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Neutrosophic Hyperideals of Semihyperrings

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Abstract. In this paper, we have introduced neutrosophic hyperideals of a semihyperring and considered some operations on them to study its basic notions and properties.

Keywords: Cartesian Product, Composition, Ideal, Intersection, Neutrosophic, Semihyperrimg.

1 Introduction

Hyperrings extend the classical notion of rings, substituting both or only one of the binary operations of addition and multiplication by hyperoperations. Hyperrings were introduced by several authors in different ways. If only the addition is a hyperoperation and the multiplication is a binary operation, then we say that R is a Krasner hyperring [4]. Davvaz [5] has defined some relations in hyperrings and proved isomorphism theorems. For a more comprehensive introduction about hyperrings, we refer to [9]. As a generalization of a ring, semiring was introduced by Vandiver [17] in 1934. A semiring is a structure $(R;+;\cdot;0)$ with two binary operations $+$ and $\cdot$ such that $(R;+;0)$ is a commutative semigroup, $(R;\cdot)$ a semigroup, multiplication is distributive from both sides over addition and $0\cdot x = 0 = x\cdot 0$ for all $x \in R$. In [18], Vougiouklis generalizes the notion of hyperring and named it as semihyperring, where both the addition and multiplication are hyperoperation. Semihyperrings are a generalization of Krasner hyperrings. Note that a semiring with zero is a semihyperring. Davvaz in [12] studied the notion of semihyperrings in a general form.

Hyperstructures, in particular hypergroups, were introduced in 1934 by Marty [11] at the eighth congress of Scandinavian Mathematicians. The notion of algebraic hyperstructure has been developed in the following decades and nowadays by many authors, especially Corsini [2, 3], Davvaz [5, 6, 7, 8, 9], Mittas [12], Spartalis [15], Stratigopoulos [16] and Vougiouklis [19]. Basic definitions and notions concerning hyperstructure theory can be found in [2].

The concept of a fuzzy set, introduced by Zadeh in his classical paper [20], provides a natural framework for generalizing some of the notions of classical algebraic structures.As a generalization of fuzzy sets, the intuitionistic fuzzy set was introduced by Atanassov [1] in 1986, where besides the degree of membership of each element there was considered a degree of non-membership with (membership value + non-membership value)≤ 1. There are also several well-known theories, for instances, rough sets, vague sets, interval-valued sets etc. which can be considered as mathematical tools for dealing with uncertainties.

In 2005, inspired from the sport games (winning/tie/ defeating), votes, from (yes /NA /no),from decision making(making a decision/ hesitating/not making), from (accepted /pending /rejected) etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, F. Smarandache [14] combined the non-standard analysis [8,18] with a tri-component logic/set/probability theory and with philosophy and introduced Neutrosophic set which represents the main distinction between fuzzy and intuitionistic fuzzy logic/set. Here he included the middle component, i.e., the neutral/ indeterminate/ unknown part (besides the truth/membership and falsehood/non-membership components that both appear in fuzzy logic/set) to distinguish between 'absolute membership and relative membership' or 'absolute non-membership and relative non-membership'.

Using this concept, in this paper, we have defined neutrosophic ideals of semihyperrings and study some of its basic properties.

2 Preliminaries

Let H be a non-empty set and let $P(H)$ be the set of all non-empty subsets of H . A hyperoperation on H is a map $\circ : H \times H \to P(H)$ and the couple $(H,\circ)$ is called a hypergroupoid.
If A and B are non-empty subsets of H and $x \in H$, then we denote $A\circ B = \bigcup_{a\in A, b\in B} a\circ b$,

$x \circ A = \{x\} \circ A$ and $A \circ x = A \circ \{x\}$. A hypergroupoid $(H, \circ)$ is called a semihypergroup if for all $x, y, z \in H$ we have $(x \circ y) \circ z = x \circ (y \circ z)$ which means that $\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$.

A semihyperring is an algebraic structure $(R;+;\cdot)$ which satisfies the following properties:
(i) $(R;+)$ is a commutative semihypergroup
(ii) $(R;\cdot)$ is a semihypergroup
(iii) Multiplication is distributive with respect to hyperoperation $+$ that is $x \cdot (y+z) = x \cdot y + x \cdot z$, $(x+y) \cdot z = x \cdot z + y \cdot z$
(iv) $0 \cdot x = 0 = x \cdot 0$ for all $x \in R$.

A semihyperring $(R;+;\cdot)$ is called commutative if and only if $a \cdot b = b \cdot a$ for all $a, b \in R$.
Vougiouklis in [18] and Davvaz in [6] studied the notion of semihyperrings in a general form, i.e., both the sum and product are hyperoperations.

A semihyperring $(R;+;\cdot)$ with identity $1_R \in R$ means that $1_R \cdot x = x \cdot 1_R = x$ for all $x \in R$.
An element $x \in R$ is called unit if there exists $y \in R$ such that $1_R = x \cdot y = y \cdot x$.
A nonempty subset S of a semihyperring $(R;+;\cdot)$ is called a sub-semihyperring if $a + b \subseteq S$ and $a \cdot b \subseteq S$ for all $a, b \in S$. A left hyperideal of a semihyperring R is a non-empty subset I of R satisfying

(i) If $a, b \in I$ then $a + b \subseteq I$
(ii) If $a \in I$ and $s \in R$ then $s \cdot a \subseteq I$
(iii) $I \neq R$.

A right hyperideal of R is defined in an analogous manner and an hyperideal of R is a nonempty subset which is both a left hyperideal and a right hyperideal of R.
For more results on semihyperrings and neutrosophic sets we refer to [6, 10] and [14] respectively.

3. Main Results

Definition 3.1. [14] A neutrosophic set A on the universe of discourse X is defined as $A = \{< x : A^T(x), A^I(x), A^F(x) >, x \in X\}$ where $A^T, A^I, A^F : X \to]^-0,1[^+$ and $^-0 \leq A^T(x) + A^I(x) + A^F(x) \leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $^-]0,1[^+$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $^-]0,1[^+$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].
Throughout this section unless otherwise mentioned R denotes a semihyperring.

Definition 3.2. Let $\mu = (\mu^T, \mu^I, \mu^F)$ be a non empty neutrosophic subset of a semihyperring R (i.e. anyone of $\mu^T(x)$, $\mu^I(x)$ or $\mu^F(x)$ not equal to zero for some $x \in R$). Then μ is called a neutrosophic left hyperideal of R if

(i) $\inf_{z \in x+y} \mu^T(z) \geq \min\{\mu^T(x), \mu^T(y)\}$,

(ii) $\inf_{z \in x+y} \mu^I(z) \geq \dfrac{\mu^I(x) + \mu^I(y)}{2}$,

(iii) $\sup_{z \in x+y} \mu^F(z) \leq \max\{\mu^F(x), \mu^F(y)\}$,

(iv) $\inf_{z \in xy} \mu^T(z) \geq \mu^T(y)$,

(v) $\inf_{z \in xy} \mu^I(z) \geq \mu^I(y)$,

(vi) $\sup_{z \in xy} \mu^F(z) \leq \mu^F(y)$.

for all $x, y \in R$.
Similarly we can define neutrosophic right hyperideal of R.

Example 3.3. Let $R = \{0, a, b, c\}$ be a set with the hyperoperation $\oplus$ and the multiplication $\bullet$ defined as follows:

$\oplus$	0	a	b	c
0	0	a	b	c
a	a	{a,b}	b	c
b	b	b	{0,b}	c
c	c	c	c	{0,c}

and

$\bullet$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	b	c

c	0	a	c	c

Then $(R,\oplus,\bullet)$ is a semihyperring.

Define neutrosophic subset μ of R by $\mu(0)=(1,0.6,0.1)$, $\mu(a)=(0.7,0.4,0.3)$, $\mu(b)=(0.8,0.5,0.2)$ $\mu(c)=(0.6,0.2,0.4)$. Then μ is a neutrosophic left hyperideal of R.

Theorem 3.4. A neutrosophic set μ of R is a neutrosophic left hyperideal of R if and only if any level subsets $\mu_t^T := \{x\in R:\mu^T(x)\geq t, t\in[0.1]\}$, $\mu_t^I := \{x\in R:\mu^I(x)\geq t, t\in[0.1]\}$ and $\mu_t^F := \{x\in R:\mu^T(x)\leq t, t\in[0.1]\}$ are left hyperideals of R.

Proof. Assume that the neutrosophic set μ of R is a neutrosophic left hyperideal of R.

Then anyone of μ^T, μ^I or μ^F is not equal to zero for some $x\in R$ i.e., in other words anyone of μ_t^T, μ_t^I or μ_t^F is not empty for some $t\in[0,1]$. So, it is sufficient to consider that all of them are not empty.

Suppose $x,y\in\mu_t=(\mu_t^T,\mu_t^I,\mu_t^F)$ and $s\in R$.Then

$$\inf_{z\in x+y}\mu^T(z)\geq\min\{\mu^T(x),\mu^T(y)\}=\min\{t,t\}=t$$

$$\inf_{z\in x+y}\mu^I(z)\geq\frac{\mu^I(x)+\mu^I(y)}{2}\geq\frac{t+t}{2}=t$$

$$\sup_{z\in x+y}\mu^F(z)\leq\max\{\mu^F(x),\mu^F(y)\}\leq\max\{t,t\}=t$$

which implies $x+y\subseteq\mu_t^T,\mu_t^I,\mu_t^F$ i.e., $x+y\subseteq\mu_t$.

Also

$$\inf_{z\in sx}\mu^T(z)\geq\mu^T(x)\geq t,$$

$$\inf_{z\in sx}\mu^I(z)\geq\mu^I(x)\geq t,$$

$$\sup_{z\in sx}\mu^F(z)\leq\mu^F(x)\leq t,$$

Hence $sx\subseteq\mu_t$.

Therefore μ_t is a left hyperideal of R.

Conversely, suppose $\mu_t(\neq\phi)$ is a left hyperideal of R. If possible μ is not a neutrosophic left hyperideal. Then for $x,y\in R$ anyone of the following inequality is true.

$$\inf_{z\in x+y}\mu^T(z)<\min\{\mu^T(x),\mu^T(y)\}$$

$$\inf_{z\in x+y}\mu^I(z)<\frac{\mu^I(x)+\mu^I(y)}{2}$$

$$\sup_{z\in x+y}\mu^F(z)>\max\{\mu^F(x),\mu^F(y)\}$$

For the first inequality, choose $t_1=\frac{1}{2}[\inf_{z\in x+y}\mu^T(z)+\min\{\mu^T(x),\mu^T(y)\}]$. Then $\inf_{z\in x+y}\mu^T(z)<t_1<\min\{\mu^T(x),\mu^T(y)\}$ which implies $x,y\in\mu_{t_1}^T$ but $x+y\not\subseteq\mu_{t_1}^T$ - a contradiction.

For the second inequality, choose $t_2=\frac{1}{2}[\inf_{z\in x+y}\mu^I(z)+\min\{\mu^I(x),\mu^I(y)\}]$. Then $\inf_{z\in x+y}\mu^I(z)<t_2<\frac{\mu^I(x)+\mu^I(y)}{2}$ which implies $x,y\in\mu_{t_2}^I$ but $x+y\not\subseteq\mu_{t_2}^I$ - a contradiction.

For the third inequality, choose $t_3=\frac{1}{2}[\sup_{z\in x+y}\mu^F(z)+\max\{\mu^F(x),\mu^F(y)\}]$. Then $\sup_{z\in x+y}\mu^F(z)>t_3>\max\{\mu^F(x),\mu^F(y)\}$ which implies $x,y\in\mu_{t_3}^F$ but $x+y\not\subseteq\mu_{t_3}^F$ - a contradiction.

So, in any case we have a contradiction to the fact that μ_t is a left hyperideal of R.

Hence the result follows.

Definition 3.5. Let μ and ν be two neutrosophic subsets of R. The intersection of μ and is ν defined by

$$(\mu^T\cap\nu^T)(x)=\min\{\mu^T(x),\nu^T(x)\}$$

$$(\mu^I\cap\nu^I)(x)=\min\{\mu^I(x),\nu^I(x)\}$$

$$(\mu^F\cap\nu^F)(x)=\max\{\mu^F(x),\nu^F(x)\}$$

for all $x\in R$.

Proposition 3.6. Intersection of a nonempty collection of neutrosophic left hyperideals is a neutrosophic left hyperideal of R.

Proof. Let $\{\mu_i : i\in I\}$ be a non-empty family of neutrosophic left hyperideals of R and $x,y\in R$. Then

$$\inf_{z\in x+y}(\bigcap_{i\in I}\mu_i^T)(z)$$

$$=\inf_{z\in x+y}\inf_{i\in I}\mu_i^T(z)$$

$\geq \inf_{i\in I}\{\min\{\mu_i^T(x),\mu_i^T(y)\}\}$
$=\min\{\inf_{i\in I}\mu_i^T(x),\inf_{i\in I}\mu_i^T(y)\}$
$=\min\{\bigcap_{i\in I}\mu_i^T(x),\bigcap_{i\in I}\mu_i^T(y)\}$

$\inf_{z\in x+y}(\bigcap_{i\in I}\mu_i^I)(z)$
$=\inf_{z\in x+y}\inf_{i\in I}\mu_i^I(z)$
$\geq \inf_{i\in I}\dfrac{\mu_i^I(x)+\mu_i^I(y)}{2}$
$=\dfrac{\inf_{i\in I}\mu_i^I(x)+\inf_{i\in I}\mu_i^I(y)}{2}$
$=\dfrac{\bigcap_{i\in I}\mu_i^I(x)+\bigcap_{i\in I}\mu_i^I(y)}{2}$.

$\sup_{z\in x+y}(\bigcap_{i\in I}\mu_i^F)(z)$
$=\sup_{z\in x+y}\sup_{i\in I}\mu_i^F(z)$
$\leq \sup_{i\in I}\{\max\{\mu_i^F(x),\mu_i^F(y)\}\}$
$=\max\{\sup_{i\in I}\mu_i^F(x),\sup_{i\in I}\mu_i^F(y)\}$
$=\max\{\bigcap_{i\in I}\mu_i^F(x),\bigcap_{i\in I}\mu_i^F(y)\}$

$\inf_{z\in sx}(\bigcap_{i\in I}\mu_i^T)(z)$
$=\inf_{z\in sx}\inf_{i\in I}\mu_i^T(z)$
$\geq \inf_{i\in I}\mu_i^T(x)$
$=\bigcap_{i\in I}\mu_i^T(x)$
$\inf_{z\in sx}(\bigcap_{i\in I}\mu_i^I)(z)$
$=\inf_{z\in sx}\inf_{i\in I}\mu_i^I(z)$
$\geq \inf_{i\in I}\mu_i^I(x)$
$=\bigcap_{i\in I}\mu_i^I(x)$

$\sup_{z\in sx}(\bigcap_{i\in I}\mu_i^F)(z)$
$=\sup_{z\in sx}\sup_{i\in I}\mu_i^F(z)$
$\leq \sup_{i\in I}\mu_i^F(x)=\bigcap_{i\in I}\mu_i^F(x)$

Hence $\bigcap_{i\in I}\mu_i$ is a neutrosophic left hyperideal of R.

Definition 3.7. Let R, S be semihyperrings and $f:R\to S$ be a function. Then f is said to be a homomorphism if for all $a,b\in R$
$(i)\, f(a+b)\subseteq f(a)+f(b)$
$(ii)\, f(ab)\subseteq f(a)f(b)$
$(iii)\, f(0_R)=0_S$
where 0_R and 0_S are the zeros of R and S respectively.

Proposition 3.8. Let $f:R\to S$ be a morphism of semihyperrings. Then
(i) If ϕ is a neutrosophic left hyperideal of S, then $f^{-1}(\phi)$ [13] is a neutrosophic left hyperideal of R.
(ii) If f is surjective morphism and μ is a neutronsophic left hyperideal of R, then $f(\mu)$ [13] is a neutrosophic left hyperideal of S.

Proof. Let $f:R\to S$ be a morphism of semihyperrings. Let ϕ be a neutrosophic left hyperideal of S and $r,s\in R$.
$\inf_{z\in r+s} f^{-1}(\phi^T)(z)$
$=\inf_{z\in r+s}\phi^T(f(z))$
$=\inf_{f(z)\subseteq f(r)+f(s)}\phi^T(f(z))$
$\geq \min\{\phi^T(f(r)),\phi^T(f(s))\}$
$=\min\{f^{-1}(\phi^T)(r),f^{-1}(\phi^T)(s)\}$.
$\inf_{z\in r+s} f^{-1}(\phi^I)(z)$
$=\inf_{z\in r+s}\phi^I(f(z))$
$=\inf_{f(z)\subseteq f(r)+f(s)}\phi^I(f(z))$

$$\geq \frac{\phi^I(f(r)) + \phi^I(f(s))}{2}$$
$$= \frac{f^{-1}(\phi^I)(r) + f^{-1}(\phi^I)(s)}{2}.$$
$$\sup_{z \in r+s} f^{-1}(\phi^F)(z)$$
$$= \sup_{z \in r+s} \phi^F(f(z))$$
$$= \sup_{f(z) \subseteq f(r)+f(s)} \phi^F(f(z))$$
$$\leq \max\{\phi^F(f(r)), \phi^F(f(s))\}$$
$$\leq \max\{f^{-1}(\phi^F)(r), f^{-1}(\phi^F)(s)\}.$$

Again

$$\inf_{z \in rs} f^{-1}(\phi^T)(z)$$
$$= \inf_{z \in rs} \phi^T(f(z))$$
$$= \inf_{f(z) \subseteq f(r)f(s)} \phi^T(f(z))$$
$$\geq \phi^T(f(s)) = f^{-1}(\phi^T)(s).$$

$$\inf_{z \in rs} f^{-1}(\phi^I)(z)$$
$$= \inf_{z \in rs} \phi^I(f(z))$$
$$= \inf_{f(z) \subseteq f(r)f(s)} \phi^I(f(z))$$
$$\geq \phi^I(f(s)) = f^{-1}(\phi^I)(s).$$

$$\sup_{z \in rs} f^{-1}(\phi^F)(z)$$
$$= \sup_{z \in rs} \phi^F(f(z))$$
$$= \sup_{f(z) \subseteq f(r)f(s)} \phi^F(f(z))$$
$$\leq \phi^F(f(s)) = f^{-1}(\phi^F)(s).$$

Thus $f^{-1}(\phi)$ is a neutrosophic left hyperideal of R.

(ii) Suppose μ be a neutrosophic left hyperideal of R and $x', y' \in S$. Then

$$\inf_{z' \in x'+y'} (f(\mu^T))(z')$$
$$= \inf_{z' \in x'+y'} \sup_{z \in f^{-1}(z')} \mu^T(z)$$
$$= \inf_{z' \in x'+y'} \sup_{x \in f^{-1}(x'), y \in f^{-1}(y')} \mu^T(z)$$
$$\geq \sup_{x \in f^{-1}(x'), y \in f^{-1}(y')} \{\min\{\mu^T(x), \mu^T(y)\}\}$$
$$= \min\{\sup_{x \in f^{-1}(x')} \mu^T(x), \sup_{y \in f^{-1}(y')} \mu^T(y)\}$$
$$= \min\{(f(\mu^T))(x'), (f(\mu^T))(y')\}.$$
$$\inf_{z' \in x'+y'} (f(\mu^I))(z')$$
$$= \inf_{z' \in x'+y'} \sup_{z \in f^{-1}(z')} \mu^I(z)$$
$$= \inf_{z' \in x'+y'} \sup_{x \in f^{-1}(x'), y \in f^{-1}(y')} \mu^I(z)$$
$$\geq \sup_{x \in f^{-1}(x'), y \in f^{-1}(y')} \frac{\mu^I(x) + \mu^I(y)}{2}$$
$$= \frac{1}{2}[\sup_{x \in f^{-1}(x')} \mu^I(x) + \sup_{y \in f^{-1}(y')} \mu^I(y)]$$
$$= \frac{1}{2}[(f(\mu^I))(x') + (f(\mu^I))(y')].$$
$$\sup_{z' \in x'+y'} (f(\mu^F))(z')$$
$$= \sup_{z' \in x'+y'} \inf_{z \in f^{-1}(z')} \mu^F(z)$$
$$\leq \sup_{z' \in x'+y'} \inf_{x \in f^{-1}(x'), y \in f^{-1}(y')} \mu^F(z)$$
$$\leq \inf_{x \in f^{-1}(x'), y \in f^{-1}(y')} \{\max\{\mu^F(x), \mu^F(y)\}\}$$
$$= \max\{\inf_{x \in f^{-1}(x')} \mu^F(x), \inf_{y \in f^{-1}(y')} \mu^F(y)\}$$
$$= \max\{(f(\mu^F))(x'), (f(\mu^F))(y')\}.$$

Again

$$\inf_{z' \in x'y'} (f(\mu^T))(z')$$
$$= \inf_{z' \in x'y'} \sup_{z \in f^{-1}(z')} \mu^T(z)$$
$$= \sup_{x \in f^{-1}(x'), y \in f^{-1}(y')} \mu^T(z)$$
$$\geq \sup_{y \in f^{-1}(y')} \mu^T(y) = (f(\mu^T))(y').$$

$$\inf_{z' \in x'y'} (f(\mu^I))(z')$$
$$= \inf_{z' \in x'y'} \sup_{z \in f^{-1}(z')} \mu^I(z)$$

$$= \sup_{x\in f^{-1}(x'),y\in f^{-1}(y')} \mu^I(z)$$
$$\geq \sup_{y\in f^{-1}(y')} \mu^I(y) = (f(\mu^I))(y').$$
$$\sup_{z'\in x'y'} (f(\mu^F))(z')$$
$$= \sup_{z'\in x'y'} \inf_{z\in f^{-1}(z')} \mu^F(z)$$
$$\leq \inf_{x\in f^{-1}(x'),y\in f^{-1}(y')} \mu^F(z)$$
$$\leq \inf_{y\in f^{-1}(y')} \mu^F(y) = (f(\mu^F))(y')$$

Thus $f(\mu)$ is a neutrosophic left hyperideal of S.

Definition 3.9. Let μ and ν be two neutrosophic subsets of R. Then the Cartesian product of μ and ν is defined by

$$(\mu^T \times \nu^T)(x,y) = \min\{\mu^T(x), \nu^T(y)\}$$
$$(\mu^I \times \nu^I)(x,y) = \frac{\mu^I(x)+\nu^I(y)}{2}$$
$$(\mu^F \times \nu^F)(x,y) = \max\{\mu^F(x), \nu^F(y)\}$$

for all $x, y \in R$.

Theorem 3.10. Let μ and ν be two neutrosophic left hyperideals of R. Then $\mu\times\nu$ is a neutrosophic left hyperideal of $R\times R$.

Proof. Let (x_1,x_2), $(y_1,y_2) \in R\times R$. Then

$$\inf_{(z_1,z_2)\in(x_1,x_2)+(y_1,y_2)} (\mu^T\times\nu^T)(z_1,z_2)$$
$$= \inf_{z_1\in(x_1+y_1),z_2\in(x_2+y_2)} (\mu^T\times\nu^T)(z_1,z_2)$$
$$= \inf_{z_1\in(x_1+y_1),z_2\in(x_2+y_2)} \min\{\mu^T(z_1),\nu^T(z_2)\}$$
$$\geq \min\{\min\{\mu^T(x_1),\mu^T(y_1)\},\min\{\nu^T(x_2),\nu^T(y_2)\}\}$$
$$= \min\{\min\{\mu^T(x_1),\nu^T(x_2)\},\min\{\mu^T(y_1),\nu^T(y_2)\}\}$$
$$= \min\{(\mu^T\times\nu^T)(x_1,x_2),(\mu^T\times\nu^T)(y_1,y_2)\}.$$

$$\inf_{(z_1,z_2)\in(x_1,x_2)+(y_1,y_2)} (\mu^I\times\nu^I)(z_1,z_2)$$
$$= \inf_{z_1\in(x_1+y_1),z_2\in(x_2+y_2)} (\mu^I\times\nu^I)(z_1,z_2)$$
$$= \inf_{z_1\in(x_1+y_1),z_2\in(x_2+y_2)} \frac{\mu^I(z_1)+\nu^I(z_2)}{2}$$
$$\geq \frac{1}{2}\left[\frac{\mu^I(x_1)+\mu^I(y_1)}{2}+\frac{\nu^I(x_2)+\nu^I(y_2)}{2}\right]$$
$$= \frac{1}{2}\left[\frac{\mu^I(x_1)+\nu^I(x_2)}{2}+\frac{\mu^I(y_1)+\nu^I(y_2)}{2}\right]$$
$$= \frac{1}{2}[(\mu^I\times\nu^I)(x_1,x_2)+(\mu^I\times\nu^I)(y_1,y_2)].$$

$$\sup_{(z_1,z_2)\in(x_1,x_2)+(y_1,y_2)} (\mu^F\times\nu^F)(z_1,z_2)$$
$$= \sup_{z_1\in(x_1+y_1),z_2\in(x_2+y_2)} (\mu^F\times\nu^F)(z_1,z_2)$$
$$= \sup_{z_1\in(x_1+y_1),z_2\in(x_2+y_2)} \max\{\mu^F(z_1),\nu^F(z_2)\}$$
$$\leq \max\{\max\{\mu^F(x_1),\mu^F(y_1)\},\max\{\nu^F(x_2),\nu^F(y_2)\}\}$$
$$= \max\{\max\{\mu^F(x_1),\nu^F(x_2)\},\max\{\mu^F(y_1),\nu^F(y_2)\}\}$$
$$= \max\{(\mu^F\times\nu^F)(x_1,x_2),(\mu^F\times\nu^F)(y_1,y_2)\}.$$

$$\inf_{(z_1,z_2)\in(x_1,x_2)(y_1,y_2)} (\mu^T\times\nu^T)(z_1,z_2)$$
$$= \inf_{z_1\in x_1y_1,z_2\in x_2y_2} (\mu^T\times\nu^T)(z_1,z_2)$$
$$= \inf_{z_1\in x_1y_1,z_2\in x_2y_2} \min\{\mu^T(z_1),\nu^T(z_2)\}$$
$$\geq \min\{\mu^T(y_1),\nu^T(y_2)\} = (\mu^T\times\nu^T)(y_1,y_2).$$

$$\inf_{(z_1,z_2)\in(x_1,x_2)(y_1,y_2)} (\mu^I\times\nu^I)(z_1,z_2)$$
$$= \inf_{z_1\in x_1y_1,z_2\in x_2y_2} (\mu^I\times\nu^I)(z_1,z_2)$$
$$= \inf_{z_1\in x_1y_1,z_2\in x_2y_2} \frac{\mu^I(z_1)+\nu^I(z_2)}{2}$$
$$\geq \frac{\mu^I(y_1)+\nu^I(y_2)}{2} = (\mu^I\times\nu^I)(y_1,y_2).$$
$$\sup_{(z_1,z_2)\in(x_1,x_2)(y_1,y_2)} (\mu^F\times\nu^F)(z_1,z_2)$$
$$= \sup_{z_1\in x_1y_1,z_2\in x_2y_2} (\mu^F\times\nu^F)(z_1,z_2)$$
$$= \sup_{z_1\in x_1y_1,z_2\in x_2y_2} \max\{\mu^F(z_1),\nu^F(z_2)\}$$

$\leq \max\{\mu^F(y_1), \nu^F(y_2)\} = (\mu^F \times \nu^F)(y_1, y_2)$.

Hence $\mu \times \nu$ is a neutrosophic left hyperideal of $R \times R$.

Definition 3.11. Let μ and ν be two neutrosophic sets of a semiring R. Define composition of μ and ν by

$$(\mu^T o \nu^T)(x) = \sup_{x \in \sum_{i=1}^{n} a_i b_i} \{\min_i \{\mu^T(a_i), \nu^T(b_i)\}\}$$

$= 0$ if x cannot be expressed as above

$$(\mu^I o \nu^I)(x) = \sup_{x \in \sum_{i=1}^{n} a_i b_i} \sum_{i=1}^{n} \frac{\mu^I(a_i) + \nu^I(b_i)}{2n}$$

$= 0$ if x cannot be expressed as above

$$(\mu^F o \nu^F)(z) = \inf_{x \in \sum_{i=1}^{n} a_i b_i} \{\max_i \{\mu^F(a_i), \nu^F(b_i)\}\}$$

$= 0$ if x cannot be expressed as above

where $x, a_i, b_i \in R$ for $i = 1, \ldots, n.$

Theorem 3.12. If μ and ν be two neutrosophic left hyperideals of , R then $\mu o \nu$ is a neutrosophic left hyperideal of R.

Proof. Suppose μ, ν be two neutrosophic hyperideals of R and $x, y \in R$. If $x + y \notin \sum_{i=1}^{n} a_i b_i$ for $a_i, b_i \in R,$ then there is nothing to proof. So, assume that $x + y \in \sum_{i=1}^{n} a_i b_i$ for $a_i, b_i \in R,$. Then

$$\inf_{z \in x+y} (\mu^T o \nu^T)(z)$$

$$= \inf_{z \in x+y} \sup_{x+y \in \sum_{i=1}^{n} a_i b_i} \{\min_i \{\mu^T(a_i), \nu^T(b_i)\}\}$$

$$\geq \sup_{x \in \sum_{i=1}^{n} c_i d_i, y \in \sum_{i=1}^{n} e_i f_i} \{\min_i \{\mu^T(c_i), \nu^T(d_i), \mu^T(e_i), \nu^T(f_i)\}\}$$

$$= \min\{\sup_{x \in \sum_{i=1}^{n} c_i d_i} \{\min_i \{\mu^T(c_i), \nu^T(d_i)\}\},$$

$$\sup_{y \in \sum_{i=1}^{n} e_i f_i} \{\min_i \{\mu^T(e_i), \nu^T(f_i)\}\}\}$$

$$= \min\{(\mu^T o \nu^T)(x), (\mu^T o \nu^T)(y)\}$$

$$\inf_{z \in x+y} (\mu^I o \nu^I)(z)$$

$$= \inf_{z \in x+y} \sup_{x+y \in \sum_{i=1}^{n} a_i b_i} \sum_{i=1}^{n} \frac{\mu^I(a_i) + \nu^I(b_i)}{2n}$$

$$\geq \sup_{x \in \sum_{i=1}^{n} c_i d_i, y \in \sum_{i=1}^{n} e_i f_i} \sum_{i=1}^{n} \frac{\mu^I(c_i) + \nu^I(d_i) + \mu^I(e_i) + \nu^I(f_i)}{4n}$$

$$\geq \frac{1}{2}[\sup_{x \in \sum_{i=1}^{n} c_i d_i,} \sum_{i=1}^{n} \frac{\mu^I(c_i) + \nu^I(d_i)}{2n},$$

$$\sup_{y \in \sum_{i=1}^{n} e_i f_i,} \sum_{i=1}^{n} \frac{\mu^I(e_i) + \nu^I(f_i)}{2n}]$$

$$= \frac{(\mu^I o \nu^I)(x) + (\mu^I o \nu^I)(y)}{2}$$

$$\sup_{z \in x+y} (\mu^F o \nu^F)(z)$$

$$= \sup_{z \in x+y} \inf_{x+y \in \sum_{i=1}^{n} a_i b_i} \{\max_i \{\mu^F(a_i), \nu^F(b_i)\}\}$$

$$\leq \inf_{x \in \sum_{i=1}^{n} c_i d_i, y \in \sum_{i=1}^{n} e_i f_i} \{\max_i \{\mu^F(c_i), \nu^F(d_i), \mu^F(e_i), \nu^F(f_i)\}\}$$

$$= \max\{\inf_{x \in \sum_{i=1}^{n} c_i d_i} \{\max_i \{\mu^F(c_i), \nu^F(d_i)\}\},$$

$$\inf_{y \in \sum_{i=1}^{n} e_i f_i} \{\max_i \{\mu^F(e_i), \nu^F(f_i)\}\}\}$$

$$= \max\{(\mu^F o \nu^F)(x), (\mu^F o \nu^F)(y)\}$$

$$\inf_{z \in xy} (\mu^T o \nu^T)(z)$$

$$= \inf_{z \in xy} \sup_{xy \in \sum_{i=1}^{n} a_i b_i} \{\min_i \{\mu^T(a_i), \nu^T(b_i)\}\}$$

$$\geq \sup_{z \in xy \in \sum_{i=1}^{n} xe_i f_i} \{\min_i \{\mu^T(xe_i), \nu^T(f_i)\}\}$$

$$\geq \sup_{y \in \sum_{i=1}^{n} e_i f_i} \{\min_i \{\mu^T(e_i), \nu^T(f_i)\}\}$$

$$= (\mu^T o \nu^T)(y)$$

$$\inf_{z \in xy} (\mu^I o \nu^I)(z)$$

$$= \inf_{z \in xy} \sup_{xy \in \sum_{i=1}^{n} a_i b_i} \sum_{i=1}^{n} \frac{\mu^I(a_i) + \nu^I(b_i)}{2n}$$

$$\geq \sup_{z \in xy \in \sum_{i=1}^{n} xe_i f_i} \sum_{i=1}^{n} \frac{\mu^I(xe_i) + \nu^I(f_i)}{2n}$$

$$\geq \sup_{y \in \sum_{i=1}^{n} e_i f_i,_i} \sum_{i=1}^{n} \frac{\mu^I(e_i) + \nu^I(f_i)}{2n}]$$

$$= (\mu^I o \nu^I)(y)$$

$$\sup_{z \in xy} (\mu^F o \nu^F)(z)$$

$$= \sup_{z \in xy} \inf_{xy \in \sum_{i=1}^{n} a_i b_i} \{\max_i \{\mu^F(a_i), \nu^F(b_i)\}\}$$

$$\leq \inf_{xy \in \sum_{i=1}^{n} xe_i f_i} \{\max_i \{\mu^F(xe_i), \nu^F(f_i)\}\}$$

$$\leq \inf_{y \in \sum_{i=1}^{n} e_i f_i} \{\max_i \{\mu^F(e_i), \nu^F(f_i)\}\}\}$$

$$= (\mu^F o \nu^F)(y).$$

Hence $\mu o \nu$ is a neutrosophic left hyperideal of R.

Conclusion

This is the introductory paper on neutrosophic hyperideals of semihyperrings in the sense of Smarandache[14]. Our next aim to use these results to study some other properties such prime neutrosophic hyperideal, semiprime neutrosophic hyperideal,neutrosophic bi-hyperideal, neutrosophic quasi-hyperideal, radicals etc.

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