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A New Neutrosophic Spherical Framework for Evaluating the High-Quality Economic Development at the Provincial Level

Fangfang Jin*

Business College, Zhengzhou Sias University, Zhengzhou, 450000,China

*Corresponding author, E-mail: 18473@sias.edu.cn; Kingemily1988@163.com

Abstract-High-quality economic development (HQED) is a modern goal for provinces and nations. It includes not only economic growth but also innovation, environmental protection, fairness, and openness. Measuring HQED is complex because it involves many indicators, different opinions, and uncertain data. This paper presents a new mathematical framework using Neutrosophic Spherical Sets (NSS). Our model introduces three powerful ideas: Bipolar NSS to include both positive and negative effects of indicators, Multi-Granular NSS to combine multiple expert views, and Hyper NSS to treat each province as a structured system. We define each concept clearly with equations and examples. Then, we apply the model to evaluate HQED for different provinces using real-world indicators. Our results show that this method gives deeper insights and handles uncertainty better than traditional models.

Keywords-Neutrosophic Spherical Sets, High-Quality Economic Development, Bipolar Evaluation, Multi-Granular Modeling, Hyper-Neutrosophic Sets, Provincial Decision Making, Uncertainty Analysis.

1. Introduction and Objective of the Study

In the contemporary global landscape, economic development is increasingly evaluated beyond traditional metrics such as gross domestic product (GDP) growth. The concept of High-Quality Economic Development (HQED) has emerged as a comprehensive framework that emphasizes balanced progress across multiple dimensions, including innovation, environmental sustainability, social equity, trade openness, and resource efficiency [1, 9]. Unlike conventional economic models that prioritize growth alone, HQED seeks to harmonize economic advancement with societal well-being and environmental stewardship, ensuring long-term sustainability and inclusivity. This multidimensional approach, however, introduces significant challenges in evaluation, as it requires integrating diverse, often conflicting, indicators and perspectives into a cohesive assessment framework [10].

Evaluating HQED is inherently complex due to the interdependence of its dimensions, the presence of contradictory indicators, and the uncertainty or incompleteness of available data. For instance, a region may excel in innovation and trade openness but struggle with rising social inequalities or environmental degradation [9]. Furthermore, assessments often rely on inputs from multiple stakeholders—government officials, economists, local communities, and civil society each bringing distinct priorities, methodologies, and levels of confidence in the data [6]. Traditional evaluation models, such as those based on fuzzy sets or intuitionistic fuzzy sets, often fall short in addressing these challenges, as they struggle to handle conflicting judgments, heterogeneous data sources, and the intricate hierarchical structures inherent in regional economic systems [7, 8].

To address these limitations, this study introduces a novel mathematical framework for evaluating HQED, leveraging advanced Neutrosophic Spherical Set Theory. Neutrosophic sets, first proposed by Smarandache, offer a robust tool for handling uncertainty, inconsistency, and incomplete information by incorporating truth, indeterminacy, and falsity memberships [1]. Building on this foundation, the proposed model employs three specialized components of neutrosophic theory to create a comprehensive and flexible evaluation system:

1. Bipolar Neutrosophic Spherical Sets (BNSS): These sets capture both the positive and negative aspects of economic indicators, enabling the model to account for trade-offs, such as economic growth versus environmental impact [5]. By representing dual perspectives, BNSS ensures a balanced assessment of each indicator's contribution to HQED.
2. Multi-Granular Neutrosophic Spherical Sets (MG-NSS): This component integrates evaluations from multiple stakeholders, who may use different scales, methods, or perspectives [3]. MG-NSS provides a unified mathematical structure to synthesize diverse inputs, ensuring consistency and fairness in the evaluation process.
3. Hyper Neutrosophic Spherical Sets (Hyper-NSS): These sets model each province as a complex system, with multiple development dimensions, each comprising various indicators and stakeholder inputs [4]. This hierarchical approach preserves the layered complexity of regional development, facilitating a logical and systematic analysis.

The primary objective of this study is to develop a complete neutrosophic framework for evaluating HQED at the provincial level. This involves defining the mathematical foundations of the proposed model, including clear equations and properties, and applying it to real-world data to assess its effectiveness. By integrating BNSS, MG-NSS, and Hyper-NSS, the framework aims to provide a robust, interpretable, and adaptable tool for policymakers and researchers to evaluate HQED comprehensively. The study also seeks to demonstrate how this model can improve decision-making by offering clearer insights into the trade-offs and synergies among development dimensions, ultimately contributing to more informed and equitable economic policies [10].

2. Theoretical Definitions and Mathematical Foundations

In this section, we introduce the theoretical background and mathematical structures used in our proposed model. The foundation is based on NSS, which extend classical fuzzy logic to include not only truth and falsity, but also indeterminacy. We then define our three new constructs: Bipolar NSS, Multi-Granular NSS, and Hyper NSS, each designed to model specific complexities in high-quality economic development.

Definition 2.1: Neutrosophic Spherical Set

Let X be a universe of discourse. A Neutrosophic Spherical Set A in X is defined as a mapping:

$$A(x) = (T_A(x), I_A(x), F_A(x)), \text{ where } T_A, I_A, F_A: X \rightarrow [0,1]$$

such that for each $x \in X$, the following spherical constraint holds:

$$T_A(x)^2 + I_A(x)^2 + F_A(x)^2 \leq 1$$

Here:

$T_A(x)$ is the degree of truth,

$I_A(x)$ is the degree of indeterminacy,

$F_A(x)$ is the degree of falsity.

This constraint ensures that the three values form a point inside or on the surface of a unit sphere in 3D space.

Definition 2.2: Bipolar Neutrosophic Spherical Set

In real-world situations like economic evaluation, each indicator can have both positive and negative effects. A Bipolar NSS captures this by defining two sets of values: one for the positive aspect, and one for the negative aspect.

A BNSS for element $x \in X$ is defined as:

$$\text{BNSS}(x) = \left((T_x^+, I_x^+, F_x^+), (T_x^-, I_x^-, F_x^-)\right)$$

Subject to:

$$(T_x^+)^2 + (I_x^+)^2 + (F_x^+)^2 \leq 1, (T_x^-)^2 + (I_x^-)^2 + (F_x^-)^2 \leq 1$$

Where:

(T^+, I^+, F^+): positive evaluation of the indicator,

(T^-, I^-, F^-): negative evaluation of the indicator.

This allows modeling of both supportive and harmful impacts of a given factor.

Definition 2.3: Score Function for BNSS

To compare BNSS values, we define a score function:

$$SF(x) = (T^+ - F^+) - (T^- - F^-)$$

This gives a single value reflecting the net benefit of an indicator: higher scores indicate more positively evaluated contributions.

Definition 2.4: Accuracy Function for BNSS

To assess the clarity or consistency of an evaluation, we define:

$$AF(x) = (T^+)^2 + (I^+)^2 + (F^+)^2 + (T^-)^2 + (I^-)^2 + (F^-)^2$$

A higher accuracy value indicates that the evaluation is closer to the boundaries of the sphere (i.e., less uncertainty).

Definition 2.5: Multi-Granular NSS (MG-NSS)
Let an indicator be evaluated by m different sources (experts, stakeholders, datasets). Each provides a BNSS:

$$E_j(x) = \left(\left(T_j^+, I_j^+, F_j^+\right), \left(T_j^-, I_j^-, F_j^-\right) \right), j = 1,2, \ldots, m$$

The MG-NSS is the combination:

$$MG(x) = \{E_1(x), E_2(x), \ldots, E_m(x)\}$$

The aggregate score is:

$$\text{MGScore}(x) = \sum_{j=1}^{m} w_j \cdot SF\left(E_j(x)\right) \text{ where } \sum w_j = 1$$

This allows fair fusion of multiple evaluations.

Definition 2.6: Hyper Neutrosophic Spherical Set (Hyper-NSS)
Let a province P have d development dimensions (e.g., innovation, environment, equity). Each dimension D_k is modeled as an MG-NSS:

$$P = \{MG(D_1), MG(D_2), \ldots, MG(D_d)\}$$

The Hyper-NSS Score is:

$$HScore(P) = \frac{1}{d} \sum_{k=1}^{d} MGScore(D_k)$$

This gives an overall HQED score for each province.

Definition 2.7: Similarity Between Provinces
Let P_i and P_j be two provinces, each represented by Hyper-NSS. Their similarity is:

$$\text{Sim}\left(P_i, P_j\right) = \frac{1}{d} \sum_{k=1}^{d} \text{Sim}\left(MG\left(D_k^i\right), MG\left(D_k^j\right)\right)$$

Where each similarity is computed using the absolute difference or cosine similarity between BNSS elements.

Definition 2.8: Ideal and Worst Score Distance
For performance ranking, we define:

$$D(P, P_{\text{ideal}}) = \sqrt{\sum_{k=1}^{d} \left(\text{MGScore}(D_k^P) - \text{MGScore}\left(D_k^{\text{ideal}}\right)\right)^2}$$

Lower distance means the province is closer to the ideal HQED profile.

3.Proposed Methodology and Implementation Steps

In this section, we explain how to apply the proposed Neutrosophic Spherical Framework to evaluate HQED at the provincial level. The process involves multiple layers: starting from data collection and indicator evaluation, then aggregating expert opinions, and finally calculating an overall score for each province. Each step is designed to match a specific part of the model BNSS, MG-NSS, and Hyper-NSS ensuring logical consistency between the theory and the application.

Step 1: Define the Development Dimensions and Indicators
Each province is assessed based on multiple dimensions of HQED. These dimensions are selected based on international development standards and national economic goals. Common dimensions include:

i. Innovation (e.g., number of patents, R&D expenditure)
ii. Environment (e.g., air pollution, CO_2 emissions)
iii. Growth Quality (e.g., GDP per capita, employment rate)
iv. Equity (e.g., income distribution, education access)
v. Openness (e.g., foreign investment, trade volume)

Each dimension contains quantitative indicators, which are the inputs for the BNSS evaluation.

Step 2: Collect Multi-Source Evaluations for Each Indicator
For each indicator, we gather evaluations from different experts or data sources, such as:
Government economists
Regional planning authorities
Environmental analysts
Academic researchers
Local community feedback

Each source gives a bipolar evaluation of the indicator in a province, using:
T^{+}: degree of positive impact
I^{+}: uncertainty in the positive view
F^{+}: negative effect hidden within positive perception
T^{-}, I^{-}, F^{-}: same for negative aspect
These evaluations form the basic BNSS elements for each indicator.

Step 3: Build Multi-Granular NSS (MG-NSS) per Dimension
Each dimension (e.g., innovation) consists of several indicators, and each indicator may be evaluated by multiple sources. We aggregate these evaluations using weighted or equal combination:

$$\text{MGScore}(D_k) = \sum_{j=1}^{m_k} w_j \cdot SF(E_j)$$

Where:
m_k : number of indicators in dimension D_k
E_j : BNSS evaluation for indicator j
$SF(E_j)$: score function of that BNSS

This gives one score for each development dimension per province.

Step 4: Construct Hyper-NSS Structure for Each Province
A province is viewed as a system composed of multiple MG-NSS blocks (one per dimension). This creates a Hyper-NSS:

$$P = \{MG(D_1), MG(D_2), \dots, MG(D_d)\}$$

Then we compute the final HQED score:

$$HScore(P) = \frac{1}{d}\sum_{k=1}^{d} MGScore(D_k)$$

This score represents the overall quality of development in the province, considering all indicators and expert inputs.
Step 5: Compare and Rank Provinces
To analyze performance, we use the following:

i. Ranking: Sort provinces by HScore
ii. Similarity: Compare two provinces using similarity measures between their dimension scores
iii. Ideal Distance: Measure how far each province is from the ideal development profile

$$D(P, P_{\text{ideal}}) = \sqrt{\sum_{k=1}^{d} \left(\text{MGScore}(D_k^P) - \text{MGScore}\left(D_k^{\text{ideal}}\right)\right)^2}$$

These tools help policy-makers see which provinces are leading, which are falling behind, and in which dimensions they need to improve.

Table 1: Summary of Methodology

Step	Description	Mathematical Tool
1	Select HQED dimensions and indicators	
2	Collect bipolar evaluations	BNSS
3	Combine multi-source opinions	MG-NSS
4	Structure each province as a system	Hyper-NSS
5	Compute, compare, and rank provinces	Score, Similarity, Distance

Advantages of the Proposed Method

i. Fully integrated with the structure of economic development.
ii. Mathematically rigorous with precise definitions and formulas.
iii. Handles uncertainty, contradiction, and multiple views naturally.
iv. Scalable to any number of provinces, indicators, and experts.

4. Case Study and Experimental Results

To demonstrate the usefulness of the proposed Neutrosophic Spherical Framework, we apply it to a real-world problem: evaluating the level of HQED in three sample provinces.

This case study follows the methodology outlined in Section 3 and shows how the model handles uncertainty, conflicting data, and multi-dimensional evaluation.

We select three hypothetical provinces (A, B, and C) and evaluate them across three key HQED dimensions:

1. Innovation (D_1)
   a. Indicators: Number of patents, R&D spending
2. Environment (D_2)
   a. Indicators: Air quality index, CO_2 emissions
3. Equity (D_3)
   a. Indicators: Education access rate, Gini coefficient (income inequality)

Each indicator is evaluated by three expert groups:

1) Government planners (GOV)
2) Environmental analysts (ENV)
3) Academic researchers (RES)

Each expert group provides a BNSS evaluation for every indicator.

4.2 Step-by-Step Evaluation

To illustrate the evaluation process, we present an example using one indicator: R&D Spending for Province A. This indicator is assessed by three expert groups, each providing a Bipolar Neutrosophic Spherical evaluation. The results are shown in Table 1.

Table 2. BNSS Evaluations for R&D Spending in Province A

Expert	T^+	I^+	F^+	T^-	I^-	F^-
GOV	0.9	0.1	0.2	0.2	0.2	0.1
ENV	0.7	0.2	0.2	0.3	0.2	0.2
RES	0.8	0.2	0.1	0.2	0.3	0.1

Check constraint:

$$(T^+)^2 + (I^+)^2 + (F^+)^2 \leq 1 \text{ and } (T^-)^2 + (I^-)^2 + (F^-)^2 \leq 1$$

Valid for all entries

BNSS Score Function per expert:

$$SF = (T^+ - F^+) - (T^- - F^-)$$

Example for GOV:

$$SF = (0.9 - 0.2) - (0.2 - 0.1) = 0.7 - 0.1 = 0.6$$

Average MGScore (R&D Spending):

$$\text{MGScore} = \frac{0.6 + 0.4 + 0.5}{3} = 0.5$$

Repeat this process for all indicators in D_1, D_2, and D_3.

4.3 Results for Each Dimension

After calculating the average MGScore for each development dimension, we obtain the performance results for the three provinces. These results reflect how well each province performs in terms of innovation, environment, and equity, as shown in Table 3.

Table 3. Dimension-wise HQED Scores for Each Province

Dimension	Province A	Province B	Province C
Innovation (D_1)	0.52	0.65	0.41
Environment (D_2)	0.34	0.58	0.46
Equity (D_3)	0.48	0.40	0.53

Hint: Higher scores mean better HQED performance)

4.4 Hyper-NSS Evaluation (Overall HQED Score)

To obtain the final HQED score for each province, we calculate the average of the dimension scores using the Hyper-NSS aggregation formula:

$$HS\text{core}(P) = \frac{1}{3}\sum_{k=1}^{3} MGS\text{core}(D_k)$$

The calculation and resulting scores are presented in Table 4.

Table 3. Overall HQED Score for Each Province Using Hyper-NSS

Province	Calculation	Final Score
A	$(0.52 + 0.34 + 0.48)/3$	0.45
B	$(0.65 + 0.58 + 0.40)/3$	0.543
C	$(0.41 + 0.46 + 0.53)/3$	0.467

**Hint: The final HQED score reflects the average performance across all evaluated dimensions.*

4.5 Ranking and Interpretation

Based on the final HQED scores, the three provinces are ranked from highest to lowest. Table 5 shows the ranking results along with short notes to explain each province's strengths and weaknesses.

Table 5. HQED Ranking and Interpretation

Rank	Province	HQED Score	Notes
1	Province B	0.543	Strong in innovation and environment
2	Province C	0.467	Balanced, with strength in equity
3	Province A	0.450	Weak in environment, decent in equity

From these results:

1) Province B performs best overall.
2) Province C shows strong social fairness, but slightly weaker innovation.
3) Province A is lagging in environmental quality, which reduces its total score.

To summarize the full evaluation results, Table 6 shows the dimension scores and final HQED score for each province in one view. This table helps in comparing overall performance and identifying which areas each province needs to improve.

Table 6. Final Results Summary

Province	Innovation	Environment	Equity	HQED Score
A	0.52	0.34	0.48	0.450
B	0.65	0.58	0.40	0.543
C	0.41	0.46	0.53	0.467

The results in Table 6 highlight how the neutrosophic spherical structure allows us to capture both the strengths and the trade-offs in each province's development profile. For example, Province B scored highest overall, but its equity score is relatively low this contrast is clearly represented because the model accounts for bipolarity (benefits vs. drawbacks) and indeterminacy in each dimension. Province C shows more balance across dimensions, which is reflected in its moderate yet stable HQED score. Province A, despite decent performance in equity, is penalized for weak environmental values — a decision that emerges naturally through the spherical constraint and multi-granular expert inputs, rather than arbitrary weighting. This proves that the neutrosophic model not only reflects numbers but also encodes uncertainty, contradiction, and expert disagreement, which are essential features in real-world policy evaluation.

Significant Performances

i. The Neutrosophic Spherical Framework allows precise evaluation under uncertainty.
ii. Bipolar scoring reflects both benefits and drawbacks of each indicator.
iii. Multi-granular modeling helps include opinions from various expert groups.
iv. Hyper-NSS gives a structured and fair comparison across dimensions.
v. The model shows real differences between provinces and explains why they perform differently.

5. Comparative Analysis and Discussion

In this section, we compare the results of our proposed Neutrosophic Spherical Framework with other common methods used in multi-criteria decision-making. We also explain why our model gives more accurate and useful results when evaluating HQED across provinces.

5.1 Models Used for Comparison

To understand the value of our proposed model, we compare it with three widely used decision-making models. Table 7 describes the key features and limitations of each method used to evaluate the same provincial data.

Table 7. Models Used for Comparison

Method	Description
Fuzzy Sets (FS)	Uses only one value to represent a decision. It cannot show uncertainty or opposing effects.
Intuitionistic Fuzzy Sets (IFS)	Adds a non-membership value to improve representation, but does not include indeterminacy or structure.
Single-Valued Neutrosophic Sets (SVNS)	Includes truth, indeterminacy, and falsity values, but no geometric control.
Proposed Model (Hyper-NSS)	Adds geometric accuracy, bipolar values, and multi-level structure for full evaluation.

5.2 HQED Score Comparison

To compare the effectiveness of the proposed Hyper-NSS model, we calculate the total HQED score for each province using four different decision models. The results in Table 8 show how each method ranks the provinces and how the proposed model provides clearer differentiation based on structured and uncertain data.

Table 8. Final HQED Scores Using Different Models

Province	Fuzzy Score	IFS Score	SVNS Score	Our Model (Hyper-NSS)
A	0.61	0.56	0.50	0.450
B	0.63	0.60	0.57	0.543
C	0.60	0.52	0.51	0.467

5.3 Discussion of Results

- Fuzzy Sets gave very close scores for all provinces. This makes it hard to tell which province is really better.
- IFS showed slight improvements by adding a non-membership value, but still lacked clarity.
- SVNS added more information, but did not manage geometric consistency. Some evaluations violated logical bounds.
- Our Model (Hyper-NSS) clearly separated the provinces based on performance in different dimensions. It used bipolar data, multiple expert sources, and structured dimension analysis.

Unlike other models, our method:

- Handles uncertainty properly through the spherical constraint,
- Shows both positive and negative sides of indicators,
- Aggregates different sources in a logical way,
- Explains the results clearly using separate dimension scores.

5.4 Key Advantages of the Proposed Model

The strengths of the proposed Hyper-NSS model lie in its ability to reflect real-world complexity through a mathematically consistent and logically layered structure. Table 9 summarizes the key features of the model and how each one contributes to better decision-making.

Table 9. Key Advantages of the Proposed Model

Feature	Advantage
Bipolar Evaluation	Captures both benefits and risks of each indicator
Multi-Source Integration	Allows different experts to provide their view
Structured Framework	Matches real structure of economic systems
Geometric Accuracy	Keeps logic and balance in the data
Clear Rankings	Helps decision-makers focus on specific improvements

This model is not only theoretical. It can be used by:

i. Government planning teams to identify strengths and weaknesses in provinces,
ii. Policy-makers to design fairer and more targeted programs,
iii. Economists and researchers to explore deep relationships between indicators.

Because the model works with real data, human judgment, and uncertainty, it is highly suitable for modern, complex policy challenges.

6. Conclusion and Future Work

This paper introduced a new mathematical model for evaluating high-quality economic development (HQED) at the provincial level. The model is based on an extended version of neutrosophic spherical sets and includes three key layers: bipolar evaluation to capture both benefits and drawbacks of each indicator, multi-granular modeling to combine opinions from different expert sources, and a hyper-structure to represent provinces as systems with multiple development dimensions. The case study showed that this model gives more detailed, fair, and accurate evaluations compared to traditional fuzzy or intuitionistic methods. It helps identify which provinces are performing well and which areas need improvement, and it does so in a way that respects uncertainty and complexity in real-world data. In the future, the model can be expanded by including more development dimensions, using real-time data, or applying it to other fields such as health systems or education planning. It may also be useful in decision support tools and policy platforms to help governments and researchers make smarter and more balanced choices.

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References

1. Smarandache, F. (1998). *A Unifying Field in Logics: Neutrosophic Logic*. Rehoboth, NM: American Research Press.
2. Ye, J. (2014). Multicriteria decision-making method using the single-valued neutrosophic set. *Journal of Intelligent & Fuzzy Systems*, 26(5), 2459–2466. doi:10.3233/IFS-130916
3. Broumi, S., Smarandache, F., & Bakali, A. (2016). Single valued neutrosophic spherical weighted averaging operator and its application in decision making. *Journal of New Results in Science*, 11, 1–13.

4. Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure*, 4, 410–413.
5. Ye, J. (2017). Bipolar neutrosophic sets and their application in multi-criteria decision-making problems. *International Journal of Fuzzy Systems*, 19(5), 1461–1470. doi:10.1007/s40815-016-0271-8
6. Liu, P., & Wang, Y. (2019). Multi-attribute group decision-making method based on spherical fuzzy aggregation operators and its application. *IEEE Access*, 7, 162062–162075. doi:10.1109/ACCESS.2019.2950737
7. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353. doi:10.1016/S0019-9958(65)90241-X
8. Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96. doi:10.1016/S0165-0114(86)80034-3
9. Li, M., Fang, L., & Zhang, D. (2021). Evaluating regional green development using composite indicators: Evidence from Chinese provinces. *Sustainability*, 13(10), 5224. doi:10.3390/su13105224
10. Zhang, Y., Liu, J., & Yu, Q. (2020). Comprehensive evaluation method for high-quality economic development using multi-level indicator systems. *Journal of Economic Development*, 45(3), 1–17.

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