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Study on Neutrosophic $\alpha_{(\gamma,\beta)}$- Contra Continuous Functions in Neutrosophic Topological Spaces

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Abstract: In this article, we explore the concept of neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous functions within the framework of neutrosophic topological spaces. These functions represent a novel generalization in the realm of neutrosophic topology, extending classical continuity concepts under uncertain, indeterminate, and inconsistent conditions. The study includes a rigorous examination of their definitional structure, behavioural characteristics, and the interrelationships with other types of continuity and contra-continuity functions. We investigate how these functions interact with neutrosophic open and closed sets, and provide conditions under which these functions preserve certain topological properties. Additionally, we establish several propositions and theorems highlighting the essential nature of neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous functions, supported by proofs and illustrative examples. Special attention is given to the comparative analysis with related concepts such as neutrosophic continuous functions, neutrosophic contra-continuous functions, and their α, γ, and β variants. The findings contribute to a deeper understanding of function behavior in neutrosophic topologies and open avenues for further research in applied mathematics, logic, and information theory.

Keywords: neutrosophic γ -open set, neutrosophic γ -closed set, neutrosophic γT_i s paces, neutrosophic α_γ-open set, neutrosophic α_γ-closed set, neutrosophic α_γ-interior, neutrosophic α_γ-closure, neutrosophic $\alpha_\gamma T_i$ paces, neutrosophic $(\alpha_\gamma, \beta_{ne})$-contra continuous functions, neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous functions.

1. Introduction

Numerous perspectives have been proposed for handling uncertainty, including probability theory, fuzzy set theory, intuitionistic fuzzy sets, rough set theory, and related approaches. Although many innovative methods have emerged as extensions of these concepts, several challenges persist—most notably, the inadequacy of parameters remains a key issue.

The concept of Fuzzy Topological Spaces was initially introduced by Chang [5]. A neutrosophic set generalizes classical sets, fuzzy sets, and intuitionistic fuzzy sets. The term neutrosophy refers to the study of neutralities. The neutrosophic approach has its origins in fuzzy logic and intuitionistic fuzzy logic. Zadeh [26] pioneered the theory of fuzzy sets, while Atanassov [2-4] developed the intuitionistic fuzzy set framework. Lugojan [15] contributed by proposing a generalization of topology. In 1999, Smarandache [16-19] introduced the novel field of neutrosophy, which incorporates an independent indeterminacy-membership function to extend previous theories.

In a neutrosophic set, the indeterminacy component is addressed through a structured approach. "The true-, indeterminacy-, and false-membership" functions exhibit self-regulating behavior, autonomously adapting to variations in input data and contextual uncertainties within the neutrosophic framework. Wang [24] familiarized the idea of the single-valued neutrosophic set (SVNS), providing a set-theoretic formulation along with various representations and forms of SVNSs to support practical applications and theoretical developments.

Salama [20-23] introduced neutrosophic topological spaces, which generalize the intuitionistic fuzzy topological spaces which were previously studied by the author Coker [6]. In this framework, a neutrosophic set is characterized not only by the degree of membership but also by the degrees of indeterminacy and non-membership associated with each element.

Hanan Ali Hussein [7] conducted studies on γ-regular, γ-normal, and γ-connected spaces. Kalaivani et al. [8–12] explored various concepts within neutrosophic topology, including α_γ-open sets, α_γ-compact spaces, regular spaces, normal spaces, neutrosophic γ-open sets, neutrosophic α_γ-open sets, as well as neutrosophic $(\alpha_\gamma,\ \beta_{ne})$-continuous functions, $(\alpha_\gamma,\ \beta_{ne})$-open (closed) functions, and $(\alpha_\gamma,\ \beta_{ne})$-contra continuous functions. Wang [24] and Turnali [25] focused their research on single-valued neutrosophic sets, while Zahraa Nabil Kazam and Raad Aziz Hussain Al-Abdulla [27] investigated certain topological properties and axioms of counting using semi-feebly open sets.

Khalid et al [13] introduced and investigated several novel neutrosophic objects within neutrosophic topological spaces. These objects are generalizations of classical and fuzzy topological entities, refined to include neutrosophic characteristics. Among the key contributions are the definitions and properties of the "neutrosophic point, neutrosophic quasi-concomitant, neutrosophic quasi-neighbourhood, neutrosophic ideal, and neutrosophic local function". Additionally, the work formalizes a neutrosophic closure operator and introduces methods for constructing generated neutrosophic topologies.

Kungumaraj E et al [14] introduced the perception of "Heptagonal Neutrosophic Topology", utilizing heptagonal neutrosophic numbers. They defined operations like union, intersection, and complement within this framework and examines properties such as interior, closure, exterior, and boundary in neutrosophic topological spaces.

In Chapter Three, the idea of neutrosophic $\alpha_{(\gamma,\beta)}$ - contra continuous functions are presented, besides their properties are thoroughly examined. The relationships among these contra continuous functions are also explored in detail. All over this section, let $\mathcal{Z}_{ne}$ designate the "Topological space" $(\mathcal{Z}_{ne}, \tau_{ne})$and $\gamma: \tau_{ne} \to P(\mathcal{Z}_{ne})$ be an "operation" on τ_{ne}.

2. Prefaces

The philosophy of neutrosophic sets is a strategy for studying the uncertainties was disclosed by "Smarandache" [14,15,16]. "Salama" [17,18,19,20], Alblowi [1] informed the thought of "Neutrosophic Topological Spaces". "The Neutrosophic set, Neutrosophic complement, Neutrosophic inclusion relation, Neutrosophic union, Neutrosophic intersection, Neutrosophic Topology, Neutrosophic open set, Neutrosophic closed set were familiarized by Salama et al" [17,18,19,20]. The Neutrosophic functions was exposed by Turnali as well Coker [22].

2.1. Definition A neutrosophic subset B of a Neutrosophic Topological space is assumed to be a neutrosophic γ-open set if for individual $z \in B$, there occurs a neutrosophic open set V,such that $z \in V$ along with $V^{\gamma} \subseteq B$. $\tau_{(ne)\gamma}$denotes the collection of all neutrosophic γ-open sets.

2.2. Definition A neutrosophic subset B of a Neutrosophic Topological space is supposed to be a neutrosophic γ-closed set in$(\mathcal{Z}_{ne}, \tau_{ne})$ if $\mathcal{Z}_{ne} - B$ is a neutrosophic γ -open set in $(\mathcal{Z}_{ne}, \tau_{ne})$.

2.3. Definition A space $(\mathcal{Z}_{ne}, \tau_{ne})$ is termed a neutrosophic γ T_0space if for individual different point $s, t \in \mathcal{Z}_{ne}$there occurs a neutrosophic γ -open set M such that $s \in M$ and $t \notin M^{\gamma}$ or $t \in M$ and $s \notin M^{\gamma}$.

2.4. Definition A space$(\mathcal{Z}_{ne}, \tau_{ne})$ is labeled a neutrosophic γ T_1space if for each distinct point $s, t \in \mathcal{Z}_{ne}$there endures neutrosophic γ -open sets M, N containing s, t respectively aforesaid that $t \notin M^{\gamma}$ and $s \notin N^{\gamma}$.

2.5. Definition A space $(\mathcal{Z}_{ne}, \tau_{ne})$is termed a neutrosophic γ T_2space if for each distinct point $s, t \in \mathcal{Z}_{ne}$there occurs neutrosophic γ -open sets M, N like that $s \in M$, $t \in N$ and $M^{\gamma} \cap N^{\gamma} = \emptyset$.

2.6. Definition Let $(\mathcal{Z}_{ne}, \tau_{ne})$ be a Neutrosophic Topological space. Then a neutrosophic subset member H of $\mathcal{Z}_{ne}$is aforesaid to be a neutrosophic γ generalized Closed set (ne γ g-Closed set) if $\tau_{(ne)\gamma}$ -cl $(H) \subseteq M$ whenever $H \subseteq M$ and M is a neutrosophic γ -open set in $(\mathcal{Z}_{ne}, \tau_{ne})$.

2.7. Definition A neutrosophic set H in a Neutrosophic Topological space $(\mathcal{Z}_{ne}, \tau_{ne})$ is concluded as a neutrosophic α_{γ} -open set if and only if $H \subseteq \tau_{(ne)\gamma}$ -int($\tau_{(ne)\gamma}$ -cl ($\tau_{(ne)\gamma}$ -int (H))).

2.8. Definition Endorse $(\mathcal{Z}_{ne}, \tau_{ne})$ as a Neutrosophic Topological space along with Q as a neutrosophic subset of $(\mathcal{Z}_{ne}, \tau_{ne})$. Then neutrosophic $\tau_{(ne)\alpha_{\gamma}}$-interior of Q is the union of entire neutrosophic α_γ -open sets housed in Q and it is denoted by $\tau_{(ne)\alpha_\gamma}$ -int(Q).

That is, $\tau_{(ne)\alpha_\gamma}$ -int(Q) $= \cup \{ U : U$ is a neutrosophic α_γ - open set and $U \subseteq Q \}$

2.9. Definition Confer $(\mathcal{Z}_{ne}, \tau_{ne})$ to be a Neutrosophic Topological space as well as $\mathcal{H}$ be a neutrosophic subset of $(\mathcal{Z}_{ne}, \tau_{ne})$. At that juncture, $\tau_{(ne)\alpha_\gamma}$ -closure of $\mathcal{H}$ is the intersection of all neutrosophic α_γ -closed sets consisting of $\mathcal{H}$ and it is indicated by $\tau_{(ne)\alpha_\gamma}$ -cl($\mathcal{H}$).

That is, $\tau_{(ne)\alpha_\gamma}$ -cl $(\mathcal{H}) = \cap \{\mathcal{F}: \mathcal{F}$ is a $n$eutrosophic α_γ -closed set besides $\mathcal{H} \subseteq \mathcal{F} \}$.

2.10. Definition A Neutrosophic Topological space $(\mathcal{Z}_{ne}, \tau_{ne})$is authorized as a neutrosophic $\alpha_\gamma T_0$ space if for respective dissimilar points p, $q \in \mathcal{Z}_{ne}$ nearby exists a neutrosophic α_γ - open set, P such that p $\in P$ and q $\notin P$ or $q \in P$ and $p \notin P$.

2.11. Definition A Neutrosophic Topological space $(\mathcal{Z}_{ne}, \tau_{ne})$is labelled a neutrosophic α_γ T_1 space if for individual dissimilar points p , $q \in \mathcal{Z}_{ne}$ there exists neutrosophic α_γ -open sets, P , Q inclosing p and q recurrently such that $q \notin P$ and $p \notin Q$.

2.12. Definition A Neutrosophic Topological space $(\mathcal{Z}_{ne}, \tau_{ne})$is designated a neutrosophic α_γ T_2 space if for individual distinctive points p, $q \in \mathcal{Z}_{ne}$ there exists neutrosophic α_γ -open sets, P , Q such that $p \in P$,$q \in Q$ and $P \cap Q = \emptyset$.

2.13. Definition Let $(\mathcal{Z}_{ne}, \tau_{ne})$be a Neutrosophic Topological space. Formerly a neutrosophic subset member M of $\mathcal{Z}_{ne}$ is forenamed to be a neutrosophic α_γg- closed set if $\tau_{(ne)\alpha_\gamma}$ -cl (M) $\subseteq P$ whenever $M \subseteq P$ and P is a neutrosophic α_γ - closed set in $(\mathcal{Z}_{ne}, \tau_{ne})$.

2.14. Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$is said to be a neutrosophic $\alpha_{(\gamma,\beta)}$ -continuous function specified for each neutrosophic α_β- open set U of $\mathcal{Y}_{ne}$, the inverse image $f_{ne}^{-1}(U)$ is a neutrosophic α_γ - open set in $\mathcal{Z}_{ne}$.

2.15. Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ is assumed to be a neutrosophic $(\alpha_\gamma, \beta_{ne})$-continuous function if the inverse image of each one neutrosophic β_{ne}-open set in $(\mathcal{Y}_{ne}, \sigma_{ne})$ abides to be a neutrosophic α_γ-open set in $(\mathcal{Z}_{ne}, \tau_{ne})$.

2.16. Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ is supposed to be a neutrosophic $(\gamma_{ne}, \beta_{ne})$-continuous function if the inverse image of each neutrosophic β_{ne}-open set in $(\mathcal{Y}_{ne}, \sigma_{ne})$ continues to be a neutrosophic γ_{ne}-open set in $(\mathcal{Z}_{ne}, \tau_{ne})$.

2.17. Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neutrosophic $\alpha_{(\gamma,\beta)}$-$h$omeomorphism, if f_{ne} is a bijective, neutrosophic $\alpha_{(\gamma,\beta)}$ -continuous function and f_{ne}^{-1} is a neutrosophic $\alpha_{(\beta,\gamma)}$ -continuous function .

2.18. Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ is supposed to be a neutrosophic $(\alpha_\gamma, \beta_{ne})$ -contra continuous function if the inverse image of each one neutrosophic β_{ne}- open set in $\mathcal{Y}_{ne}$ is a neutrosophic α_γ-closed set in $\mathcal{Z}_{ne}$.

3. Neutrosophic $\alpha_{(\gamma,\beta)}$- Contra Continuous Functions

3.1. Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ is thought to be a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function in order that for a neutrosophic α_β -open set U of $\mathcal{Y}_{ne}$, $f_{ne}^{-1}(U)$ is a neutrosophic α_γ-closed set in $\mathcal{Z}_{ne}$.

3.1. Example Given $\mathcal{Z}_{ne} = \{h_1, h_2, h_3\}$, $\tau_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_3, F_4, F_5\}$, $\mathcal{Y}_{ne} = \{g_1, g_2, g_3\}$ and $\sigma_{ne} = \{1_{ne}, \mathcal{Y}_{ne}, H_1, H_3, H_{4,}H_5\}$ were

$F_1 = \{z, (0.1,0.6,0.1), (0.1,0.1,0.1), (0.3,0.1,0.1)\}$, $F_3 = \{z, (0.8,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.3)\}$,

$F_4 = \{z, (0.8,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.1)\}$, $F_5 = \{z, (0.7,0.6,0.3), (0.1,0.1,0.1), (0.1,0.1,0.1)\}$

$H_1 = \{y, (0.2,0.6,0.1), (0.1,0.1,0.1), (0.3,0.1,0.1)\}$, $H_3 = \{y, (0.7,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.3)\}$,

$H_4 = \{y, (0.7,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.1)\}$, $H_5 = \{y, (0.8,0.6,0.3), (0.1,0.1,0.1), (0.1,0.1,0.1)\}$

Define an operation γ on τ_{ne} such that $(U)^\gamma = \begin{cases} U \; if \; U = \{h_1\} \\ U \cup h_3 \; if \; U \neq \{h_1\} \end{cases}$. Define an operation β on σ_{ne} such that $(U)^\beta = \begin{cases} U \cup g_3 \; if \; U = \{g_1\} \\ cl(U) if \; U \neq \{g_1\} \end{cases}$. Define $f_{ne}: \mathcal{Z}_{ne} \to \mathcal{Y}_{ne}$ as $f_{ne}(h_1) = g_2$, $f_{ne}(h_2) = g_1$ and $f_{ne}(h_3) = g_3$ Then the reversed image of each neutrosophic α_β-open set acts as a neutrosophic α_γ-open set under f_{ne}. Thence f_{ne} occurs as a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function.

3.1. Remark Every neutrosophic $\alpha_{(\gamma,\beta)}$-contra continuous function is a neutrosophic$(\alpha_\gamma, \beta_{ne})$- contra continuous function. The consequent illustration shows that the converse of the directly above remark need not be true.

3.2. Example Given $\mathcal{Z}_{ne} = \{h_1, h_2, h_3\}$, $\tau_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_3, F_4, F_5\}$, $\mathcal{Y}_{ne} = \{g_1, g_2, g_3\}$ and $\sigma_{ne} = \{1_{ne}, \mathcal{Y}_{ne}, H_2, H_5\}$ were

$F_1 = \{z, (0.1,0.6,0.1), (0.1,0.1,0.1), (0.3,0.1,0.1)\}$, $F_3 = \{z, (0.8,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.3)\}$,

$F_4 = \{z, (0.8,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.1)\}$, $F_5 = \{z, (0.7,0.6,0.3), (0.1,0.1,0.1), (0.1,0.1,0.1)\}$

$H_2 = \{y, (0.7,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.3)\}$, $H_5 = \{y, (0.7,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.1)\}$

Determine an operation γ on τ_{ne} such that $(U)^\gamma = cl(U)$ for each U in τ_{ne}. Suggest an operation β on σ_{ne} such that $(V)^\beta = cl(V)$ for each V in σ_{ne}.

Assume a mapping $f_{ne}: \mathcal{Z}_{ne} \to \mathcal{Y}_{ne}$ as $f_{ne}(h_1) = g_1$, $f_{ne}(h_2) = g_3$ and $f_{ne}(h_3) = g_2$ Then the reversed image of each neutrosophic β_{ne} -open set is not an neutrosophic α_γ - closed set underneath f_{ne}. Hence f_{ne} endures to be a neutrosophic$(\alpha_\gamma, \beta_{ne})$- contra continuous function. Here $f_{ne}^{-1}(\{g_3\}) = \{h_2\}$ also $f_{ne}^{-1}(\{g_1\}) = \{h_1\}$ are not α_γ -closed sets. Thereafter f_{ne} is not a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function.

3.2. Remark The observation of neutrosophic $\alpha_{(\gamma,\beta)}$ -contra continuous functions in addition neutrosophic $(\gamma_{ne}, \beta_{ne})$- contra irresolute functions are self-governing. The succeeding illustration validates the significance.

3.3. Example Given $\mathcal{Z}_{ne} = \{h_1, h_2, h_3\}$, $\tau_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_3, F_4, F_5\}$, $\mathcal{Y}_{ne} = \{g_1, g_2, g_3\}$and $\sigma_{ne} = \{1_{ne}, \mathcal{Y}_{ne}, H_1, H_3, H_4, H_5\}$ were

$F_1 = \{z, (0.1,0.6,0.1), (0.1,0.1,0.1), (0.3,0.1,0.1)\}$, $F_3 = \{z, (0.8,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.3)\}$,

$F_4 = \{z, (0.8,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.1)\}$, $F_5 = \{z, (0.7,0.6,0.3), (0.1,0.1,0.1), (0.1,0.1,0.1)\}$

$H_1 = \{y, (0.2,0.6,0.1), (0.1,0.1,0.1), (0.3,0.1,0.1)\}$, $H_3 = \{y, (0.7,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.3)\}$,

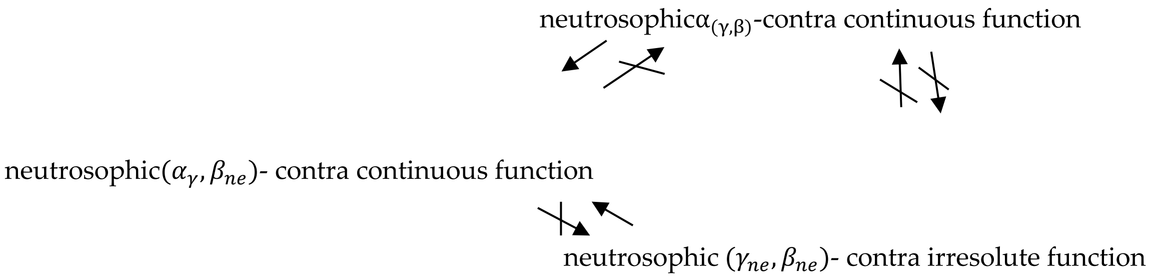
$H_4 = \{y, (0.7,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.1)\}$, $H_5 = \{y, (0.8,0.6,0.3), (0.1,0.1,0.1), (0.1,0.1,0.1)\}$

Determine an operation γ on τ_{ne} such that $(U)^{\gamma} = \begin{cases} U \ if \ U = \{h_1\} \\ U \cup \{h_3\} \ if \ U \neq \{h_1\} \end{cases}$.

Accomplish an operation β on σ_{ne} such that $(U)^{\beta} = \begin{cases} U \cup \{g_3\} \ if \ U \neq \{g_1\} \\ U \ if \ U = \{g_1\} \end{cases}$.

Consider a mapping $f_{ne}: \mathcal{Z}_{ne} \to \mathcal{Y}_{ne}$ as $f_{ne}(h_1) = g_1$, $f_{ne}(h_2) = g_2$ and $f_{ne}(h_3) = g_3$. Thence, f_{ne} endures to be a neutrosophic $(\gamma_{ne}, \beta_{ne})$- contra irresolute function. But then again $f_{ne}^{-1}(\{g_1, g_3\})$= $\{h_1, h_3\}$ is not a α_{γ}-closed set underneath f_{ne}. Thereafter, f_{ne} is not a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function. Through the Example 4.2, f_{ne} is a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function but not a $(\gamma_{ne}, \beta_{ne})$- contra irresolute function.

3.3. Remark As of the 2.16.,2.18, 3.1., Definitions, 3.1. besides 3.2. Remarks, the succeeding graphic implications 3.1. Figure is conquered.



$M \to N$ suggest M denote N, $M \nrightarrow N$indicates M does not denote N.

3.1. Figure: Connotation among ℵeutrosophic contra continuous function

3.4. Remark Consider $\mathcal{Z}_{ne}$ as a neutrosophic γ_{ne}-regular space and $\mathcal{Y}_{ne}$ as a neutrosophic β_{ne}-regular space, in an instance the neutrosophic(γ_{ne}, β_{ne})- contra irresoluteness in addition neutrosophic $\alpha_{(\gamma,\beta)}$ -contra continuity are similar.

3.1. Theorem Assume $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ as a function. Formerly the subsequent declarations are comparable:

(i) f_{ne} is a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function;

(ii) for each point z in $\mathcal{Z}_{ne}$ and each neutrosophic α_β - closed set V in $\mathcal{Y}_{ne}$ such that $f_{ne}(z) \in V$, there exists a neutrosophic α_γ- open set W in $\mathcal{Z}_{ne}$ alike that $z \in W$, $f_{ne}(W) \widetilde{\subseteq} V$;

(iii) for all z in $\mathcal{Z}_{ne}$ and a neutrosophic α_β- closed set F in $\mathcal{Y}_{ne}$, $f_{ne}^{-1}(F) \in \tau_{ne\alpha_\gamma}$;

(iv) $f_{ne}(\tau_{ne\alpha_\gamma}$-cl $(A))\subseteq \sigma_{ne\alpha_\beta}$-ker $f_{ne}(f_{ne}(A))$ for all subsets A of $\mathcal{Z}_{ne}$;

(v)$\tau_{ne\alpha_\gamma}$-cl $(f_{ne}^{-1}(B)) \subseteq f_{ne}^{-1}(\sigma_{ne\alpha_\beta}$-ker $(B))$

Proof. (i) $\Rightarrow$ (ii) Assume z in $\mathcal{Z}_{ne}$ and V be any neutrosophic β_{ne} -closed set of Y_s encompassing f_{ne} (z). Established that $W = f_{ne}^{-1}(V)$ then by 3.1 Definition, W is a neutrosophic α_γ- open set comprising z besides $f_{ne}(W) = f_{ne}(f_{ne}^{-1}(V))\subseteq V$.

(ii) $\Rightarrow$ (iii) Agree that V be a neutrosophic β_{ne}-closed set in $\mathcal{Y}_{ne}$. Let $F = \mathcal{Y}_{ne} - V$, formerly F is a $n$eutrosophic β_{ne}-open set in $\mathcal{Y}_{ne}$. Assume z $\in f_{ne}^{-1}(V)$, by utilizing the fact of (ii) there arises a neutrosophic open set W of $\mathcal{Z}_{ne}$ such that $f_{ne}(W)\subseteq V$. Consequently z $\in W \subseteq \tau_{ne\gamma}$ - int ($\tau_{ne\gamma}$ - cl ($\tau_{ne\gamma}$ -int (W))) $\subseteq \tau_{ne\gamma}$ -int ($\tau_{ne\gamma}$ - cl ($\tau_{ne\gamma}$ -int ($f_{ne}^{-1}(F)$)))) and hence $f_{ne}^{-1}(F)\subseteq \tau_{ne\gamma}$ -int ($\tau_{ne\gamma}$- cl ($\tau_{ne\gamma}$ -int ($f_{ne}^{-1}(F)$)))). Then$f_{ne}^{-1}(F)$ is a neutrosophic α_γ - closed set in $\mathcal{Z}_{ne}$. Hence $f_{ne}^{-1}(V) = \mathcal{Z}_{ne} - f_{ne}^{-1}(Y_s - V) = \mathcal{Z}_{ne} - f_{ne}^{-1}(F)$ is a neutrosophic α_γ - open set in $\mathcal{Z}_{ne}$.

(iii) $\Rightarrow$ (i) Let B be a neutrosophic β_{ne}-open set in $\mathcal{Y}_{ne}$. Then $F = \mathcal{Y}_{ne} - B$ is a neutrosophic β_{ne}-closed set in $\mathcal{Y}_{ne}$. By (iii), $f_{ne}^{-1}(F)$ is a neutrosophic α_γ- open set in $\mathcal{Z}_{ne}$. Henceforth $f_{ne}^{-1}(B) = \mathcal{Z}_{ne} - f_{ne}^{-1}(\mathcal{Y}_{ne} - B) = \mathcal{Z}_{ne} - f_{ne}^{-1}((F)$is a neutrosophic α_γ -closed set in $\mathcal{Z}_{ne}$.

(iii) $\Rightarrow$ (iv) Let $\mathcal{B}$ be a neutrosophic subset of $\mathcal{Z}_{ne}$ then assume that y $\notin \sigma_{ne\alpha_\beta}$-ker$\left(f_{ne}(\mathcal{B})\right)$,

formerly there exists a $n$eutrosophic β_{ne}-closed set $\mathcal{D}$ in $\mathcal{Y}_{ne}$, like that y $\in \mathcal{D}$ and $f_{ne}(\mathcal{B}) \cap \mathcal{D} = \emptyset$, consequently $f_{ne}^{-1}(f_{ne}(\mathcal{B}) \cap \mathcal{D}) = \emptyset$. This deduces that $\mathcal{B} \cap \mathcal{D} = \emptyset$, and so $\tau_{ne\alpha_\gamma}$- cl $(\mathcal{B})\subseteq \mathcal{Z}_{ne} - f_{ne}^{-1}(\mathcal{D})$. It follows that $f_{ne}(\tau_{s\alpha_\gamma}$-cl $(\mathcal{B}))\widetilde{\cap}\mathcal{D}=\emptyset$, which designates that y $\notin f_{ne}(\tau_{ne\alpha_\gamma}$-cl $(\mathcal{B}))$. Henceforward, it is showed that $f_{ne}(\tau_{ne\alpha_\gamma}$-cl $(\mathcal{B}))\subseteq \sigma_{ne\alpha_\beta}$-ker $(f_{ne}(\mathcal{B}))$ for all neutrosophic subset $\mathcal{B}$ of $\mathcal{Z}_{ne}$.

(iv) $\Rightarrow$ (v) Assume B be a neutrosophic subset of $\mathcal{Y}_{ne}$. At that time $f_{ne}^{-1}(B) \subseteq \mathcal{Z}_{ne}$. By supposition $f_{ne}(\tau_{ne\alpha_\gamma}$-cl $(f_{ne}^{-1}(B)))\subseteq \sigma_{ne\alpha_\beta}$-ker $(f_{ne}(f_{ne}^{-1}(B)))$. It trails that $f_{ne}(\tau_{ne\alpha_\gamma}$-cl $(f_{ne}^{-1}(B)))\subseteq \sigma_{sne}$-ker (B) , consequently, $f_{ne}(\tau_{ne\alpha_\gamma}$-cl $(f_{ne}^{-1}(B)))\subseteq f_{ne}^{-1}(\sigma_{ne\alpha_\beta}$-ker $(B))$.

(v) $\Rightarrow$ (i) Consider V be any neutrosophic β_{ne}-open set in $\mathcal{Y}_{ne}$. By the given declaration, $\tau_{ne\alpha_\gamma}$-cl $(V)\subseteq f_{ne}^{-1}(\sigma_{s\alpha_\beta}$-ker $(V))$, since V is a neutrosophic β_{ne}-open set formerly, $\sigma_{ne\alpha_\beta}$-ker $(V) = V$. In consequence $\tau_{ne\alpha_\gamma}$ -cl $(f_{ne}^{-1}(V))\subseteq f_{ne}^{-1}(V)$.It follows that $\tau_{ne\alpha_\gamma}$ -cl $(f_{ne}^{-1}(V)) = f_{ne}^{-1}(V)$.

3.2. Theorem Let $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ be a function. γ_{ne} be a monotone operator. The collection of all points z in $\mathcal{Z}_{ne}$ such that f_{ne} is not a neutrosophic $(\alpha_\gamma, \beta_{ne})$- contra continuous function is accurately the union of the neutrosophic α_γ- frontiers of the inverse images of the neutrosophic β_{ne}- closed sets in $\mathcal{Y}_{ne}$ that comprises $f_{ne}(z)$.

Proof. Consider that f_{ne} is not a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function at the point $z \in \mathcal{Z}_{ne}$, formerly there exists a neutrosophic α_β - closed set F such that $f_{ne}(z) \in F$ and $f_{ne}(U) \cap (\mathcal{Y}_{ne} - F) \neq \emptyset$ for a neutrosophic α_γ - open $s_e$et U, such that x $\in U$ .It trails that $U \cap f_{ne}^{-1}((\mathcal{Y}_{ne} - F) \neq \emptyset$, but this means that $z \in \tau_{ne\alpha_\gamma}$-cl (f_{ne}^{-1} $(\mathcal{Y}_{ne}-F)$) = $\tau_{ne\alpha_\gamma}$-cl ($\mathcal{Z}_{ne}-$ (f_{ne}^{-1} $(F$))). Since $z \in f_{ne}^{-1}(F)$, then $z \in \tau_{ne\alpha_\gamma}$-cl ($f_{ne}^{-1}(F)) \cap \tau_{ne\alpha_\gamma}$-cl ($\mathcal{Z}_{ne} - f_{ne}^{-1}(F)$) . Therefore {z $\in \mathcal{Z}_{ne}$: f_{ne} is not a neutrosophic $\alpha_{(\gamma,\beta)}$-contra continuous function at the point $\{z\}$ is confined in $\tau_{ne\alpha_\gamma}$-Fr (f_{ne}^{-1} (F))}.

On the contrary, consider that $z \in \tau_{ne\alpha_\gamma}$-Fr (f_{ne}^{-1} (F)), where F is a neutrosophic α_β-closed set in $\mathcal{Y}_{ne}$, $f_{ne}(z) \in F$ and f_{ne} is a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function . Formerly there occurs a neutrosophic α_γ-open set U such that $z \in U$ and f_{ne} $(U) \subseteq F$, therefore $z \in U \subseteq f_{ne}^{-1}(F)$. From this, $z \in \tau_{ne\alpha_\gamma}$-int (f_{ne}^{-1} $(F)) \subseteq \tau_{ne\alpha_\gamma}$-Fr (f_{ne}^{-1} (F)) is obtained. Hence, $z \notin \tau_{ne\alpha_\gamma}$-Fr ($f_{ne}^{-1}(F)$), which is an incongruity. Consequently f_{ne} is not a neutrosophic $\alpha_{(\gamma,\beta)}$- contra continuous function. Similarly, the succeeding theorem is attained.

3.3. Theorem Consider $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ be a function. The collection of all points z in $\mathcal{Z}_{ne}$ such that $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \to (\mathcal{Y}_{ne}, \sigma_{ne})$ is not an neutrosophic $\alpha_{(\gamma,\beta)}$-contra continuous function is precisely the union of all neutrosophic α_γ-frontier of the contrary image of the neutrosophic α_β-closed set in $\mathcal{Y}_{ne}$ that comprises $f_{ne}(z)$.

Proof. The proof can be supported by applying the practice described in directly above 3.2 Theorem.

4. Conclusion

In this article, the author developed and analyzed several advanced concepts within the framework of Neutrosophic topology. By utilizing the notions of Neutrosophic γ-open sets, Neutrosophic α_γ-open sets, Neutrosophic $\alpha_{(\gamma,\beta)}$)-continuous functions, and the Neutrosophic α_γ-kernel, the concept of Neutrosophic $\alpha_{(\gamma,\beta)}$ -contra continuous functions were introduced and thoroughly explored. Their structural properties were elaborated in detail.

Furthermore, the notion of a Neutrosophic α_γ-locally indiscrete space was established, providing a new perspective on topological discreteness within the neutrosophic setting. The interrelationships among Neutrosophic (α_γ, β_{ne})-contra continuous functions, Neutrosophic $\alpha_{(\gamma,\beta)}$-contra continuous functions, and Neutrosophic (γ_{ne}, β_{ne})-contra irresolute functions were examined and characterized using illustrative diagrams. Their performance under numerous conditions was examined, highlighting their probable implication in the learning of uncertainty, general topology, and "decision-making models" based on neutrosophic logic.

5.Future Work

Future research can explore extensions of Neutrosophic contra continuous functions in more generalized topological spaces, such as bitopological, fuzzy bitopological, or soft neutrosophic spaces. Additionally, the integration of these function types into multi-criteria decision-making (MCDM) frameworks and artificial intelligence applications may provide more robust models for handling complex, uncertain data. Further investigation into the algebraic structure, continuity preservation, and compactness-related properties of these functions may also yield deeper insights and practical implications in theoretical computer science and applied systems modeling.

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