I_{\bar{R}}(x_i, y_j, z_k), F_{\bar{R}}(x_i, y_j, z_k)\rangle$$

$$= \langle T_{\bar{R}_1}(x_i, y_j, z_k) \vee T_{\bar{R}_2}(x_i, y_j, z_k), I_{\bar{R}_2}(x_i, y_j, z_k) \wedge I_{\bar{R}_2}(x_i, y_j, z_k), F_{\bar{R}_1}(x_i, y_j, z_k) \wedge F_{\bar{R}_2}(x_i, y_j, z_k)\rangle$$

to obtain the relation $\bar{R}$ *inference.*

Step 4 *Given a situation represented by the neutrosophic matrix* $\bar{A}$ *and* $\bar{B}$ *derive the result in the form* $\bar{C} = \bar{A}\hat{\circ}(\bar{B}\hat{\circ}\bar{R})$

Step 5 *According to the central definitions of neutrosophic sets by Hanafy et al. [66] and Deli and Öztürk [67] defuzzification make decision.*

$$\frac{\sum_{i=1}^{r} z_i T_{\bar{C}}(z_i) - \sum_{i=1}^{r} z_i I_{\bar{C}}(z_i) + \sum_{i=1}^{r} z_i F_{\bar{C}}(z_i)}{\sum_{i=1}^{r} T_{\bar{C}}(z_i) - \sum_{i=1}^{r} I_{\bar{C}}(z_i) + \sum_{i=1}^{r} F_{\bar{C}}(z_i)}$$

Definition 2.9. *[68] Consider data set* $X = \{x_1, x_2, \ldots, X_N\}$*, where* N *is the number of data inputs, and for each input vector* $x_k = \{x_1(k), x_2(k), \ldots, x_n(k)\}$*,* n *is the dimension of* x_k*. Typically, the* i*th rule of the TSK fuzzy model can be expressed as:*

$$\begin{aligned} &\textit{Rule } i\textit{:} \quad \textit{IF } x_1(k) \textit{ is } A_1^i \textit{ AND } x_2(k) \textit{ is } A_2^i \textit{ AND } \ldots \textit{AND } x_n(k) \textit{ is } A_n^i \\ &\qquad\qquad \textit{THEN } y^i(k) = a_0^i + a_1^i x_1(k) + a_2^i x_2(k) \ldots + a_n^i x_n(k),\ i = 1, 2, \ldots, c \end{aligned} \tag{6}$$

where i *is the* i*th fuzzy rule,* c *is the number of fuzzy rules,* A_q^i *is the fuzzy set,* $q = 1, 2, \ldots, n$*,* $x_q(k)$ *is the input variable,* $y^i(k)$ *is the output of the* i*th fuzzy rule, and* a_q^i *is the consequent parameter of the* i*th output. The full representation of the fuzzy model can be given as:*

$$\hat{y}(k) = \frac{\sum_{i=1}^{c} w_i(k) y^i(k)}{\sum_{i=1}^{c} w_i(k)} = \sum_{i=1}^{c} \lambda_i y^i(k) \tag{7}$$

where $\hat{y}(k)$ *is the output of the fuzzy system,*

$$w_i(k) = \prod_{q=1}^{n} A_q^i(x_q(k)) \tag{8}$$

is the weight,

$$\lambda_i = \frac{w_i(k)}{\sum_{i=1}^{c} w_i(k) y^i(k)} \textit{ and } \sum_{i=1}^{c} \lambda_i = 1. \tag{9}$$

3. Neutrosophic TSK Inference System

Lets $\bar{\mathcal{N}}_1, \bar{\mathcal{N}}_2, \ldots, \bar{\mathcal{N}}_n$ be neutrosophic sets in X.

$$\bar{\bar{x}}_1 \in \bar{\mathcal{N}}_1 \text{ AND } \bar{\bar{x}}_2 \in \bar{\mathcal{N}}_2 \text{ AND } \ldots \text{AND } \bar{\bar{x}}_n \in \bar{\mathcal{N}}_n \text{ THEN } \begin{cases} \dot{x}_1 = & f_1(x_1, x_2, \ldots, x_n) \\ \dot{x}_2 = & f_2(x_1, x_2, \ldots, x_n) \\ \quad\vdots & \quad\vdots \\ \dot{x}_n & f_n(x_1, x_2, \ldots, x_n). \end{cases}$$

where $x_i = ((T(x_i), I(x_i), F(x_i))$ and $f_i(x_1, x_2, \ldots, x_n)$ is non-linear function for $i = 1, 2, \ldots, n$.

3.1. *Two Inputs Neutrosophic TSK Method*

Consider the following system:

$$\begin{aligned} \dot{x}_1 =& f_1(x_1, x_2) \\ \dot{x}_2 =& f_2(x_1, x_2) \end{aligned} \tag{10}$$

In order to simplify, we assume that $x_1 \in (a', b')$ and $x_2 \in (c', d')$. We can represent equation 10 as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} \tag{11}$$

For the nonlinear terms $z_1 = a_{11}$, $z_2 = a_{12}$, $z_3 = a_{21}$ and $z_4 = a_{22}$. We obtain the following;

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} \tag{12}$$

We define the membership functions for linguistic values as "$\bar{A}_1$", "$\bar{A}_2$", …and "$\bar{B}_1$", "$\bar{B}_2, \ldots$". As a result, the non-linear system 10 is described by the following neutrosophic model for $i \in \{1, 2, ...\}$ and $m, n \in \{1, 2, ...\}$;

Rule ***i*:** IF $\bar{\bar{x}}_1$ is "$\bar{A}_m$" and $\bar{\bar{x}}_2$ is "$\bar{B}_n$" THEN $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = M_i \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

where

$$k_1 = \begin{cases} \max_{x_1 \in (a', b'), x_2 \in (c', d')} z_1, & \bar{A}_m \text{ is "HIGH" neutrosophic linguistic value} \\ \text{mid}_{x_1 \in (a', b'), x_2 \in (c', d')} z_1, & \bar{A}_m \text{ is "MIDDLE" neutrosophic linguistic value} \\ \min_{x_1 \in (a', b'), x_2 \in (c', d')} z_1, & \bar{A}_m \text{ is "LOW" neutrosophic linguistic value} \end{cases},$$

$$k_2 = \begin{cases} \max_{x_1 \in (a', b'), x_2 \in (c', d')} z_2, & \bar{B}_n \text{ is "HIGH" neutrosophic linguistic value} \\ \text{mid}_{x_1 \in (a', b'), x_2 \in (c', d')} z_2, & \bar{B}_n \text{ is "MIDDLE" neutrosophic linguistic value} \\ \min_{x_1 \in (a', b'), x_2 \in (c', d')} z_2, & \bar{B}_n \text{ is "LOW" neutrosophic linguistic value} \end{cases},$$

$$k_3 = \begin{cases} \max_{x_1 \in (a', b'), x_2 \in (c', d')} z_3, & \bar{A}_m \text{ is "HIGH" neutrosophic linguistic value} \\ \text{mid}_{x_1 \in (a', b'), x_2 \in (c', d')} z_3, & \bar{A}_m \text{ is "MIDDLE" neutrosophic linguistic value} \\ \min_{x_1 \in (a', b'), x_2 \in (c', d')} z_3, & \bar{A}_m \text{ is "LOW" neutrosophic linguistic value} \end{cases}$$

and

$$k_4 = \begin{cases} \max_{x_1\in(a',b'),x_2\in(c',d')} z_4, & \bar{B}_n \text{ is "HIGH" neutrosophic linguistic value} \\ \text{mid}_{x_1\in(a',b'),x_2\in(c',d')} z_4, & \bar{B}_n \text{ is "MIDDLE" neutrosophic linguistic value} \\ \min_{x_1\in(a',b'),x_2\in(c',d')} z_4, & \bar{B}_n \text{ is "LOW" neutrosophic linguistic value} \end{cases}$$

and where

$$\text{mid}_{x_1\in(a',b'),x_2\in(c',d')} z_k = \frac{\max_{x_1\in(a',b'),x_2\in(c',d')} z_k + \min_{x_1\in(a',b'),x_2\in(c',d')} z_k}{2} \qquad \text{for} \quad k = 1,2,3,4$$

According to above Rule $i \in \{1, 2 \ldots, p\}$ and $m, n \in \{1, 2, \ldots\}$, we can construct Model Rule i as;

Model Rule i: IF $\bar{\bar{x}}_1$ is "$\bar{A}_m$" and $\bar{\bar{x}}_2$ is "$\bar{B}_n$" THEN $\begin{cases} \dot{x}_1 = k_1 x_1 + k_2 x_2, \\ \dot{x}_2 = k_3 x_1 + k_4 x_2, \end{cases}$.

Then for final outputs we need to calculate $\dot{x}_1$ and $\dot{x}_2$. For $i = 1, 2$ and each interval like $x_1 \in (a', b')$ and $x_2 \in (c', d')$ calculate as following:

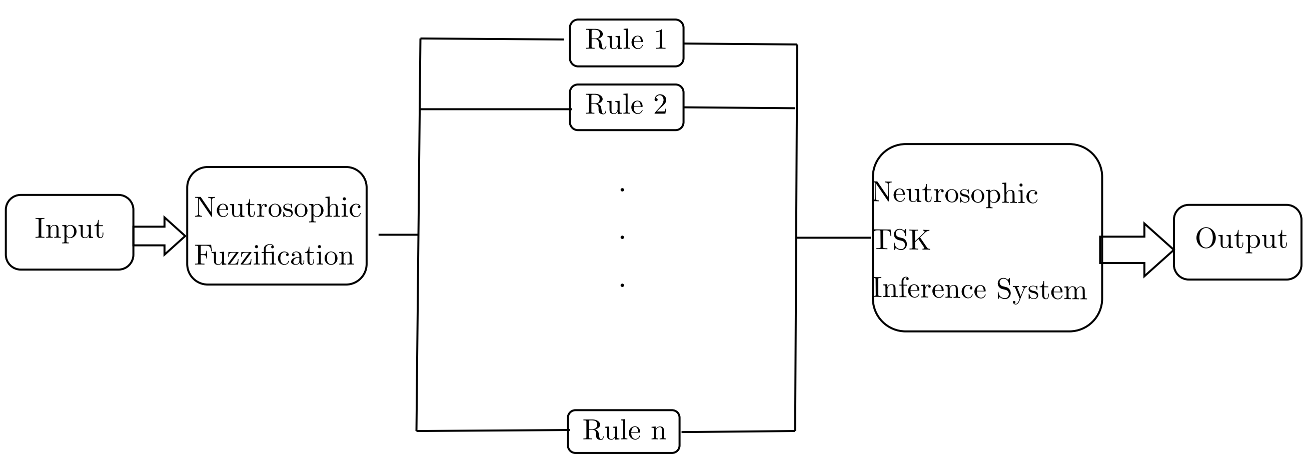
$$\dot{x}_i = \frac{1}{4\cdot p}\left(\frac{2 + h_1(x) - h_2(x) - h_3(x)}{3} \cdot M_1[x_1, x_2]^T + \frac{2 + h_4(x) - h_5(x) - h_6(x)}{3} \cdot M_2[x_1, x_2]^T + \ldots\right) \tag{13}$$

where

$$h_1(x) = t(T_{\bar{A}_{m_1}}(x_1), T_{\bar{B}_{n_1}}(x_2)) \quad h_2(x) = s(I_{\bar{A}_{m_1}}(x_1), I_{\bar{B}_{n_1}}(x_2)) \quad h_3(x) = s(F_{\bar{A}_{m_1}}(x_1), F_{\bar{B}_{n_1}}(x_2))$$
$$h_4(x) = t(T_{\bar{A}_{m_2}}(x_1), T_{\bar{B}_{n_2}}(x_2)) \quad h_5(x) = s(I_{\bar{A}_{m_2}}(z_1), I_{\bar{B}_{n_2}}(x_2)) \quad h_6(x) = s(F_{\bar{A}_{m_2}}(x_1), F_{\bar{B}_{n_2}}(x_2))$$

and where $(m_1, m_2, ...), (n_1, n_2, ...) \in \{1, 2, ...\}$

Thus the output values for $\dot{x}_1$ and $\dot{x}_2$ are obtained.



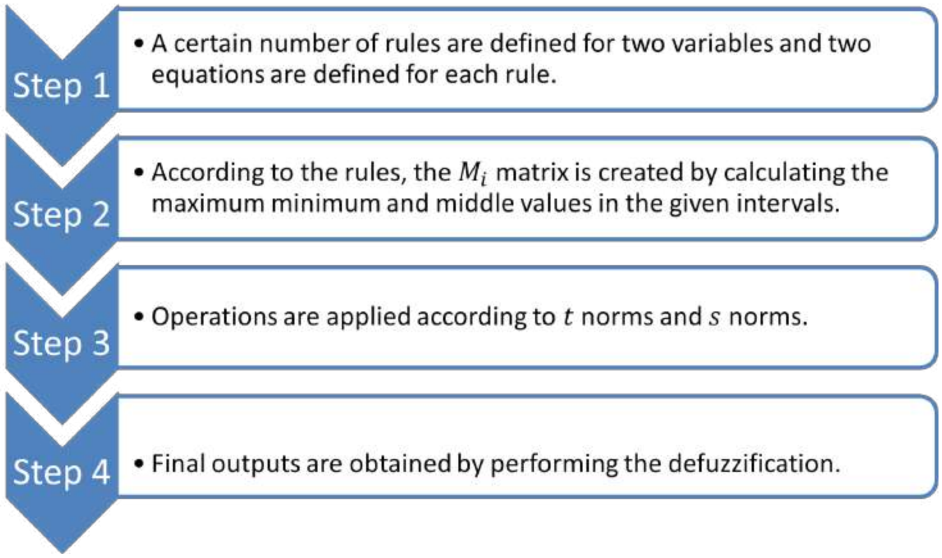


FIGURE 1. Flowchart of Proposed Inference System

4. **Application of Neutrosophic TSK Method**

The proposed neutrosophic TSK inference model for optimizing energy and heat consumption aims to increase efficiency and improve decision processes in the current conditions of industrial and commercial systems. The ability of neutrosophic logic to handle truth, indeterminacy and falsity simultaneously makes it possible to model complex systems more realistically. For ease of implementation and interpretability, instead of an **n** input, the model focuses on **2** input variables: x_1 = heating time (hours) and x_2 = energy consumption (kWh). In response to these inputs, the outputs of the system are energy efficiency and the amount of carbon emissions. These two output variables provide the opportunity to evaluate both the performance and environmental impacts of the system in a holistic approach. The main purpose of the developed model is to represent the non-linear systems with relationship between energy consumption and heating time with the flexible structure offered by neutrosophic logic; and to reveal system configurations that will achieve maximum efficiency with minimum energy. At the same time, it is aimed to contribute to environmental sustainability by analyzing energy usage habits that have a direct impact on carbon emissions.

Let us examine the non-linear system defined as follows:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_1^2 x_2 + 2x_1 x_2 + 2x_2 \\ x_1 x_2^3 + 2x_1^2 x_2 + 28x_2 \end{pmatrix} \tag{14}$$

To simplify, we assume that $x_1 \in [0,6]$ and $x_2 \in [0,20]$. Clearly, any range for x_1 and x_2 can be assumed to build a neutrosophic model. Equation 14 can be written as

$$\dot{x} = \begin{bmatrix} x_1x_2 + 2x_2 & 2 \\ x_2^3 + 2x_1x_2 & 28 \end{bmatrix} X,$$

where $X = [x_1 \quad x_2]^T$ and $x_1x_2+2x_2$ and $x_2^3+2x_1x_2$ are non-linear terms. Define the non-linear terms as follows $z_1 \equiv x_1x_2 + 2x_2$ and $z_2 \equiv x_2^3 + 2x_1x_2$. Then, we have

$$\dot{x} = \begin{bmatrix} z_1 & 2 \\ z_2 & 28 \end{bmatrix} X.$$

Now, we must determine the maximum, minimum and middle values of z_1 and z_2 for $x_1 \in [0,6]$, $x_2 \in [0,20]$. The results are obtained as follows:

$$\max_{x_1,x_2} z_1 = 160, \qquad \min_{x_1,x_2} z_1 = 0, \qquad \operatorname*{mid}_{x_1,x_2} z_1 = 80$$
$$\max_{x_1,x_2} z_2 = 8240, \qquad \min_{x_1,x_2} z_2 = 0, \qquad \operatorname*{mid}_{x_1,x_2} z_2 = 4120$$

The membership functions are labeled as "$\bar{A}_1$ =high," "$\bar{A}_2$ =low," "$\bar{B}_1$ =high," "$\bar{B}_2$ =low" and "$\bar{B}_3$ =middle". The non-linear system 14 is consequently represented by the neutrosophic model outlined below:

***Model Rule 1:*:** IF $\bar{\bar{x}}_1$ is "low" AND $\bar{\bar{x}}_2$ is "high" THEN $\dot{x} = M_1X$.

***Model Rule 2:*:** IF $\bar{\bar{x}}_1$ is "low" AND $\bar{\bar{x}}_2$ is "middle" THEN $\dot{x} = M_2X$.

***Model Rule 3:*:** IF $\bar{\bar{x}}_1$ is "high" AND $\bar{\bar{x}}_2$ is "low" THEN $\dot{x} = M_3X$.

***Model Rule 4:*:** IF $\bar{\bar{x}}_1$ is "high" AND $\bar{\bar{x}}_2$ is "middle" THEN $\dot{x} = M_4X$.

Here,

$$M_1 = \begin{bmatrix} 0 & 2 \\ 0 & 28 \end{bmatrix} \quad M_2 = \begin{bmatrix} 0 & 2 \\ 4120 & 28 \end{bmatrix} \quad M_3 = \begin{bmatrix} 160 & 2 \\ 0 & 28 \end{bmatrix} \quad M_4 = \begin{bmatrix} 160 & 2 \\ 8240 & 28 \end{bmatrix}.$$

Based on the above rules in the Neutrosophic TSK model, the output of each rule is determined by a non-linear function. In this model, the output of each rule can be defined as follows:

Rule 1: IF heating time is "LOW" AND energy consumption is "HIGH", THEN energy efficiency dependent carbon emission $= \begin{cases} \dot{x}_1 = & 2x_2 \\ \dot{x_2} = & 28x_2, \end{cases}$.

Rule 2: IF heating time is "LOW" AND energy consumption is "MIDDLE", THEN energy efficiency dependent carbon emission $= \begin{cases} \dot{x}_1 = & 2x_2 \\ \dot{x_2} = & 4120x_1 + 28x_2, \end{cases}.$

Rule 3: IF heating time is "HIGH" AND energy consumption is "LOW", THEN energy efficiency dependent carbon emission $= \begin{cases} \dot{x}_1 = & 160x_1 + 2x_2 \\ \dot{x_2} = & 28x_2, \end{cases}$

Rule 4: IF heating time is "HIGH" AND energy consumption is "MIDDLE", THEN energy efficiency dependent carbon emission $= \begin{cases} \dot{x}_1 = & 160x_1 + 2x_2 \\ \dot{x_2} = & 4120x_1 + 28x_2, \end{cases}.$

Let neutrosophic linguistic values $\bar{A}_1 =< (0,1,5,6); 0.1, 0.5, 0.9 >$, $\bar{A}_2 =< (0,1,5,6); 0.9, 0.5, 0.1 >$, $\bar{B}_1 =< (0,5,10,20); 0.1, 0.5, 0.9 >$, $\bar{B}_2 =< (0,5,10,20); 0.5, 0.5, 0.5 >$ and $\bar{B}_3 =< (0,5,10,20); 0.9, 0.5, 0.1 >$ be "HIGH", "LOW", "HIGH", "MIDDLE" and "LOW" respectively. Given these values assume that $x_1 \in [1,5]$ and $x_2 \in (10,20)$. Then final output for these intervals calculate using Definition 2.6 and Definition 2.7 as for algebraic product and algebraic sum:

$$
\begin{aligned}
\dot{x}_1 =& \frac{1}{4}\Bigg(\frac{2 + (0.1 \cdot \frac{(20-x_2)0.9}{10}) - (0.5 + (\frac{x_2-10+0.5(20-x_2)}{10}) - (0.5 \cdot (\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \\
& \frac{-(0.9 + (\frac{x_2-10+0.1(20-x_2)}{10}) - (0.9 \cdot (\frac{x_2-10+0.1(20-x_2)}{10})))}{3} \cdot (2x_2) \\
& + \frac{2 + (0.1 \cdot \frac{(20-x_2)0.5}{10}) - (0.5 + (\frac{x_2-10+0.5(20-x_2)}{10}) - (0.5 \cdot (\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \\
& \frac{-(0.9 + (\frac{x_2-10+0.5(20-x_2)}{10}) - (0.9 \cdot (\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \cdot (2x_2) \\
& + \frac{2 + (0.9 \cdot \frac{(20-x_2)0.1}{10}) - (0.5 + (\frac{x_2-10+0.5(20-x_2)}{10}) - (0.5 \cdot (\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \\
& \frac{-(0.1 + (\frac{x_2-10+0.9(20-x_2)}{10}) - (0.1 \cdot (\frac{x_2-10+0.9(20-x_2)}{10})))}{3} \cdot (160x_1 + 2x_2) \\
& + \frac{2 + (0.9 \cdot \frac{(20-x_2)0.5}{10}) - (0.5 + (\frac{x_2-10+0.5(20-x_2)}{10}) - (0.5 \cdot (\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \\
& \frac{-(0.1 + (\frac{x_2-10+0.5(20-x_2)}{10}) - (0.1 \cdot (\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \cdot (160x_1 + 2x_2)\Bigg) \\
& = -7.176x_1x_2 - 0.2357x_2^2 + 40.8x_1 + 1.27665x_2
\end{aligned}
$$

and

$$
\begin{aligned}
\dot{x_2} =& \frac{1}{4}\Bigg(\frac{2+(0.1\cdot\frac{(20-x_2)0.9}{10})-(0.5+(\frac{x_2-10+0.5(20-x_2)}{10})-(0.5\cdot(\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \\
& \frac{-(0.9+(\frac{x_2-10+0.1(20-x_2)}{10})-(0.9\cdot(\frac{x_2-10+0.1(20-x_2)}{10})))}{3}\cdot(8240x_1+28x_2) \\
& +\frac{2+(0.1\cdot\frac{(20-x_2)0.5}{10})-(0.5+(\frac{x_2-10+0.5(20-x_2)}{10})-(0.5\cdot(\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \\
& \frac{-(0.9+(\frac{x_2-10+0.5(20-x_2)}{10})-(0.9\cdot(\frac{x_2-10+0.5(20-x_2)}{10})))}{3}\cdot(4120x_1+28x_2) \\
& +\frac{2+(0.9\cdot\frac{(20-x_2)0.1}{10})-(0.5+(\frac{x_2-10+0.5(20-x_2)}{10})-(0.5\cdot(\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \\
& \frac{-(0.1+(\frac{x_2-10+0.9(20-x_2)}{10})-(0.1\cdot(\frac{x_2-10+0.9(20-x_2)}{10})))}{3}\cdot(28x_2) \\
& +\frac{2+(0.9\cdot\frac{(20-x_2)0.5}{10})-(0.5+(\frac{x_2-10+0.5(20-x_2)}{10})-(0.5\cdot(\frac{x_2-10+0.5(20-x_2)}{10})))}{3} \\
& \frac{-(0.1+(\frac{x_2-10+0.5(20-x_2)}{10})-(0.1\cdot(\frac{x_2-10+0.5(20-x_2)}{10})))}{3}\cdot(4120x_1+28x_2)\Bigg) \\
& = -586.52x_1x_2+4085.598x_1+17.8731x_2-3.2998x_2^2
\end{aligned}
$$

Similarly the values of $\dot{x}_1$ and $\dot{x_2}$ are calculated for each interval;

- $x_1 \in (0,1)$, $x_2 \in (0,5)$,

$$\dot{x}_1 = -1.336x_1^2x_2 - 0.03335x_1x_2^2 + 74.664x_1^2 + 9.86398x_1x_2 + 0.2683x_2^2 - 106.664x_1 - 2.6665x_2$$

$$\dot{x_2} = 2271.233x_1^2 + 10.5231x_2^2 + 881.6331x_1x_2 - 360.5703x_1^2x_2 - 1.3862x_1x_2^2 - 825.2457x_1 - 56.2497x_2$$

- $x_1 \in [1,5]$, $x_2 \in [5,10]$,

$$\dot{x}_1 = 21.06665x_1 + 0.393275x_2$$

$$\dot{x_2} = 2190.398x_1 + 12.8793x_2$$

- $x_1 \in (5,6)$, $x_2 \in (10,20)$,

$$\dot{x}_1 = -0.664x_1^2x_2 - 0.0166x_1x_2^2 - 43.15x_1^2 - 9.21x_1x_2 - 0.12365x_2^2 - 53.868x_1 + 0.326675x_2$$

$$\dot{x_2} = -1378.842x_1^2 - 4.52845x_2^2 - 208.7149x_1x_2 - 179.529x_1^2x_2 - 0.7678x_1x_2^2 + 4420.018x_1 + 3.6372x_2$$

- $x_1 \in (0,1)$, $x_2 \in [5,10]$,

$$\dot{x}_1 = -0.2433x_1x_2 - 33.464x_1^2 - 1.16665x_2 - 64x_1$$

$$\dot{x_2} = -1854x_1 + 643.6055x_1x_2 - 22.8662x_2 + 2.2169x_2^2 - 360.5x_1^2$$

- $x_1 \in [1,5]$, $x_2 \in (0,5)$,

$$\dot{x}_1 = 7.2x_1x_2 + 0.23335x_2^2 - 32x_1 - 1.3333x_2$$

$$\dot{x_2} = 590.602x_1x_2 - 3420.897x_1 - 25.1993x_2 + 3.2669x_2^2$$

- $x_1 \in [1, 5]$, $x_2 \in (10, 20)$,

$$\dot{x}_1 = 7.176x_1x_2 - 0.2357x_2^2 + 40.8x_1 + 1.27665x_2$$
$$\dot{x_2} = -586.52x_1x_2 + 4085.598x_1 + 17.8731x_2 - 3.2998x_2^2$$

- $x_1 \in (5, 6)$, $x_2 \in [5, 10]$,

$$\dot{x}_1 = -0.6334x_1x_2 - 25.336x_1^2 + 0.400025x_2 + 4.668x_1$$
$$\dot{x_2} = 1407.804x_1 - 3577.402x_1^2 + 5.6007x_2 - 652.402x_2^2 - 8.8676x_1x_2$$

- $x_1 \in (0, 1)$, $x_2 \in (10, 20)$,

$$\dot{x}_1 = 0.3812x_1^2x_2 + 0.01305x_1x_2^2 + 16.008x_1^2 - 1.2308x_1x_2 - 0.06665x_2^2 - 21.332x_1 - 0.14665x_2$$
$$\dot{x_2} = 494.4x_1 + 71.791x_1^2 + 532.607x_2^2 - 184.9292x_1x_2 + 24.102x_1^2x_2 + 0.1827x_1x_2^2 - 2.0531x_2$$

- $x_1 \in (10, 20)$, $x_2 \in (0, 1)$,

$$\dot{x}_1 = 12.04535x_1x_2 + 0.19535x_2^2 - 0.0357x_1x_2^2 - 74.664x_1^2 - 1.816x_1^2x_2 - 234.664x_1 - 3.266625x_2$$
$$\dot{x_2} = -3433.196x_1 - 9828.299x_1^2 + 703.2566x_1x_2 - 73.542x_1^2x_2 - 45.7324x_2 + 2.7304x_2^2 - 0.4998x_1x_2^2$$

and the following graphs are obtained based on these functions:

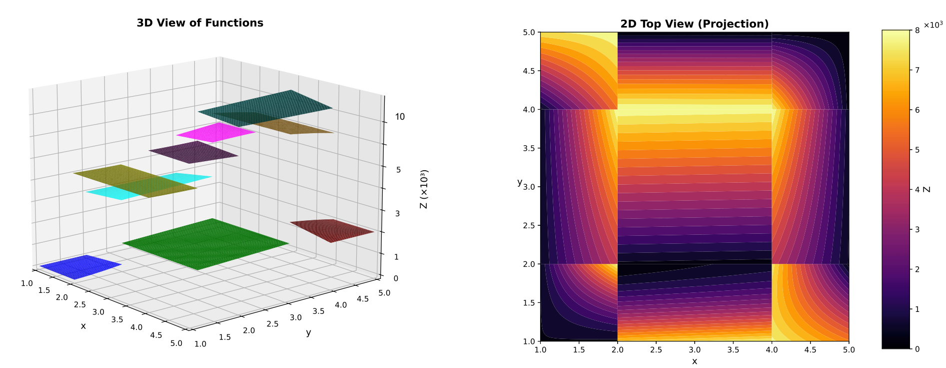


FIGURE 2. value of $\dot{x_1}$ based on $x_1 \in (0, 6)$

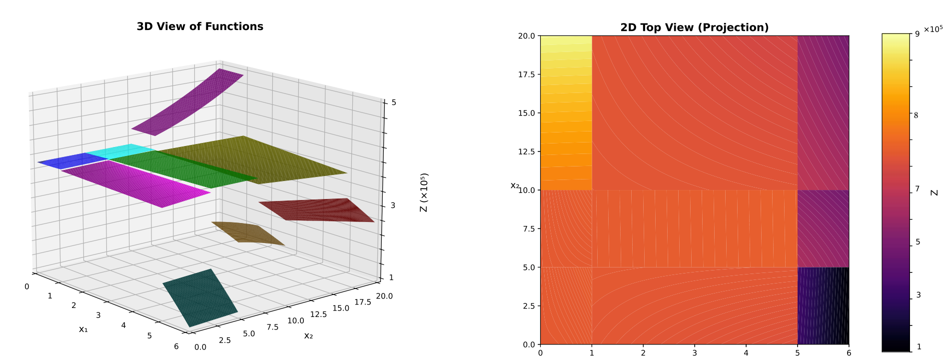


FIGURE 3. value of $\dot{x_2}$ based on $x_2 \in (0, 20)$

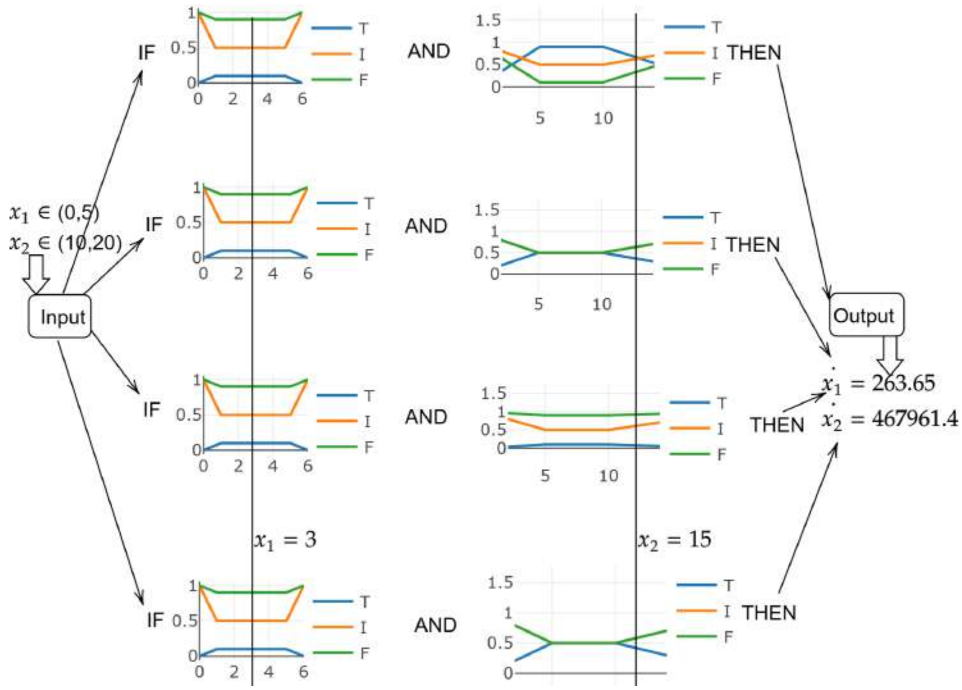


FIGURE 4. Neutrosophic TSK Inference System

5. Comparison and Analysis Discussion

The neutrosophic TSK model gives more precision, flexibility and compatibility compared to the classical, fuzzy and/or intuitionistic fuzzy models. The feasibility and effectiveness of the proposed decision-making approach are verified by a comparison analysis using intuitionistic environments with those methods used by Mehran [61], [54, 58, 59] as shown in Table 1.

TABLE 1. Comparison of the different TSK model

Model	Co-Domain	Non-membership	Indeterminacy	Membership	t and s norms
The model in [61]	$[0,1]$	No	No	Yes	No
The model in [54]	$[0,1]$	No	No	Yes	No
The model in [58]	$[0,1]$	No	No	Yes	No
The model in [59]	$[0,1]$	No	No	Yes	No
The model in [69]	$[0,1]^2$	Yes	No	Yes	Yes
Proposed Model	$[0,1]^3$	Yes	Yes	Yes	Yes

6. Conclusions

The proposed neutrosophic TSK model provides a powerful framework for handling uncertainty in real-world scenarios. The model effectively builds inferential relationships using past experiences and data and provides meaningful solutions for future events using IF-THEN rules. The included t and s norms enhance the model's capability in nonlinear systems, making it an invaluable tool for complex decision-making processes. The application to energy consumption demonstrated the validity and practical importance of the model. Moreover, its extension to n-input systems opens up wider application opportunities in many different fields.

Further, the MCDM problems under other neutrosophic environments would also be studied near future. In future, we plan to extend our research work to the extensions of Neutrosophic TSK model , and so on.

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