Q-Neutrosophic Type-2 Soft Set (T2-QNSS) for Tourism Development Potential Analysis of "Intangible Cultural Heritage" Resources from the Perspective of Cultural and Tourism Integrations

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Abstract—the process of intangible cultural heritage (ICH) evaluation within tourism has been involving diversity of complex linguistic contexts, vague expert opinions, and uncertain decision parameters—challenging the existing decision-making solutions. In response, and to overcome these limitations, we propose a novel Neutrosophic Modeling solution called the Q-Neutrosophic Type-2 Soft Set (T2-QNSS) that integrated Type-2 Neutrosophic Sets, bipolar logic, and multi-context soft set to represent multi-dimensional uncertainty and hesitation in a more expressive manner. The contributions of T2-QNSS are four-folded. First, it formally define the T2-QNSS structure and its associated set-theoretic and algebraic operations. Second, we introduce a Credibility-Weighted Nonlinear Aggregation (CWNA) operator that accommodates varying levels of expert reliability. Third, it offers a novel scoring approach through the Bipolar Divergence-Aware Certainty Index (BDCI), which accounts for uncertainty in bipolar evaluations. Finally, it introduces a decision-making algorithm. Based on applicability test on a realistic ICH tourism case study involving multiple experts, alternatives, and parameters; T2-QNSS demonstrate that it not only advances neutrosophic theory but also offers a powerful and flexible framework for multi-expert decision-making in tourism.

Keywords—Uncertainty Modeling, Neutrosophic Set Theory, Q-Neutrosophic Type-2 Soft Sets (T2-QNSS), Neutrosophic Logic Credibility-Weighted Aggregation, Bipolar Divergence-Aware Certainty Index (BDCI), Intangible Cultural Heritage.

1. Introduction

Intangible Cultural Heritage (ICH), encompassing traditions, oral histories, performing artistics , and local knowledge, which played pivotal role in shaping cultural identity and fostering sustainable tourism [1]. As global tourism increasily emphasizes authentic and immersive experiences, ICH had evolved as a valuable asset for destinations seeking to differentiate themselves while preserving cultural legacies [2]. The integration of ICH into tourism development not only improve visitor experiences but also supports communital empowerment (as shown in Figure 1) and economic growth [3]. Nevertheless, dynamic and multifaceted nature of ICH, coupled with the complexities of tourism systems, demands innovative approaches to model and manage the interplay of cultural, social, and economic factors effectively [4].

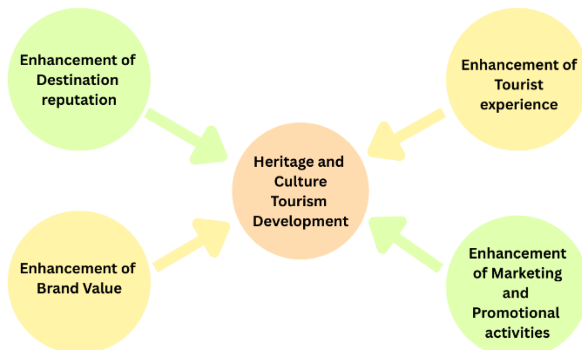

Figure 1. Key benefits of Heritage and Culture Tourism Development.

Despite the recognized importance of ICH in tourism, existing frameworks for its integration often fail to account for uncertaintie in stakeholder perspectives and cultural valuations [5]. qualitative assessments or fuzzy logic-based models [6], struggle to capture the nuanced and conflicting opinions of diverse stakeholders, including local communities, tourists, and policymakers. This gap in the literature results in incomplete models that overlook critical aspects of ICH, like its subjective value or evolving significance, which lead to suboptimal strategies for sustainable tourism development.

Neutrosophic Sets (NS), introduced by Smarandache in 1998[5], provide a robust mathematical framework for handling uncertainty by incorporating three independent dimensions: truth-membership, indeterminacy-membership, and falsity-membership. Unlike old set theories, NS enable simultaneous representation of uncertainties making it uniquely suited for modeling ICH settings in tourism [7]. NS comes to be ideal tool for addressing the identified gap, enabling a more comprehensive and inclusive approach to integrating ICH into tourism development.

This paper introduces a novel neutrosophic soft set model, termed the Q-Neutrosophic Type-2 Soft Set (T2-QNSS), to address complex decision support characterized by high degrees of uncertainty, subjective evaluations. Our contributions are fourfold:

(1) We formally define the structure and algebraic operations of T2-QNSS by integrating Type-2 neutrosophic logic with bipolar modeling under multi-context (Q-set) environments;
(2) We propose a Credibility-Weighted Nonlinear Aggregation (CWNA) operator that enabled fusion of multi-expert evaluations by considering expert trustworthiness and nonlinear fusion of Type-2 values.
(3) We develop a novel Bipolar Divergence-Aware Certainty Index (BDCI) that quantifies truth-falsity divergence, and contradiction across bipolar dimensions for robust scoring and ranking.
(4) We demonstrate the practical applicablity of our framework through a realistic multi-parameter case study on Intangible Cultural Heritage (ICH) in Tourism, involving multiple alternatives, linguistic contexts, and expert judgments.

Experimental results validate the effectiveness of our proposed T2-QNSS model in handling fuzzy, hesitant, and linguistically diverse knowledge under conflicting expert views.

2. Core Concepts

In this part of our article, we introduce a concise but insightful overview on essential concepts of neutrosophic theory.

Definition 2.1 ([8], [9]**)**. Consider $\mathbb{U}$ as universal set, N as NS that is expressed as:

$$\begin{gathered} N = \{\langle \varpi, (\mathfrak{T}_{\aleph}(\varpi), \mathfrak{I}_{\aleph}(\varpi), \mathfrak{F}_{\aleph}(\varpi))\rangle : \varpi \in \mathbb{U}\}, \\ \text{where} \\ \mathfrak{T}_{\aleph}(\varpi), \mathfrak{I}_{\aleph}(\varpi), \mathfrak{F}_{\aleph}(\varpi): \mathbb{U} \to]^{-}0, 1^{+}[\\ \&\& \\ ^{-}0 \leq \mathfrak{T}_{\aleph}(\varpi) + \mathfrak{I}_{\aleph}(\varpi) + \mathfrak{F}_{\aleph}(\varpi) \leq 3^{+} \end{gathered} \tag{1}$$

Definition 2.2 ([10], [11]). Consider $\mathbb{U}$ as universal set, an Interval-Valued NS A over $\mathbb{U}$ is defined as:

$$N_{intervaled} = \{\langle \varpi, \mathfrak{T}_{A}(\varpi), \mathfrak{I}_{A}(\varpi), \mathfrak{F}_{A}(\varpi)\rangle : \varpi \in \mathbb{U}\} \tag{2}$$

where $\mathfrak{T}_{A}(\varpi) \subseteq [0,1]$, $\mathfrak{I}_{A}(\varpi) \subseteq [0,1]$, and $\mathfrak{F}_{A}(\varpi) \subseteq [0,1]$, and for all $\varpi \in \mathbb{U}$:

$$0 \leq \sup(\mathfrak{T}_{A}(\varpi)) + \sup(\mathfrak{I}_{A}(\varpi)) + \sup(\mathfrak{F}_{A}(\varpi)) \leq 3 \tag{3}$$

This model allows uncertainty to be represented more flexibly by using intervals instead of precise membership values.

Definition 2.3 ([12]). Given a universe of discourse $\mathbb{U}$, A Bipolar NS B is defined as:

$$N_{bipolar} = \left\{ \begin{matrix} \langle \varpi, \mathfrak{T}^{+}(\varpi), \mathfrak{I}^{+}(\varpi), \mathfrak{F}^{+}(\varpi), \mathfrak{T}^{-}(\varpi), \mathfrak{I}^{-}(\varpi), \mathfrak{F}^{-}(\varpi) \rangle : \\ \varpi \in \mathbb{U} \end{matrix} \right\} \quad (4)$$

Where $\mathfrak{T}^{+}(\varpi), \mathfrak{I}^{+}(\varpi), \mathfrak{F}^{+}(\varpi) \in [0,1]$ denotes positive membership functions, and $\mathfrak{T}^{-}(\varpi), \mathfrak{I}^{-}(\varpi), \mathfrak{F}^{-}(\varpi) \in [-1,0]$ denotes negative membership functions,

For each $\varpi \in \mathbb{U}$, the constraints are:

$$0 \leq \mathfrak{T}^{+}(\varpi) + \mathfrak{I}^{+}(\varpi) + \mathfrak{F}^{+}(\varpi) \leq 3, -3 \leq \mathfrak{T}^{-}(\varpi) + \mathfrak{I}^{-}(\varpi) + \mathfrak{F}^{-}(\varpi) \leq 0 \quad (5)$$

This structure is particularly useful when modeling opposing opinions or dual-perspective evaluations, such as in contradictory interior design preferences.

Definition 2.4 ([13]). Consider $\mathbb{U}$ as universal set and Q as a nonempty set, then the Q- NS, $\aleph_Q$, is articulated follows:

$$\aleph_Q = \left\{ \begin{matrix} \langle (\varpi, q), \mathfrak{T}_{\aleph_Q}(\varpi, q), \mathfrak{I}_{\aleph_Q}(\varpi, q), \mathfrak{F}_{\aleph_Q}(\varpi, q) \rangle : \\ \varpi \in \mathbb{U}, q \in Q \end{matrix} \right\}, \\ \text{where} \\ \mathfrak{T}_{\aleph_Q}, \mathfrak{I}_{\aleph_Q}, \mathfrak{F}_{\aleph_Q} : \mathbb{U} \times Q \to]^{-}0, 1^{+}[\text{ and } {}^{-}0 \leq \mathfrak{T}_{\aleph_Q} + \mathfrak{I}_{\aleph_Q} + \mathfrak{F}_{\aleph_Q} \leq 3^{+} \quad (6)$$

Definition 2.5 ([13]). Consider $\mathbb{U}$ as universal set and Q as a nonempty set, l be any positive integer and $\mathfrak{I}$ be a unit interval [0,1], then multi Q- NS $\widetilde{\aleph}_Q$ in $\mathbb{U}$ and Q is a set of ordered sequences

$$\widetilde{\aleph}_Q = \left\{ \left\langle (\varpi, q), \mathfrak{T}_{\widetilde{\aleph}_{Q_i}}(\varpi, q), \mathfrak{I}_{\widetilde{\aleph}_{Q_i}}(\varpi, q), \mathfrak{F}_{\widetilde{\aleph}_{Q_i}}(\varpi, q) \right\rangle : \varpi \in \mathbb{U}, q \in Q \right\} \\ \text{where} \\ \mathfrak{T}_{\widetilde{\aleph}_{Q_i}}, \mathfrak{I}_{\widetilde{\aleph}_{Q_i}}, \mathfrak{F}_{\widetilde{\aleph}_{Q_i}} : \mathbb{U} \times Q \to \mathfrak{I}^{l} \\ 0 \leq \mathfrak{T}_{\widetilde{\aleph}_{Q_i}} + \mathfrak{I}_{\widetilde{\aleph}_{Q_i}} + \mathfrak{F}_{\widetilde{\aleph}_{Q_i}} \leq 3 \quad (7)$$

Definition 2.6 ([14]). Consider $\mathbb{U}$ as universal set and a non-empty set Q, and parameters set E, and set of all multi Q- NS s, $\mu^{l}Q_{NS}(\mathbb{U})$, with dimension $l = 1$, then, a Q-neutrosophic soft set (QNSS) can be define as follows:

$$\aleph_Q : Z \to \mu^{l}Q_{NS}(\mathbb{U}) \text{ such that } Z \subseteq E \text{ and } \aleph_Q(e) = \phi \text{ if } e \notin Z \quad (8)$$

QNSS can be represented by the set of ordered pairs

$$(\aleph_Q, Z) = \{(e, \aleph_Q(e)) : e \in Z, \aleph_Q \in \mu^{l}Q_{NS}(\mathbb{U})\}. \quad (9)$$

Definition 2.7 ([13],[14]**).** Let's have subsets $\left(\aleph_Q, Z\right)=\left\{\begin{pmatrix}\mathfrak{T}_{\aleph_Q(e_j)}(\varpi,q)_i, \mathfrak{I}_{\aleph_Q(e_j)}(\varpi,q)_i, \\ \mathfrak{F}_{\aleph_Q(e_j)}(\varpi,q)_i\end{pmatrix}: \\ \forall e_j \in Z,(\varpi,q)_i \in \mathbb{U} \times Q\right\} \in$ $Q_{NSS}(\mathbb{U})$ and $\left(\varphi_Q, \Xi\right)=\left\{\begin{pmatrix}\mathfrak{T}_{\varphi_Q(e_j)}(\varpi,q)_i, \mathfrak{I}_{\varphi_Q(e_j)}(\varpi,q)_i, \\ \mathfrak{F}_{\varphi_Q(e_j)}(\varpi,q)_i\end{pmatrix}: \\ \forall e_j \in \Xi,(\varpi,q)_i \in \mathbb{U} \times Q\right\} \in Q_{NSS}(\mathbb{U})$, the $\left(\varphi_Q, \Xi\right)$ is called a subset of $\left(\aleph_{\mathrm{Q}}, \mathrm{Z}\right)$ if $\Xi \subseteq Z$ and $\varphi_Q(\varpi) \subseteq \aleph_Q(\varpi)$ for all $\varpi \in \mathbb{U}$.

$$\begin{gathered} Z \subseteq \Xi \,\&\, \mathrm{N}_Q(e) \subseteq \varphi\Phi_Q(e) \forall e \in Z \\ \textit{where} \\ \mathfrak{T}_{\mathrm{N}_Q(e)}(\varpi,q) \leq \mathfrak{T}_{\Phi_Q(e)}(\varpi,q), \\ \mathfrak{I}_{\mathrm{N}_Q(e)}(\varpi,q) \geq \mathfrak{I}_{\Phi_Q(e)}(\varpi,q), \mathfrak{F}_{\mathrm{N}_Q(e)}(\varpi,q) \\ \geq \mathfrak{F}_{\Phi_Q(e)}(\varpi,q), \forall(\varpi,q) \in \mathbb{U} \times Q \end{gathered} \tag{10}$$

Definition 2.8 ([13],[14]**).** Consider $\mathbb{U}$ as universal set, QNSSs $\left(\mathrm{N_Q}, \mathrm{X}\right)= \left\{\left(\mathrm{T}_{\mathrm{N_Q(e)}}(\varpi,\mathrm{q}), \mathrm{I}_{\mathrm{N_Q(e)}}(\varpi,\mathrm{q}), \mathrm{F}_{\mathrm{N_Q(e)}}(\varpi,\mathrm{q})\right): \\ \forall \mathrm{e} \in \mathrm{X},(\varpi,\mathrm{q}) \in \mathbb{U} \times \mathrm{Q}\right\}$ and $\left(\varphi_Q, Y\right)= \left\{\left(\mathfrak{T}_{\varphi_Q(e)}(\varpi,q), \mathfrak{I}_{\varphi_Q(e)}(\varpi,q), \mathfrak{F}_{\varphi_Q(e)}(\varpi,q)\right): \\ \forall e \in Y,(\varpi,q) \in \mathbb{U} \times Q\right\}$, union is denoted as $\left(\cup_{\mathrm{Q}}, \mathrm{C}\right)= (\mathrm{N_Q}, \mathrm{A}) \cup (\Phi_{\mathrm{Q}}, \mathrm{B})$, and is calculated as follows:

$$\mathfrak{T}_{\cup_Q(c)}(\varpi,q)=\begin{cases}\mathfrak{T}_{\mathrm{N}_Q(c)}(\varpi,q) & if c \in Z-\Xi, \\ \mathfrak{T}_{\Phi_Q(c)}(\varpi,q) & if c \in \Xi-Z, \\ max\left\{\mathfrak{T}_{\Lambda_Q(c)}(\varpi,q), \mathfrak{T}_{\Phi_Q(c)}(\varpi,q)\right\} & if c \in Z \cap \Xi,\end{cases} \tag{11}$$

$$\mathfrak{I}_{\cup_Q(c)}(\varpi,q)=\begin{cases}\mathfrak{I}_{\mathrm{N}_Q(c)}(\varpi,q) & if c \in Z-\Xi, \\ \mathfrak{I}_{\Phi_Q(c)}(\varpi,q) & if c \in \Xi-Z, \\ \min\left\{\mathfrak{I}_{\mathrm{N}_Q(c)}(\varpi,q), \mathfrak{I}_{\Phi_Q(c)}(\varpi,q)\right\} & if c \in Z \cap \Xi,\end{cases} \tag{12}$$

$$\mathfrak{F}_{\cup_Q(c)}(\varpi,q)=\begin{cases}\mathfrak{F}_{\mathrm{N}_Q(c)}(\varpi,q) & if c \in Z-\Xi, \\ \mathfrak{F}_{\Phi_Q(c)}(\varpi,q) & if c \in \Xi-Z, \\ min\{\mathfrak{F}_{\mathrm{N}_Q(c)}(\varpi,q), \mathfrak{F}_{\Phi_Q(c)}(\varpi,q)\} & if c \in Z \cap \Xi.\end{cases} \tag{13}$$

where $C=A \cup B$ and $c \in C,(\varpi,q) \in \mathbb{U} \times Q$.

Definition 2.9 ([13]**).** Consider QNSSs $(\aleph_Q, Z) = \left\{\begin{matrix}\left(\mathfrak{T}_{\aleph_{Q(e)}}(\varpi, q), \mathfrak{I}_{\aleph_{Q(e)}}(\varpi, q), \mathfrak{F}_{\aleph_{Q(e)}}(\varpi, q)\right): \\ \forall e \in Z, (\varpi, q) \in \mathbb{U} \times Q\end{matrix}\right\}$ as well as $(\varphi_Q, \Xi) = \left\{\begin{matrix}\left(\mathfrak{T}_{\varphi_{Q(e)}}(\varpi, q), \mathfrak{I}_{\varphi_{Q(e)}}(\varpi, q), \mathfrak{F}_{\varphi_{Q(e)}}(\varpi, q)\right): \\ \forall e \in \Xi, (\varpi, q) \in \mathbb{U} \times Q\end{matrix}\right\}$, their intersection of is denoted as $(\cap_Q, C) = (\mathrm{N}_Q, Z) \cap (\Phi_Q, \Xi)$, and is calculated as bellows:

$$\mathfrak{T}_{\cap_{Q(c)}}(\varpi, q) = min\left\{\begin{matrix}\mathfrak{T}_{\mathrm{N}_{Q(c)}}(\varpi, q), \\ \mathfrak{T}_{\Phi_{Q(c)}}(\varpi, q)\end{matrix}\right\}, \mathfrak{I}_{\cap_{Q(c)}}(\varpi, q) = max\left\{\begin{matrix}\mathfrak{I}_{\mathrm{N}_{Q(c)}}(\varpi, q), \\ \mathfrak{I}_{\Phi_{Q(c)}}(\varpi, q)\end{matrix}\right\}, \mathfrak{F}_{\cap_{Q(c)}}(\varpi, q) = max\left\{\mathfrak{F}_{\mathrm{N}_{Q(c)}}(\varpi, q), \mathfrak{F}_{\Phi_{Q(c)}}(\varpi, q)\right\}. \quad (14)$$

Definition 2.10 ([14]**).** Consider QNSS $(\aleph_Q, Z) = \left\{\begin{matrix}\left(\mathfrak{T}_{\aleph_{Q(e)}}(\varpi, q), \mathfrak{I}_{\aleph_{Q(e)}}(\varpi, q), \mathfrak{F}_{\aleph_{Q(e)}}(\varpi, q)\right): \\ \forall e \in Z, (\varpi, q) \in \mathbb{U} \times Q\end{matrix}\right\}$, the complement $(\mathrm{N}_Q, Z)^c = (\mathrm{N}_Q^c, Z)$ is defined as:

$$(\mathrm{N}_Q, Z)^c = \left\{\langle e, \mathfrak{T}^c_{\mathrm{N}_{Q(e)}}(\varpi, q), \mathfrak{I}^c_{\mathrm{N}_{Q(e)}}(\varpi, q), \mathfrak{F}^c_{\mathrm{N}_{Q(e)}}(\varpi, q)\rangle : e \in Z, (\varpi, q) \in \mathbb{U} \times Q\right\}, \quad (15)$$

where $\forall e \in Z, (\varpi, q) \in \mathbb{U} \times Q$

$$\begin{aligned}\mathfrak{T}^c_{\mathrm{N}_{Q(e)}}(\varpi, q) &= 1 - \mathfrak{T}_{\mathrm{N}_{Q(e)}}(\varpi, q), \\ \mathfrak{I}^c_{\mathrm{N}_{Q(e)}}(\varpi, q) &= 1 - \mathfrak{I}_{\mathrm{N}_{Q(e)}}(\varpi, q), \\ \mathfrak{F}^c_{\mathrm{N}_{Q(e)}}(\varpi, q) &= 1 - \mathfrak{F}_{\mathrm{N}_{Q(e)}}(\varpi, q).\end{aligned} \quad (16)$$

Definition 2.11 ([15]**).** Consider QNSSs $(\aleph_Q, Z)$, and (φ_Q, Ξ), with $\mathbb{U} = \{\varpi_1, \varpi_2, \ldots, \varpi_m\}$, $Q = \{q_1, q_2, \ldots, q_l\}$, and $Z = \{e_1, e_2, \ldots, e_n\}$, hamming distance between them is defined as:

$$dist_H^{QNSS}\left((\aleph_Q, Z), (\varphi_Q, \Xi)\right) = \sum_{j=1}^{n}\sum_{i=1}^{lm}\left(\frac{\left|\mathfrak{T}_{\aleph_Q(e_j^Z)}(\varpi, q)_i - \mathfrak{T}_{\varphi_Q(e_j^\Xi)}(\varpi, q)_i\right| + \left|\mathfrak{I}_{\aleph_Q(e_j^Z)}(\varpi, q)_i - \mathfrak{I}_{\varphi_Q(e_j^\Xi)}(\varpi, q)_i\right| + \left|\mathfrak{F}_{\aleph_Q(e_j^Z)}(\varpi, q)_i - \mathfrak{F}_{\varphi_Q(e_j^\Xi)}(\varpi, q)_i\right|}{3}\right), \quad (17)$$

Definition 2.12 ([15]**).** Consider QNSSs $(\aleph_Q, Z)$, and (φ_Q, Ξ), with $\mathbb{U} = \{\varpi_1, \varpi_2, \ldots, \varpi_m\}$, $Q = \{q_1, q_2, \ldots, q_l\}$, and $Z = \{e_1, e_2, \ldots, e_n\}$, the excluding distance between these QNSSs is measred as :

$$dist_E^{QNSS}\left((\aleph_Q, Z),(\Phi_Q, \Xi)\right) = \sum_{j=1}^{n}\sum_{i=1}^{lm}\sqrt{\frac{\left(\mathfrak{T}_{\aleph_Q(e_j^Z)}(\varpi,q)_i - \mathfrak{T}_{\Phi_Q(e_j^\Xi)}(\varpi,q)_i\right)^2 + \left(\mathfrak{I}_{\aleph_Q(e_j^Z)}(\varpi,q)_i - \mathfrak{I}_{\Phi_Q(e_j^\Xi)}(\varpi,q)_i\right)^2 + \left(\mathfrak{F}_{\aleph_Q(e_j^Z)}(\varpi,q)_i - \mathfrak{F}_{\Phi_Q(e_j^\Xi)}(\varpi,q)_i\right)^2}{3}}, \quad (18)$$

Definition 2.13 ([13],[14]**)**. The possibility operation of QNSS $(\aleph_Q, Z) = \left\{\begin{pmatrix}\mathfrak{T}_{\aleph_Q(e)}(\varpi,q), \mathfrak{I}_{\aleph_Q(e)}(\varpi,q), \\ \mathfrak{F}_{\aleph_Q(e)}(\varpi,q)\end{pmatrix}: \forall e \in Z, (\varpi,q) \in \mathbb{U}\times Q\right\}$, is defined as:

$$\otimes\left(\mathrm{N}_Q, Z\right) = \left\{\begin{matrix}\langle e,[(\varpi,q), 1-\mathfrak{F}_{\mathrm{N}_Q}(\varpi,q), \mathfrak{I}_{\mathrm{N}_Q}(\varpi,q), \mathfrak{F}_{\mathrm{N}_Q}(\varpi,q)]\rangle: \\ (\varpi,q)\in\mathbb{U}\times Q\end{matrix}\right\}. \quad (19)$$

Definition 2.14 ([13],[14]**)**. Given $(\aleph_Q, Z) \in QNSS(\mathbb{U})$, $\aleph_Q(e) = \phi$ for all $e \in Z$, then $(\aleph_Q, Z)$ can be declared as a null $QNSS(\mathbb{U})$, and is referred to as (ϕ, Z).

Definition 2.15 ([13],[14]). The necessity operation of QNSS $(\aleph_Q, Z) = \left\{\begin{matrix}\left(\mathfrak{T}_{\aleph_Q(e)}(\varpi,q), \mathfrak{I}_{\aleph_Q(e)}(\varpi,q), \mathfrak{F}_{\aleph_Q(e)}(\varpi,q)\right): \\ \forall e \in Z, (\varpi,q) \in \mathbb{U}\times Q\end{matrix}\right\}$ is defined as:

$$\oplus\left(\mathrm{N}_Q, Z\right) = \left\{\begin{matrix}\langle e,[(\varpi,q), \mathfrak{T}_{\mathrm{N}_Q}(\varpi,q), \mathfrak{I}_{\mathrm{N}_Q}(\varpi,q), 1-\mathfrak{T}_{\mathrm{N}_Q}(\varpi,q)]\rangle: \\ (\varpi,q)\in\mathbb{U}\times Q\end{matrix}\right\}. \quad (20)$$

3. Q-Neutrosophic Type-2 Soft Set (T2-QNSS)

In classic QNSS, each evaluation tuple consists of $\mathfrak{T}, \mathfrak{I}, \mathfrak{F} \in [0,1]$ representing degrees of memberships. In Type-2 logic, uncertainty is captured at a higher level — not just uncertainty *about the truth* (as in intervals), but co-existing, opposing assessments. Thus, we propose T2-QNSS in which each soft set evaluation is expanded to include positive and negative neutrosophic components — modeling dual subjective perspectives.

Definition 3.1. Let U be Universe of discourse, Q is Set of qualitative labels, E is Set of parameters, and $\mathcal{Z} \subseteq E$ is parameter subset; then T2-QNSS is a mapping:

$$\Gamma_Q^{(2)}: \mathcal{Z} \to \mu^{l}_{Q^{(2)}NS}(U) \quad (21)$$

Where for each parameter $e \in \mathcal{Z}$, and each $(u,q) \in U \times Q$, the evaluation is given by a tuple:

$$\langle (u,q), \mathfrak{T}^{+}(u,q), \mathfrak{I}^{+}(u,q), \mathfrak{F}^{+}(u,q), \mathfrak{T}^{-}(u,q), \mathfrak{I}^{-}(u,q), \mathfrak{F}^{-}(u,q) \rangle \quad (22)$$

Where: $\mathfrak{T}^{+}(u,q), \mathfrak{I}^{+}(u,q), \mathfrak{F}^{+}(u,q) \in [0,1]$: positive neutrosophic components.
$\mathfrak{T}^{-}(u,q), \mathfrak{I}^{-}(u,q), \mathfrak{F}^{-}(u,q) \in [-1,0]$: negative neutrosophic components (opposing view)

With constraints:

$$0 \le \mathfrak{T}^{+}(u,q) + \mathfrak{I}^{+}(u,q) + \mathfrak{F}^{+}(u,q) \le 3, -3 \le \mathfrak{T}^{-}(u,q) + \mathfrak{I}^{-}(u,q) + \mathfrak{F}^{-}(u,q) \le 0 \quad (23)$$

Example 1. Consider government evaluating tourists' perceptions of different ICH experiences. Each experience is assessed by two opposing perspectives namely Positive dimension such as appreciation, perceived authenticity, cultural immersion; and Negative dimension like discomfort, irrelevance, perceived exploitation. U= {u_1= Traditional Dance, u_2 = Folk Storytelling}, with qualitative labels Q={ q_1 =Local Tourist, q_2 = Foreign Tourist}, and evaluation aspects $E = \{e_1 = Cultural\ Authenticity,\ e_2 = Emotional\ impact\}$. Table 1 shows Type-2 QNSS evaluations, where each row gives a 6-component bipolar neutrosophic tuple

Table 1. T2-QNSS representation of tourists' bipolar evaluations of ICH experiences

Parameter	(ICH Activity, Tourist Type)	$\mathfrak{T}^{+}$	$\mathfrak{I}^{+}$	$\mathfrak{F}^{+}$	$\mathfrak{T}^{-}$	$\mathfrak{I}^{-}$	$\mathfrak{F}^{-}$
e_1	(Dance, Local)	0.8	0.1	0.1	-0.1	-0.2	-0.1
e_1	(Dance, Foreign)	0.6	0.2	0.2	-0.3	-0.3	-0.2
e_1	(Storytelling, Local)	0.7	0.2	0.1	-0.2	-0.1	-0.2
e_1	(Storytelling, Foreign)	0.5	0.3	0.2	-0.4	-0.2	-0.2
e_2	(Dance, Local)	0.6	0.2	0.2	-0.3	-0.2	-0.1
e_2	(Dance, Foreign)	0.5	0.3	0.2	-0.4	-0.2	-0.2
e_2	(Storytelling, Local)	0.9	0.05	0.05	-0.1	-0.1	-0.2
e_2	(Storytelling, Foreign)	0.4	0.3	0.3	-0.5	-0.2	-0.3

To make things simpler, we visualize the bipolar evaluations for parameters in Figure 2.

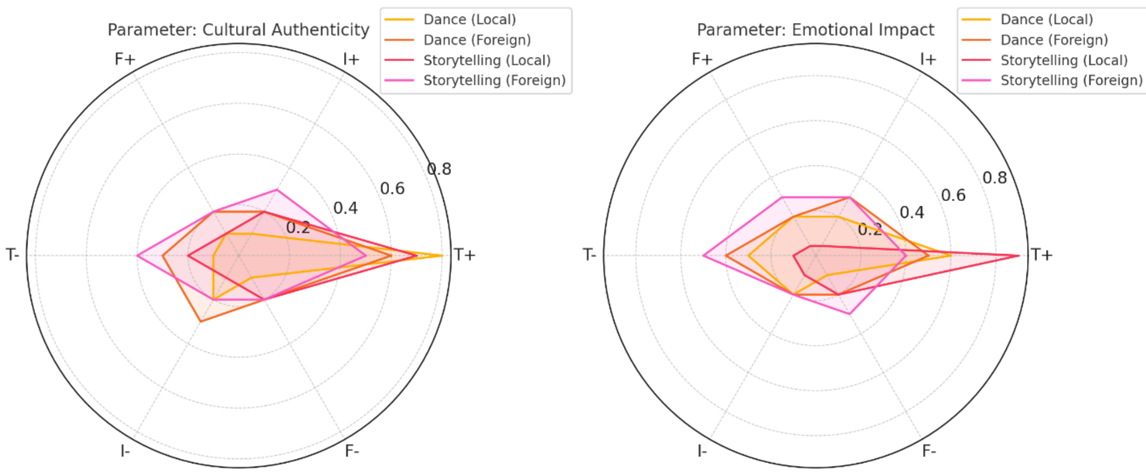

Figure 2. T2-QNSS Bipolar Evaluation for ICH Tourism

4. Operations

In this section, we define the fundamental operations over T2-QNSS, including subset, union, intersection, complement, and related algebraic operations. These operations extend traditional QNSS logic to accommodate the bipolar structure of T2-QNSS evaluations.

Consider $\left(\Gamma_Q^{(2)}, \mathcal{Z}\right)$ and $\left(\Phi_Q^{(2)}, \Xi\right)$ be two T2-QNSSs over U, where $(u,q) \in U \times Q$, where each mapping contains:

$$\langle (u,q), \mathfrak{T}^+, \mathfrak{I}^+, \mathfrak{F}^+, \mathfrak{T}^-, \mathfrak{I}^-, \mathfrak{F}^- \rangle \tag{24}$$

Definition 4.1. Consider $\left(\Gamma_Q^{(2)}, \mathcal{Z}\right)$ and $\left(\Phi_Q^{(2)}, \Xi\right)$ be two T2-QNSSs over U, then $\left(\Gamma_Q^{(2)}, \mathcal{Z}\right) \subseteq \left(\Phi_Q^{(2)}, \Xi\right)^{**}$ if $\mathcal{Z} \subseteq \Xi$, and for all $e \in \mathcal{Z}, (u,q) \in U \times Q$:

$$\begin{array}{lll} \mathfrak{T}_{\Gamma^+}(u,q) \leq \mathfrak{T}_{\Phi^+}(u,q), & \mathfrak{I}_{\Gamma^+}(u,q) \geq \mathfrak{I}_{\Phi^+}(u,q), & \mathfrak{F}_{\Gamma^+}(u,q) \geq \mathfrak{F}_{\Phi^+}(u,q) \\ \mathfrak{T}_{\Gamma^-}(u,q) \geq \mathfrak{T}_{\Phi^-}(u,q), & \mathfrak{I}_{\Gamma^-}(u,q) \leq \mathfrak{I}_{\Phi^-}(u,q), & \mathfrak{F}_{\Gamma^-}(u,q) \leq \mathfrak{F}_{\Phi^-}(u,q) \end{array} \tag{25}$$

Definition 4.2. Consider $\left(\Gamma_Q^{(2)}, \mathcal{Z}\right)$ and $\left(\Phi_Q^{(2)}, \Xi\right)$ be two T2-QNSSs over U, $\left(\Theta_Q^{(2)}, \mathcal{C}\right) = \left(\Gamma_Q^{(2)}, \mathcal{Z}\right) \cup \left(\Phi_Q^{(2)}, \Xi\right)$, where $\mathcal{C} = \mathcal{Z} \cup \Xi$. For all $e \in \mathcal{C}, (u,q) \in U \times Q$:

$$\begin{aligned}
\mathfrak{T}_{\Theta^+}(u,q) &= \max\{\mathfrak{T}_{\Gamma^+}(u,q), \mathfrak{T}_{\Phi^+}(u,q)\}\\
\mathfrak{J}_{\Theta^+}(u,q) &= \min\{\mathfrak{J}_{\Gamma^+}(u,q), \mathfrak{J}_{\Phi^+}(u,q)\}\\
\mathfrak{F}_{\Theta^+}(u,q) &= \min\{\mathfrak{F}_{\Gamma^+}(u,q), \mathfrak{F}_{\Phi^+}(u,q)\}\\
\mathfrak{T}_{\Theta^-}(u,q) &= \min\{\mathfrak{T}_{\Gamma^-}(u,q), \mathfrak{T}_{\Phi^-}(u,q)\}\\
\mathfrak{J}_{\Theta^-}(u,q) &= \max\{\mathfrak{J}_{\Gamma^-}(u,q), \mathfrak{J}_{\Phi^-}(u,q)\}\\
\mathfrak{F}_{\Theta^-}(u,q) &= \max\{\mathfrak{F}_{\Gamma^-}(u,q), \mathfrak{F}_{\Phi^-}(u,q)\}
\end{aligned} \tag{26}$$

Definition 4.3. Consider $\left(\Gamma_Q^{(2)}, \mathcal{Z}\right)$ and $\left(\Phi_Q^{(2)}, \Xi\right)$ be two T2-QNSSs over U, $\left(\Lambda_Q^{(2)}, \mathcal{C}\right) = \left(\Gamma_Q^{(2)}, \mathcal{Z}\right) \cap \left(\Phi_Q^{(2)}, \Xi\right)$, where $\mathcal{C} = \mathcal{Z} \cap \Xi$. For all $e \in \mathcal{C}, (u,q) \in U \times Q$:

$$\begin{aligned}
\mathfrak{T}_{\Lambda^+}(u,q) &= \min\{\mathfrak{T}_{\Gamma^+}(u,q), \mathfrak{T}_{\Phi^+}(u,q)\}\\
\mathfrak{J}_{\Lambda^+}(u,q) &= \max\{\mathfrak{J}_{\Gamma^+}(u,q), \mathfrak{J}_{\Phi^+}(u,q)\}\\
\mathfrak{F}_{\Lambda^+}(u,q) &= \max\{\mathfrak{F}_{\Gamma^+}(u,q), \mathfrak{F}_{\Phi^+}(u,q)\}\\
\mathfrak{T}_{\Lambda^-}(u,q) &= \max\{\mathfrak{T}_{\Gamma^-}(u,q), \mathfrak{T}_{\Phi^-}(u,q)\}\\
\mathfrak{J}_{\Lambda^-}(u,q) &= \min\{\mathfrak{J}_{\Gamma^-}(u,q), \mathfrak{J}_{\Phi^-}(u,q)\}\\
\mathfrak{F}_{\Lambda^-}(u,q) &= \min\{\mathfrak{F}_{\Gamma^-}(u,q), \mathfrak{F}_{\Phi^-}(u,q)\}
\end{aligned} \tag{27}$$

Definition 4.4. Consider $\left(\Gamma_Q^{(2)}, \mathcal{Z}\right)$ be an T2-QNSS over U, Let $\left(\Gamma_Q^{(2)}, \mathcal{Z}\right)^c = \left(\Gamma_Q^{(2)c}, \mathcal{Z}\right)$. For each $e \in \mathcal{Z}, (u,q) \in U \times Q$.

$$\begin{aligned}
&\mathfrak{T}^c_{\Gamma^+}(u,q) = \mathfrak{F}_{\Gamma^+}(u,q), \quad \mathfrak{F}^c_{\Gamma^+}(u,q) = \mathfrak{T}_{\Gamma^+}(u,q), \quad \mathfrak{J}^c_{\Gamma^+}(u,q) = 1 - \mathfrak{J}_{\Gamma^+}(u,q)\\
&\mathfrak{T}^c_{\Gamma^-}(u,q) = \mathfrak{F}_{\Gamma^-}(u,q), \quad \mathfrak{F}^c_{\Gamma^-}(u,q) = \mathfrak{T}_{\Gamma^-}(u,q), \quad \mathfrak{J}^c_{\Gamma^-}(u,q) = 1 - |\mathfrak{J}_{\Gamma^-}(u,q)|
\end{aligned} \tag{28}$$

According to the above definition T2-QNSS, we can drive the algebraic operators on T2-QNSS**.** These operators extend classical soft set algebra to the bipolar neutrosophic domain**,** enabling advanced decision modeling, reasoning, and fusion under both positive and negative evaluations**.**

Definition 4.5. Let $\alpha \in [0,1]$ be a scalar (weight or importance factor). The scalar multiplication of a T2-QNSS with α is defined component-wise. For each $e \in \mathcal{Z}, (u,q) \in U \times Q$:

$$\begin{aligned}
&\alpha \cdot \mathfrak{T}^+(u,q) = \alpha \cdot \mathfrak{T}^+(u,q), \quad \alpha \cdot \mathfrak{J}^+(u,q) = \alpha \cdot \mathfrak{J}^+(u,q), \quad \alpha \cdot \mathfrak{F}^+(u,q) = \alpha \cdot \mathfrak{F}^+(u,q)\\
&\alpha \cdot \mathfrak{T}^-(u,q) = \alpha \cdot \mathfrak{T}^-(u,q), \quad \alpha \cdot \mathfrak{J}^-(u,q) = \alpha \cdot \mathfrak{J}^-(u,q), \quad \alpha \cdot \mathfrak{F}^-(u,q) = \alpha \cdot \mathfrak{F}^-(u,q)
\end{aligned} \tag{29}$$

Note: For the negative components ($\mathfrak{T}^-, \mathfrak{J}^-, \mathfrak{F}^-$), which lie in $[-1,0]$, multiplication with $\alpha \in [0,1]$ will preserve negativity.

Definition 4.6. The algebraic sum of two T2-QNSSs, $\Gamma^{(2)} \oplus \Phi^{(2)} = \Theta^{(2)}$, captures combined evaluations. For all $e \in \mathcal{Z} \cap \Xi$, and $(u,q) \in U \times Q$:

$$\begin{aligned} \mathfrak{T}_\Theta^+(u,q) &= \min\{1, \mathfrak{T}_\Gamma^+(u,q) + \mathfrak{T}_\Phi^+(u,q)\} \\ \mathfrak{I}_\Theta^+(u,q) &= \min\{1, \mathfrak{I}_\Gamma^+(u,q) + \mathfrak{I}_\Phi^+(u,q)\} \\ \mathfrak{F}_\Theta^+(u,q) &= \min\{1, \mathfrak{F}_\Gamma^+(u,q) + \mathfrak{F}_\Phi^+(u,q)\} \end{aligned} \tag{30}$$

and,

$$\begin{aligned} \mathfrak{T}_\Theta^-(u,q) &= \max\{-1, \mathfrak{T}_\Gamma^-(u,q) + \mathfrak{T}_\Phi^-(u,q)\} \\ \mathfrak{I}_\Theta^-(u,q) &= \max\{-1, \mathfrak{I}_\Gamma^-(u,q) + \mathfrak{I}_\Phi^-(u,q)\} \\ \mathfrak{F}_\Theta^-(u,q) &= \max\{-1, \mathfrak{F}_\Gamma^-(u,q) + \mathfrak{F}_\Phi^-(u,q)\} \end{aligned} \tag{31}$$

Definition 4.7. The algebraic product, $\Gamma^{(2)} \otimes \Phi^{(2)} = \Theta^{(2)}$, is defined for multiplicative fusion - often used when both sources must support a fact. For all $e \in \mathcal{Z} \cap \Xi$, and $(u,q) \in U \times Q$:

$$\begin{aligned} \mathfrak{T}_\Theta^+(u,q) &= \mathfrak{T}_\Gamma^+(u,q) \cdot \mathfrak{T}_\Phi^+(u,q) \\ \mathfrak{I}_\Theta^+(u,q) &= \mathfrak{I}_\Gamma^+(u,q) \cdot \mathfrak{I}_\Phi^+(u,q) \\ \mathfrak{F}_\Theta^+(u,q) &= \mathfrak{F}_\Gamma^+(u,q) \cdot \mathfrak{F}_\Phi^+(u,q) \end{aligned} \tag{32}$$

and,

$$\begin{aligned} \mathfrak{T}_\Theta^-(u,q) &= \mathfrak{T}_\Gamma^-(u,q) \cdot \mathfrak{T}_\Phi^-(u,q) \\ \mathfrak{I}_\Theta^-(u,q) &= \mathfrak{I}_\Gamma^-(u,q) \cdot \mathfrak{I}_\Phi^-(u,q) \\ \mathfrak{F}_\Theta^-(u,q) &= \mathfrak{F}_\Gamma^-(u,q) \cdot \mathfrak{F}_\Phi^-(u,q) \end{aligned} \tag{33}$$

Definition 4.8. Dual negation, $\mathcal{N}\left(\Gamma_Q^{(2)}\right) = \Gamma_Q^{(2)'}$, helps evaluate inverse belief. It is defined as follows:

$$\begin{aligned} &\mathfrak{T}^{+'}(u,q) = \mathfrak{F}^+(u,q), \quad \mathfrak{F}^{+'}(u,q) = \mathfrak{T}^+(u,q), \quad \mathfrak{I}^{+'}(u,q) = 1 - \mathfrak{I}^+(u,q) \\ &\mathfrak{T}^{-'}(u,q) = \mathfrak{F}^-(u,q), \quad \mathfrak{F}^{-'}(u,q) = \mathfrak{T}^-(u,q), \quad \mathfrak{I}^{-'}(u,q) = 1 - |\mathfrak{I}^-(u,q)| \end{aligned} \tag{34}$$

5. Algorithmic Decision Based on T2-QNSS

This section proposes a structured decision-making framework based on the T2-QNSS for evaluating and ranking a finite set of alternatives under bipolar uncertainty and multi-criteria subjectivity.

In step 1, we collect expert evaluations. For every expert r and every ordered pair (x_i, e_j) the expert reports

$$\Gamma_r^{(2)}(e_j)(x_i) = \langle \mathfrak{T}_{rij}^+, \mathfrak{I}_{rij}^+, \mathfrak{F}_{rij}^+, \mathfrak{T}_{rij}^-, \mathfrak{I}_{rij}^-, \mathfrak{F}_{rij}^- \rangle. \tag{35}$$

These k individual T2-QNSSs constitute an information matrix of dimension $m \times n \times 6$.

In step 2, we present a novel aggregation for T2-QNSS named Credibility-Weighted Nonlinear Aggregation (CWNA). It define a nonlinear, credibility-sensitive aggregation rule using a soft-maximum kernel for positive evidence and a soft-minimum kernel for negative evidence. Let $\rho = \{\rho_1, \ldots, \rho_k\}$ be expert credibility weights such that $\sum_{r=1}^{k} \rho_r = 1$. We define positive truth aggregation T^+

$$\mathfrak{T}_{ij}^+ = \frac{\sum_{r=1}^{k} \rho_r \cdot \left(\mathfrak{T}_{rij}^+\right)^\alpha}{\sum_{r=1}^{k} \rho_r} \quad \text{where } \alpha > 1 \tag{36}$$

where $\alpha \to 1$, reduces to standard weighted average, while $\alpha \to \infty$, becomes crisp maximum. For positive indeterminacy $\mathfrak{I}^+$:

$$\mathfrak{I}_{ij}^+ = \frac{1}{1 + \sum_{r=1}^{k} \rho_r \cdot \log\left(1 + \mathfrak{I}_{rij}^+\right)} \tag{37}$$

Then, We define positive falsity $\mathfrak{F}^+$

$$\mathfrak{F}_{ij}^+ = \frac{\prod_{r=1}^{k} \left(1 + \rho_r \cdot \mathfrak{F}_{rij}^+\right) - 1}{\prod_{r=1}^{k} (1 + \rho_r) - 1} \tag{38}$$

Then, we apply mirrored logic for negative evaluation, using $\beta > 1$ for contrast:

$$\mathfrak{T}_{ij}^- = -\left(\frac{\sum_{r=1}^{k} \rho_r \cdot \left|\mathfrak{T}_{rij}^-\right|^\beta}{\sum_{r=1}^{k} \rho_r}\right) \; \mathfrak{F}_{ij}^- = -\left(\frac{\prod_{r=1}^{k} \left(1 + \rho_r \cdot \left|\mathfrak{F}_{rij}^-\right|\right) - 1}{\prod_{r=1}^{k} (1 + \rho_r) - 1}\right)$$
$$\mathfrak{I}_{ij}^- = -\left(\frac{1}{1 + \sum_{r=1}^{k} \rho_r \cdot \log\left(1 + \left|\mathfrak{I}_{rij}^-\right|\right)}\right) \tag{39}$$

Thus, we represent final aggregated tuple

$$\Gamma^{(2)}(e_j)(x_i) = \langle \mathfrak{T}_{ij}^+, \mathfrak{I}_{ij}^+, \mathfrak{F}_{ij}^+, \mathfrak{T}_{ij}^-, \mathfrak{I}_{ij}^-, \mathfrak{F}_{ij}^- \rangle \tag{40}$$

In step 3, we introduce a new Bipolar Divergence-Aware Certainty Index (BDCI) as the scoring function:

$$S(x_i) = \sum_{j=1}^{n} w_j \cdot \left[\mathcal{A}_{ij} - \gamma \cdot \mathcal{D}_{ij} - \lambda \cdot \mathcal{U}_{ij}\right] \quad (41)$$

Where for each pair $\left(x_i, e_j\right)$, $\mathcal{A}_{ij}$ denote Bipolar Agreement Score and is defined as.

$$\mathcal{A}_{ij} = \mathfrak{T}_{ij}^{+} \cdot \left(1 + \mathfrak{T}_{ij}^{-}\right) + \mathfrak{F}_{ij}^{-} \cdot \left(1 - \mathfrak{F}_{ij}^{+}\right) \quad (42)$$

This term rewards consistency between positive and negative sides. If both support the alternative ($\mathfrak{T}^{*}$ high, T^{-}near 0), this is strong evidence. $\mathcal{D}_{ij}$ define contradiction divergence Scor, which penalizes conflict between bipolar parts:

$$\mathcal{D}_{ij} = \left|\mathfrak{T}_{ij}^{+} + \mathfrak{T}_{ij}^{-}\right| + \left|\mathfrak{F}_{ij}^{+} + \mathfrak{F}_{ij}^{-}\right| \quad (43)$$

Finally, the $\mathcal{U}_{ij}$ symbolize uncertainty Score

$$\mathcal{U}_{ij} = \mathfrak{I}_{ij}^{+} + \left|\mathfrak{I}_{ij}\right| \quad (44)$$

Where w_j : weight of parameter e_j. $\gamma \in [0,1]$: divergence sensitivity - higher = more penalty for contradiction. $\lambda \in [0,1]$: uncertainty penalty

In step 4, we - Overall Alternative Score

Aggregate over parameters with the given weights:

$$S(x_i) = \sum_{j=1}^{n} w_j S_{ij} \quad (45)$$

In step 5, we rank the alternatives in descending order of $S(x_i)$.

6. Case Study: Evaluating ICH Sites for Sustainable Cultural Tourism Development

A tourism planning committee is evaluating five ICH sites $X = \{x_1, x_2, x_3, x_4, x_5\}$ for inclusion in a national promotional campaign. The evaluation is based on six decision parameters relevant to cultural sustainability: e_1 : Cultural Significance, e_2 : Visitor Engagement, e_3 : Preservation Readiness, e_4 : Accessibility, e_5 : Local Community Involvement, e_6 : Economic Impact Potential. Experts consider each site under three linguistic cultural contexts: $Q = \{q_1 =$ "Localrelevance" , $q_2 =$ "Nationalsymbolism" , $q_3 =$ "Globalappeal" $\}$. Three cultural heritage experts provide evaluations as T2-QNSS capturing both positive and negative dimensions of each site's features under the contexts. Expert credibility weights are given as:

$$\rho = \{0.5,0.3,0.2\}$$

Parameter importance weights (based on stakeholder consensus):

$$w = \{0.25,0.15,0.2,0.1,0.2,0.1\}$$

To model nonlinear trust and divergence effects, the committee sets aggregation parameters: $\alpha = 2.5$, $\beta = 3.0$, $\gamma = 1.2$, $\lambda = 0.8$

Tables 2 through 4 present the individual evaluations of the five intangible cultural heritage alternatives as assessed by three domain experts. These assessments utilize the T2-QNSS framework, considering both positive and negative degrees across: $e_1 \cdots : e_6$ and Q.

Table 2. T2-QNSS decision matrices provided from Expert 1.

U	Q	e_1	e_2	e_3	e_4	e_5	e_6
x_1	q_1	⟨(0.72,0.01,0.08,-0.21,-0.29,-0.2)⟩	⟨(0.85,0.04,0.13,-0.11,-0.09,-0.15)⟩	⟨(0.41,0.1,0.19,-0.37,-0.09,-0.18)⟩	⟨(0.8,0.0,0.24,-0.45,-0.14,-0.05)⟩	⟨(0.88,0.17,0.03,-0.15,-0.34,-0.18)⟩	⟨(0.8,0.36,0.16,-0.59,-0.15,-0.17)⟩
x_1	q_2	⟨(0.81,0.31,0.26,-0.39,-0.28,-0.01)⟩	⟨(0.51,0.14,0.02,-0.22,-0.04,-0.08)⟩	⟨(0.72,0.18,0.11,-0.2,-0.11,-0.28)⟩	⟨(0.72,0.3,0.05,-0.46,-0.07,-0.11)⟩	⟨(0.89,0.32,0.17,-0.44,-0.34,-0.23)⟩	⟨(0.51,0.02,0.09,-0.23,-0.08,-0.28)⟩
x_1	q_3	⟨(0.84,0.16,0.2,-0.3,-0.37,-0.14)⟩	⟨(0.53,0.12,0.17,-0.23,-0.23,-0.27)⟩	⟨(0.6,0.11,0.3,-0.35,-0.04,-0.01)⟩	⟨(0.45,0.31,0.24,-0.31,-0.03,-0.11)⟩	⟨(0.9,0.26,0.29,-0.53,-0.0,-0.22)⟩	⟨(0.74,0.27,0.08,-0.42,-0.04,-0.13)⟩
x_2	q_1	⟨(0.63,0.48,0.26,-0.23,-0.2,-0.05)⟩	⟨(0.86,0.44,0.09,-0.42,-0.24,-0.05)⟩	⟨(0.78,0.27,0.23,-0.37,-0.0,-0.1)⟩	⟨(0.41,0.46,0.26,-0.52,-0.12,-0.02)⟩	⟨(0.84,0.47,0.03,-0.34,-0.03,-0.23)⟩	⟨(0.78,0.06,0.14,-0.37,-0.11,-0.26)⟩
x_2	q_2	⟨(0.61,0.11,0.16,-0.46,-0.08,-0.09)⟩	⟨(0.9,0.32,0.13,-0.36,-0.05,-0.07)⟩	⟨(0.57,0.29,0.07,-0.21,-0.03,-0.19)⟩	⟨(0.51,0.45,0.26,-0.14,-0.1,-0.2)⟩	⟨(0.51,0.07,0.28,-0.39,-0.19,-0.24)⟩	⟨(0.8,0.1,0.03,-0.32,-0.17,-0.14)⟩
x_2	q_3	⟨(0.76,0.34,0.3,-0.15,-0.16,-0.1)⟩	⟨(0.83,0.12,0.06,-0.32,-0.17,-0.08)⟩	⟨(0.52,0.46,0.13,-0.53,-0.22,-0.02)⟩	⟨(0.9,0.42,0.29,-0.56,-0.34,-0.05)⟩	⟨(0.64,0.11,0.12,-0.13,-0.15,-0.3)⟩	⟨(0.53,0.39,0.14,-0.31,-0.38,-0.3)⟩
x_3	q_1	⟨(0.68,0.36,0.05,-0.25,-0.39,-0.17)⟩	⟨(0.67,0.37,0.02,-0.39,-0.2,-0.26)⟩	⟨(0.48,0.48,0.02,-0.19,-0.24,-0.2)⟩	⟨(0.52,0.06,0.27,-0.22,-0.24,-0.19)⟩	⟨(0.61,0.29,0.16,-0.57,-0.08,-0.21)⟩	⟨(0.52,0.2,0.2,-0.25,-0.13,-0.23)⟩
x_3	q_2	⟨(0.44,0.23,0.3,-0.6,-0.03,-0.06)⟩	⟨(0.53,0.47,0.26,-0.54,-0.15,-0.05)⟩	⟨(0.82,0.35,0.18,-0.59,-0.26,-0.0)⟩	⟨(0.81,0.15,0.2,-0.57,-0.05,-0.03)⟩	⟨(0.45,0.28,0.08,-0.4,-0.29,-0.06)⟩	⟨(0.72,0.13,0.15,-0.55,-0.34,-0.03)⟩
x_3	q_3	⟨(0.61,0.14,0.0,-0.49,-0.25,-0.08)⟩	⟨(0.77,0.28,0.13,-0.1,-0.03,-0.26)⟩	⟨(0.85,0.27,0.25,-0.39,-0.06,-0.04)⟩	⟨(0.55,0.45,0.24,-0.53,-0.36,-0.06)⟩	⟨(0.52,0.05,0.23,-0.54,-0.16,-0.19)⟩	⟨(0.48,0.46,0.26,-0.59,-0.32,-0.26)⟩
x_4	q_1	⟨(0.41,0.37,0.1,-0.57,-0.32,-0.26)⟩	⟨(0.81,0.13,0.24,-0.15,-0.35,-0.26)⟩	⟨(0.51,0.41,0.14,-0.25,-0.32,-0.07)⟩	⟨(0.41,0.1,0.1,-0.53,-0.39,-0.08)⟩	⟨(0.72,0.2,0.29,-0.37,-0.38,-0.03)⟩	⟨(0.89,0.09,0.29,-0.23,-0.04,-0.13)⟩
x_4	q_2	⟨(0.76,0.16,0.18,-0.36,-0.15,-0.17)⟩	⟨(0.53,0.35,0.0,-0.56,-0.22,-0.22)⟩	⟨(0.77,0.34,0.11,-0.13,-0.27,-0.1)⟩	⟨(0.56,0.42,0.22,-0.25,-0.12,-0.12)⟩	⟨(0.6,0.15,0.04,-0.31,-0.38,-0.2)⟩	⟨(0.85,0.31,0.09,-0.37,-0.0,-0.09)⟩
x_4	q_3	⟨(0.61,0.29,0.2,-0.33,-0.18,-0.06)⟩	⟨(0.64,0.45,0.24,-0.18,-0.03,-0.15)⟩	⟨(0.72,0.17,0.25,-0.48,-0.27,-0.07)⟩	⟨(0.5,0.01,0.07,-0.34,-0.34,-0.02)⟩	⟨(0.61,0.31,0.06,-0.45,-0.2,-0.07)⟩	⟨(0.73,0.0,0.23,-0.49,-0.04,-0.13)⟩
x_5	q_1	⟨(0.49,0.48,0.16,-0.13,-0.1,-0.25)⟩	⟨(0.63,0.4,0.2,-0.59,-0.24,-0.29)⟩	⟨(0.85,0.31,0.22,-0.35,-0.33,-0.16)⟩	⟨(0.85,0.37,0.14,-0.23,-0.1,-0.19)⟩	⟨(0.78,0.26,0.19,-0.24,-0.03,-0.09)⟩	⟨(0.54,0.16,0.16,-0.17,-0.09,-0.21)⟩
x_5	q_2	⟨(0.75,0.03,0.12,-0.37,-0.17,-0.06)⟩	⟨(0.61,0.45,0.18,-0.45,-0.34,-0.23)⟩	⟨(0.59,0.0,0.11,-0.48,-0.34,-0.29)⟩	⟨(0.61,0.37,0.16,-0.4,-0.09,-0.07)⟩	⟨(0.62,0.01,0.1,-0.44,-0.16,-0.05)⟩	⟨(0.63,0.06,0.19,-0.11,-0.16,-0.17)⟩
x_5	q_3	⟨(0.41,0.32,0.04,-0.33,-0.02,-0.11)⟩	⟨(0.51,0.16,0.23,-0.29,-0.3,-0.25)⟩	⟨(0.53,0.04,0.01,-0.37,-0.4,-0.1)⟩	⟨(0.73,0.39,0.2,-0.48,-0.38,-0.06)⟩	⟨(0.41,0.08,0.04,-0.43,-0.23,-0.07)⟩	⟨(0.75,0.38,0.05,-0.4,-0.3,-0.03)⟩

Table 3. T2-QNSS decision matrices provided from Expert 2.

U	Q	e_1	e_2	e_3	e_4	e_5	e_6
x_1	q_1	⟨(0.81,0.48,0.03,-0.11,-0.12,-0.2)⟩	⟨(0.88,0.2,0.21,-0.14,-0.28,-0.19)⟩	⟨(0.45,0.39,0.26,-0.4,-0.05,-0.3)⟩	⟨(0.79,0.17,0.13,-0.29,-0.2,-0.1)⟩	⟨(0.82,0.41,0.03,-0.58,-0.25,-0.25)⟩	⟨(0.75,0.22,0.22,-0.58,-0.11,-0.24)⟩
x_1	q_2	⟨(0.67,0.24,0.13,-0.47,-0.11,-0.26)⟩	⟨(0.82,0.04,0.26,-0.22,-0.19,-0.18)⟩	⟨(0.59,0.01,0.26,-0.19,-0.08,-0.24)⟩	⟨(0.57,0.44,0.21,-0.24,-0.0,-0.28)⟩	⟨(0.44,0.36,0.15,-0.48,-0.28,-0.19)⟩	⟨(0.65,0.4,0.03,-0.21,-0.28,-0.09)⟩
x_1	q_3	⟨(0.69,0.24,0.16,-0.31,-0.3,-0.1)⟩	⟨(0.75,0.14,0.08,-0.16,-0.08,-0.04)⟩	⟨(0.67,0.38,0.06,-0.21,-0.19,-0.22)⟩	⟨(0.89,0.26,0.08,-0.15,-0.08,-0.07)⟩	⟨(0.49,0.01,0.16,-0.24,-0.39,-0.17)⟩	⟨(0.75,0.06,0.26,-0.35,-0.35,-0.17)⟩

x_2	q_1	⟨(0.63,0.22,0.06,-0.13,-0.38,-0.14)⟩	⟨(0.81,0.2,0.02,-0.41,-0.02,-0.04)⟩	⟨(0.68,0.15,0.3,-0.16,-0.31,-0.18)⟩	⟨(0.8,0.11,0.16,-0.33,-0.18,-0.26)⟩	⟨(0.9,0.15,0.19,-0.4,-0.3,-0.28)⟩	⟨(0.5,0.11,0.2,-0.18,-0.07,-0.02)⟩
x_2	q_2	⟨(0.4,0.23,0.18,-0.25,-0.09,-0.21)⟩	⟨(0.75,0.23,0.21,-0.56,-0.32,-0.19)⟩	⟨(0.73,0.47,0.13,-0.37,-0.26,-0.27)⟩	⟨(0.81,0.04,0.05,-0.25,-0.3,-0.17)⟩	⟨(0.54,0.06,0.21,-0.45,-0.38,-0.15)⟩	⟨(0.65,0.04,0.01,-0.32,-0.13,-0.08)⟩
x_2	q_3	⟨(0.45,0.48,0.25,-0.39,-0.38,-0.3)⟩	⟨(0.74,0.13,0.01,-0.48,-0.19,-0.2)⟩	⟨(0.86,0.09,0.18,-0.42,-0.2,-0.03)⟩	⟨(0.57,0.17,0.2,-0.53,-0.13,-0.21)⟩	⟨(0.54,0.47,0.24,-0.38,-0.18,-0.09)⟩	⟨(0.56,0.49,0.12,-0.36,-0.4,-0.2)⟩
x_3	q_1	⟨(0.67,0.21,0.06,-0.28,-0.3,-0.19)⟩	⟨(0.78,0.1,0.16,-0.56,-0.18,-0.21)⟩	⟨(0.46,0.49,0.18,-0.22,-0.06,-0.17)⟩	⟨(0.68,0.05,0.3,-0.56,-0.18,-0.04)⟩	⟨(0.82,0.25,0.21,-0.35,-0.11,-0.25)⟩	⟨(0.89,0.12,0.17,-0.29,-0.37,-0.15)⟩
x_3	q_2	⟨(0.84,0.43,0.08,-0.5,-0.17,-0.28)⟩	⟨(0.65,0.41,0.08,-0.25,-0.23,-0.3)⟩	⟨(0.64,0.07,0.16,-0.27,-0.22,-0.16)⟩	⟨(0.63,0.16,0.06,-0.45,-0.23,-0.07)⟩	⟨(0.79,0.02,0.22,-0.45,-0.32,-0.12)⟩	⟨(0.73,0.41,0.29,-0.35,-0.01,-0.15)⟩
x_3	q_3	⟨(0.7,0.43,0.26,-0.32,-0.21,-0.14)⟩	⟨(0.76,0.2,0.2,-0.18,-0.19,-0.29)⟩	⟨(0.57,0.35,0.19,-0.53,-0.34,-0.26)⟩	⟨(0.59,0.16,0.22,-0.48,-0.35,-0.01)⟩	⟨(0.43,0.32,0.28,-0.6,-0.3,-0.13)⟩	⟨(0.45,0.32,0.26,-0.32,-0.28,-0.27)⟩
x_4	q_1	⟨(0.42,0.4,0.09,-0.29,-0.06,-0.16)⟩	⟨(0.68,0.4,0.05,-0.14,-0.35,-0.19)⟩	⟨(0.52,0.46,0.04,-0.33,-0.1,-0.08)⟩	⟨(0.4,0.4,0.27,-0.44,-0.06,-0.13)⟩	⟨(0.57,0.29,0.19,-0.31,-0.1,-0.25)⟩	⟨(0.5,0.19,0.14,-0.22,-0.23,-0.17)⟩
x_4	q_2	⟨(0.9,0.15,0.29,-0.43,-0.11,-0.17)⟩	⟨(0.74,0.37,0.01,-0.4,-0.2,-0.27)⟩	⟨(0.54,0.4,0.18,-0.28,-0.25,-0.19)⟩	⟨(0.74,0.36,0.2,-0.52,-0.25,-0.27)⟩	⟨(0.72,0.15,0.13,-0.39,-0.29,-0.03)⟩	⟨(0.55,0.37,0.05,-0.17,-0.22,-0.29)⟩
x_4	q_3	⟨(0.67,0.46,0.25,-0.23,-0.33,-0.14)⟩	⟨(0.8,0.37,0.1,-0.16,-0.39,-0.04)⟩	⟨(0.88,0.43,0.22,-0.59,-0.39,-0.24)⟩	⟨(0.58,0.4,0.0,-0.37,-0.18,-0.2)⟩	⟨(0.74,0.29,0.25,-0.57,-0.04,-0.07)⟩	⟨(0.41,0.44,0.17,-0.56,-0.09,-0.02)⟩
x_5	q_1	⟨(0.81,0.45,0.09,-0.3,-0.06,-0.28)⟩	⟨(0.55,0.25,0.03,-0.54,-0.05,-0.14)⟩	⟨(0.74,0.37,0.28,-0.31,-0.3,-0.05)⟩	⟨(0.61,0.05,0.15,-0.3,-0.38,-0.01)⟩	⟨(0.59,0.22,0.29,-0.53,-0.04,-0.21)⟩	⟨(0.67,0.49,0.11,-0.3,-0.08,-0.04)⟩
x_5	q_2	⟨(0.82,0.23,0.2,-0.42,-0.24,-0.01)⟩	⟨(0.79,0.12,0.04,-0.38,-0.03,-0.23)⟩	⟨(0.5,0.11,0.26,-0.26,-0.06,-0.27)⟩	⟨(0.4,0.43,0.04,-0.16,-0.1,-0.05)⟩	⟨(0.73,0.01,0.0,-0.49,-0.1,-0.1)⟩	⟨(0.49,0.03,0.22,-0.36,-0.3,-0.14)⟩
x_5	q_3	⟨(0.79,0.26,0.03,-0.35,-0.38,-0.01)⟩	⟨(0.79,0.43,0.16,-0.33,-0.39,-0.02)⟩	⟨(0.64,0.2,0.21,-0.35,-0.36,-0.02)⟩	⟨(0.44,0.3,0.02,-0.24,-0.25,-0.16)⟩	⟨(0.56,0.5,0.16,-0.33,-0.24,-0.03)⟩	⟨(0.75,0.43,0.2,-0.48,-0.29,-0.06)⟩

Table 4. T2-QNSS decision matrices provided from Expert 3.

U	Q	e_1	e_2	e_3	e_4	e_5	e_6
x_1	q_1	⟨(0.63,0.11,0.1,-0.33,-0.17,-0.03)⟩	⟨(0.61,0.33,0.11,-0.18,-0.37,-0.02)⟩	⟨(0.82,0.05,0.03,-0.47,-0.32,-0.17)⟩	⟨(0.69,0.28,0.1,-0.16,-0.14,-0.2)⟩	⟨(0.78,0.43,0.22,-0.58,-0.24,-0.11)⟩	⟨(0.69,0.11,0.2,-0.21,-0.04,-0.25)⟩
x_1	q_2	⟨(0.58,0.38,0.17,-0.5,-0.34,-0.29)⟩	⟨(0.81,0.31,0.19,-0.11,-0.37,-0.25)⟩	⟨(0.53,0.09,0.21,-0.25,-0.14,-0.0)⟩	⟨(0.83,0.28,0.12,-0.17,-0.25,-0.01)⟩	⟨(0.77,0.11,0.13,-0.27,-0.15,-0.22)⟩	⟨(0.79,0.28,0.03,-0.13,-0.06,-0.19)⟩
x_1	q_3	⟨(0.74,0.14,0.2,-0.34,-0.18,-0.08)⟩	⟨(0.78,0.06,0.13,-0.24,-0.27,-0.15)⟩	⟨(0.73,0.02,0.12,-0.4,-0.0,-0.09)⟩	⟨(0.51,0.07,0.08,-0.26,-0.0,-0.22)⟩	⟨(0.49,0.19,0.21,-0.35,-0.33,-0.24)⟩	⟨(0.44,0.43,0.01,-0.11,-0.37,-0.26)⟩
x_2	q_1	⟨(0.69,0.29,0.21,-0.31,-0.05,-0.01)⟩	⟨(0.56,0.4,0.19,-0.52,-0.37,-0.03)⟩	⟨(0.82,0.12,0.18,-0.36,-0.16,-0.09)⟩	⟨(0.57,0.17,0.05,-0.36,-0.05,-0.15)⟩	⟨(0.85,0.17,0.22,-0.51,-0.33,-0.07)⟩	⟨(0.47,0.1,0.18,-0.48,-0.26,-0.05)⟩
x_2	q_2	⟨(0.79,0.25,0.23,-0.48,-0.18,-0.28)⟩	⟨(0.68,0.32,0.19,-0.53,-0.25,-0.05)⟩	⟨(0.43,0.22,0.09,-0.24,-0.02,-0.15)⟩	⟨(0.56,0.23,0.02,-0.52,-0.03,-0.26)⟩	⟨(0.83,0.31,0.15,-0.33,-0.22,-0.24)⟩	⟨(0.85,0.22,0.24,-0.43,-0.13,-0.14)⟩
x_2	q_3	⟨(0.48,0.03,0.03,-0.55,-0.14,-0.21)⟩	⟨(0.65,0.09,0.07,-0.32,-0.18,-0.16)⟩	⟨(0.48,0.19,0.08,-0.3,-0.14,-0.18)⟩	⟨(0.79,0.32,0.02,-0.15,-0.27,-0.09)⟩	⟨(0.76,0.33,0.27,-0.54,-0.13,-0.17)⟩	⟨(0.47,0.17,0.29,-0.45,-0.16,-0.18)⟩
x_3	q_1	⟨(0.87,0.15,0.11,-0.5,-0.33,-0.2)⟩	⟨(0.81,0.37,0.21,-0.36,-0.26,-0.13)⟩	⟨(0.58,0.18,0.05,-0.21,-0.38,-0.15)⟩	⟨(0.51,0.07,0.02,-0.52,-0.04,-0.23)⟩	⟨(0.82,0.44,0.01,-0.27,-0.31,-0.04)⟩	⟨(0.59,0.08,0.25,-0.49,-0.32,-0.05)⟩
x_3	q_2	⟨(0.62,0.21,0.2,-0.22,-0.18,-0.09)⟩	⟨(0.77,0.22,0.16,-0.25,-0.32,-0.14)⟩	⟨(0.82,0.18,0.28,-0.59,-0.18,-0.08)⟩	⟨(0.59,0.26,0.29,-0.51,-0.32,-0.04)⟩	⟨(0.53,0.32,0.26,-0.38,-0.04,-0.25)⟩	⟨(0.83,0.14,0.23,-0.24,-0.36,-0.04)⟩
x_3	q_3	⟨(0.62,0.47,0.07,-0.33,-0.14,-0.01)⟩	⟨(0.43,0.25,0.07,-0.6,-0.15,-0.01)⟩	⟨(0.87,0.42,0.19,-0.5,-0.06,-0.09)⟩	⟨(0.81,0.35,0.04,-0.45,-0.18,-0.0)⟩	⟨(0.44,0.13,0.25,-0.37,-0.29,-0.16)⟩	⟨(0.46,0.14,0.09,-0.12,-0.17,-0.24)⟩
x_4	q_1	⟨(0.63,0.06,0.27,-0.4,-0.01,-0.15)⟩	⟨(0.52,0.07,0.13,-0.41,-0.1,-0.12)⟩	⟨(0.73,0.04,0.29,-0.13,-0.21,-0.15)⟩	⟨(0.89,0.28,0.12,-0.34,-0.25,-0.29)⟩	⟨(0.53,0.01,0.24,-0.27,-0.29,-0.19)⟩	⟨(0.79,0.37,0.1,-0.12,-0.22,-0.24)⟩
x_4	q_2	⟨(0.49,0.39,0.14,-0.45,-0.25,-0.24)⟩	⟨(0.43,0.39,0.14,-0.25,-0.02,-0.06)⟩	⟨(0.42,0.47,0.15,-0.59,-0.22,-0.08)⟩	⟨(0.78,0.1,0.11,-0.49,-0.35,-0.1)⟩	⟨(0.46,0.18,0.27,-0.47,-0.36,-0.12)⟩	⟨(0.89,0.25,0.15,-0.56,-0.21,-0.24)⟩
x_4	q_3	⟨(0.76,0.04,0.18,-0.51,-0.22,-0.1)⟩	⟨(0.44,0.33,0.09,-0.4,-0.17,-0.21)⟩	⟨(0.58,0.02,0.26,-0.28,-0.4,-0.08)⟩	⟨(0.89,0.47,0.02,-0.42,-0.15,-0.24)⟩	⟨(0.74,0.48,0.04,-0.4,-0.31,-0.01)⟩	⟨(0.43,0.39,0.11,-0.29,-0.23,-0.18)⟩
x_5	q_1	⟨(0.74,0.47,0.11,-0.48,-0.23,-0.16)⟩	⟨(0.6,0.32,0.07,-0.16,-0.29,-0.15)⟩	⟨(0.59,0.28,0.08,-0.23,-0.18,-0.3)⟩	⟨(0.54,0.46,0.15,-0.16,-0.34,-0.14)⟩	⟨(0.85,0.22,0.03,-0.44,-0.34,-0.1)⟩	⟨(0.57,0.03,0.16,-0.55,-0.34,-0.21)⟩
x_5	q_2	⟨(0.86,0.32,0.24,-0.35,-0.05,-0.06)⟩	⟨(0.47,0.4,0.01,-0.38,-0.15,-0.24)⟩	⟨(0.68,0.31,0.03,-0.25,-0.4,-0.22)⟩	⟨(0.66,0.38,0.25,-0.14,-0.39,-0.19)⟩	⟨(0.62,0.34,0.1,-0.54,-0.31,-0.19)⟩	⟨(0.49,0.48,0.13,-0.56,-0.02,-0.04)⟩

x_5	q_3	⟨(0.48,0.08,0.1,-0.45,-0.14,-0.28)⟩	⟨(0.85,0.42,0.08,-0.42,-0.22,-0.04)⟩	⟨(0.55,0.27,0.15,-0.18,-0.38,-0.05)⟩	⟨(0.73,0.36,0.18,-0.52,-0.23,-0.25)⟩	⟨(0.41,0.02,0.19,-0.39,-0.26,-0.23)⟩	⟨(0.61,0.32,0.15,-0.41,-0.12,-0.29)⟩

By aggregating above metrics using CWNA, we obtain decision matrix in Table 5.

Table 5. CWNA Aggregated T2-QNSS Decision Matrix based on $\rho = \{0.5,0.3,0.2\}$.

U	Q	e_1	e_2	e_3	e_4	e_5	e_6
x_1	q_1	⟨(0.73,0.17,0.07,-0.20,-0.21,-0.17)⟩	⟨(0.81,0.15,0.15,-0.13,-0.20,-0.14)⟩	⟨(0.50,0.18,0.18,-0.40,-0.12,-0.21)⟩	⟨(0.78,0.11,0.18,-0.34,-0.16,-0.10)⟩	⟨(0.84,0.29,0.07,-0.36,-0.29,-0.19)⟩	⟨(0.76,0.27,0.19,-0.51,-0.12,-0.21)⟩
x_1	q_2	⟨(0.72,0.30,0.20,-0.44,-0.24,-0.14)⟩	⟨(0.66,0.14,0.13,-0.20,-0.15,-0.14)⟩	⟨(0.64,0.11,0.18,-0.21,-0.11,-0.21)⟩	⟨(0.70,0.34,0.11,-0.34,-0.09,-0.14)⟩	⟨(0.73,0.29,0.16,-0.42,-0.28,-0.22)⟩	⟨(0.61,0.19,0.06,-0.20,-0.14,-0.21)⟩
x_1	q_3	⟨(0.78,0.18,0.19,-0.31,-0.31,-0.12)⟩	⟨(0.65,0.11,0.14,-0.21,-0.19,-0.18)⟩	⟨(0.65,0.17,0.19,-0.32,-0.08,-0.09)⟩	⟨(0.59,0.25,0.16,-0.25,-0.04,-0.12)⟩	⟨(0.69,0.17,0.24,-0.41,-0.18,-0.21)⟩	⟨(0.68,0.24,0.12,-0.34,-0.20,-0.17)⟩
x_2	q_1	⟨(0.64,0.36,0.19,-0.22,-0.22,-0.07)⟩	⟨(0.79,0.36,0.09,-0.44,-0.20,-0.04)⟩	⟨(0.76,0.20,0.24,-0.30,-0.12,-0.12)⟩	⟨(0.56,0.30,0.19,-0.43,-0.12,-0.12)⟩	⟨(0.86,0.31,0.12,-0.39,-0.17,-0.21)⟩	⟨(0.63,0.08,0.17,-0.33,-0.13,-0.15)⟩
x_2	q_2	⟨(0.58,0.17,0.18,-0.40,-0.10,-0.16)⟩	⟨(0.81,0.29,0.17,-0.45,-0.17,-0.10)⟩	⟨(0.59,0.33,0.09,-0.26,-0.10,-0.21)⟩	⟨(0.61,0.28,0.15,-0.25,-0.15,-0.20)⟩	⟨(0.58,0.12,0.23,-0.40,-0.25,-0.21)⟩	⟨(0.77,0.11,0.07,-0.34,-0.15,-0.12)⟩
x_2	q_3	⟨(0.61,0.32,0.23,-0.30,-0.22,-0.18)⟩	⟨(0.77,0.12,0.05,-0.37,-0.18,-0.13)⟩	⟨(0.61,0.30,0.14,-0.45,-0.20,-0.05)⟩	⟨(0.78,0.33,0.21,-0.47,-0.26,-0.11)⟩	⟨(0.63,0.26,0.19,-0.29,-0.15,-0.21)⟩	⟨(0.53,0.38,0.16,-0.35,-0.34,-0.25)⟩
x_3	q_1	⟨(0.72,0.27,0.07,-0.31,-0.35,-0.18)⟩	⟨(0.73,0.29,0.10,-0.43,-0.21,-0.22)⟩	⟨(0.49,0.42,0.07,-0.20,-0.21,-0.18)⟩	⟨(0.57,0.06,0.23,-0.38,-0.18,-0.15)⟩	⟨(0.71,0.31,0.15,-0.44,-0.14,-0.19)⟩	⟨(0.65,0.15,0.20,-0.31,-0.24,-0.17)⟩
x_3	q_2	⟨(0.60,0.29,0.21,-0.49,-0.10,-0.13)⟩	⟨(0.61,0.40,0.19,-0.40,-0.21,-0.14)⟩	⟨(0.77,0.23,0.19,-0.49,-0.23,-0.06)⟩	⟨(0.71,0.17,0.18,-0.52,-0.16,-0.04)⟩	⟨(0.57,0.21,0.16,-0.41,-0.25,-0.12)⟩	⟨(0.74,0.22,0.21,-0.43,-0.24,-0.07)⟩
x_3	q_3	⟨(0.64,0.29,0.09,-0.41,-0.22,-0.08)⟩	⟨(0.70,0.25,0.14,-0.22,-0.10,-0.22)⟩	⟨(0.77,0.32,0.22,-0.45,-0.14,-0.12)⟩	⟨(0.61,0.34,0.19,-0.50,-0.32,-0.03)⟩	⟨(0.48,0.15,0.25,-0.52,-0.23,-0.17)⟩	⟨(0.47,0.35,0.23,-0.42,-0.28,-0.26)⟩
x_4	q_1	⟨(0.46,0.32,0.13,-0.45,-0.18,-0.21)⟩	⟨(0.71,0.20,0.16,-0.20,-0.30,-0.21)⟩	⟨(0.56,0.35,0.14,-0.25,-0.23,-0.09)⟩	⟨(0.50,0.23,0.15,-0.47,-0.26,-0.14)⟩	⟨(0.64,0.19,0.25,-0.33,-0.28,-0.13)⟩	⟨(0.75,0.18,0.21,-0.20,-0.13,-0.16)⟩
x_4	q_2	⟨(0.75,0.20,0.20,-0.40,-0.16,-0.18)⟩	⟨(0.57,0.36,0.03,-0.45,-0.17,-0.20)⟩	⟨(0.63,0.38,0.14,-0.27,-0.25,-0.12)⟩	⟨(0.66,0.34,0.19,-0.38,-0.21,-0.16)⟩	⟨(0.61,0.16,0.11,-0.37,-0.35,-0.13)⟩	⟨(0.77,0.32,0.09,-0.35,-0.11,-0.18)⟩
x_4	q_3	⟨(0.66,0.29,0.21,-0.34,-0.23,-0.09)⟩	⟨(0.65,0.40,0.17,-0.22,-0.17,-0.13)⟩	⟨(0.74,0.22,0.24,-0.47,-0.33,-0.12)⟩	⟨(0.60,0.22,0.04,-0.37,-0.25,-0.12)⟩	⟨(0.68,0.34,0.11,-0.48,-0.17,-0.06)⟩	⟨(0.57,0.21,0.19,-0.47,-0.09,-0.11)⟩
x_5	q_1	⟨(0.64,0.47,0.13,-0.25,-0.11,-0.24)⟩	⟨(0.60,0.34,0.12,-0.49,-0.19,-0.22)⟩	⟨(0.77,0.32,0.21,-0.31,-0.29,-0.15)⟩	⟨(0.72,0.29,0.15,-0.24,-0.23,-0.13)⟩	⟨(0.74,0.24,0.19,-0.37,-0.10,-0.13)⟩	⟨(0.58,0.23,0.15,-0.29,-0.14,-0.16)⟩
x_5	q_2	⟨(0.79,0.15,0.17,-0.38,-0.17,-0.04)⟩	⟨(0.64,0.34,0.10,-0.41,-0.21,-0.23)⟩	⟨(0.58,0.10,0.14,-0.37,-0.27,-0.27)⟩	⟨(0.56,0.39,0.14,-0.28,-0.15,-0.09)⟩	⟨(0.65,0.08,0.07,-0.47,-0.17,-0.09)⟩	⟨(0.56,0.14,0.19,-0.28,-0.17,-0.14)⟩
x_5	q_3	⟨(0.54,0.25,0.05,-0.36,-0.15,-0.11)⟩	⟨(0.66,0.29,0.18,-0.33,-0.31,-0.14)⟩	⟨(0.57,0.13,0.10,-0.33,-0.38,-0.07)⟩	⟨(0.64,0.36,0.14,-0.42,-0.31,-0.13)⟩	⟨(0.46,0.19,0.11,-0.39,-0.24,-0.09)⟩	⟨(0.72,0.38,0.11,-0.43,-0.26,-0.09)⟩

To assess the robustness and reliability of the **CWNA** method under varying expert influence, a sensitivity analysis was conducted using four distinct credibility weight configurations: (i) Equal weights for all experts, (ii) Expert 1 dominant, (iii) Expert 2 dominant, and (iv) Expert 3 dominant. Each configuration re-weights the three expert decision matrices and re-aggregates T2-QNSS values accordingly. This analysis enables a better understanding of how much individual expert bias can affect the final aggregation results. The sensitivity outcomes (showing Table 6-9) indicate noticeable variations in aggregated values across scenarios. These variations may influence the subsequent scoring and ranking outcomes, demonstrating the importance of carefully assigning credibility weights based on expert reliability and experience.

Table 6. CWNA Aggregated T2-QNSS Decision Matrix under Equal Expert Weights.

U	Q	e_1	e_2	e_3	e_4	e_5	e_6
x_1	q_1	⟨(0.72,0.20,0.07,-0.22,-0.19,-0.14)⟩	⟨(0.78,0.19,0.15,-0.14,-0.25,-0.12)⟩	⟨(0.56,0.18,0.16,-0.41,-0.15,-0.22)⟩	⟨(0.76,0.15,0.16,-0.30,-0.16,-0.12)⟩	⟨(0.83,0.34,0.09,-0.44,-0.28,-0.18)⟩	⟨(0.75,0.23,0.19,-0.46,-0.10,-0.22)⟩
x_1	q_2	⟨(0.69,0.31,0.19,-0.45,-0.24,-0.19)⟩	⟨(0.71,0.16,0.16,-0.18,-0.20,-0.17)⟩	⟨(0.61,0.09,0.19,-0.21,-0.11,-0.17)⟩	⟨(0.71,0.34,0.13,-0.29,-0.11,-0.13)⟩	⟨(0.70,0.26,0.15,-0.40,-0.26,-0.21)⟩	⟨(0.65,0.23,0.05,-0.19,-0.14,-0.19)⟩
x_1	q_3	⟨(0.76,0.18,0.19,-0.32,-0.28,-0.11)⟩	⟨(0.69,0.11,0.13,-0.21,-0.19,-0.15)⟩	⟨(0.67,0.17,0.16,-0.32,-0.08,-0.11)⟩	⟨(0.62,0.21,0.13,-0.24,-0.04,-0.13)⟩	⟨(0.63,0.15,0.22,-0.37,-0.24,-0.21)⟩	⟨(0.64,0.25,0.12,-0.29,-0.25,-0.19)⟩
x_2	q_1	⟨(0.65,0.33,0.18,-0.22,-0.21,-0.07)⟩	⟨(0.74,0.35,0.10,-0.45,-0.21,-0.04)⟩	⟨(0.76,0.18,0.24,-0.30,-0.16,-0.12)⟩	⟨(0.59,0.25,0.16,-0.40,-0.12,-0.14)⟩	⟨(0.86,0.26,0.15,-0.42,-0.22,-0.19)⟩	⟨(0.58,0.09,0.17,-0.34,-0.15,-0.11)⟩
x_2	q_2	⟨(0.60,0.20,0.19,-0.40,-0.12,-0.19)⟩	⟨(0.78,0.29,0.18,-0.48,-0.21,-0.10)⟩	⟨(0.58,0.33,0.10,-0.27,-0.10,-0.20)⟩	⟨(0.63,0.24,0.11,-0.30,-0.14,-0.21)⟩	⟨(0.63,0.15,0.21,-0.39,-0.26,-0.21)⟩	⟨(0.77,0.12,0.09,-0.36,-0.14,-0.12)⟩
x_2	q_3	⟨(0.56,0.28,0.19,-0.36,-0.23,-0.20)⟩	⟨(0.74,0.11,0.05,-0.37,-0.18,-0.15)⟩	⟨(0.62,0.25,0.13,-0.42,-0.19,-0.08)⟩	⟨(0.75,0.30,0.17,-0.41,-0.25,-0.12)⟩	⟨(0.65,0.30,0.21,-0.35,-0.15,-0.19)⟩	⟨(0.52,0.35,0.18,-0.37,-0.31,-0.23)⟩
x_3	q_1	⟨(0.74,0.24,0.07,-0.34,-0.34,-0.19)⟩	⟨(0.75,0.28,0.13,-0.44,-0.21,-0.20)⟩	⟨(0.51,0.38,0.08,-0.21,-0.23,-0.17)⟩	⟨(0.57,0.06,0.20,-0.43,-0.15,-0.15)⟩	⟨(0.75,0.33,0.13,-0.40,-0.17,-0.17)⟩	⟨(0.67,0.13,0.21,-0.34,-0.27,-0.14)⟩
x_3	q_2	⟨(0.63,0.29,0.19,-0.44,-0.13,-0.14)⟩	⟨(0.65,0.37,0.17,-0.35,-0.23,-0.16)⟩	⟨(0.76,0.20,0.21,-0.48,-0.22,-0.08)⟩	⟨(0.68,0.19,0.18,-0.51,-0.20,-0.05)⟩	⟨(0.59,0.21,0.19,-0.41,-0.22,-0.14)⟩	⟨(0.76,0.23,0.22,-0.38,-0.24,-0.07)⟩
x_3	q_3	⟨(0.64,0.35,0.11,-0.38,-0.20,-0.08)⟩	⟨(0.65,0.24,0.13,-0.29,-0.12,-0.19)⟩	⟨(0.76,0.35,0.21,-0.47,-0.15,-0.13)⟩	⟨(0.65,0.32,0.17,-0.49,-0.30,-0.02)⟩	⟨(0.46,0.17,0.25,-0.50,-0.25,-0.16)⟩	⟨(0.46,0.31,0.20,-0.34,-0.26,-0.26)⟩
x_4	q_1	⟨(0.49,0.28,0.15,-0.42,-0.13,-0.19)⟩	⟨(0.67,0.20,0.14,-0.23,-0.27,-0.19)⟩	⟨(0.59,0.30,0.16,-0.24,-0.21,-0.10)⟩	⟨(0.57,0.26,0.16,-0.44,-0.23,-0.17)⟩	⟨(0.61,0.17,0.24,-0.32,-0.26,-0.16)⟩	⟨(0.73,0.22,0.18,-0.19,-0.16,-0.18)⟩
x_4	q_2	⟨(0.72,0.23,0.20,-0.41,-0.17,-0.19)⟩	⟨(0.57,0.37,0.05,-0.40,-0.15,-0.18)⟩	⟨(0.58,0.40,0.15,-0.33,-0.25,-0.12)⟩	⟨(0.69,0.29,0.18,-0.42,-0.24,-0.16)⟩	⟨(0.59,0.16,0.15,-0.39,-0.34,-0.12)⟩	⟨(0.76,0.31,0.10,-0.37,-0.14,-0.21)⟩
x_4	q_3	⟨(0.68,0.26,0.21,-0.36,-0.24,-0.10)⟩	⟨(0.63,0.38,0.14,-0.25,-0.20,-0.13)⟩	⟨(0.73,0.21,0.24,-0.45,-0.35,-0.13)⟩	⟨(0.66,0.29,0.03,-0.38,-0.22,-0.15)⟩	⟨(0.70,0.36,0.12,-0.47,-0.18,-0.05)⟩	⟨(0.52,0.28,0.17,-0.45,-0.12,-0.11)⟩
x_5	q_1	⟨(0.68,0.47,0.12,-0.30,-0.13,-0.23)⟩	⟨(0.59,0.32,0.10,-0.43,-0.19,-0.19)⟩	⟨(0.73,0.32,0.19,-0.30,-0.27,-0.17)⟩	⟨(0.67,0.29,0.15,-0.23,-0.27,-0.11)⟩	⟨(0.74,0.23,0.17,-0.40,-0.14,-0.13)⟩	⟨(0.59,0.23,0.14,-0.34,-0.17,-0.15)⟩
x_5	q_2	⟨(0.81,0.19,0.19,-0.38,-0.15,-0.04)⟩	⟨(0.62,0.32,0.08,-0.40,-0.17,-0.23)⟩	⟨(0.59,0.14,0.13,-0.33,-0.27,-0.26)⟩	⟨(0.56,0.39,0.15,-0.23,-0.19,-0.10)⟩	⟨(0.66,0.12,0.07,-0.49,-0.19,-0.11)⟩	⟨(0.54,0.19,0.18,-0.34,-0.16,-0.12)⟩
x_5	q_3	⟨(0.56,0.22,0.06,-0.38,-0.18,-0.13)⟩	⟨(0.72,0.34,0.16,-0.35,-0.30,-0.10)⟩	⟨(0.57,0.17,0.12,-0.30,-0.38,-0.06)⟩	⟨(0.63,0.35,0.13,-0.41,-0.29,-0.16)⟩	⟨(0.46,0.20,0.13,-0.38,-0.24,-0.11)⟩	⟨(0.70,0.38,0.13,-0.43,-0.24,-0.13)⟩

Table 7. CWNA Aggregated T2-QNSS Decision Matrix under Expert 1 Dominance.

U	Q	e_1	e_2	e_3	e_4	e_5	e_6
x_1	q_1	⟨(0.73,0.14,0.07,-0.20,-0.23,-0.17)⟩	⟨(0.82,0.12,0.15,-0.13,-0.18,-0.14)⟩	⟨(0.48,0.17,0.18,-0.39,-0.11,-0.21)⟩	⟨(0.78,0.08,0.19,-0.37,-0.15,-0.08)⟩	⟨(0.85,0.27,0.06,-0.32,-0.30,-0.19)⟩	⟨(0.77,0.29,0.18,-0.53,-0.12,-0.20)⟩
x_1	q_2	⟨(0.74,0.30,0.21,-0.43,-0.25,-0.11)⟩	⟨(0.63,0.14,0.11,-0.20,-0.13,-0.13)⟩	⟨(0.66,0.12,0.16,-0.20,-0.11,-0.23)⟩	⟨(0.70,0.33,0.10,-0.36,-0.08,-0.14)⟩	⟨(0.76,0.30,0.16,-0.42,-0.30,-0.22)⟩	⟨(0.59,0.15,0.07,-0.21,-0.13,-0.22)⟩
x_1	q_3	⟨(0.79,0.18,0.19,-0.31,-0.32,-0.12)⟩	⟨(0.62,0.12,0.14,-0.21,-0.20,-0.19)⟩	⟨(0.64,0.16,0.21,-0.32,-0.07,-0.07)⟩	⟨(0.57,0.26,0.18,-0.26,-0.04,-0.12)⟩	⟨(0.74,0.19,0.25,-0.43,-0.15,-0.21)⟩	⟨(0.70,0.24,0.11,-0.36,-0.17,-0.16)⟩
x_2	q_1	⟨(0.64,0.39,0.20,-0.22,-0.22,-0.07)⟩	⟨(0.80,0.37,0.09,-0.43,-0.20,-0.04)⟩	⟨(0.76,0.22,0.24,-0.32,-0.10,-0.12)⟩	⟨(0.53,0.33,0.20,-0.45,-0.12,-0.10)⟩	⟨(0.86,0.34,0.10,-0.38,-0.14,-0.22)⟩	⟨(0.66,0.08,0.16,-0.34,-0.12,-0.17)⟩
x_2	q_2	⟨(0.58,0.16,0.18,-0.41,-0.10,-0.15)⟩	⟨(0.83,0.30,0.16,-0.44,-0.15,-0.10)⟩	⟨(0.59,0.32,0.09,-0.25,-0.09,-0.20)⟩	⟨(0.59,0.31,0.17,-0.22,-0.14,-0.20)⟩	⟨(0.57,0.10,0.24,-0.40,-0.24,-0.22)⟩	⟨(0.77,0.10,0.06,-0.34,-0.15,-0.12)⟩
x_2	q_3	⟨(0.64,0.33,0.25,-0.27,-0.21,-0.17)⟩	⟨(0.78,0.12,0.05,-0.36,-0.18,-0.12)⟩	⟨(0.60,0.33,0.14,-0.47,-0.20,-0.05)⟩	⟨(0.80,0.34,0.23,-0.49,-0.28,-0.10)⟩	⟨(0.63,0.23,0.17,-0.25,-0.15,-0.23)⟩	⟨(0.53,0.38,0.16,-0.34,-0.35,-0.26)⟩
x_3	q_1	⟨(0.71,0.29,0.06,-0.29,-0.36,-0.18)⟩	⟨(0.72,0.30,0.08,-0.43,-0.20,-0.23)⟩	⟨(0.49,0.44,0.06,-0.20,-0.22,-0.18)⟩	⟨(0.56,0.06,0.24,-0.35,-0.20,-0.16)⟩	⟨(0.69,0.30,0.15,-0.47,-0.12,-0.19)⟩	⟨(0.62,0.16,0.20,-0.30,-0.22,-0.18)⟩
x_3	q_2	⟨(0.57,0.28,0.23,-0.52,-0.09,-0.12)⟩	⟨(0.60,0.42,0.20,-0.42,-0.20,-0.13)⟩	⟨(0.77,0.25,0.19,-0.51,-0.24,-0.05)⟩	⟨(0.73,0.17,0.18,-0.53,-0.14,-0.04)⟩	⟨(0.55,0.22,0.14,-0.41,-0.26,-0.10)⟩	⟨(0.74,0.20,0.20,-0.45,-0.26,-0.06)⟩
x_3	q_3	⟨(0.63,0.26,0.08,-0.42,-0.22,-0.08)⟩	⟨(0.72,0.26,0.14,-0.20,-0.09,-0.23)⟩	⟨(0.78,0.31,0.23,-0.44,-0.13,-0.10)⟩	⟨(0.60,0.36,0.20,-0.51,-0.33,-0.04)⟩	⟨(0.49,0.13,0.25,-0.53,-0.21,-0.17)⟩	⟨(0.47,0.38,0.23,-0.45,-0.29,-0.26)⟩
x_4	q_1	⟨(0.45,0.33,0.12,-0.47,-0.21,-0.22)⟩	⟨(0.73,0.19,0.18,-0.19,-0.31,-0.22)⟩	⟨(0.55,0.37,0.14,-0.25,-0.25,-0.08)⟩	⟨(0.48,0.20,0.15,-0.48,-0.29,-0.12)⟩	⟨(0.65,0.19,0.26,-0.34,-0.30,-0.11)⟩	⟨(0.78,0.16,0.22,-0.21,-0.11,-0.16)⟩

x_4	q_2	⟨(0.75,0.19,0.20,-0.39,-0.15,-0.18)⟩	⟨(0.57,0.36,0.02,-0.47,-0.18,-0.21)⟩	⟨(0.66,0.37,0.13,-0.24,-0.26,-0.12)⟩	⟨(0.64,0.36,0.20,-0.35,-0.19,-0.15)⟩	⟨(0.61,0.15,0.10,-0.35,-0.35,-0.15)⟩	⟨(0.78,0.32,0.09,-0.35,-0.09,-0.16)⟩
x_4	q_3	⟨(0.65,0.29,0.21,-0.33,-0.22,-0.09)⟩	⟨(0.65,0.41,0.18,-0.21,-0.14,-0.13)⟩	⟨(0.74,0.21,0.24,-0.48,-0.32,-0.11)⟩	⟨(0.58,0.18,0.05,-0.36,-0.27,-0.10)⟩	⟨(0.66,0.33,0.10,-0.47,-0.18,-0.06)⟩	⟨(0.60,0.17,0.20,-0.48,-0.08,-0.11)⟩
x_5	q_1	⟨(0.61,0.47,0.14,-0.22,-0.11,-0.24)⟩	⟨(0.61,0.35,0.14,-0.51,-0.20,-0.23)⟩	⟨(0.78,0.32,0.21,-0.32,-0.30,-0.15)⟩	⟨(0.74,0.30,0.14,-0.24,-0.21,-0.14)⟩	⟨(0.74,0.24,0.19,-0.34,-0.08,-0.12)⟩	⟨(0.58,0.22,0.15,-0.26,-0.12,-0.17)⟩
x_5	q_2	⟨(0.78,0.12,0.16,-0.38,-0.17,-0.05)⟩	⟨(0.63,0.36,0.12,-0.42,-0.23,-0.23)⟩	⟨(0.58,0.07,0.14,-0.39,-0.28,-0.27)⟩	⟨(0.56,0.39,0.14,-0.30,-0.14,-0.08)⟩	⟨(0.65,0.06,0.07,-0.47,-0.17,-0.08)⟩	⟨(0.57,0.12,0.19,-0.24,-0.17,-0.14)⟩
x_5	q_3	⟨(0.52,0.27,0.05,-0.35,-0.13,-0.11)⟩	⟨(0.63,0.27,0.19,-0.32,-0.31,-0.16)⟩	⟨(0.56,0.11,0.08,-0.34,-0.39,-0.07)⟩	⟨(0.66,0.36,0.15,-0.43,-0.32,-0.11)⟩	⟨(0.45,0.18,0.09,-0.40,-0.24,-0.08)⟩	⟨(0.73,0.38,0.10,-0.42,-0.27,-0.08)⟩

Table 8. CWNA Aggregated T2-QNSS Decision Matrix under Expert 2 Dominance.

U	Q	e_1	e_2	e_3	e_4	e_5	e_6
x_1	q_1	⟨(0.76,0.31,0.05,-0.17,-0.16,-0.17)⟩	⟨(0.82,0.19,0.17,-0.14,-0.26,-0.15)⟩	⟨(0.52,0.26,0.20,-0.41,-0.11,-0.25)⟩	⟨(0.77,0.16,0.15,-0.30,-0.18,-0.11)⟩	⟨(0.82,0.37,0.07,-0.49,-0.27,-0.21)⟩	⟨(0.75,0.23,0.20,-0.51,-0.10,-0.23)⟩
x_1	q_2	⟨(0.68,0.28,0.16,-0.46,-0.19,-0.22)⟩	⟨(0.76,0.11,0.20,-0.20,-0.20,-0.17)⟩	⟨(0.60,0.06,0.22,-0.20,-0.10,-0.20)⟩	⟨(0.65,0.38,0.16,-0.27,-0.06,-0.19)⟩	⟨(0.60,0.30,0.15,-0.43,-0.27,-0.20)⟩	⟨(0.65,0.30,0.04,-0.20,-0.20,-0.15)⟩
x_1	q_3	⟨(0.73,0.20,0.18,-0.31,-0.29,-0.10)⟩	⟨(0.71,0.12,0.11,-0.19,-0.15,-0.11)⟩	⟨(0.67,0.25,0.12,-0.28,-0.12,-0.15)⟩	⟨(0.73,0.23,0.11,-0.20,-0.05,-0.11)⟩	⟨(0.57,0.10,0.20,-0.32,-0.30,-0.19)⟩	⟨(0.69,0.18,0.17,-0.32,-0.29,-0.18)⟩
x_2	q_1	⟨(0.64,0.29,0.13,-0.19,-0.28,-0.10)⟩	⟨(0.77,0.29,0.07,-0.43,-0.13,-0.04)⟩	⟨(0.73,0.17,0.26,-0.24,-0.22,-0.15)⟩	⟨(0.68,0.19,0.16,-0.37,-0.14,-0.19)⟩	⟨(0.88,0.22,0.16,-0.41,-0.25,-0.23)⟩	⟨(0.55,0.10,0.18,-0.28,-0.12,-0.07)⟩
x_2	q_2	⟨(0.52,0.21,0.19,-0.34,-0.11,-0.20)⟩	⟨(0.77,0.27,0.19,-0.51,-0.25,-0.14)⟩	⟨(0.64,0.38,0.11,-0.31,-0.17,-0.23)⟩	⟨(0.70,0.16,0.09,-0.28,-0.21,-0.19)⟩	⟨(0.59,0.11,0.21,-0.41,-0.31,-0.19)⟩	⟨(0.72,0.09,0.06,-0.34,-0.14,-0.10)⟩
x_2	q_3	⟨(0.52,0.36,0.22,-0.37,-0.29,-0.24)⟩	⟨(0.74,0.12,0.03,-0.42,-0.18,-0.17)⟩	⟨(0.72,0.18,0.15,-0.42,-0.19,-0.06)⟩	⟨(0.68,0.25,0.18,-0.46,-0.20,-0.15)⟩	⟨(0.60,0.37,0.22,-0.36,-0.16,-0.15)⟩	⟨(0.54,0.41,0.16,-0.37,-0.35,-0.22)⟩
x_3	q_1	⟨(0.71,0.23,0.07,-0.32,-0.32,-0.19)⟩	⟨(0.76,0.21,0.14,-0.49,-0.20,-0.20)⟩	⟨(0.49,0.43,0.12,-0.21,-0.16,-0.17)⟩	⟨(0.61,0.06,0.24,-0.48,-0.16,-0.11)⟩	⟨(0.78,0.30,0.16,-0.38,-0.14,-0.20)⟩	⟨(0.76,0.13,0.19,-0.32,-0.31,-0.15)⟩
x_3	q_2	⟨(0.72,0.35,0.15,-0.46,-0.14,-0.20)⟩	⟨(0.65,0.38,0.13,-0.31,-0.23,-0.22)⟩	⟨(0.71,0.15,0.19,-0.40,-0.22,-0.11)⟩	⟨(0.66,0.18,0.13,-0.49,-0.21,-0.06)⟩	⟨(0.67,0.13,0.20,-0.43,-0.26,-0.13)⟩	⟨(0.75,0.30,0.25,-0.37,-0.15,-0.10)⟩
x_3	q_3	⟨(0.67,0.38,0.17,-0.36,-0.20,-0.10)⟩	⟨(0.70,0.23,0.16,-0.25,-0.15,-0.23)⟩	⟨(0.69,0.35,0.20,-0.50,-0.23,-0.18)⟩	⟨(0.63,0.26,0.19,-0.48,-0.32,-0.02)⟩	⟨(0.45,0.23,0.26,-0.54,-0.27,-0.15)⟩	⟨(0.46,0.31,0.23,-0.33,-0.27,-0.26)⟩
x_4	q_1	⟨(0.46,0.33,0.13,-0.37,-0.10,-0.18)⟩	⟨(0.67,0.28,0.10,-0.20,-0.30,-0.19)⟩	⟨(0.56,0.37,0.11,-0.27,-0.17,-0.09)⟩	⟨(0.50,0.32,0.21,-0.44,-0.16,-0.15)⟩	⟨(0.59,0.22,0.22,-0.31,-0.19,-0.19)⟩	⟨(0.64,0.21,0.16,-0.20,-0.19,-0.18)⟩
x_4	q_2	⟨(0.79,0.20,0.24,-0.42,-0.15,-0.18)⟩	⟨(0.64,0.37,0.03,-0.40,-0.17,-0.22)⟩	⟨(0.56,0.40,0.16,-0.31,-0.25,-0.15)⟩	⟨(0.71,0.32,0.19,-0.46,-0.24,-0.21)⟩	⟨(0.64,0.16,0.14,-0.39,-0.32,-0.08)⟩	⟨(0.68,0.33,0.08,-0.29,-0.17,-0.24)⟩
x_4	q_3	⟨(0.68,0.34,0.23,-0.31,-0.28,-0.12)⟩	⟨(0.70,0.38,0.13,-0.21,-0.27,-0.10)⟩	⟨(0.79,0.30,0.23,-0.51,-0.37,-0.17)⟩	⟨(0.63,0.34,0.02,-0.37,-0.21,-0.17)⟩	⟨(0.71,0.33,0.17,-0.51,-0.13,-0.06)⟩	⟨(0.48,0.34,0.17,-0.49,-0.11,-0.07)⟩
x_5	q_1	⟨(0.73,0.46,0.11,-0.30,-0.10,-0.25)⟩	⟨(0.58,0.29,0.07,-0.47,-0.14,-0.17)⟩	⟨(0.73,0.34,0.23,-0.30,-0.28,-0.12)⟩	⟨(0.64,0.20,0.15,-0.26,-0.32,-0.07)⟩	⟨(0.68,0.23,0.22,-0.45,-0.10,-0.16)⟩	⟨(0.62,0.33,0.13,-0.32,-0.13,-0.11)⟩
x_5	q_2	⟨(0.81,0.21,0.19,-0.40,-0.19,-0.03)⟩	⟨(0.69,0.24,0.06,-0.39,-0.12,-0.23)⟩	⟨(0.55,0.13,0.18,-0.30,-0.18,-0.26)⟩	⟨(0.49,0.41,0.11,-0.20,-0.16,-0.08)⟩	⟨(0.69,0.08,0.04,-0.49,-0.15,-0.11)⟩	⟨(0.52,0.13,0.20,-0.35,-0.22,-0.13)⟩
x_5	q_3	⟨(0.65,0.24,0.05,-0.37,-0.26,-0.08)⟩	⟨(0.75,0.37,0.16,-0.34,-0.34,-0.07)⟩	⟨(0.60,0.18,0.16,-0.32,-0.37,-0.04)⟩	⟨(0.56,0.33,0.09,-0.34,-0.27,-0.16)⟩	⟨(0.50,0.32,0.14,-0.36,-0.24,-0.08)⟩	⟨(0.72,0.40,0.16,-0.45,-0.26,-0.10)⟩

Table 9. CWNA Aggregated T2-QNSS Decision Matrix under Expert 3 Dominance.

U	Q	e_1	e_2	e_3	e_4	e_5	e_6
x_1	q_1	⟨(0.68,0.16,0.08,-0.26,-0.18,-0.10)⟩	⟨(0.71,0.25,0.13,-0.16,-0.30,-0.08)⟩	⟨(0.66,0.13,0.11,-0.44,-0.22,-0.20)⟩	⟨(0.73,0.20,0.13,-0.24,-0.15,-0.15)⟩	⟨(0.81,0.37,0.14,-0.49,-0.26,-0.15)⟩	⟨(0.72,0.18,0.20,-0.36,-0.08,-0.23)⟩
x_1	q_2	⟨(0.64,0.34,0.18,-0.47,-0.28,-0.23)⟩	⟨(0.75,0.22,0.17,-0.15,-0.27,-0.20)⟩	⟨(0.58,0.09,0.20,-0.23,-0.12,-0.10)⟩	⟨(0.76,0.32,0.12,-0.24,-0.16,-0.08)⟩	⟨(0.73,0.20,0.14,-0.35,-0.21,-0.22)⟩	⟨(0.71,0.25,0.04,-0.17,-0.11,-0.19)⟩

x_1	q_3	⟨(0.75,0.16,0.19,-0.33,-0.24,-0.10)⟩	⟨(0.72,0.09,0.13,-0.22,-0.22,-0.15)⟩	⟨(0.69,0.11,0.14,-0.35,-0.05,-0.10)⟩	⟨(0.57,0.16,0.11,-0.25,-0.02,-0.17)⟩	⟨(0.57,0.17,0.22,-0.36,-0.28,-0.22)⟩	⟨(0.56,0.32,0.07,-0.22,-0.30,-0.22)⟩
x_2	q_1	⟨(0.67,0.31,0.19,-0.26,-0.15,-0.04)⟩	⟨(0.67,0.37,0.14,-0.48,-0.27,-0.04)⟩	⟨(0.78,0.16,0.21,-0.32,-0.16,-0.11)⟩	⟨(0.58,0.22,0.11,-0.39,-0.09,-0.15)⟩	⟨(0.86,0.23,0.18,-0.45,-0.26,-0.14)⟩	⟨(0.54,0.09,0.18,-0.40,-0.19,-0.09)⟩
x_2	q_2	⟨(0.68,0.22,0.21,-0.43,-0.14,-0.23)⟩	⟨(0.74,0.30,0.18,-0.50,-0.22,-0.08)⟩	⟨(0.52,0.28,0.09,-0.26,-0.07,-0.18)⟩	⟨(0.60,0.24,0.07,-0.39,-0.10,-0.23)⟩	⟨(0.71,0.21,0.19,-0.37,-0.25,-0.22)⟩	⟨(0.80,0.16,0.15,-0.39,-0.14,-0.13)⟩
x_2	q_3	⟨(0.53,0.18,0.13,-0.44,-0.19,-0.21)⟩	⟨(0.70,0.10,0.06,-0.35,-0.18,-0.15)⟩	⟨(0.56,0.22,0.11,-0.37,-0.17,-0.12)⟩	⟨(0.77,0.31,0.11,-0.31,-0.26,-0.11)⟩	⟨(0.69,0.31,0.23,-0.43,-0.14,-0.18)⟩	⟨(0.50,0.28,0.23,-0.40,-0.25,-0.21)⟩
x_3	q_1	⟨(0.79,0.20,0.09,-0.41,-0.34,-0.19)⟩	⟨(0.78,0.32,0.16,-0.41,-0.23,-0.17)⟩	⟨(0.54,0.30,0.07,-0.21,-0.29,-0.16)⟩	⟨(0.55,0.06,0.13,-0.47,-0.11,-0.18)⟩	⟨(0.78,0.37,0.08,-0.35,-0.22,-0.12)⟩	⟨(0.64,0.11,0.22,-0.40,-0.29,-0.11)⟩
x_3	q_2	⟨(0.63,0.26,0.20,-0.35,-0.15,-0.12)⟩	⟨(0.70,0.31,0.16,-0.31,-0.27,-0.15)⟩	⟨(0.78,0.19,0.24,-0.53,-0.20,-0.08)⟩	⟨(0.64,0.22,0.23,-0.51,-0.25,-0.04)⟩	⟨(0.57,0.25,0.22,-0.40,-0.15,-0.19)⟩	⟨(0.79,0.19,0.23,-0.32,-0.29,-0.06)⟩
x_3	q_3	⟨(0.63,0.40,0.09,-0.36,-0.18,-0.05)⟩	⟨(0.56,0.25,0.11,-0.42,-0.13,-0.12)⟩	⟨(0.81,0.38,0.20,-0.48,-0.12,-0.11)⟩	⟨(0.71,0.33,0.12,-0.47,-0.25,-0.01)⟩	⟨(0.45,0.15,0.25,-0.45,-0.27,-0.16)⟩	⟨(0.46,0.24,0.16,-0.25,-0.22,-0.25)⟩
x_4	q_1	⟨(0.54,0.19,0.20,-0.41,-0.08,-0.17)⟩	⟨(0.61,0.15,0.14,-0.30,-0.20,-0.16)⟩	⟨(0.64,0.20,0.21,-0.19,-0.21,-0.12)⟩	⟨(0.70,0.27,0.15,-0.40,-0.24,-0.22)⟩	⟨(0.58,0.10,0.24,-0.30,-0.27,-0.17)⟩	⟨(0.75,0.28,0.15,-0.16,-0.19,-0.20)⟩
x_4	q_2	⟨(0.63,0.30,0.18,-0.43,-0.20,-0.21)⟩	⟨(0.51,0.38,0.09,-0.34,-0.10,-0.13)⟩	⟨(0.51,0.43,0.15,-0.44,-0.24,-0.11)⟩	⟨(0.73,0.22,0.15,-0.45,-0.28,-0.14)⟩	⟨(0.54,0.17,0.20,-0.42,-0.35,-0.12)⟩	⟨(0.81,0.29,0.12,-0.44,-0.17,-0.22)⟩
x_4	q_3	⟨(0.71,0.17,0.20,-0.42,-0.23,-0.10)⟩	⟨(0.55,0.36,0.12,-0.31,-0.19,-0.16)⟩	⟨(0.67,0.13,0.25,-0.38,-0.37,-0.11)⟩	⟨(0.75,0.36,0.03,-0.39,-0.19,-0.19)⟩	⟨(0.71,0.41,0.09,-0.44,-0.23,-0.03)⟩	⟨(0.49,0.32,0.15,-0.38,-0.16,-0.14)⟩
x_5	q_1	⟨(0.70,0.47,0.12,-0.37,-0.17,-0.20)⟩	⟨(0.60,0.32,0.09,-0.32,-0.23,-0.18)⟩	⟨(0.67,0.30,0.15,-0.27,-0.23,-0.22)⟩	⟨(0.62,0.36,0.15,-0.20,-0.30,-0.12)⟩	⟨(0.78,0.23,0.11,-0.42,-0.22,-0.12)⟩	⟨(0.58,0.15,0.15,-0.42,-0.24,-0.18)⟩
x_5	q_2	⟨(0.83,0.24,0.21,-0.37,-0.11,-0.05)⟩	⟨(0.56,0.35,0.05,-0.39,-0.16,-0.24)⟩	⟨(0.63,0.21,0.09,-0.30,-0.32,-0.24)⟩	⟨(0.60,0.39,0.19,-0.20,-0.27,-0.14)⟩	⟨(0.64,0.21,0.08,-0.51,-0.24,-0.14)⟩	⟨(0.52,0.31,0.16,-0.43,-0.10,-0.09)⟩
x_5	q_3	⟨(0.53,0.16,0.07,-0.41,-0.16,-0.19)⟩	⟨(0.77,0.37,0.13,-0.38,-0.27,-0.08)⟩	⟨(0.56,0.21,0.13,-0.25,-0.38,-0.05)⟩	⟨(0.67,0.35,0.15,-0.46,-0.26,-0.19)⟩	⟨(0.44,0.13,0.15,-0.39,-0.25,-0.16)⟩	⟨(0.67,0.35,0.14,-0.42,-0.19,-0.19)⟩

The BAS quantifies the level of consensus in expert evaluations. A higher BAS reflects stronger agreement toward the suitability of an alternative. Figure 3 presented the BAS distributions for each alternative, which identify the one the most favorable and least contradictory evaluations under diverse parameter-linguistic combinations.

Figure 3. BAS for each alternative under different context-parameter evaluations using the aggregated T2-QNSS matrix.

The Uncertainty Score (US) captures the degree of hesitation in evaluation with averaged indeterminacy values from bipolar factors. It reflects the level of ambiguity or vagueness present in expert assessments. Lower US values imply more confidnt judgments. Figure 4 depicts uncertainty profiles for each alternative, highlight which option exhibit higher cognitive hesitation across multiple parameters and linguistic values.

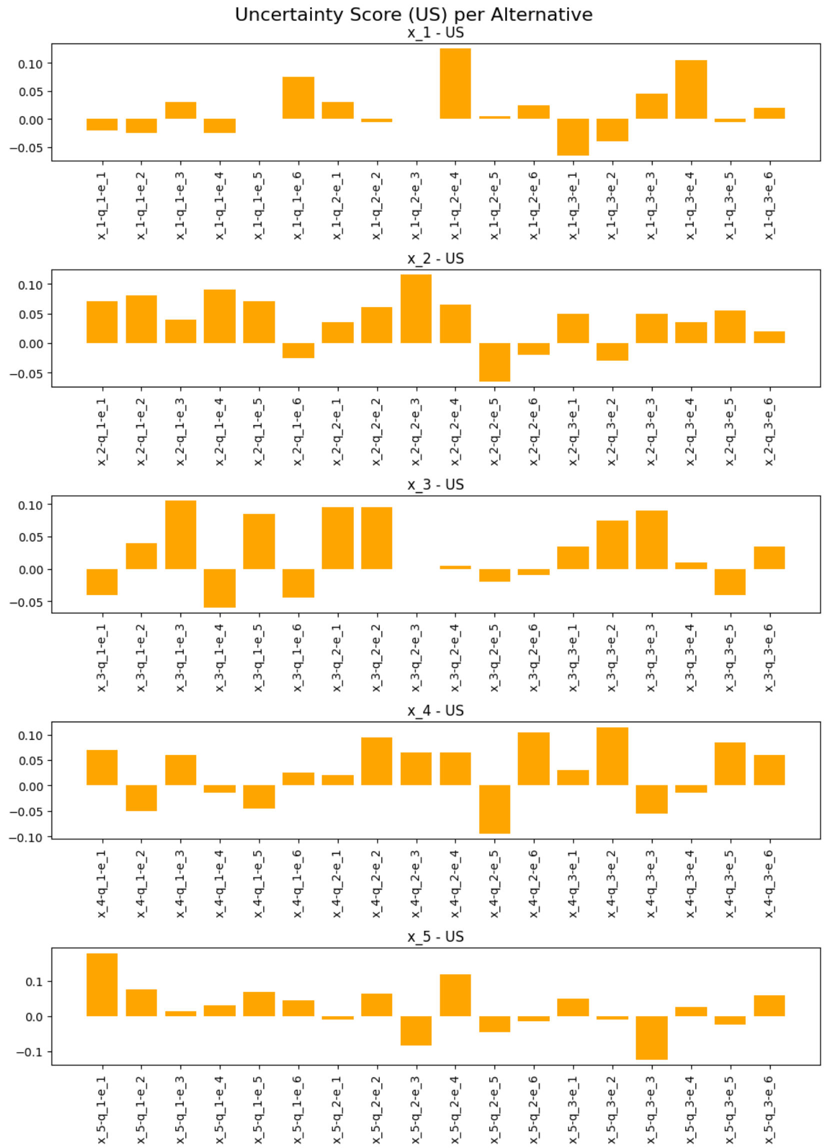

Figure 4. Uncertainty Score (US) across all parameters and contexts for each alternative, representing expert ambiguity levels in T2-QNSS assessments.

The CDS reveals inconsistencies between positive and negative memberships. Higher CDS values indicate conflicting assessments and reduced decision confidence. Figure 5 illustrate CDS values for each alternative, aiding in detecting which alternatives are subject to more contradictory evaluations and may require further expert clarification.

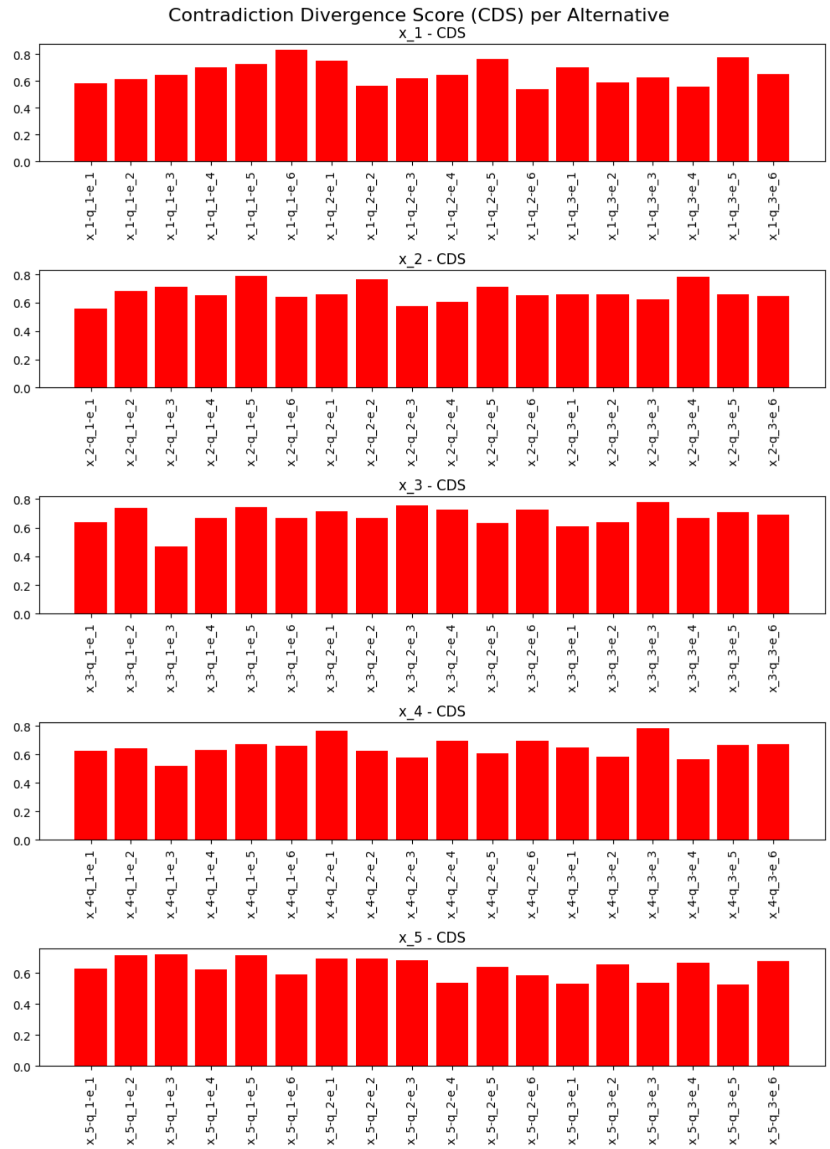

Figure 5. CDS showing bipolar inconsistency levels in the expert evaluations of each alternative under the T2-QNSS model.

The final ranking of alternatives based on the BDCI reveals that *alternative* x_2 *in linguistic context* q_3 achieved the highest score. This indicates strong expert agreement (high BAS), minimal uncertainty (low US), and relatively low contradiction (moderate CDS), affirming its suitability in evaluating ICH for tourism. These findings emphasize the effectiveness of the proposed T2-QNSS-based model in capturing nuanced expert judgments across multiple dimensions.

Table 10. Final BDCI scores for each alternative-context pair, ranked in descending order based on bipolar agreement, uncertainty, and contradiction metric

U	Q	BAS	US	CDS	BDCI_Score
x_1	q_1	0.953333	0.005833	0.685000	0.337917
x_1	q_2	0.926667	0.030000	0.647500	0.324833
x_1	q_3	0.910833	0.010000	0.650833	0.322250
x_2	q_1	0.913333	0.014167	0.654167	0.321583
x_2	q_2	0.886667	0.005000	0.636667	0.314500
x_2	q_3	0.911667	0.031667	0.660833	0.314167
x_3	q_1	0.880000	0.007500	0.625000	0.312750
x_3	q_2	0.913333	0.042500	0.661667	0.311583
x_3	q_3	0.905000	0.030000	0.672500	0.309000
x_4	q_1	0.923333	0.069167	0.665000	0.307917
x_4	q_2	0.851667	-0.004167	0.597500	0.307583
x_4	q_3	0.912500	0.054167	0.671667	0.305667
x_5	q_1	0.877500	0.036667	0.653333	0.297083
x_5	q_2	0.879167	0.034167	0.682500	0.292833
x_5	q_3	0.880000	0.027500	0.703333	0.291083

In Figure 6, we displays the average BDCI score for each alternative across all linguistic contexts and parameters. This helps identify which alternative offers the most favorable certainty profile overall, considering agreement, uncertainty, and contradiction.

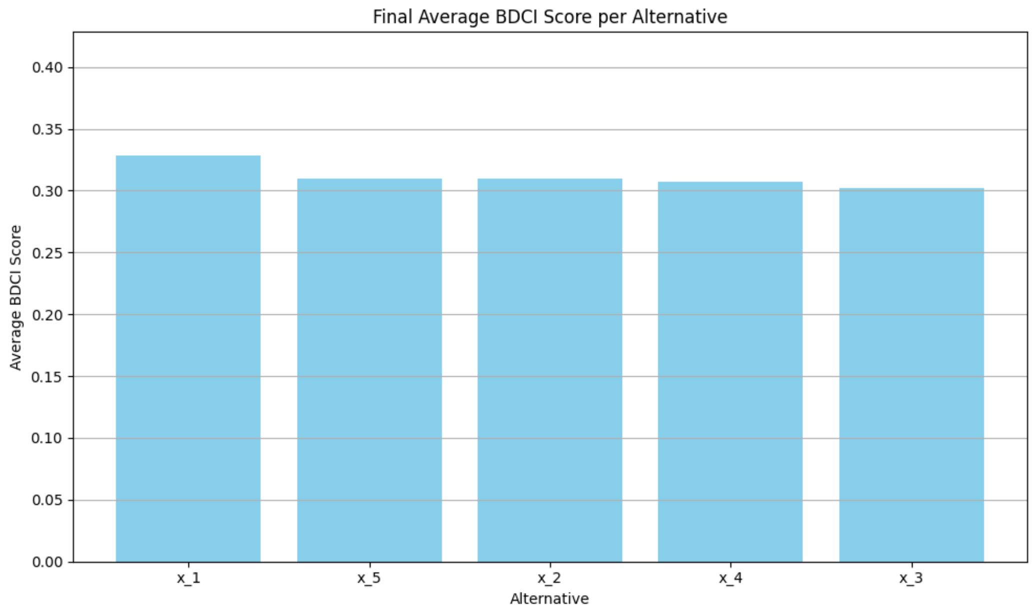

Figure 6. Final Average BDCI Score per Alternative

7. Conclusion & Future Work

This research developed Q-Neutrosophic Type-2 Soft Sets (T2-QNSS) to provide a robust mathematical neutrosophic frameworks that integrating bipolar truth modeling and soft set theory, to handle complex, uncertain, and subjective evaluations such as those found in ICH within tourism. We developed a comprehensive decision-support algorithm leveraging Credibility-Weighted Nonlinear Aggregation (CWNA) and a new Bipolar Divergence-Aware Certainty Index (BDCI) to aggregate expert evaluations and rank alternatives effectively. Proof of concept is performed using a realistic case study, and the findings demonstrated the effectiveness of the T2-QNSS model in evaluating tourism heritage parameters. Future work may explore hybridization with machine learning and the integration of dynamic linguistic contexts.

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