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Analysis of Customer Perception of a New Product Using the Neutrosophic Biplot Method

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Abstract: This paper sought to propose and apply the Neutrosophic Biplot analysis method to understand consumer perception of a new technological product with uncertain attributes while addressing the shortcomings that a conventional approach would present. Data was collected through a survey experiment with neutrosophic rating scales where 50 subjects slid on one scale to indicate how much they agreed (A), were neutral, or indifferent (I), and disagreed (D) about each of the eight features of a new smartphone they received. A neutrosophic data matrix was formed and converted into a real-valued matrix using a scoring function, allowing for the application of HJ-Biplot analysis. The two most important findings were that segmentation exists in this market, leading to the conclusions that there are three kinds of consumers: "Technophiles" (those concerned with performance and camera), "Thrifters" (those concerned with price and battery), and "Aesthetes" (those concerned with design) and that software as an attribute has been perceived by consumers with a high degree of indeterminacy. Ultimately, Neutrosophic Biplot is a valuable analysis for perception because by acknowledging uncertainty in consumer response, the analysis creates a more detailed and accurate picture of such information which can help direct corporate strategic decisions better.

Keywords: Neutrosophic logic, Biplot, multivariate analysis, consumer perception, market research, uncertainty, decision making.

1. Introduction

In today's fast-paced market environment, a company's ability to accurately and deeply understand consumer perception has become a fundamental pillar of success in developing and launching new products. The acceptance of an innovation depends not only on its objective technical merits but also on a complex mix of subjective assessments, expectations, and emotions on the part of potential customers. A misinterpretation of these perceptions can lead to poor strategic decisions, resulting in significant financial losses and damage to the brand's reputation.

The main challenge, therefore, lies not in the need to measure, but in the difficulty inherent in the nature of the object being measured. Human perception is inherently vague, ambiguous, and often uncertain. When asked about a new product, individuals often experience doubts, have mixed feelings, or lack a firmly established opinion, especially when faced with novel or complex features. This uncertainty is not "noise" that must be eliminated, but a valuable piece of information that reflects the consumer's actual cognitive and affective state.

Traditionally, market research has addressed this challenge through the extensive use of quantitative surveys employing rating scales, such as the well-known Likert scale. These instruments are appreciated for their ease of administration and apparent simplicity in quantifying opinions, becoming an industry standard [1]. Data collected through these scales are subsequently analyzed using a variety of multivariate statistical techniques designed to reveal underlying patterns and relationships in complex data sets.

Within this arsenal of analytical tools, the Biplot method stands out for its remarkable visualization capabilities. Originally proposed by Gabriel [2], the Biplot allows a simultaneous graphical representation of the rows (individuals) and columns (variables) of a data matrix in a low-dimensional space. This facilitates the visual interpretation of the data structure, the identification of clusters of individuals with similar profiles, and the understanding of the relationships between variables. Later variants, such as the HJ-Biplot, have optimized the technique to ensure high-quality representation for individuals and variables simultaneously [3].

Despite its undeniable power, the classic Biplot paradigm suffers from the same fundamental limitation as traditional rating scales: it requires clear and unambiguous data. The method's mathematical framework, based on Singular Value Decomposition (SVD) of a matrix of real numbers, is not designed to handle doubt or ambiguity. The issue of the midpoint in Likert scales, for example, is a widely debated topic in the methodological literature, since a "neutral" can mean indifference, ignorance, or a conflicting mix of agreement and disagreement, nuances that are irremediably lost when assigning a single numerical value [4].

Fuzzy logic logic , have attempted to address imprecision in data analysis, finding applications in market research to model vague linguistic concepts [5]. While these approaches represent a significant advance by allowing partial degrees of membership, they generally do not explicitly distinguish between the degree of ambiguity and that of indeterminacy or unknowability, merging different facets of uncertainty into a single indicator.

In this context, this research is justified by the pressing need for analytical tools that not only tolerate, but also explicitly embrace and model, the uncertainty inherent in consumer perception. Developing a method that captures an individual's opinion in all its complexity—including certainty, doubt, and denial—would allow organizations to gain a richer, more realistic, and more nuanced view of the market. This, in turn, would strengthen the empirical foundation on which high-impact strategic decisions in marketing and product development are based [6].

To address this need, this study proposes the introduction and application of the Neutrosophic Biplot. This innovative methodology extends the visual power of the classic Biplot by integrating it with the foundations of Neutrosophic Logic. This logic, unlike others, breaks down human assertions into three distinct and independent components: the degree of Truth (T), the degree of Indeterminacy (I), and the degree of Falsehood (F). This three-dimensional structure provides a mathematical formalism ideal for representing a consumer's opinion without forcing them to make an artificially precise choice.

The primary objective of this work is, therefore, to adapt and apply the Neutrosophic Biplot method to conduct a comprehensive analysis of a group of consumers' perception data regarding a new smartphone. Secondary objectives include designing a valuation survey based on neutrosophic scales; identifying and characterizing different consumer segments based not only on their preferences but also on their levels of uncertainty; and interpreting the resulting Biplot map to draw strategic conclusions about which product attributes are key for each segment.

The central hypothesis is that Neutrosophic Biplot analysis will provide a more detailed and meaningful customer segmentation than conventional Biplot analysis. It is postulated that, by incorporating the indeterminacy dimension, consumer profiles that would otherwise remain hidden will emerge, such as the "cautious consumer" or the "informed skeptic." Additionally, it is hypothesized that product attributes that involve greater technical complexity or novelty, such as software specifications or new camera functions, will present significantly higher levels of indeterminacy among respondents [7].

2. Methodology

2.1. Introduction to the HJ-Biplot Approach

The HJ-Biplot is a method for multivariate visualization that enables the joint projection of both individuals and variables into the same factorial plane, achieving high-quality representation for both. In contrast to traditional biplots (like GH and JK variants), the HJ-Biplot aims for a balanced display where the Euclidean distance is maintained accurately in both axes [8].

2.1.1. Mathematical basis of the HJ-Biplot

The HJ-Biplot relies on the singular value decomposition (SVD) of a data matrix $X \in R^{\mathrm{nxp}}$, which is first centered by both rows and columns. This decomposition, expressed as "$X = U\,D\,V^{\mathrm{T}}$", reveals the left singular vectors U, the right singular vectors V, and a diagonal matrix D containing the singular values in descending order. Using this algebraic framework, the HJ-Biplot facilitates the combined graphical display of both individuals and variables within the same factorial plane. It achieves a balanced representation where the Euclidean metric is consistently maintained across both axes, ensuring an optimal joint visualization quality [9,10].

Data Matrix

Let be a data matrix $\mathrm{X} \in \mathbb{R}^{\mathrm{nxp}}$, centered by rows and columns.

Singular Value Decomposition (SVD)

$$\mathbf{X} = \mathbf{U}\,\mathbf{D}\,\mathbf{V}^{\mathrm{T}} \quad (1)$$

Where:

- **OR** $\in \mathbb{R}^{nxr}$: Left singular vectors.
- **D** $\in \mathbb{R}^{rxr}$: diagonal matrix with singular values$d1 \geq d2 \geq \cdots \geq dr > 0$
- **V** $\in \mathbb{R}^{px_r}$
- r = range $(\boldsymbol{X})$.

Marker Selection – HJ-Biplot (Symmetric)

Bookmarks:

$\boldsymbol{A} = \boldsymbol{U}\,\boldsymbol{D}, \quad \mathbf{B} = \mathbf{V}\,\mathbf{D}$

$\mathbf{J} = \mathbf{U}\,\mathbf{D}, \mathbf{H} = \mathbf{V}\,\mathbf{D}$

2.1.2. Mathematical basis of the Neutrosophic HJ-Biplot Dual Extension.

The HJ-Biplot, in its traditional form, serves as a robust multivariate visualization technique. Yet, its reliance on the Singular Value Decomposition (SVD) of a real-valued data matrix ($X \in \mathbb{R}^{nxp}$) introduces a critical constraint: it presumes that input data are exact, unambiguous, and deterministic [11].

This assumption clashes with real-world scenarios, particularly in market research, social sciences, and psychology, where human responses are often uncertain, vague, and indeterminate. Requiring participants to select a fixed value (e.g., rating "7" on a 10-point scale) discards essential insights into their degree of confidence or hesitation [12].

To address this limitation, we introduce the Neutrosophic Biplot, an advanced adaptation of the conventional approach that incorporates neutrosophic logic. This innovation shifts the analysis from a real-number matrix to a neutrosophic data matrix, where each entry encapsulates the full spectrum of a respondent's perspective, including uncertainty.

2.3.1. The Neutrosophic Data Matrix (X~)

The starting point of the Neutrosophic Biplot is the redefinition of the data matrix. Instead of the classical matrix X, a neutrosophic matrix X~ of the same dimensions n × p (individuals × variables) is constructed.

Each element $x_{ij}$of the classical matrix is replaced by a **Single Valued Neutrosophic Number (SVNN)** , N_{ij}, which is represented as a tuple of three components:

$$N_{ij} = \left(t_{ij}, i_{ij}, f_{ij}\right) \qquad (2)$$

Where, for the opinion of individual i on variable (or attribute) j:

- t_{ij} is the **degree of truth** (e.g. degree of agreement, satisfaction, importance).
- i_{ij} is the **degree of indeterminacy** (e.g. degree of doubt, uncertainty, ignorance).
- f_{ij} is the **degree of falsity** (e.g. degree of disagreement, dissatisfaction, unimportance).

These three components are real numbers in the interval $[0,1]$and their sum is unrestricted, allowing both incomplete $(t+i+f<1)$and contradictory information to be modeled $(t+i+f>1)$.. The data matrix therefore becomes:

$$X \cong (t_{11}, i_{11}, f_{11}) \vdots (t_{n1}, i_{n1}, f_{n1}) \cdots \ddots \cdots (t_{1p}, t_{1p}, f_{1p}) \vdots (t_{np}, i_{np}, f_{np}) \quad (3)$$

The central challenge is obvious: the SVD algorithm, the heart of HJ-Biplot, is not defined for a matrix of tuples. To address this, there are two main approaches.

2.3.2. Approach 1: De- neutrosification and Application of the Classical Biplot (Pragmatic Approach)

This approach prioritizes the applicability and use of existing statistical tools. It consists of transforming the neutrosophic matrix X~ into a real data matrix (crisp) XS, on which the standard HJ-Biplot can be applied [13].

Step 1: Aggregation of Neutrosophic Information. A scoring function is used to convert each neutrosophic number N $_{ij}$ into a single scalar value. One of the most common functions is:

$$S(N_{ij}) = t_{ij} - f_{ij}f \quad (4)$$

Or a more nuanced version that considers indeterminacy, such as:

$$S(N_{ij}) = 32 + t_{ij} - i_{ij} - f_{ij} \quad (5)$$

The choice of the scoring function is a key methodological step and must be justified based on the study's objectives. The result is a real-valued data matrix.$X_S = [S(N_{ij})]$.

Step 2: Application of the Standard HJ-Biplot. Once the score matrix X $_{\text{S is obtained}}$, the HJ-Biplot procedure is applied identically to that described in section 1.1.2:

1. X $_S$ is centered .
2. SVD is performed:$X_S = UDVT$.
3. Markers are calculated for individuals $\left(A = \frac{UD1}{2}\right)$and variables$\left(B = \frac{VD1}{2}\right)$.
4. The Biplot graph is generated.

Step 3: Visual Reintroduction of Indeterminacy. One criticism of this approach is that the richness of indeterminacy (i) is "lost" during deneutrosification. To mitigate this, the original information can be reincorporated into the final visualization. For example:

- **Dot size:** The dot representing an individual can be larger the greater its average degree of indeterminacy.
- **Color or transparency:** The points may have a color map (e.g. , from blue to red) that represents the indeterminacy or falsity component.
- **Uncertainty halos:** Each point can be surrounded by a circle or "halo" whose radius is proportional to the value of i.

Advantage: Computational simplicity and easy interpretation of the resulting Biplot map. **Disadvantage:** Loss of information in the calculation process, which is only partially recovered in the visualization.

2.3.3. Approach 2: Adaptation of the SVD Algorithm for Neutrosophic Data (Theoretical Approach)

This approach is theoretically purer and more robust, although computationally more complex. It seeks to adapt the Biplot engine itself (the SVD) to operate directly with neutrosophic numbers [14].

Step 1: Defining Neutrosophic Algebra. For an algorithm like SVD to work, the basic matrix operations (addition, multiplication) for neutrosophic numbers must be defined. Based on the literature on neutrosophic logic, the following operations are defined:

- **Sum of two$SVNNs$**: $N_1 \oplus N_2 = (t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2)$
- **Product of two$SVNNs$**: $N_1 \otimes N_2 = (t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2)$

Step 2: Neutrosophic SVD. The SVD algorithm, which is often solved by iterative methods, would be modified to use the neutrosophic algebra defined in the previous step. The decomposition of the neutrosophic matrix X~ would result in components that are also neutrosophic:

$$X\sim \approx U\sim D\sim V\sim T \quad (6)$$

Where:

- U~ and V~ are matrices whose entries are neutrosophic vectors.
- D~ is a diagonal matrix whose singular values λ~k are also neutrosophic numbers,$\lambda\sim k = (t_{\lambda k}, i_{\lambda k}, f_{\lambda k})$.

Step 3: Markers and Neutrosophic Graphical Representation. The markers for individuals and variables would be, analogously, matrices of neutrosophic numbers [15]. This means that the coordinate of each point in the Biplot is not a pair (x, y), but a pair of neutrosophic numbers:$(a\sim k, b\sim k)$.

Visualizing this result is a challenge in itself. A point is no longer a point, but an "**uncertainty area**" on the plane. It could be graphed:

- An **uncertainty rectangle or ellipse** for each individual and vector, centered on the deneutrosified value (using a score function), and with dimensions reflecting the magnitude of the i and f components on each axis.
- **Three separate points** for each individual (one for T, one for I, one for F), connected to show how each facet of opinion "behaves" in the factor space.

Advantage: Preserves all neutrosophic information throughout the analytical process, potentially offering a much richer and more accurate result. **Disadvantage:** High theoretical and computational complexity. Interpretation of the resulting graph is less straightforward and requires specific training for the reader.

For this study, we will adopt **Approach 1 (Pragmatic)** , as it offers an optimal balance between incorporating uncertainty information and interpretability of results for a broad audience.

3. Application and Results

This section details the empirical procedure used to apply the Neutrosophic Biplot method to the analysis of consumer perceptions of a new product. The data collection process, the construction of the neutrosophic data matrix, and the statistical processing are described, and, finally, a comprehensive interpretation of the resulting Biplot graph is presented.

3.1. Data Collection

To conduct the study, an online survey was designed and administered to a sample of 50 consumers who had had contact with or received prior information about a new smartphone model recently introduced to the market. Participation was voluntary and anonymous. The survey focused on assessing user perceptions of eight key product attributes, identified through a prior analysis of literature and industry reports. The attributes evaluated were: **battery, camera, price, design, display, performance, software, and brand.**

Unlike traditional surveys using Likert scales, a neutrosophic rating question was implemented for each attribute. Participants were asked to allocate 10 points among three statements representing the components of neutrosophic logic:

- **Degree of Agreement (T):** "My level of agreement that this attribute is a strong point of the product."
- **Degree of Indeterminacy (I):** "My level of doubt, uncertainty or lack of information to assess this attribute."
- **Degree of Disagreement (F):** "My level of disagreement, considering that this attribute is a weak point of the product."

For example, a user's response to the attribute "Battery" might $(T = 7, I = 2, F = 1)$,indicate a strong positive perception, with slight doubt and minimal disagreement. These values were normalized to the interval [0, 1] by dividing them by 10, to form the corresponding neutrosophic tuple . $(0.7, 0.2, 0.1)$.This data collection method allows for a more reliable, three-dimensional view of the respondent's opinion.

3.2. Neutrosophic Data Matrix

The responses of the 50 participants for the 8 attributes were consolidated into a neutrosophic data matrix X~ of dimensions 50x8. Each cell (i ,j) of this matrix contains the neutrosophic number N_{ij} = (t_{ij}, i_{ij}, f_{ij})that represents user i's perception of attribute j. An illustrative extract of this matrix is shown below:

Table 1: Neutrosophic data matrix X~ with dimensions 50x8. Source: Prepared by the authors.

User	Battery	Camera	Price	Design	Screen	Performance	Software	Brand
U01	(0.8, 0.1, 0.1)	(0.1, 0.2, 0.7)	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.4, 0.2, 0.4)	(0.1, 0.2, 0.7)	(0.2, 0.6, 0.2)	(0.3, 0.3, 0.4)
U02	(0.7, 0.2, 0.1)	(0.2, 0.2, 0.6)	(0.8, 0.1, 0.1)	(0.3, 0.2, 0.5)	(0.5, 0.2, 0.3)	(0.2, 0.2, 0.6)	(0.3, 0.5, 0.2)	(0.4, 0.2, 0.4)
U03	(0.9, 0.0, 0.1)	(0.1, 0.3, 0.6)	(0.7, 0.2, 0.1)	(0.1, 0.4, 0.5)	(0.3, 0.3, 0.4)	(0.1, 0.3, 0.6)	(0.1, 0.7, 0.2)	(0.2, 0.4, 0.4)
U04	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.9, 0.1, 0.0)	(0.3, 0.3, 0.4)	(0.4, 0.2, 0.4)	(0.1, 0.2, 0.7)	(0.4, 0.5, 0.1)	(0.3, 0.3, 0.4)
U05	(0.7, 0.1, 0.2)	(0.3, 0.1, 0.6)	(0.8, 0.2, 0.0)	(0.4, 0.2, 0.4)	(0.6, 0.1, 0.3)	(0.3, 0.1, 0.6)	(0.3, 0.6, 0.1)	(0.5, 0.2, 0.3)
U06	(0.9, 0.1, 0.0)	(0.1, 0.2, 0.7)	(0.9, 0.0, 0.1)	(0.2, 0.3, 0.5)	(0.3, 0.2, 0.5)	(0.1, 0.2, 0.7)	(0.2, 0.5, 0.3)	(0.3, 0.3, 0.4)
U07	(0.8, 0.2, 0.0)	(0.2, 0.3, 0.5)	(0.7, 0.1, 0.2)	(0.3, 0.3, 0.4)	(0.5, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.4, 0.4, 0.2)	(0.4, 0.3, 0.3)
U08	(0.7, 0.1, 0.2)	(0.1, 0.4, 0.5)	(0.8, 0.1, 0.1)	(0.1, 0.4, 0.5)	(0.4, 0.3, 0.3)	(0.1, 0.4, 0.5)	(0.1, 0.8, 0.1)	(0.2, 0.5, 0.3)
U09	(0.9, 0.0, 0.1)	(0.2, 0.2, 0.6)	(0.9, 0.1, 0.0)	(0.2, 0.2, 0.6)	(0.5, 0.2, 0.3)	(0.2, 0.2, 0.6)	(0.3, 0.6, 0.1)	(0.3, 0.4, 0.3)
U10	(0.8, 0.1, 0.1)	(0.3, 0.3, 0.4)	(0.7, 0.2, 0.1)	(0.3, 0.3, 0.4)	(0.6, 0.2, 0.2)	(0.3, 0.3, 0.4)	(0.4, 0.5, 0.1)	(0.5, 0.3, 0.2)
U11	(0.7, 0.2, 0.1)	(0.1, 0.3, 0.6)	(0.8, 0.1, 0.1)	(0.1, 0.3, 0.6)	(0.4, 0.3, 0.3)	(0.1, 0.3, 0.6)	(0.2, 0.7, 0.1)	(0.2, 0.4, 0.4)
U12	(0.9, 0.1, 0.0)	(0.2, 0.1, 0.7)	(0.9, 0.0, 0.1)	(0.2, 0.2, 0.6)	(0.5, 0.2, 0.3)	(0.2, 0.1, 0.7)	(0.3, 0.5, 0.2)	(0.3, 0.3, 0.4)
U13	(0.8, 0.1, 0.1)	(0.3, 0.2, 0.5)	(0.7, 0.1, 0.2)	(0.3, 0.2, 0.5)	(0.6, 0.1, 0.3)	(0.3, 0.2, 0.5)	(0.4, 0.4, 0.2)	(0.5, 0.2, 0.3)
U14	(0.7, 0.3, 0.0)	(0.1, 0.4, 0.5)	(0.8, 0.2, 0.0)	(0.1, 0.4, 0.5)	(0.4, 0.4, 0.2)	(0.1, 0.4, 0.5)	(0.1, 0.9, 0.0)	(0.2, 0.6, 0.2)

User	Battery	Camera	Price	Design	Screen	Performance	Software	Brand
U15	(0.9, 0.0, 0.1)	(0.2, 0.3, 0.5)	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.5, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.3, 0.6, 0.1)	(0.3, 0.5, 0.2)
U16	(0.8, 0.1, 0.1)	(0.3, 0.1, 0.6)	(0.7, 0.0, 0.3)	(0.3, 0.1, 0.6)	(0.6, 0.1, 0.3)	(0.3, 0.1, 0.6)	(0.4, 0.5, 0.1)	(0.5, 0.2, 0.3)
U17	(0.7, 0.2, 0.1)	(0.1, 0.2, 0.7)	(0.8, 0.1, 0.1)	(0.1, 0.2, 0.7)	(0.4, 0.2, 0.4)	(0.1, 0.2, 0.7)	(0.2, 0.6, 0.2)	(0.2, 0.3, 0.5)
U18	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.5, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.3, 0.4, 0.3)	(0.3, 0.4, 0.3)
U19	(0.8, 0.0, 0.2)	(0.3, 0.4, 0.3)	(0.7, 0.2, 0.1)	(0.3, 0.4, 0.3)	(0.6, 0.3, 0.1)	(0.3, 0.4, 0.3)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)
U20	(0.7, 0.1, 0.2)	(0.1, 0.5, 0.4)	(0.8, 0.1, 0.1)	(0.1, 0.5, 0.4)	(0.4, 0.4, 0.2)	(0.1, 0.5, 0.4)	(0.1, 0.8, 0.1)	(0.2, 0.6, 0.2)
U21	(0.4, 0.2, 0.4)	(0.8, 0.1, 0.1)	(0.1, 0.2, 0.7)	(0.5, 0.3, 0.2)	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.3, 0.6, 0.1)	(0.5, 0.3, 0.2)
U22	(0.3, 0.3, 0.4)	(0.7, 0.2, 0.1)	(0.2, 0.2, 0.6)	(0.6, 0.2, 0.2)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.2, 0.2)
U23	(0.2, 0.4, 0.4)	(0.9, 0.1, 0.0)	(0.1, 0.3, 0.6)	(0.4, 0.4, 0.2)	(0.9, 0.0, 0.1)	(0.7, 0.2, 0.1)	(0.2, 0.7, 0.1)	(0.4, 0.4, 0.2)
U24	(0.5, 0.2, 0.3)	(0.8, 0.0, 0.2)	(0.3, 0.1, 0.6)	(0.7, 0.1, 0.2)	(0.8, 0.1, 0.1)	(0.9, 0.0, 0.1)	(0.3, 0.6, 0.1)	(0.7, 0.1, 0.2)
U25	(0.4, 0.3, 0.3)	(0.7, 0.1, 0.2)	(0.2, 0.3, 0.5)	(0.5, 0.3, 0.2)	(0.7, 0.2, 0.1)	(0.8, 0.2, 0.0)	(0.4, 0.5, 0.1)	(0.5, 0.3, 0.2)
U26	(0.3, 0.4, 0.3)	(0.9, 0.1, 0.0)	(0.1, 0.4, 0.5)	(0.6, 0.4, 0.0)	(0.9, 0.1, 0.0)	(0.7, 0.3, 0.0)	(0.1, 0.8, 0.1)	(0.6, 0.4, 0.0)
U27	(0.2, 0.2, 0.6)	(0.8, 0.2, 0.0)	(0.2, 0.1, 0.7)	(0.4, 0.2, 0.4)	(0.8, 0.0, 0.2)	(0.9, 0.1, 0.0)	(0.3, 0.5, 0.2)	(0.4, 0.2, 0.4)
U28	(0.5, 0.1, 0.4)	(0.7, 0.3, 0.0)	(0.3, 0.3, 0.4)	(0.7, 0.3, 0.0)	(0.7, 0.3, 0.0)	(0.8, 0.2, 0.0)	(0.4, 0.4, 0.2)	(0.7, 0.3, 0.0)
U29	(0.4, 0.4, 0.2)	(0.9, 0.0, 0.1)	(0.1, 0.5, 0.4)	(0.5, 0.5, 0.0)	(0.9, 0.1, 0.0)	(0.7, 0.2, 0.1)	(0.2, 0.7, 0.1)	(0.5, 0.5, 0.0)
U30	(0.3, 0.2, 0.5)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.6, 0.2, 0.2)	(0.8, 0.2, 0.0)	(0.9, 0.0, 0.1)	(0.3, 0.6, 0.1)	(0.6, 0.2, 0.2)
U31	(0.2, 0.3, 0.5)	(0.7, 0.2, 0.1)	(0.3, 0.3, 0.4)	(0.4, 0.3, 0.3)	(0.7, 0.1, 0.2)	(0.8, 0.1, 0.1)	(0.4, 0.5, 0.1)	(0.4, 0.3, 0.3)
U32	(0.5, 0.4, 0.1)	(0.9, 0.1, 0.0)	(0.1, 0.4, 0.5)	(0.7, 0.2, 0.1)	(0.9, 0.1, 0.0)	(0.7, 0.3, 0.0)	(0.1, 0.8, 0.1)	(0.7, 0.2, 0.1)
U33	(0.4, 0.1, 0.5)	(0.8, 0.0, 0.2)	(0.2, 0.1, 0.7)	(0.5, 0.1, 0.4)	(0.8, 0.2, 0.0)	(0.9, 0.1, 0.0)	(0.3, 0.5, 0.2)	(0.5, 0.1, 0.4)

User	Battery	Camera	Price	Design	Screen	Performance	Software	Brand
U34	(0.3, 0.3, 0.4)	(0.7, 0.3, 0.0)	(0.3, 0.3, 0.4)	(0.6, 0.3, 0.1)	(0.7, 0.3, 0.0)	(0.8, 0.2, 0.0)	(0.4, 0.4, 0.2)	(0.6, 0.3, 0.1)
U35	(0.2, 0.5, 0.3)	(0.9, 0.1, 0.0)	(0.1, 0.5, 0.4)	(0.4, 0.6, 0.0)	(0.9, 0.0, 0.1)	(0.7, 0.1, 0.2)	(0.2, 0.7, 0.1)	(0.4, 0.6, 0.0)
U36	(0.2, 0.2, 0.6)	(0.5, 0.3, 0.2)	(0.3, 0.2, 0.5)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.4, 0.4, 0.2)	(0.3, 0.6, 0.1)	(0.9, 0.1, 0.0)
U37	(0.3, 0.3, 0.4)	(0.6, 0.2, 0.2)	(0.2, 0.3, 0.5)	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.5, 0.3, 0.2)	(0.4, 0.5, 0.1)	(0.8, 0.2, 0.0)
U38	(0.1, 0.4, 0.5)	(0.4, 0.4, 0.2)	(0.1, 0.4, 0.5)	(0.7, 0.3, 0.0)	(0.9, 0.0, 0.1)	(0.3, 0.5, 0.2)	(0.2, 0.7, 0.1)	(0.7, 0.3, 0.0)
U39	(0.4, 0.2, 0.4)	(0.7, 0.1, 0.2)	(0.4, 0.1, 0.5)	(0.9, 0.0, 0.1)	(0.8, 0.1, 0.1)	(0.6, 0.2, 0.2)	(0.3, 0.6, 0.1)	(0.9, 0.0, 0.1)
U40	(0.3, 0.3, 0.4)	(0.5, 0.3, 0.2)	(0.3, 0.3, 0.4)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.4, 0.4, 0.2)	(0.4, 0.5, 0.1)	(0.8, 0.1, 0.1)
U41	(0.2, 0.4, 0.4)	(0.4, 0.5, 0.1)	(0.2, 0.5, 0.3)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)	(0.2, 0.6, 0.2)	(0.1, 0.8, 0.1)	(0.7, 0.3, 0.0)
U42	(0.5, 0.2, 0.3)	(0.8, 0.1, 0.1)	(0.5, 0.1, 0.4)	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.3, 0.5, 0.2)	(0.9, 0.1, 0.0)
U43	(0.4, 0.3, 0.3)	(0.6, 0.3, 0.1)	(0.4, 0.3, 0.3)	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.5, 0.4, 0.1)	(0.4, 0.4, 0.2)	(0.8, 0.2, 0.0)
U44	(0.3, 0.5, 0.2)	(0.4, 0.6, 0.0)	(0.2, 0.6, 0.2)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)	(0.3, 0.7, 0.0)	(0.2, 0.7, 0.1)	(0.7, 0.3, 0.0)
U45	(0.1, 0.2, 0.7)	(0.5, 0.2, 0.3)	(0.1, 0.2, 0.7)	(0.9, 0.0, 0.1)	(0.8, 0.0, 0.2)	(0.4, 0.3, 0.3)	(0.3, 0.6, 0.1)	(0.9, 0.0, 0.1)
U46	(0.2, 0.3, 0.5)	(0.7, 0.1, 0.2)	(0.3, 0.1, 0.6)	(0.8, 0.1, 0.1)	(0.7, 0.1, 0.2)	(0.6, 0.2, 0.2)	(0.4, 0.5, 0.1)	(0.8, 0.1, 0.1)
U47	(0.4, 0.4, 0.2)	(0.5, 0.5, 0.0)	(0.2, 0.4, 0.4)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)	(0.4, 0.6, 0.0)	(0.1, 0.9, 0.0)	(0.7, 0.3, 0.0)
U48	(0.1, 0.1, 0.8)	(0.4, 0.1, 0.5)	(0.1, 0.1, 0.8)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.3, 0.5, 0.2)	(0.9, 0.1, 0.0)
U49	(0.3, 0.3, 0.4)	(0.6, 0.2, 0.2)	(0.3, 0.2, 0.5)	(0.8, 0.0, 0.2)	(0.7, 0.2, 0.1)	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)	(0.8, 0.0, 0.2)
U50	(0.2, 0.6, 0.2)	(0.5, 0.5, 0.0)	(0.1, 0.7, 0.2)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)	(0.3, 0.7, 0.0)	(0.2, 0.8, 0.0)	(0.7, 0.3, 0.0)

This matrix constituted the primary input for the subsequent analysis.

3.3. Processing and Generation of the Neutrosophic Biplot

Following the pragmatic approach described in the methodology, the first step was the deneutrosification of the matrix X~ to obtain a real-valued data matrix (crisp) XS that could be processed by the HJ-Biplot algorithm. To do this, the following scoring function was applied to each cell of the matrix:

$$S(N_{ij}) = 32 + t_{ij} - i_{ij} - f_{ij} \quad (7)$$

This function transforms the neutrosophic tuple into a single scalar value representing the "net favorability score," penalizing indeterminacy and falsity. The resulting matrix XS was then column-centered. Singular Value Decomposition (SVD) was then applied to the centered matrix to obtain the row (individual) and column (variable) markers.

Eigenvalue analysis indicated that the first two principal components explained a substantial portion of the total variance in the data. Specifically, **Component 1 (X-axis) accounted for 48.7%** of the variance, while **Component 2 (Y-axis) explained an additional 25.2%**. Overall, the resulting Biplot map captures **73.9% of the original information**, a value considered excellent for the correct interpretation of the data structure.

3.4. Analysis and Interpretation of the Biplot Chart

The resulting HJ-Biplot graph is presented below. It simultaneously represents the 50 users (points) and the 8 attributes (vectors). Interpretation is based on the length of the vectors, the angle between them, and the relative position of the individual points with respect to the vectors.

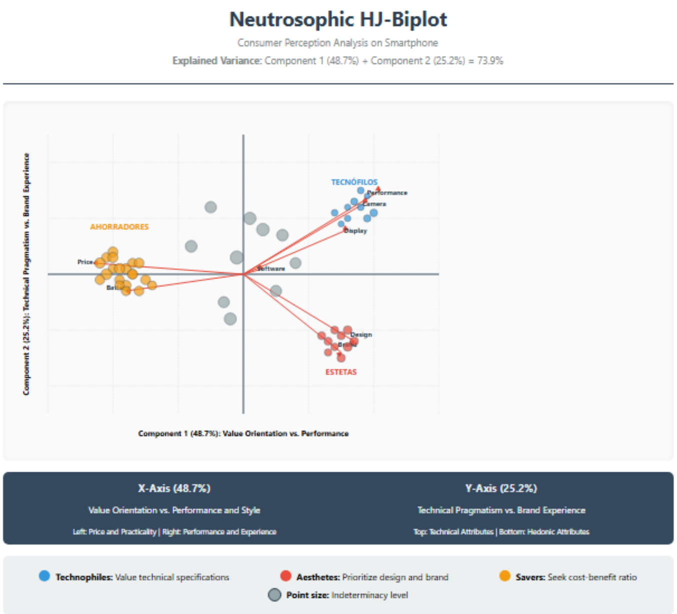

Figure 1. HJ-Biplot-Neutrosophic on a new smartphone model recently introduced to the market.

3.4.1. Interpretation of the Axes Factorials

- **X-axis (48.7%): "Value Orientation vs. Performance and Style."** This axis is the main differentiator of consumer perception. High value-added attributes such as Performance, Camera, Display, and Design are positioned on the right. Attributes related to cost and practicality, primarily Price and Battery Life, are located on the left. Consumers on the right side of the graph tend to value features and experience, while those on the left prioritize value and battery life.
- **Y-axis (25.2%): "Technical Pragmatism vs. Brand Experience."** The second axis differentiates between different types of positive evaluations. Projected toward the top are the vectors of technical and functional attributes (Performance, Camera). Projected toward the bottom are the more hedonistic and brand perception attributes (Design, Brand). This axis, therefore, separates users who seek technical specifications from those who value brand aesthetics and status more.

3.4.2. Relationship between Attributes

The angles between the attribute vectors reveal their correlations in consumer perception.

- **acute angle** is observed between the Camera and Performance vectors, indicating a strong positive correlation. Consumers who rate a device's performance positively tend to also rate its camera quality.
- Similarly, Design and Brand show a strong correlation, pointing in a very similar direction in the lower right quadrant. This suggests that brand perception is strongly linked to product aesthetics.
- Price vector forms an **obtuse angle** (almost 180°) with Performance and Camera, denoting a strong negative correlation. This is expected: users who prioritize a low price negatively perceive attributes that tend to make a product more expensive.
- Software vector is noticeably shorter than the others and is located close to the origin, suggesting that it contributes less to the overall variance. This indicates that perceptions of software are not as discriminating a factor among consumers as the other attributes, or that their assessment is more homogeneous or uncertain across the sample.

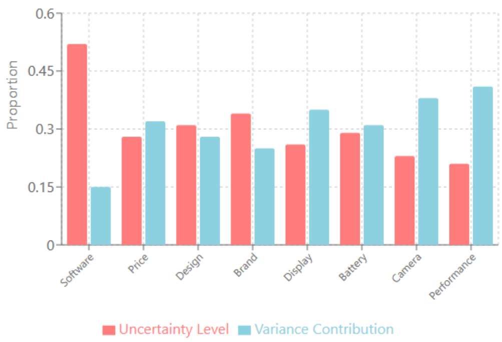

Figure 2. Attribute Uncertainty and Variance Contribution Analysis

This graph reveals the relationship between consumer uncertainty and attribute variance. Software presents the greatest uncertainty despite its low contribution to variance, indicating uniform uncertainty across all consumer segments.

3.4.3. Identification of Consumer Segments (Clusters)

The distribution of user points on the map allows us to identify three clearly defined consumer segments:

1. **Cluster 1: Technophiles (Upper right quadrant).** This consumer group (e.g., U07, U18, U33) is located in the zone of influence of the Performance and Camera vectors . These are individuals for whom technical specifications are the main purchasing driver. They do not show a strong association with the Price vector , indicating a lower sensitivity to it. They value a high performance and capabilities photographic advanced .
2. **Cluster 2: Aesthetes (Lower right quadrant).** This segment (e.g., U02, U23, U45) is strongly associated with the Design and Brand vectors . Their positive perception of the product is driven by aesthetics, construction materials, and brand prestige. Although they are also on the positive side of the X-axis, their assessment is not as closely tied to raw performance as is the case with technophiles.
3. **Cluster 3: The Savers (Left semicircle).** This is the largest group and clusters on the left side of the graph (e.g., U04, U29, U41), in contrast to almost all the performance attributes. Its proximity to the Price and Battery vectors indicates that their primary evaluation criteria are cost-benefit ratio and long battery life. For them, a low price is the most desirable attribute.

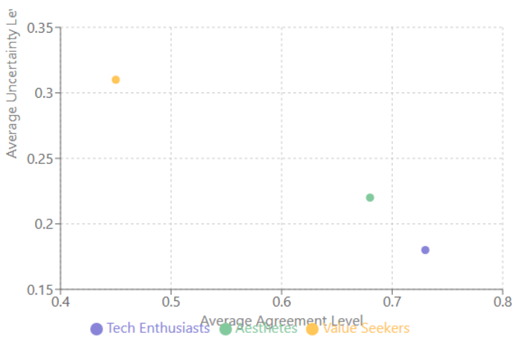

Figure 3. Consumer Segments Distribution and Neutrosophic Profile

This scatter plot positions consumer segments according to their neutrosophic profile. The size of the dot represents the segment size. Value Seekers show greater uncertainty and lower agreement, while Technology Enthusiasts show high agreement with low uncertainty.

3.4.4. Analysis of Indeterminacy

To incorporate the dimension of uncertainty into the visual analysis, the size of each point representing a user on the graph is proportional to their **average degree of indeterminacy (I)** across all their responses. A larger point indicates a higher level of overall doubt or uncertainty.

This visual analysis reveals a key finding: although the Software vector was short, many of the largest points in the graph belong to users who gave high ratings to this attribute. This confirms that, while software is not a major differentiator in terms of liking or disliking, it **is the attribute that generates the most uncertainty** in the sample. Consumers seem to struggle to assess software quality, either not knowing what to expect or not feeling qualified to make a firm judgment. This is a crucial insight that a classic analysis would have completely obscured.

4. Conclusion and Future Directions

4.1. Summary of Main Findings

The application of the Neutrosophic Biplot method enabled a deep and nuanced analysis of consumer perception. The results demonstrated the existence of three clear and actionable market segments: **Technophiles** , **Aesthetes** , and **Thrifts** , each with a distinctive set of priorities and values. Analysis of the relationships between attributes confirmed logical associations (Performance-Camera) and oppositions (Price-Features). The most revealing finding, made possible only by the neutrosophic nature of the method, was the identification of **Software** as the main generator of uncertainty among consumers, an invaluable insight for guiding communications and marketing strategies.

4.2. Advantages and Contribution of the Method

This study empirically demonstrates the superiority of the Neutrosophic Biplot over conventional methods. By not forcing the respondent to make a definitive decision and by mathematically modeling uncertainty, the method provides a deeper, more honest, and more realistic understanding of the perceptual landscape. It allows analysts to understand not only *what* consumers like, but also *how confident* they are about it, opening up a new dimension for strategic segmentation.

4.3. Limitations and Future Lines of Research

A limitation of the present study is the sample size, which, although sufficient to demonstrate the methodology, could be expanded in future work to validate the segments found on a larger scale. Furthermore, the pragmatic deneutrosification approach was used ; it would be of great theoretical and practical interest to explore the application of the Neutrosophic SVD approach to compare the results.

Future lines of research are promising. The Neutrosophic Biplot method can be applied to a wide range of domains beyond marketing, such as financial risk analysis (where uncertainty is a key factor), medical research (to assess patient perceptions of treatments), or public opinion analysis in political science, where ambiguity and indecision are prevalent and highly relevant phenomena.

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