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A Neutrosophic Approach for Evaluating the Multimedia-Aided College Piano Instruction Evaluation Techniques: Dombi-Based Heronian Mean Operators

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Abstract—This paper contributes to the body of neutrosophic knowledge through proposing an advanced neutrosophic aggregation paradigm through the development of the Cubic Bipolar Neutrosophic Soft Set (CBNSS) aggregation operator based on the combination of Dombi operation as well as Weighted Heronian Mean. It is named CBNS-D-GWHM. CBNSS is formally defined to extend the expressive capacity of neutrosophic science by integrating the cubic and bipolar structures within soft set theory to enable simultaneous modeling of interval-valued, precise, positive, and negative membership information. With established theoretical properties, our CBNS-D-GWHM operator can provide a highly flexible and parameterized aggregation mechanism, capable of capturing complex scenarios. An illustrative case study in the domain of multimedia-aided college piano instruction demonstrated the applicability and robustness of CBNS-D-GWHM, which highlighted its capacity to synthesize nuanced expert knowledge. The findings not only advance the state of neutrosophic aggregation theory but also provide a novel foundation for further extensions in complex decision modeling and information fusion tasks.

Keywords—Neutrosophic Sets (NS), Single-Valued Neutrosophic Sets (SVNS), Interval-Valued Neutrosophic Sets (IVNS), Neutrosophic Aggregation Operators, Dombi Neutrosophic Operators, Neutrosophic Ranking Method.

1. Introduction

1.1. Background

Multimedia-aided college piano instruction techniques have transformed music education by integrating digital tools, such as video tutorials, interactive software, and augmented reality, to enhance learning outcomes for students [1]. This approach, increasingly adopted in educational settings as of July 24, 2025, offers personalized feedback, visual demonstrations, and flexible learning paces, making it highly relevant in preparing aspiring pianists for a competitive and technology-driven music industry [2]. The growing accessibility of multimedia resources underscores the need for effective evaluation methods to ensure these techniques meet diverse learner needs and pedagogical goals [3].

A key challenge in the existing literature and practice is the lack of robust frameworks to evaluate the effectiveness of multimedia-aided college piano instruction [2], [4], particularly when confronted with subjective, incomplete, or contradictory data [5]. Standardized tests or qualitative reviews often fail to account for the uncertainties inherent in student progress, varying teaching styles, and the inconsistent impact of multimedia elements. This gap hinders the ability to optimize instructional strategies, limiting their potential to fully support college piano education across different skill levels and cultural contexts.

In this context, Neutrosophic Set Theory has offered a powerful framework for modeling such uncertainty [6], [7], [8]. Unlike classical set theory or fuzzy set theory, a Single-Valued Neutrosophic Set (SVNS) explicitly characterizes an element by three independent membership functions: truth (T), indeterminacy (I), and falsity (F) [9], [10]. Cubic Bipolar Neutrosophic Soft Sets (CBNSS) are an advanced extension of neutrosophic soft set theory [11], which was designed to provide a more expressive and comprehensive mathematical tool for modeling complex, uncertain, and bipolar information in decision-making environments [12].

The Dombi operational laws, known for their adjustable flexibility through a parameter λ, provide a robust mechanism for modeling interaction between criteria in neutrosophic environments [13]. Supplementing this, the Heronian Mean (HM) operator offers an effective means to aggregate correlated criteria, capturing interrelationships often ignored by traditional averaging techniques. Despite their strengths, no existing study has integrated Dombi's operational laws with the HM under a neutrosophic framework to address the evaluation of multimedia-aided college piano instruction techniques.

1.2. Objectives and Contributions

The primary aim of this paper is to advance neutrosophic aggregation theory by proposing a novel Cubic Bipolar Neutrosophic Soft Dombi-based Generalized Weighted HM (CBNSS-D-GWHM) operator, specifically designed to address the complex uncertainty and bipolarity encountered in the evaluation of multimedia-aided college piano instruction techniques. The main contributions of this work are as follows. First, we develop a comprehensive aggregation operator that integrates cubic, bipolar, and soft set structures, allowing for the simultaneous representation of interval-valued and precise positive/negative membership information. Second, we introduce the Dombi t-norm/t-conorm within a custom HM, providing a highly flexible and parameterized approach to neutrosophic aggregation. Third, we formally establish key theoretical properties of the proposed operator, including idempotency, boundedness, monotonicity, and commutativity. We apply the developed framework to a realistic case study on multimedia-aided college piano instruction, demonstrating its effectiveness in aggregating nuanced expert evaluations and supporting robust, interpretable decisions.

2. Preliminaries

This section presents the fundamental concepts and mathematical formulations used in this study, including SVNS, Dombi t-norm and t-conorm, the HM operator, and other preliminaries shaping the theoretical foundation of our work

Definition 2.1. Let U be a universal set of discourse. A Single-Valued Neutrosophic Set (SVNS) A in U is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in U\}, \tag{2.1}$$

where $T_A: U \to [0,1]$, $I_A: U \to [0,1]$, $F_A: U \to [0,1]$ respectively representing membership of truth, indeterminacy, and falsity, such that:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3, \forall x \in U \tag{2.2}$$

$T_A(x), I_A(x)$, and $F_A(x)$ are independent, enabling SVNS to model uncertainty more flexibly than fuzzy or intuitionistic fuzzy sets.

Definition 2.2. Let $A_1 = \langle T_1, I_1, F_1\rangle$ and $A_2 = \langle T_2, I_2, F_2\rangle$ There are two SVNS values. The basic algebraic operations are defined as follows:

i. Complement

$$A_1^c = \langle F_1, 1 - I_1, T_1\rangle \tag{2.3}$$

ii. Addition

$$A_1 \oplus A_2 = \langle T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2\rangle \tag{2.4}$$

iii. Multiplication

$$A_1 \otimes A_2 = \langle T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2\rangle \tag{2.5}$$

iv. Scalar Multiplication (for $\lambda > 0$)

$$\lambda A_1 = \left\langle 1 - (1 - T_1)^\lambda, (I_1)^\lambda, (F_1)^\lambda \right\rangle \tag{2.6}$$

v. Power (for $\lambda > 0$)

$$A_1^\lambda = \left\langle (T_1)^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda \right\rangle \tag{2.7}$$

Definition 2.3. Let U be a universal set of discourse. A bipolar neutrosophic set (BiNS) is defined as [14]:

$$A = \{\langle x, T^{+}(x), I^{+}(x), F^{+}(x), T^{-}(x), I^{-}(x), F^{-}(x)\rangle : x \in X\}, \tag{2.8}$$

where

$$T^{+}, I^{+}, F^{+} : X \to [1,0] \text{ and } T^{-}, I^{-}, F^{-} : X \to [-1,0].$$

Definition 2.4. Let $A_1 = \langle T_1^{+}, I_1^{+}, F_1^{+}, T_1^{-}, I_1^{-}, F_1^{-}\rangle$, and $A_2 = \langle T_2^{+}, I_2^{+}, F_2^{+}, T, I_2^{-}, F_2^{-}\rangle$ be two BNS elements, and let $\lambda > 0$. The fundamental operations are defined as follows:

i. Addition

$$A_1 \oplus A_2 = \left\langle \begin{matrix} T_1^{+}T_2^{+}, I_1^{+} + I_2^{+} - I_1^{+}I_2^{+}, F_1^{+} + F_2^{+} - F_1^{+}F_2^{+}, \\ -(-T_1^{-} - T_2^{-} - T_1^{-}T_2^{-}), -I_1^{-}I_2^{-}, -F_1^{-}F_2^{-} \end{matrix} \right\rangle \tag{2.9}$$

ii. Multiplication

$$A_1 \otimes A_2 = \left\langle \begin{matrix} T_1^{+}T_2^{+}, I_1^{+} + I_2^{+} - I_1^{+}I_2^{+}, F_1^{+} + F_2^{+} - F_1^{+}F_2^{+}, \\ -(-T_1^{-} - T_2^{-} - T_1^{-}T_2^{-}), -I_1^{-}I_2^{-}, -F_1^{-}F_2^{-} \end{matrix} \right\rangle \tag{2.10}$$

iii. Scalar Multiplication ($\lambda > 0$)

$$\lambda A_1 = \left\langle \begin{matrix} 1 - (1 - T_1^{+})^{\lambda}, (I_1^{+})^{\lambda}, (F_1^{+})^{\lambda}, -\left((-T_1^{-})^{\lambda}\right), \\ -\left((-I_1^{-})^{\lambda}\right), -\left(1 - \left(1 - (-F_1^{-})\right)^{\lambda}\right) \end{matrix} \right\rangle \tag{2.11}$$

iv. Power Operation ($\lambda > 0$)

$$A_1^{\lambda} = \left\langle \begin{matrix} (T_1^{+})^{\lambda}, 1 - (1 - I_1^{+})^{\lambda}, 1 - (1 - F_1^{+})^{\lambda}, \\ -\left(1 - \left(1 - (-T_1^{-})\right)^{\lambda}\right), -(-I_1^{-})^{\lambda}, -(-F_1^{-})^{\lambda} \end{matrix} \right\rangle \tag{2.12}$$

Definition 2.5. Score, Accuracy, and Certainty Functions for BNS**.** Let $A_1 = \langle T_1^{+}, I_1^{+}, F_1^{+}, T_1^{-}, I_1^{-}, F_1^{-}\rangle$ be a BNS element. The score, accuracy, and certainty functions are defined as:

i. Score function:

$$s(A_1) = \frac{T_1^{+} + (1 - I_1^{+}) + (1 - F_1^{+}) + (1 - T_1^{-}) + I_1^{-} + F_1^{-}}{6} \tag{2.13}$$

ii. Accuracy function:

$$a(A_1) = T_1^{+} - F_1^{+} + F_1^{-} - T_1^{-} \tag{2.14}$$

iii. Certainty function:

$$c(A_1) = T_1^+ - F_1^- \tag{2.15}$$

Definition 2.6 — Generalized Weighted HM (GWHM) Operator**.** Given **a** weight vector of a collection of non-negative numbers

$$\mathbf{w} = (w_1, w_2, \dots, w_n)^\top \text{ and } x_i \ (i = 1,2,\dots,n)$$

such that $p, q \geq 0$ are real parameters, $w_i \in [0,1]$ for all i, and $\sum_{i=1}^{n} w_i = 1$. Then the GWHM operator is defined as:

$$\mathrm{GWHM}^{(p,q)}(x_1, x_2, \dots, x_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left((w_i x_i)^p \otimes \left(w_j x_j\right)^q\right)\right)^{\frac{1}{p+q}} \tag{2.16}$$

where ⊗ denotes a binary product operation.

Definition 2.7 - Improved GWHM (IGWHM) Operator. Given **a** weight vector of a collection of non-negative numbers

$$\mathbf{w} = (w_1, w_2, \dots, w_n)^\top \text{ and } x_i \ (i = 1,2,\dots,n)$$

satisfying $p, q \geq 0$, $w_i \in [0,1]$, $\sum_{i=1}^{n} w_i = 1$. Then IGWHM operator is defined as:

$$\mathrm{IGWHM}^{(p,q)}(x_1, x_2, \dots, x_n) = \frac{\left(\sum_{i=1}^{n} \sum_{j=i}^{n} w_i w_j x_i^p \otimes x_j^q\right)^{\frac{1}{p+q}}}{\left(\sum_{i=1}^{n} \sum_{j=i}^{n} w_i w_j\right)^{\frac{1}{p+q}}} \tag{2.17}$$

where ⊗ again denotes the chosen product rule (to be specified according to the application domain).

Definition 2.8 - Dombi t-Norm [15] and t-Conorm. Let $\gamma > 0$ be a real parameter and $x, y \in [0,1]$. The, Dombi t-norm $T_{D,\gamma}: [0,1]^2 \to [0,1]$ is defined as:

$$T_{D,\gamma}(x, y) = \left(1 + \left[\left(\frac{1-x}{x}\right)^\gamma + \left(\frac{1-y}{y}\right)^\gamma\right]^{\frac{1}{\gamma}}\right)^{-1} \tag{2.18}$$

The Dombi t-conorm $T_{D,\gamma}^*: [0,1]^2 \to [0,1]$ is defined as:

$$T_{D,\gamma}^*(x, y) = 1 - \left(1 + \left[\left(\frac{x}{1-x}\right)^\gamma + \left(\frac{y}{1-y}\right)^\gamma\right]^{\frac{1}{\gamma}}\right)^{-1} \tag{2.19}$$

Definition 2.9. Let $A_1 = \langle T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^- \rangle$ and $A_2 = \langle T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^- \rangle$ be two BNSs and γ, δ be two positive real numbers. Then, the Dombi sums and product operations are defined as follows:

$$A_1 \oplus_D A_2 = \left\langle \begin{array}{c} 1-\left(1+\left(\left(\frac{T_1^+}{1-T_1^+}\right)^{\gamma}+\left(\frac{T_2^+}{1-T_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \left(1+\left(\left(\frac{1-I_1^+}{I_1^+}\right)^{\gamma}+\left(\frac{1-I_2^+}{I_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \\ \left(1+\left(\left(\frac{1-F_1^+}{F_1^+}\right)^{\gamma}+\left(\frac{1-F_2^+}{F_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, -\left(1+\left(\left(\frac{1-T_1^-}{-T_1^-}\right)^{\gamma}+\left(\frac{1-T_2^-}{-T_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \\ \left(1+\left(\left(\frac{-I_1^-}{1+I_1^-}\right)^{\gamma}+\left(\frac{-I_2^-}{1+I_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}-1, \left(1+\left(\left(\frac{-F_1^-}{1+F_1^-}\right)^{\gamma}+\left(\frac{-F_2^-}{1+F_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}-1 \end{array} \right\rangle \tag{2.20}$$

$$A_1 \otimes_D A_2 = \left\langle \begin{array}{c} \left(1+\left(\left(\frac{1-T_1^+}{T_1^+}\right)^{\gamma}+\left(\frac{1-T_2^+}{T_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, 1-\left(1+\left(\left(\frac{I_1^+}{1-I_1^+}\right)^{\gamma}+\left(\frac{I_2^+}{1-I_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \\ 1-\left(1+\left(\left(\frac{F_1^+}{1-F_1^+}\right)^{\gamma}+\left(\frac{F_2^+}{1-F_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \left(1+\left(\left(\frac{-T_1^-}{1+T_1^-}\right)^{\gamma}+\left(\frac{-T_2^-}{1+T_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}-1, \\ -\left(1+\left(\left(\frac{1+I_1^-}{-I_1^-}\right)^{\gamma}+\left(\frac{1+I_2^-}{-I_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, -\left(1+\left(\left(\frac{1+F_1^-}{-F_1^-}\right)^{\gamma}+\left(\frac{1+F_2^-}{-F_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1} \end{array} \right\rangle \tag{2.21}$$

$$\delta \cdot A_1 = \left\langle \begin{array}{c} 1-\left(1+\left(\delta\left(\frac{T_1^+}{1-T_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \left(1+\left(\delta\left(\frac{1-I_1^+}{I_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \\ \left(1+\left(\delta\left(\frac{1-F_1^+}{F_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, -\left(1+\left(\delta\left(\frac{1+T_1^-}{-T_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \\ \left(1+\left(\delta\left(\frac{-I_1^-}{1+I_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}-1, \left(1+\left(\delta\left(\frac{-F_1^-}{1+F_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}-1 \end{array} \right\rangle \tag{2.22}$$

$$A_1^{\delta}=\left\langle \begin{array}{c} \left(1+\left(\delta\left(\frac{1-T_1^{+}}{T_1^{+}}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1},1-\left(1+\left(\delta\left(\frac{I_1^{+}}{1-I_1^{+}}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}, \\ 1-\left(1+\left(\delta\left(\frac{F_1^{+}}{1-F_1^{+}}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1},\left(1+\left(\delta\left(\frac{-T_1^{-}}{1+T_1^{-}}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1}-1, \\ -\left(1+\left(\delta\left(\frac{1+I_1^{-}}{-I_1^{-}}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1},-\left(1+\left(\delta\left(\frac{1+F_1^{-}}{-F_1^{-}}\right)^{\gamma}\right)^{\frac{1}{\gamma}}\right)^{-1} \end{array}\right\rangle \tag{2.23}$$

Definition 2.10. Let $p,q\geq 0$, and a collection of Bipolar Neutrosophic Sets (BNSs) $A_i=\langle T_i^{+},I_i^{+},F_i^{+},T_i^{-},I_i^{-},F_i^{-}\rangle, i=1,2,\ldots,n,$ and weight vector $w=(w_1,w_2,\ldots,w_n)^{\top}, w_i\in[0,1],\sum_{i=1}^{n} w_i=1$. The BNS Dombi-based Generalized Weighted HM (BND-GWHM) is defined as:

$$\mathrm{BND-GWHM}^{(p,q)}(A_1,\ldots,A_n)=\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left((w_iA_i)^p\otimes_D\left(w_jA_j\right)^q\right)\right)^{\frac{1}{p+q}}, \tag{2.24}$$

where the Dombi product $\otimes_D$ is applied component-wise:
For the positive membership components:

$$\begin{aligned} T_{ij}^{+}&=\left[1+\left(\left(\frac{1-(w_iT_i^{+})^p}{(w_iT_i^{+})^p}\right)^{\gamma}+\left(\frac{1-\left(w_jT_j^{+}\right)^q}{\left(w_jT_j^{+}\right)^q}\right)^{\gamma}\right)^{1/\gamma}\right]^{-1}, \\ I_{ij}^{+}&=1-\left[1+\left(\left(\frac{(w_iI_i^{+})^p}{1-(w_iI_i^{+})^p}\right)^{\gamma}+\left(\frac{\left(w_jI_j^{+}\right)^q}{1-\left(w_jI_j^{+}\right)^q}\right)^{\gamma}\right)^{1/\gamma}\right]^{-1}, \\ F_{ij}^{+}&=1-\left[1+\left(\left(\frac{(w_iF_i^{+})^p}{1-(w_iF_i^{+})^p}\right)^{\gamma}+\left(\frac{\left(w_jF_j^{+}\right)^q}{1-\left(w_jF_j^{+}\right)^q}\right)^{\gamma}\right)^{1/\gamma}\right]^{-1}. \end{aligned} \tag{2.25}$$

For the negative membership components:

$$\begin{aligned} T_{ij}^{-}&=\left[1+\left(\left(\frac{-(w_iT_i^{-})^p}{1+(w_iT_i^{-})^p}\right)^{\gamma}+\left(\frac{-\left(w_jT_j^{-}\right)^q}{1+\left(w_jT_j^{-}\right)^q}\right)^{\gamma}\right)^{1/\gamma}\right]^{-1}-1, \\ I_{ij}^{-}&=-\left[1+\left(\left(\frac{1+(w_iI_i^{-})^p}{-(w_iI_i^{-})^p}\right)^{\gamma}+\left(\frac{1+\left(w_jI_j^{-}\right)^q}{-\left(w_jI_j^{-}\right)^q}\right)^{\gamma}\right)^{1/\gamma}\right]^{-1}, \end{aligned} \tag{2.26}$$

3. Aggregation of Cubic bipolar neutrosophic soft sets

In this section, we introduce the concept of CBNSS, which is the synergistic integration of the advantages of cubic sets, bipolar neutrosophic sets, and soft sets.

Definition 3.1. Cubic Bipolar Neutrosophic Set (CBNS). Let U be a universal set of discourse.
A Cubic Bipolar Neutrosophic set (CBNS) $\tilde{A}$ in U is defined as:

$$\tilde{A} = \left\{\left\langle \begin{matrix} x, \left([T_L^+(x), T_U^+(x)], t^+(x)\right), \left([I_L^+(x), I_U^+(x)], i^+(x)\right), \left([F_L^+(x), F_U^+(x)], f^+(x)\right), \\ ([T_L^-(x), T_U^-(x)], t^-(x)), ([I_L^-(x), I_U^-(x)], i^-(x)), ([F_L^-(x), F_U^-(x)], f^-(x)) \end{matrix} \right\rangle : x \in U \right\} \tag{3.1}$$

where $[T_L^+(x), T_U^+(x)] \subseteq [0,1]$ and $t^+(x) \in [0,1]$ symbolizes interval-valued and crisp positive truth. $[I_L^+(x), I_U^+(x)] \subseteq [0,1]$ and $i^+(x) \in [0,1]$ symbolizes interval-valued and crisp positive indeterminacy. $[F_L^+(x), F_U^+(x)] \subseteq [0,1]$ and $f^+(x) \in [0,1]$ symbolizes interval-valued and crisp positive falsity. $[T_L^-(x), T_U^-(x)] \subseteq [-1,0]$ and $t^-(x) \in [-1,0]$ symbolizes interval-valued and crisp negative truth. $[I_L^-(x), I_U^-(x)] \subseteq [-1,0]$ and $i^-(x) \in [-1,0]$ symbolizes interval-valued and crisp negative indeterminacy. $[F_L^-(x), F_U^-(x)] \subseteq [-1,0]$ and $f^-(x) \in [-1,0]$ symbolizes interval-valued and crisp negative falsity.

Definition 3.2. Let U be a universal set and E be a set of parameters. A Cubic Bipolar Neutrosophic Soft Set (CBNSS) (F, E) over U is a parameterized family of CBNSs, defined as:

$$(F, E) = \{\langle e, F(e)\rangle : e \in E, F(e) \subseteq \text{CBNS}(U)\} \tag{3.2}$$

where for each parameter $e \in E$ and for each element $x \in U$:

$$F(e) = \left\langle \begin{matrix} x, \left(\left[T_{L,e}^+(x), T_{U,e}^+(x)\right], t_e^+(x)\right), \left(\left[I_{L,e}^+(x), I_{U,e}^+(x)\right], i_e^+(x)\right), \left(\left[F_{L,e}^+(x), F_{U,e}^+(x)\right], f_e^+(x)\right), \\ \left(\left[T_{L,e}^-(x), T_{U,e}^-(x)\right], t_e^-(x)\right), \left(\left[I_{L,e}^-(x), I_{U,e}^-(x)\right], i_e^-(x)\right), \left(\left[F_{L,e}^-(x), F_{U,e}^-(x)\right], f_e^-(x)\right) \end{matrix} \right\rangle \tag{3.3}$$

such that:

$$T_{L,e}^+(x), T_{U,e}^+(x), t_e^+(x), I_{L,e}^+(x), I_{U,e}^+(x), i_e^+(x), F_{L,e}^+(x), F_{U,e}^+(x), f_e^+(x) \in [0,1]$$
$$T_{L,e}^-(x), T_{U,e}^-(x), t_e^-(x), I_{L,e}^-(x), I_{U,e}^-(x), i_e^-(x), F_{L,e}^-(x), F_{U,e}^-(x), f_e^-(x) \in [-1,0]$$

Illustrative Example 3.1. Let the universe of alternatives represent three college piano instructors:

$$U = \{u_1\text{: Instructor A}, u_2\text{: Instructor B}, u_3\text{: Instructor C}\} \tag{3.4}$$

While a set of parameters represents instructional quality aspects:

$$E = \{e_1\text{: Technical Accuracy, } e_2\text{: Expressiveness, } e_3\text{: Use of Multimedia }\} \quad (3.5)$$

Then, for each parameter $e \in E$, we define a CBNS evaluation of each instructor $u \in U$, as in Table 1.

Table 3.1. CBNSS for Multimedia-Aided Piano Instruction

Parameter (e)	Instructor (u)	Positive Truth $[T_L^+, T_U^+], t^+$	Positive Indet. $[I_L^+, I_U^+], i^+$	Positive Falsity $[F_L^+, F_U^+], f^+$	Negative Truth $[T_L^-, T_U^-], t^-$	Negative Indet. $[I_L^-, I_U^-], i^-$	Negative Falsity $[F_L^-, F_U^-], f^-$
e_1 : Technical Accuracy	u_1	[0.75, 0.85], 0.80	[0.10, 0.15], 0.12	[0.05, 0.10], 0.08	[−0.25, −0.15 −0.20	[-0.10, -0.05], -0.08	[−0.05, −0.02 −0.03
	u_2	[0.70, 0.80], 0.75	[0.15, 0.20], 0.18	[0.05, 0.10], 0.07	[−0.30, −0.20 −0.25	[-0.12, -0.08], -0.10	[-0.06, -0.03], -0.04
	u_3	[0.65, 0.78], 0.70	[0.18, 0.25], 0.20	[0.07, 0.12], 0.09	[−0.35, −0.25 −0.30	[−0.15, −0.10 −0.12	[−0.08, −0.04 −0.05
	u_1	[0.80, 0.90], 0.85	[0.08, 0.12], 0.10	[0.02, 0.05], 0.03	[−0.20, −0.10 −0.15	[−0.08, −0.04 −0.06	[−0.04, −0.01 −0.02
e_2 : Expressiveness							
	u_2	[0.78, 0.88], 0.82	[0.10, 0.15], 0.12	[0.04, 0.07], 0.05	[−0.25, −0.15 −0.20	[-0.10, -0.06], -0.08	[−0.05, −0.02 −0.03
	u_3	[0.70, 0.82], 0.76	[0.15, 0.20], 0.18	[0.06, 0.10], 0.08	[−0.30, −0.18 −0.24	[−0.12, −0.08 −0.10	[-0.07, -0.03], -0.04
e_3 : Use of Multimedia	u_1	[0.85, 0.95], 0.90	[0.05, 0.08], 0.06	[0.01, 0.03], 0.02	[−0.18, −0.08 −0.12	[−0.06, −0.03 −0.04	[−0.03, −0.01 −0.02
	u_2	[0.80, 0.92], 0.86	[0.06, 0.10], 0.08	[0.02, 0.04], 0.03	[−0.20, −0.10 −0.15	[−0.08, −0.04 −0.06	[−0.04, −0.02 −0.03
	u_3	[0.78, 0.88], 0.83	[0.08, 0.12], 0.10	[0.03, 0.05], 0.04	[−0.22, −0.12 −0.17	[−0.10, −0.05 −0.07	[−0.05, −0.03 −0.04

Definition 3.3. Dombi-Based GWHM for CBNSS (CBNS-DGWHM)

Let $p, q \geq 0$, $F(e) =$
$$\left\langle \begin{matrix} x, \left(\left[T_{L,e}^{+}(x), T_{U,e}^{+}(x)\right], t_e^{+}(x)\right), \left(\left[I_{L,e}^{+}(x), I_{U,e}^{+}(x)\right], i_e^{+}(x)\right), \left(\left[F_{L,e}^{+}(x), F_{U,e}^{+}(x)\right], f_e^{+}(x)\right), \\ \left(\left[T_{L,e}^{-}(x), T_{U,e}^{-}(x)\right], t_e^{-}(x)\right), \left(\left[I_{L,e}^{-}(x), I_{U,e}^{-}(x)\right], i_e^{-}(x)\right), \left(\left[F_{L,e}^{-}(x), F_{U,e}^{-}(x)\right], f_e^{-}(x)\right) \end{matrix} \right\rangle$$ be a collection of CBNSS values for parameters $e_1, e_2, \ldots, e_n$, with associated weights $w = (w_1, w_2, \ldots, w_n)^T$, where $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. The CBNS-DGWHM operator is defined as:

$$\text{CBNSS} - \text{D} - \text{GWHM}^{(p,q)}\left(F(e_1), \ldots, F(e_n)\right) = \left[\frac{2}{n(n+1)}\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\left(w_i F(e_i)\right)^p \otimes \left(w_j F(e_j)\right)^q\right)\right]^{\frac{1}{p+q}} \tag{3.6}$$

where ⊗ represents the Dombi t-norm/t-conorm aggregation applied component-wise to both interval

Theorem 3.1. Component-wise Computation of CBNS-DGWHM. Let $p, q, \gamma \geq 0$. we define the Dombi transformations for the positive interval-valued truth-membership:

$$G_{L,i}^{+} = \frac{T_{L,e_i}^{+}(x)}{1-T_{L,e_i}^{+}(x)}, G_{U,i}^{+} = \frac{T_{U,e_i}^{+}(x)}{1-T_{U,e_i}^{+}(x)}, G_{c,i}^{+} = \frac{t_{e_i}^{+}(x)}{1-t_{e_i}^{+}(x)} \tag{3.7}$$

Similarly, for positive indeterminacy:

$$H_{L,i}^{+} = \frac{1-I_{L,\epsilon_i}^{+}(x)}{I_{L,\epsilon_i}^{+}(x)}, H_{U,i}^{+} = \frac{1-I_{U,\epsilon_i}^{+}(x)}{I_{U,\epsilon_i}^{+}(x)}, H_{c,i}^{+} = \frac{1-i_{\epsilon_i}^{+}(x)}{i_{\epsilon_i}^{+}(x)} \tag{3.8}$$

For positive falsity:

$$I_{L,i}^{+} = \frac{1-F_{L,e_i}^{+}(x)}{F_{L,e_i}^{+}(x)}, I_{U,i}^{+} = \frac{1-F_{U,e_i}^{+}(x)}{F_{U,e_i}^{+}(x)}, I_{c,i}^{+} = \frac{1-f_{e_i}^{+}(x)}{f_{e_i}^{+}(x)} \tag{3.9}$$

For negative interval-valued truth-membership:

$$J_{L,i}^{-} = \frac{1+T_{L,e_i}^{-}(x)}{-T_{L,e_i}^{-}(x)}, J_{U,i}^{-} = \frac{1+T_{U,e_i}^{-}(x)}{-T_{U,e_i}^{-}(x)}, J_{c,i}^{-} = \frac{1+t_{e_i}^{-}(x)}{-t_{e_i}^{-}(x)} \tag{3.10}$$

For negative indeterminacy:

$$K_{L,i}^{-} = \frac{-I_{L,e_i}^{-}(x)}{1+I_{L,e_i}^{-}(x)}, K_{U,i}^{-} = \frac{-I_{U,e_i}^{-}(x)}{1+I_{U,e_i}^{-}(x)}, K_{c,i}^{-} = \frac{-i_{e_i}^{-}(x)}{1+i_{e_i}^{-}(x)} \tag{3.11}$$

For negative falsity:

$$L_{L,i}^{-} = \frac{-F_{L,e_i}^{-}(x)}{1+F_{L,e_i}^{-}(x)}, L_{U,i}^{-} = \frac{-F_{U,e_i}^{-}(x)}{1+F_{U,e_i}^{-}(x)}, L_{c,i}^{-} = \frac{-f_{e_i}^{-}(x)}{1+f_{e_i}^{-}(x)} \tag{3.12}$$

The aggregated CBNSS-D-GWHM is then computed for each component (lower, upper, crisp) using:

For Lower bound of Positive Truth

$$T_L^{+*} = \left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i\left(G'_{L,i}\right)^{\gamma}} + \frac{q}{w_j\left(G'_{L,j}\right)^{\gamma}}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.13}$$

Upper bound:

$$T_U^{+*} = \left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i\left(G^{*}_{U,i}\right)^{\gamma}} + \frac{q}{w_f\left(G^{t}_{U,j}\right)^{\gamma}}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.14}$$

Crisp part:

$$t_c^{+*} = \left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i\left(G'_{c,i}\right)^{\gamma}} + \frac{q}{w_j\left(G'_{c,j}\right)^{\gamma}}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.15}$$

For the Lower bound of Positive Indeterminacy:

$$I_L^{+*} = 1 - \left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i\left(H'_{L,i}\right)^{\gamma}} + \frac{q}{w_j\left(H'_{L,j}\right)^{\gamma}}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.16}$$

(similar for I_U^{+*} and i_c^{+*} replacing H variables)

For the Lower bound of Positive Falsity:

$$F_L^{+*} = 1 - \left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i\left(I_{L,i}\right)^{\gamma}} + \frac{q}{w_j\left(I_{L,j}\right)^{\gamma}}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.17}$$

(similar for F_U^{+*} and f_c^{+*} replacing I variables)

For Lower bound of Negative Truth:

$$T_L^{-*} = \left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{p}{w_i\left(J_{L,i}\right)^{\gamma}} + \frac{q}{w_j\left(J_{L,j}\right)^{\gamma}}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} - 1 \tag{3.18}$$

(similar for T_U^{-*} and t_c^{-*} replacing J variables)

For the Lower bound of Negative Indeterminacy:

$$I_L^{-*} = -\left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{p}{w_i\left(K_{L,j}\right)^{\gamma}} + \frac{q}{w_j\left(K_{L,j}\right)^{\gamma}}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.19}$$

(similar for I_U^{-*} and i_c^{-*} replacing K variables)

For the lower bound of Negative Falsity:

$$F_L^{-*} = -\left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{p}{w_i\left(L_{L,i}\right)^{\gamma}} + \frac{q}{w_j\left(L_{L,j}\right)^{\gamma}}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.20}$$

(similar for F_U^{-*} and f_c^{-*} replacing L variables)

Theorem 3.2 (Idempotency Property). Let $F(e_i) = \left\langle \begin{matrix} x, \left(\left[T_{L,e_i}^{+}(x), T_{U,e_i}^{+}(x)\right], t_{e_i}^{+}(x)\right), \left(\left[I_{L,e_i}^{+}(x), I_{U,e_i}^{+}(x)\right], i_{e_i}^{+}(x)\right), \left(\left[F_{L,e_i}^{+}(x), F_{U,e_i}^{+}(x)\right], f_{e_i}^{+}(x)\right), \\ \left(\left[T_{L,e_i}^{-}(x), T_{U,e_i}^{-}(x)\right], t_{e_i}^{-}(x)\right), \left(\left[I_{L,e_i}^{-}(x), I_{U,e_i}^{-}(x)\right], i_{e_i}^{-}(x)\right), \left(\left[F_{L,e_i}^{-}(x), F_{U,e_i}^{-}(x)\right], f_{e_i}^{-}(x)\right) \end{matrix} \right\rangle$ be n identical CBNS values for all $i = 1,2,\dots,n$, with weights $w_i \in [0,1]$ such that $\sum_{i=1}^{n} w_i = 1$. Then, the CBNS-D-GWHM operator satisfies:

$$\text{CBNS} - \text{DGWHM}^{(p,q)}\big(F(e_1), \dots, F(e_n)\big) = F(e_1) \tag{3.21}$$

whenever $F(e_1) = F(e_2) = \cdots = F(e_n)$.

Proof:

Let us focus on one generic component X_i (for example $T_{L,e_i}^{+}(x)$), whose Dombi transformation is:

$$G_i = \frac{X_i}{1 - X_i} \tag{3.22}$$

The aggregated value is given by:

$$X^* = \left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i G_i'} + \frac{q}{w_j G_j'}\right)^{-1}}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.23}$$

If $X_i = X$ for all i, then:

$$G_i = \frac{X}{1-X} = G, \forall i, \tag{3.24}$$

hence,

$$w_i G_i^{\gamma} = w_i G^{\gamma}, \forall i, \tag{3.25}$$

The denominator inside the sum becomes:

$$\frac{p}{w_i G^{\gamma}} + \frac{q}{w_j G^{\gamma}} = \frac{p+q}{G^{\gamma}} \cdot \frac{1}{w_i + w_j} \cdot \left(w_i + w_j\right) \tag{3.26}$$

but since w_i, w_j are fixed and G^{γ} is constant:

$$\frac{p}{w_i G^{\gamma}} + \frac{q}{w_j G^{\gamma}} = \frac{p}{w_i G^{\gamma}} + \frac{q}{w_j G^{\gamma}} \tag{3.27}$$

and because X_i are equal, the entire expression depends only on w_i, w_j, not on i or j individually.

Since G^{γ} is constant:

$$\left(\frac{p}{w_i G^{\gamma}} + \frac{q}{w_j G^{\gamma}}\right)^{-1} = G^{\gamma} \left(\frac{p}{w_i} + \frac{q}{w_j}\right)^{-1} \tag{3.28}$$

thus,:

$$\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i G^{\gamma}} + \frac{q}{w_j G^{\gamma}}\right)^{-1} = G^{\gamma} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i} + \frac{q}{w_j}\right)^{-1} \tag{3.29}$$

By substitution back into aggregation formula

$$X^* = \left[1 + \left(\frac{n(n+1)}{2(p+q)} \cdot \frac{1}{G^{\gamma} \cdot C}\right)^{\frac{1}{\gamma}}\right]^{-1} \tag{3.30}$$

where $C = \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i} + \frac{q}{w_j}\right)^{-1}$.

From the definition of $G = \frac{X}{1-X}$, solving the aggregation equation for X^* yields:

$$G^* = G \;\Rightarrow\; X^* = X \tag{3.31}$$

Theorem 3.3 (Boundedness Property**).** Let $F(e_i) = \left\langle \begin{array}{l} x, \left(\left[T^+_{L,e_i}(x), T^+_{U,e_i}(x)\right], t^+_{e_i}(x)\right), \left(\left[I^+_{L,e_i}(x), I^+_{U,e_i}(x)\right], i^+_{e_i}(x)\right), \left(\left[F^+_{L,e_i}(x), F^+_{U,e_i}(x)\right], f^+_{e_i}(x)\right), \\ \left(\left[T^-_{L,e_i}(x), T^-_{U,e_i}(x)\right], t^-_{e_i}(x)\right), \left(\left[I^-_{L,e_i}(x), I^-_{U,e_i}(x)\right], i^-_{e_i}(x)\right), \left(\left[F^-_{L,e_i}(x), F^-_{U,e_i}(x)\right], f^-_{e_i}(x)\right) \end{array} \right\rangle$
be a set of n CBNS values with weights $w_i \in [0,1]$ such that $\sum_{i=1}^{n} w_i = 1$.
Then, for each component X of the aggregated CBNS obtained by the CBNS-D-GWHM operator:

$$\min_{1\le i\le n} X_i \le X^* \le \max_{1\le i\le n} X_i \tag{3.32}$$

where X_i is the i-th component value and X^* is the aggregated value for that component.

Proof:
We prove this component-wise, and the result will hold for all 18 components of the CBNS (positive/ negative, truth/indeterminacy/falsity, lower/upper/crisp parts). Take one component $X_i \in (0,1)$ (e.g., $T^+_{L,e_i}(x)$) and recall its Dombi transformation:

$$G_i = \frac{X_i}{1-X_i}, X_i = \frac{G_i}{1+G_i} \tag{3.33}$$

The CBNS-D-GWHM aggregation for this component as in eq (3.23). From Dombi aggregation theory:

- Positivity: $G_i > 0$ whenever $X_i \in (0,1)$.
- Monotonicity: If $X_i \le Y_i$ for all i_r then $G_i \le H_i$ for their Dombi transformations, and the aggregation is monotone increasing in each argument.

The inner term:

$$M = \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{p}{w_i G_i^{\gamma}} + \frac{q}{w_j G_j^{\gamma}}\right)^{-1} \tag{3.34}$$

is increasing in each G_i, since for $a > 0, 1/(a)$ decreases with a_r and thus the reciprocal sum increases with G_i.

Step 3 - Minimum Bound
Let:

$$X_{\min} = \min_{1\le i\le n} X_i \tag{3.35}$$

then:

$$G_{\min} = \frac{X_{\min}}{1-X_{\min}} \leq G_i, \forall i \tag{3.36}$$

Since the aggregation is increasing in all G_i, replacing all G_i by $G_{\min}$ yields:

$$G^* \geq G_{\min} \tag{3.37}$$

and transforming back:

$$X^* = \frac{G^*}{1+G^*} \geq \frac{G_{\min}}{1+G_{\min}} = X_{\min} \tag{3.38}$$

Similarly, let:

$$X_{\max} = \max_{1 \leq i \leq n} X_i \tag{3.39}$$

then:

$$G_{\max} = \frac{X_{\max}}{1-X_{\max}} \geq G_i, \forall i \tag{3.40}$$

By monotonicity:

$$G^* \leq G_{\max} \tag{3.41}$$

and transforming back:

$$X^* \leq \frac{G_{\max}}{1+G_{\max}} = X_{\max} \tag{3.42}$$

The above proof applies identically to all components

4. Decision Making based on CBNSS-D-GWHM Operators.

In this section, we formulate a systematic decision-making model based on the CBNSS-D-GWHM operator. CBNSS-D-GWHM proceeds with a sequence of steps on input of alternative–criterion evaluation as a CBNSS, till ranking the alternatives, as shown in Figure 4.1.

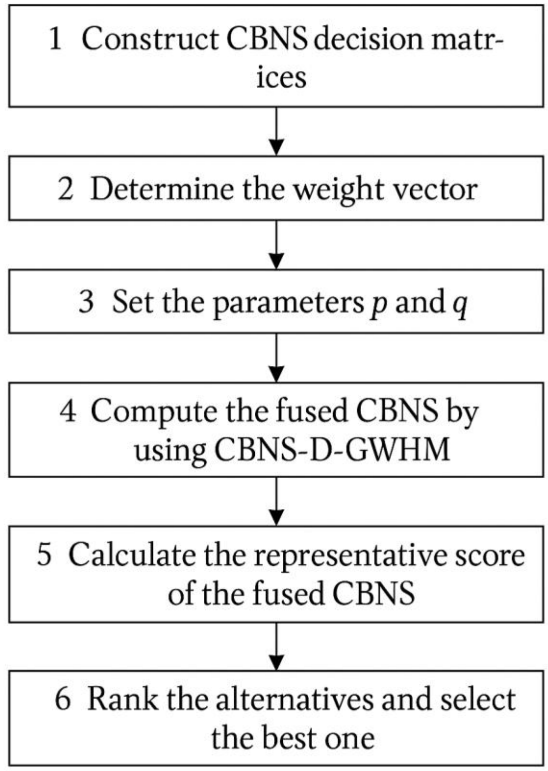

Figure 4.1. Algorithmic Decision-Making Using CBNSS-D-GWHM Operators

Given A set of alternatives $U = \{u_1, u_2, \dots, u_m\}$, A set of decision criteria $E = \{e_1, e_2, \dots, e_n\}$, Weight vector of criteria $W = (w_1, w_2, \dots, w_n)^T$, where $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k = 1$, and (CBNSS evaluation $F(e_k)$ for each criterion e_k and alternative u_i in the form:

$$F(e_k)(u_i) = \left\langle \begin{array}{c} \left(\left[T^+_{L,e_k}(u_i), T^+_{U,e_k}(u_i)\right], t^+_{e_k}(u_i)\right), \left(\left[I^+_{L,e_k}(u_i), I^+_{U,e_k}(u_i)\right], i^+_{e_k}(u_i)\right), \left(\left[F^+_{L,e_k}(u_i), F^+_{U,e_k}(u_i)\right], f^+_{e_k}(u_i)\right), \\ \left(\left[T^-_{L,e_k}(u_i), T^-_{U,e_k}(u_i)\right], t^-_{e_k}(u_i)\right), \left(\left[I^-_{L,e_k}(u_i), I^-_{U,e_k}(u_i)\right], i^-_{e_k}(u_i)\right), \left(\left[F^-_{L,e_k}(u_i), F^-_{U,e_k}(u_i)\right], f^-_{e_k}(u_i)\right) \end{array} \right\rangle$$

- Dombi parameter $\gamma > 0$ and HM parameters $p, q \geq 0$.

Step 1: Construct the Decision Matrix

Organize all expert evaluations for each u_i under each e_k into an $m \times n$ CBNSS decision matrix:

$$\mathbf{D} = \begin{bmatrix} F(e_1)(u_1) & F(e_2)(u_1) & \dots & F(e_n)(u_1) \\ F(e_1)(u_2) & F(e_2)(u_2) & \dots & F(e_n)(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ F(e_1)(u_m) & F(e_2)(u_m) & \dots & F(e_n)(u_m) \end{bmatrix} \tag{3.43}$$

Step 2: Normalize the Decision Matrix (if required)
If the criteria are of different types (benefit or cost), normalize the CBNSS values so that all criteria are converted into benefit-type using appropriate neutrosophic normalization rules.

Step 3: Apply the CBNS-D-GWHM Aggregation
For each alternative u_i, aggregate the evaluations across all criteria using the CBNSS-D-GWHM in definition 3.3.

Step 4: Compute the Overall Aggregated CBNSS
The output of Step 3 for each u_i is a fused CBNSS:

Step 5: Compute the Score Function

To keep all values in [0,1], we transform the negative membership degrees as:

$$T_{\text{adj}}^{-} = 1 + T^{-}, I_{\text{adj}}^{-} = -I^{-}, F_{\text{adj}}^{-} = -F^{-} \tag{3.44}$$

since $T^{-} \in [-1,0]$ and $I^{-}, F^{-} \in [-1,0]$.
This mapping makes all components positively comparable. For each interval-valued component $[L, U]$, take its arithmetic mean:

$$\bar{T}^{+} = \frac{T_L^{+} + T_U^{+}}{2}, \bar{I}^{+} = \frac{I_L^{+} + I_U^{+}}{2}, \bar{F}^{+} = \frac{F_L^{+} + F_U^{+}}{2}, \tag{3.45}$$

and similarly for negative components. Thus, we can compute the score as follows:

$$S_{\text{CBNS}}(u_i) = \frac{\underbrace{\bar{T}^{+} + t^{+}}_{\text{Positive truth}} + \underbrace{(1-\bar{I}^{+}) + (1-i^{+})}_{\text{Penalty for indeterminacy}} + \underbrace{(1-\bar{F}^{+}) + (1-f^{+})}_{\text{Penalty for falsity}} + \underbrace{\bar{T}_{\text{adj}}^{-} + t_{\text{adj}}^{-}}_{\text{Negative truth adjusted}} + \underbrace{\left(1-\bar{I}_{\text{adj}}^{-}\right) + \left(1-i_{\text{adj}}^{-}\right)}_{\text{Penalty from negative indeterminacy}} + \underbrace{\left(1-\bar{F}_{\text{adj}}^{-}\right) + \left(1-f_{\text{adj}}^{-}\right)}_{\text{Penalty from negative falsity}}}{12} \tag{3.46}$$

If we define:

$$P_T = \bar{T}^{+} + t^{+}, \quad P_I = (1 - \bar{I}^{+}) + (1 - i^{+}), P_F = (1 - \bar{F}^{+}) + (1 - f^{+})\ N_T = \left(1 - \bar{I}_{\text{adj}}^{-}\right) + \left(1 - i_{\text{adj}}^{-}\right), N_F = \left(1 - \bar{F}_{\text{adj}}^{-}\right) + \left(1 - f_{\text{adj}}^{-}\right)\text{的} = \bar{T}_{\text{adj}}^{-}, N_I = \left(1 - \bar{T}_{\text{adj}}\right) \tag{3.47}$$

then:

$$S_{\text{CBNS}}(u_i) = \frac{P_T + P_I + P_F + N_T + N_I + N_F}{12} \quad (3.48)$$

Step 6: Rank the Alternatives
Sort the alternatives u_i in descending order of $S(u_i)$:

$$S(u_{(1)}) \geq S(u_{(2)}) \geq \cdots \geq S(u_{(m)}) \quad (3.49)$$

where $u_{(1)}$ is the best alternative.

5. Case study

To demonstrate the practical applicability and effectiveness of CBNSS operators, we present a real-world case study, on which proposed operators are applied to aggregate CBNSS information, compute representative scores, and make ranking decision. In modern college piano pedagogy, multimedia technologies have become increasingly essential to enhance both the teaching and learning experience. With integration of traditional college piano instruction with audio-visual aids, interactive applications, and online learning platforms, educators can improve students' engagement, skill acquisition, and performance quality. However, choosing the most effective multimedia-aided college piano instruction technique is challenging due to the diversity of available tools, varying student needs, and subjective preferences of instructors. To address this, we consider seven alternative multimedia-aided college piano instruction techniques (as shown in Figure 5.1) including A_1 – Interactive Digital Sheet Music with Real-Time Feedback, A_2 – Virtual Reality (VR) Piano Performance Simulations, A_3 – AI-Driven Piano Tutoring Applications, A_4 – Online Collaborative Performance Platforms, A_5 – Video-Based Step-by-Step Piano Tutorials, A_6 – MIDI-Integrated Learning Systems with Gamification, A_7 – Hybrid In-Person and Multimedia-Enhanced Lessons. These alternatives were evaluated against five key criteria (namely C_1 – Pedagogical Effectiveness, C_2 – Student Engagement, C_3 – Technological Accessibility, C_4 – Performance Tracking & Feedback, C_5 – Adaptability to Student Needs) by a panel of expert music educators, as shown in Tables A.1-A.4, in Appendix A.

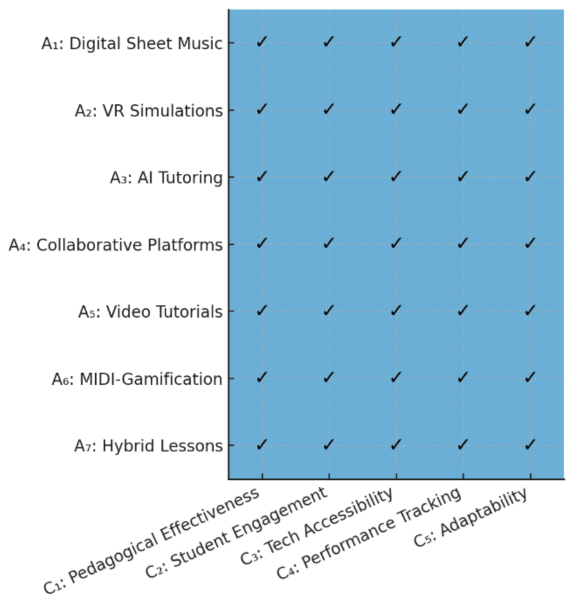

Figure 5.1. Alternatives vs Criteria Mapping

6. Results and Discussions

This section presents and analyzes the outcomes of applying the proposed CBNS-D-GWHM operator to the realistic case study of multimedia-aided college piano instruction techniques.

The results of aggregating expert evaluations using the proposed CBNS-D-GWHM operator are summarized in Table 6.1. This aggregation synthesizes the interval-valued and precise bipolar neutrosophic assessments from all participating experts for each alternative under every criterion. The aggregated decision matrix provides a unified representation that robustly integrates the perspectives of multiple decision-makers, accounts for both positive and negative evidence, and preserves the inherent uncertainty and hesitation in expert judgments.

Table 6.1. Aggregated CBNS-D-GWHM operator for Multimedia-Aided Piano Instruction Techniques.

Techniques	C1	C2	C3	C4	C5
A1	([0.528,0.934],0.766),([0.386,0.550],0.414),([0.055,0.221],0.126),([-0.894,-0.784],-0.872),([-0.195,-0.078],-0.123),([-0.242,-0.108],-0.146)	([0.684,0.922],0.745),([0.036,0.542],0.489),([0.234,0.384],0.278),([-0.889,-0.795],-0.840),([-0.213,-0.082],-0.190),([-0.155,-0.056],-0.096)	([0.762,0.815],0.791),([0.233,0.373],0.326),([0.449,0.571],0.486),([-0.911,-0.782],-0.865),([-0.208,-0.065],-0.122),([-0.188,-0.098],-0.148)	([0.782,0.779],0.806),([0.141,0.536],0.319),([0.429,0.598],0.500),([-0.820,-0.702],-0.750),([-0.234,-0.094],-0.176),([-0.114,-0.064],-0.071)	([0.694,0.774],0.725),([0.303,0.366],0.309),([0.404,0.568],0.513),([-0.854,-0.777],-0.806),([-0.126,-0.053],-0.084),([-0.206,-0.127],-0.149)
A2	([0.577,0.733],0.632),([0.187,0.538],0.384),([0.378,0.509],0.4	([0.648,0.755],0.667),([0.202,0.434],0.359),([0.108,0.421],0.2	([0.611,0.896],0.695),([0.289,0.579],0.470),([0.339,0.542],0.4	([0.808,0.925],0.841),([0.337,0.538],0.431),([0.097,0.540],0.3	([0.750,0.943],0.889),([0.451,0.634],0.577),([0.253,0.469],0.4

	14),([-0.897,-0.777],-0.823),([-0.230,-0.115],-0.149),([-0.209,-0.150],-0.199)	25),([-0.814,-0.765],-0.783),([-0.240,-0.161],-0.215),([-0.155,-0.024],-0.076)	46),([-0.831,-0.755],-0.806),([-0.198,-0.089],-0.136),([-0.128,-0.097],-0.105)	83),([-0.874,-0.781],-0.800),([-0.245,-0.137],-0.172),([-0.243,-0.140],-0.161)	36),([-0.907,-0.796],-0.881),([-0.197,-0.116],-0.182),([-0.148,-0.045],-0.119)
A3	([0.489,0.612],0.549),([0.310,0.487],0.448),([0.056,0.564],0.235),([-0.948,-0.780],-0.851),([-0.180,-0.028],-0.086),([-0.229,-0.079],-0.110)	([0.516,0.897],0.615),([0.056,0.627],0.396),([0.315,0.520],0.371),([-0.859,-0.731],-0.783),([-0.196,-0.112],-0.176),([-0.158,-0.072],-0.125)	([0.597,0.770],0.628),([0.060,0.202],0.160),([0.060,0.501],0.426),([-0.911,-0.818],-0.864),([-0.153,-0.072],-0.122),([-0.127,-0.071],-0.099)	([0.786,0.899],0.851),([0.115,0.350],0.258),([0.190,0.505],0.382),([-0.830,-0.693],-0.761),([-0.182,-0.054],-0.132),([-0.199,-0.058],-0.098)	([0.802,0.958],0.882),([0.315,0.520],0.423),([0.418,0.588],0.456),([-0.870,-0.793],-0.837),([-0.184,-0.118],-0.164),([-0.139,-0.040],-0.110)
A4	([0.732,0.926],0.778),([0.192,0.401],0.362),([0.090,0.296],0.137),([-0.937,-0.873],-0.895),([-0.133,-0.072],-0.111),([-0.178,-0.061],-0.160)	([0.825,0.945],0.866),([0.215,0.619],0.511),([0.412,0.602],0.507),([-0.843,-0.751],-0.778),([-0.166,-0.092],-0.131),([-0.232,-0.130],-0.143)	([0.554,0.875],0.705),([0.233,0.530],0.466),([0.350,0.486],0.439),([-0.874,-0.731],-0.792),([-0.190,-0.150],-0.175),([-0.266,-0.076],-0.121)	([0.641,0.744],0.719),([0.189,0.508],0.441),([0.052,0.294],0.218),([-0.819,-0.789],-0.802),([-0.224,-0.122],-0.180),([-0.232,-0.147],-0.191)	([0.584,0.803],0.732),([0.137,0.441],0.190),([0.240,0.331],0.270),([-0.892,-0.766],-0.839),([-0.203,-0.048],-0.078),([-0.125,-0.079],-0.099)
A5	([0.584,0.834],0.717),([0.160,0.629],0.591),([0.220,0.512],0.359),([-0.940,-0.879],-0.887),([-0.201,-0.056],-0.081),([-0.191,-0.109],-0.137)	([0.729,0.905],0.809),([0.319,0.534],0.481),([0.122,0.435],0.264),([-0.838,-0.765],-0.786),([-0.158,-0.053],-0.085),([-0.180,-0.108],-0.152)	([0.814,0.912],0.881),([0.442,0.525],0.486),([0.263,0.536],0.412),([-0.876,-0.736],-0.815),([-0.293,-0.212],-0.246),([-0.258,-0.160],-0.225)	([0.688,0.784],0.750),([0.190,0.530],0.455),([0.125,0.590],0.543),([-0.897,-0.801],-0.887),([-0.159,-0.130],-0.149),([-0.209,-0.103],-0.138)	([0.518,0.853],0.603),([0.341,0.567],0.449),([0.274,0.571],0.392),([-0.865,-0.748],-0.807),([-0.134,-0.069],-0.086),([-0.299,-0.152],-0.218)
A6	([0.734,0.875],0.797),([0.156,0.568],0.436),([0.257,0.389],0.353),([-0.839,-0.742],-0.777),([-0.258,-0.158],-0.189),([-0.209,-0.070],-0.099)	([0.677,0.967],0.783),([0.360,0.614],0.526),([0.132,0.462],0.419),([-0.937,-0.780],-0.871),([-0.224,-0.151],-0.189),([-0.206,-0.072],-0.157)	([0.684,0.941],0.889),([0.053,0.427],0.300),([0.160,0.522],0.506),([-0.927,-0.808],-0.859),([-0.153,-0.047],-0.093),([-0.195,-0.133],-0.167)	([0.669,0.944],0.801),([0.033,0.534],0.355),([0.110,0.566],0.452),([-0.842,-0.725],-0.806),([-0.252,-0.103],-0.158),([-0.164,-0.075],-0.126)	([0.742,0.946],0.803),([0.399,0.536],0.484),([0.343,0.422],0.398),([-0.850,-0.799],-0.821),([-0.235,-0.166],-0.185),([-0.174,-0.078],-0.102)
A7	([0.437,0.561],0.454),([0.301,0.426],0.362),([0.557,0.620],0.601),([-0.915,-0.711],-0.875),([-0.178,-0.072],-0.128),([-0.183,-0.069],-0.131)	([0.736,0.902],0.773),([0.251,0.364],0.336),([0.446,0.602],0.542),([-0.865,-0.802],-0.822),([-0.185,-0.096],-0.124),([-0.253,-0.116],-0.170)	([0.584,0.878],0.806),([0.336,0.587],0.521),([0.295,0.511],0.489),([-0.816,-0.751],-0.792),([-0.227,-0.137],-0.178),([-0.205,-0.132],-0.155)	([0.385,0.803],0.556),([0.364,0.496],0.441),([0.449,0.579],0.532),([-0.841,-0.793],-0.820),([-0.160,-0.066],-0.131),([-0.195,-0.102],-0.173)	([0.648,0.956],0.793),([0.346,0.460],0.440),([0.132,0.300],0.213),([-0.801,-0.713],-0.762),([-0.213,-0.079],-0.135),([-0.216,-0.118],-0.155)

In Table 6.2, we show representative scores for each alternative concerning every decision criterion. The scores are computed using the newly derived CBNS score function, which faithfully transforms the CBNS evaluations into interpretable scalar values. This conversion simplifies the complexity inherent in CBNS data, making it possible to quantitatively compare the performance of different instruction techniques across all evaluation aspects.

Table 6.2. Representative score matrix for each alternative under each criterion, obtained

Techniques	C1	C2	C3	C4	C5
A1	0.671625	0.663500	0.641125	0.658708	0.642167
A2	0.610250	0.670708	0.630167	0.649542	0.627708
A3	0.628250	0.637292	0.679417	0.706208	0.648292
A4	0.693792	0.637500	0.623625	0.657625	0.693208
A5	0.618542	0.675167	0.617458	0.620583	0.610542
A6	0.660375	0.622833	0.672000	0.667375	0.638000
A7	0.570000	0.634000	0.616958	0.585875	0.683417

In Table 6.3. we summarizes the decision-making outcomes by providing the aggregated scores and the resulting ranking for each alternative. The aggregated scores are obtained by averaging the criterion-wise scores, thereby reflecting the overall effectiveness of each instruction technique as perceived by the expert panel. The ranking column offers a clear basis for decision selection, with rank 1 denoting the most recommended alternative. This final step translates the sophisticated CBNS data aggregation into a straightforward and actionable decision-support outcome, facilitating the selection of the most suitable multimedia-aided college piano instruction technique.

Table 6.3. Aggregated scores and final ranking of multimedia-aided college piano instruction techniques.

Techniques	Aggregated Score	Rank
A4	0.661150	1
A3	0.659892	2
A1	0.655425	3
A6	0.652117	4
A2	0.637675	5
A5	0.628458	6
A7	0.618050	7

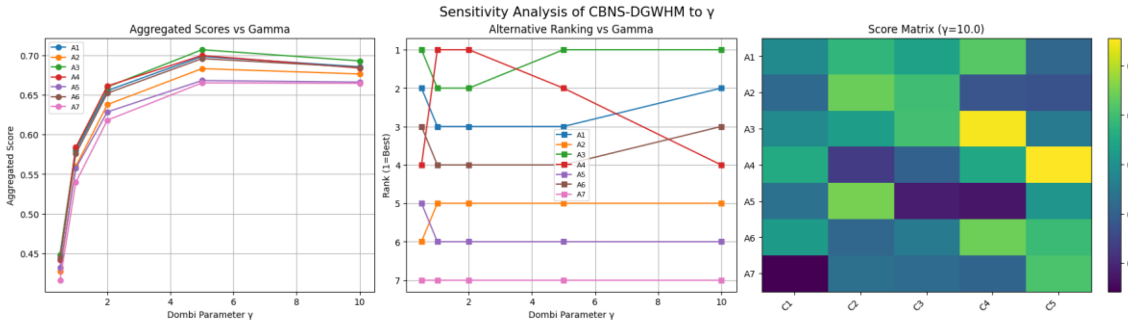

Figure 6.1. Sensitivity analysis of the CBNS-DGWHM decision-making results with respect to the Dombi parameter γ.

In Figure 6.1, we present sensitivity analysis to explores the robustness of the CBNS-DGWHM- under fluctuating Dombi parameter values (γ). In left subplot, we demonstrate that the aggregated scores of most alternatives increase relatively with the increase of γ, indicating a high degree of reliability in the fusion mechanism of the operator. This explain that extreme parameterizations may accentuate or suppress the influence of outlier expert valuations. In middle subplot, we can see that the ranking order of the top-performing alternatives is generally preserved, though some alternatives may experience minor shifts in their relative positions as γ changes. The heatmap in the tight part provided a granular view of the scoring landscape for each alternative-criterion combination at high γ, further supporting the interpretability of the aggregation process.

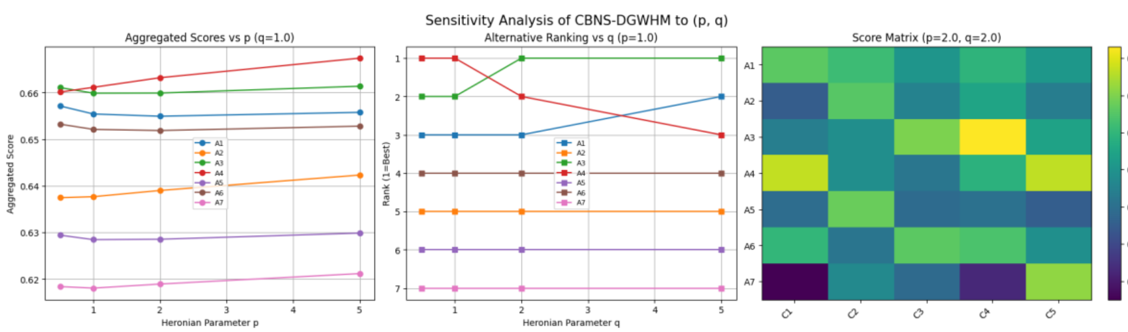

Figure 6.2**.** Sensitivity analysis of the CBNS-DGWHM operator concerning the HM parameters p and q. (Left) Aggregated alternative scores as a function of p (with $q = 1.0$). Center) Variation in alternative rankings as a function of q (with $p = 1.0$). (Right) Heatmap of the decision score matrix at $p = 2.0, q = 2.0$.

In addition, the sensitivity analysis presented in Figure 6.2 explores the adaptability of the CBNS-DGWHM operator concerning variation of HM parameters. The left subplot indicates that the aggregated scores of the alternatives exhibit a gradual and consistent change with the increase of p, which suggests that the aggregation process remained stable and does not result in abrupt shifts in preference order. The center subplot reveals that the rankings of alternatives are largely preserved across different q values, with only minor positional exchanges among close contenders. Again, the right heatmap on the provided score matrix for all alternatives and criteria at a representative parameter set to show assortment in performance crossways alternatives.

7. Conclusion

This paper introduced a novel neutrosophic framework that introduces CBNS-D-GWHM to effectively model the multifaceted uncertainty, bipolarity, and hesitancy inherent in expert evaluations of multimedia-aided college piano instruction techniques. CBNS-D-GWHM provides a mathematically robust mechanism to integrate both interval-valued and precise neutrosophic information, while the newly derived score function enables interpretable and representative ranking of alternatives. The effectiveness of the framework was demonstrated through a realistic case study, in which multiple college piano instruction techniques were evaluated under diverse pedagogical and technological criteria. The results illustrate that the CBNS-D-GWHM operators offer high flexibility and granularity in modeling expert knowledge and facilitate transparent, consistent, and information-rich decision outcomes.

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Appendix A

Table A.1. CBNSS Matrix for Expert 1's Evaluation of Multimedia-Aided Piano Instruction Techniques.

	C1	C2	C3	C4	C5
A1	([0.67,0.98],0.93),([0.33,0.44],0.35),([0.07,0.15],0.08),([-0.97,-0.56],-0.89),([-0.27,-0.08],-0.21),([-0.49,-0.01],-0.24)	([0.87,0.98],0.91),([0.0,0.26],0.25),([0.19,0.27],0.27),([-0.95,-0.79],-0.91),([-0.14,-0.09],-0.12),([-0.3,-0.09],-0.24)	([0.88,0.92],0.92),([0.08,0.32],0.26),([0.42,0.48],0.46),([-0.74,-0.71],-0.72),([-0.27,-0.0],-0.17),([-0.45,-0.21],-0.38)	([0.87,0.91],0.91),([0.05,0.37],0.18),([0.28,0.47],0.35),([-0.67,-0.53],-0.56),([-0.39,-0.31],-0.36),([-0.21,-0.04],-0.05)	([0.81,0.85],0.83),([0.41,0.49],0.47),([0.43,0.48],0.48),([-0.72,-0.62],-0.7),([-0.15,-0.02],-0.07),([-0.48,-0.46],-0.47)
A2	([0.69,0.79],0.75),([0.28,0.32],0.29),([0.13,0.27],0.14),([-0.79,-0.56],-0.57),([-0.42,-0.2],-0.21),([-0.27,-0.25],-0.25)	([0.63,0.65],0.64),([0.27,0.39],0.36),([0.02,0.16],0.05),([-0.59,-0.56],-0.57),([-0.44,-0.22],-0.36),([-0.13,-0.09],-0.1)	([0.86,0.98],0.91),([0.39,0.49],0.4),([0.23,0.37],0.31),([-0.64,-0.55],-0.64),([-0.23,-0.04],-0.14),([-0.16,-0.1],-0.13)	([0.89,0.94],0.94),([0.34,0.46],0.35),([0.05,0.46],0.18),([-0.77,-0.55],-0.56),([-0.38,-0.04],-0.06),([-0.35,-0.34],-0.34)	([0.94,0.99],0.98),([0.38,0.44],0.44),([0.19,0.28],0.27),([-0.97,-0.56],-0.87),([-0.19,-0.0],-0.1),([-0.12,-0.05],-0.11)
A3	([0.64,0.68],0.66),([0.31,0.4],0.33),([0.05,0.28],0.27),([-0.98,-0.54],-0.64),([-0.4,-0.13],-0.15),([-0.38,-0.18],-0.2)	([0.79,0.91],0.87),([0.04,0.43],0.11),([0.14,0.22],0.2),([-0.98,-0.69],-0.87),([-0.24,-0.23],-0.23),([-0.31,-0.23],-0.3)	([0.63,0.85],0.64),([0.01,0.04],0.03),([0.37,0.42],0.4),([-0.94,-0.71],-0.79),([-0.21,-0.16],-0.17),([-0.26,-0.23],-0.23)	([0.78,0.88],0.82),([0.13,0.26],0.25),([0.3,0.45],0.35),([-0.64,-0.53],-0.63),([-0.47,-0.23],-0.44),([-0.48,-0.21],-0.27)	([0.93,0.98],0.96),([0.1,0.37],0.25),([0.28,0.37],0.36),([-0.95,-0.64],-0.82),([-0.3,-0.07],-0.27),([-0.3,-0.04],-0.29)
A4	([0.77,0.98],0.8),([0.04,0.14],0.12),([0.02,0.18],0.03),([-0.83,-0.73],-0.73),([-	([0.65,0.7],0.7),([0.42,0.49],0.46),([0.12,0.37],0.2),([-0.63,-0.55],-0.6),([-0.3,-	([0.73,0.82],0.74),([0.36,0.41],0.37),([0.2,0.23],0.22),([-0.99,-0.52],-0.65),([-	([0.78,0.81],0.8),([0.04,0.37],0.35),([0.03,0.42],0.41),([-0.68,-0.66],-0.66),([-	([0.84,0.94],0.89),([0.32,0.37],0.36),([0.06,0.08],0.07),([-0.84,-0.64],-0.72),([-

	0.19,-0.12],-0.14),([-0.39,-0.17],-0.35)	0.22],-0.24),([-0.46,-0.05],-0.09)	0.18,-0.05],-0.09),([-0.34,-0.27],-0.32)	0.47,-0.01],-0.3),([-0.36,-0.22],-0.35)	0.35,-0.19],-0.21),([-0.16,-0.1],-0.1)
A5	([0.5,0.91],0.63),([0.32,0.45],0.36),([0.35,0.37],0.36),([-0.84,-0.79],-0.81),([-0.17,-0.06],-0.07),([-0.37,-0.34],-0.36)	([0.9,0.98],0.94),([0.29,0.32],0.31),([0.07,0.46],0.32),([-0.73,-0.72],-0.73),([-0.46,-0.15],-0.24),([-0.5,-0.41],-0.48)	([0.93,0.98],0.97),([0.25,0.41],0.36),([0.19,0.27],0.23),([-0.84,-0.58],-0.81),([-0.49,-0.42],-0.46),([-0.48,-0.45],-0.47)	([0.85,0.94],0.92),([0.19,0.47],0.35),([0.05,0.28],0.27),([-0.99,-0.76],-0.9),([-0.48,-0.41],-0.46),([-0.46,-0.16],-0.2)	([0.57,0.97],0.62),([0.22,0.3],0.23),([0.21,0.49],0.25),([-0.78,-0.68],-0.72),([-0.26,-0.19],-0.25),([-0.47,-0.4],-0.45)
A6	([0.55,0.87],0.65),([0.34,0.43],0.39),([0.35,0.43],0.42),([-0.76,-0.64],-0.73),([-0.49,-0.43],-0.47),([-0.49,-0.17],-0.25)	([0.65,0.91],0.88),([0.29,0.47],0.4),([0.3,0.42],0.4),([-0.96,-0.78],-0.87),([-0.4,-0.29],-0.34),([-0.13,-0.07],-0.1)	([0.58,0.77],0.59),([0.18,0.38],0.31),([0.29,0.31],0.29),([-0.83,-0.62],-0.71),([-0.31,-0.04],-0.06),([-0.25,-0.2],-0.22)	([0.6,0.99],0.85),([0.01,0.49],0.34),([0.02,0.45],0.22),([-0.76,-0.5],-0.6),([-0.47,-0.21],-0.32),([-0.13,-0.06],-0.07)	([0.9,0.95],0.91),([0.3,0.34],0.31),([0.09,0.12],0.11),([-0.92,-0.8],-0.84),([-0.43,-0.38],-0.43),([-0.45,-0.24],-0.28)
A7	([0.6,0.64],0.63),([0.13,0.2],0.19),([0.42,0.49],0.44),([-0.71,-0.53],-0.69),([-0.19,-0.15],-0.18),([-0.16,-0.07],-0.12)	([0.59,0.89],0.72),([0.13,0.15],0.15),([0.29,0.34],0.34),([-0.58,-0.5],-0.51),([-0.15,-0.09],-0.13),([-0.49,-0.03],-0.31)	([0.67,0.89],0.69),([0.21,0.37],0.32),([0.42,0.46],0.42),([-0.57,-0.51],-0.55),([-0.43,-0.39],-0.4),([-0.46,-0.44],-0.46)	([0.52,0.66],0.66),([0.43,0.49],0.49),([0.21,0.39],0.34),([-0.88,-0.8],-0.84),([-0.32,-0.2],-0.28),([-0.28,-0.25],-0.26)	([0.78,0.81],0.8),([0.39,0.49],0.43),([0.03,0.17],0.09),([-0.64,-0.53],-0.56),([-0.43,-0.33],-0.43),([-0.36,-0.18],-0.31)

Table A.2. CBNSS Matrix for Expert 2's Evaluation of Multimedia-Aided Piano Instruction Techniques.

	C1	C2	C3	C4	C5
A1	([0.82,0.98],0.85),([0.23,0.38],0.28),([0.05,0.26],0.08),([-0.76,-0.69],-0.7),([-0.4,-0.24],-0.31),([-0.49,-0.33],-0.38)	([0.78,0.9],0.82),([0.4,0.48],0.47),([0.07,0.21],0.09),([-0.89,-0.6],-0.64),([-0.49,-0.21],-0.45),([-0.21,-0.02],-0.13)	([0.51,0.57],0.55),([0.36,0.39],0.37),([0.29,0.35],0.31),([-0.96,-0.84],-0.85),([-0.23,-0.03],-0.23),([-0.18,-0.1],-0.17)	([0.61,0.81],0.75),([0.1,0.5],0.22),([0.42,0.43],0.43),([-0.68,-0.51],-0.51),([-0.43,-0.18],-0.19),([-0.15,-0.14],-0.15)	([0.81,0.83],0.83),([0.38,0.44],0.42),([0.22,0.39],0.36),([-0.63,-0.58],-0.59),([-0.24,-0.1],-0.14),([-0.4,-0.05],-0.11)
A2	([0.86,0.93],0.88),([0.04,0.29],0.29),([0.26,0.44],0.37),([-0.81,-0.76],-0.8),([-0.39,-0.24],-0.37),([-0.25,-0.1],-0.23)	([0.86,0.89],0.88),([0.1,0.39],0.22),([0.32,0.44],0.38),([-0.69,-0.58],-0.59),([-0.4,-0.26],-0.38),([-0.43,-0.04],-0.2)	([0.59,0.61],0.6),([0.08,0.33],0.18),([0.19,0.47],0.34),([-0.63,-0.58],-0.61),([-0.4,-0.24],-0.35),([-0.28,-0.2],-0.2)	([0.89,0.98],0.9),([0.14,0.44],0.23),([0.02,0.31],0.18),([-0.9,-0.84],-0.85),([-0.47,-0.14],-0.36),([-0.37,-0.1],-0.2)	([0.81,0.94],0.83),([0.28,0.45],0.42),([0.07,0.45],0.39),([-0.61,-0.6],-0.6),([-0.19,-0.16],-0.19),([-0.37,-0.09],-0.29)
A3	([0.61,0.68],0.68),([0.08,0.21],0.19),([0.08,0.46],0.39),([-0.96,-0.71],-0.9),([-0.34,-0.01],-0.28),([-0.46,-0.27],-0.33)	([0.73,0.9],0.8),([0.01,0.44],0.33),([0.4,0.47],0.47),([-0.69,-0.51],-0.52),([-0.23,-0.19],-0.21),([-0.28,-0.11],-0.26)	([0.84,0.87],0.84),([0.39,0.45],0.44),([0.4,0.48],0.48),([-0.81,-0.79],-0.8),([-0.4,-0.18],-0.36),([-0.26,-0.19],-0.24)	([0.9,0.97],0.95),([0.1,0.12],0.11),([0.39,0.4],0.39),([-0.61,-0.55],-0.57),([-0.14,-0.03],-0.04),([-0.12,-0.07],-0.07)	([0.78,1.0],0.92),([0.28,0.34],0.34),([0.29,0.41],0.3),([-0.65,-0.63],-0.63),([-0.21,-0.09],-0.13),([-0.13,-0.08],-0.13)
A4	([0.92,0.97],0.93),([0.26,0.33],0.3),([0.35,0.39],0.36),([-0.85,-0.7],-0.78),([-0.31,-0.23],-0.27),([-0.46,-0.18],-0.45)	([0.94,0.99],0.96),([0.06,0.47],0.22),([0.39,0.45],0.42),([-0.73,-0.63],-0.64),([-0.48,-0.31],-0.42),([-0.19,-0.18],-0.19)	([0.51,0.96],0.9),([0.05,0.31],0.21),([0.12,0.34],0.25),([-0.71,-0.53],-0.59),([-0.42,-0.41],-0.42),([-0.45,-0.04],-0.08)	([0.85,0.92],0.92),([0.19,0.42],0.29),([0.11,0.16],0.13),([-0.84,-0.74],-0.77),([-0.39,-0.29],-0.37),([-0.48,-0.46],-0.46)	([0.64,0.9],0.84),([0.03,0.31],0.04),([0.16,0.37],0.16),([-0.98,-0.56],-0.84),([-0.13,-0.0],-0.07),([-0.22,-0.18],-0.19)
A5	([0.65,0.71],0.67),([0.03,0.5],0.47),([0.15,0.44],0.39),([-0.89,-0.62],-0.63),([-0.21,-0.08],-0.19),([-0.32,-0.18],-0.22)	([0.83,0.91],0.83),([0.1,0.29],0.23),([0.06,0.15],0.08),([-0.78,-0.66],-0.66),([-0.15,-0.09],-0.14),([-0.28,-0.1],-0.21)	([0.69,0.81],0.78),([0.23,0.25],0.24),([0.07,0.39],0.25),([-0.96,-0.68],-0.83),([-0.41,-0.24],-0.3),([-0.39,-0.24],-0.25)	([0.85,0.87],0.86),([0.19,0.46],0.33),([0.43,0.47],0.44),([-0.72,-0.67],-0.71),([-0.12,-0.08],-0.11),([-0.14,-0.05],-0.12)	([0.81,0.9],0.87),([0.12,0.5],0.22),([0.19,0.34],0.33),([-0.93,-0.59],-0.88),([-0.24,-0.18],-0.21),([-0.47,-0.15],-0.17)
A6	([0.92,0.94],0.93),([0.03,0.38],0.21),([0.26,0.46],0.33),([-0.94,-0.59],-0.63),([-0.42,-0.34],-0.39),([-0.44,-0.08],-0.15)	([0.54,1.0],0.8),([0.23,0.39],0.24),([0.03,0.38],0.3),([-0.91,-0.54],-0.74),([-0.42,-0.33],-0.37),([-0.49,-0.25],-0.38)	([0.83,0.87],0.86),([0.01,0.15],0.08),([0.27,0.29],0.28),([-0.78,-0.61],-0.71),([-0.37,-0.08],-0.3),([-0.46,-0.42],-0.43)	([0.85,0.91],0.9),([0.03,0.49],0.13),([0.4,0.49],0.42),([-0.55,-0.53],-0.54),([-0.35,-0.08],-0.19),([-0.24,-0.19],-0.21)	([0.69,0.89],0.86),([0.18,0.28],0.23),([0.44,0.47],0.47),([-0.6,-0.57],-0.59),([-0.42,-0.4],-0.4),([-0.25,-0.15],-0.17)
A7	([0.7,0.82],0.71),([0.21,0.3],0.29),([0.34,0.43],0.4),([-0.9,-0.54],-0.78),([-0.35,-0.16],-0.32),([-0.16,-0.06],-0.08)	([0.68,0.86],0.81),([0.07,0.16],0.13),([0.39,0.49],0.39),([-0.83,-0.73],-0.81),([-0.26,-0.15],-0.23),([-0.49,-0.41],-0.44)	([0.86,0.96],0.95),([0.12,0.35],0.29),([0.16,0.2],0.19),([-0.59,-0.57],-0.58),([-0.14,-0.07],-0.09),([-0.43,-0.19],-0.23)	([0.54,0.94],0.69),([0.12,0.18],0.14),([0.4,0.5],0.47),([-0.61,-0.52],-0.57),([-0.25,-0.11],-0.24),([-0.47,-0.27],-0.43)	([0.85,0.98],0.94),([0.34,0.44],0.38),([0.06,0.08],0.06),([-0.59,-0.53],-0.56),([-0.2,-0.1],-0.19),([-0.46,-0.39],-0.42)

Table A.3. CBNSS Matrix for Expert 3's Evaluation of Multimedia-Aided Piano Instruction Techniques.

	C1	C2	C3	C4	C5
A1	([0.62,0.93],0.86),([0.14,0.31],0.15),([0.01,0.05],0.03),([-0.82,-0.66],-0.8),([-0.27,-0.18],-0.25),([-0.24,-0.03],-0.03)	([0.85,0.88],0.87),([0.44,0.47],0.46),([0.11,0.15],0.12),([-0.92,-0.7],-0.88),([-0.37,-0.06],-0.36),([-0.34,-0.25],-0.27)	([0.93,0.94],0.93),([0.09,0.1],0.09),([0.29,0.48],0.34),([-0.89,-0.53],-0.78),([-0.48,-0.03],-0.12),([-0.37,-0.32],-0.34)	([0.95,1.0],0.95),([0.24,0.46],0.31),([0.23,0.4],0.27),([-0.96,-0.61],-0.83),([-0.45,-0.08],-0.42),([-0.2,-0.06],-0.07)	([0.57,0.72],0.67),([0.33,0.42],0.37),([0.17,0.31],0.21),([-0.79,-0.56],-0.6),([-0.16,-0.01],-0.15),([-0.2,-0.13],-0.14)
A2	([0.56,0.78],0.65),([0.28,0.44],0.3),([0.25,0.26],0.26),([-0.85,-0.66],-0.72),([-0.47,-0.3],-0.33),([-0.49,-0.44],-0.48)	([0.8,0.9],0.8),([0.38,0.48],0.39),([0.26,0.47],0.35),([-0.91,-0.86],-0.86),([-0.4,-0.37],-0.39),([-0.3,-0.02],-0.16)	([0.64,0.86],0.76),([0.32,0.38],0.34),([0.19,0.24],0.24),([-0.92,-0.63],-0.78),([-0.43,-0.2],-0.24),([-0.23,-0.2],-0.21)	([0.75,0.83],0.76),([0.18,0.33],0.3),([0.27,0.39],0.32),([-0.59,-0.54],-0.55),([-0.12,-0.02],-0.03),([-0.42,-0.03],-0.11)	([0.71,0.9],0.72),([0.3,0.48],0.32),([0.29,0.42],0.38),([-0.97,-0.72],-0.95),([-0.44,-0.11],-0.42),([-0.31,-0.1],-0.31)
A3	([0.58,0.63],0.6),([0.29,0.44],0.41),([0.01,0.47],0.05),([-0.98,-0.72],-0.75),([-0.23,-0.03],-0.12),([-0.23,-0.04],-0.07)	([0.52,0.6],0.53),([0.44,0.5],0.44),([0.0,0.48],0.15),([-0.71,-0.53],-0.58),([-0.33,-0.16],-0.3),([-0.31,-0.1],-0.17)	([0.52,0.83],0.63),([0.43,0.48],0.44),([0.37,0.49],0.46),([-0.9,-0.81],-0.88),([-0.27,-0.15],-0.17),([-0.18,-0.02],-0.11)	([0.86,0.91],0.91),([0.04,0.23],0.13),([0.08,0.37],0.26),([-0.84,-0.51],-0.59),([-0.26,-0.04],-0.22),([-0.45,-0.06],-0.25)	([0.93,0.98],0.94),([0.16,0.27],0.16),([0.16,0.43],0.18),([-0.7,-0.65],-0.65),([-0.43,-0.38],-0.42),([-0.25,-0.06],-0.1)
A4	([0.7,0.9],0.74),([0.24,0.43],0.39),([0.04,0.08],0.07),([-0.89,-0.82],-0.86),([-0.17,-0.12],-0.14),([-0.13,-0.08],-0.09)	([0.94,0.95],0.95),([0.3,0.39],0.35),([0.39,0.42],0.39),([-0.69,-0.53],-0.54),([-0.14,-0.04],-0.07),([-0.31,-0.11],-0.17)	([0.8,0.83],0.82),([0.28,0.38],0.33),([0.26,0.31],0.27),([-0.67,-0.52],-0.61),([-0.42,-0.37],-0.39),([-0.49,-0.17],-0.33)	([0.68,0.74],0.71),([0.17,0.2],0.17),([0.01,0.08],0.05),([-0.57,-0.53],-0.56),([-0.13,-0.07],-0.09),([-0.38,-0.18],-0.25)	([0.78,0.87],0.87),([0.09,0.15],0.13),([0.28,0.45],0.38),([-0.71,-0.59],-0.62),([-0.48,-0.01],-0.1),([-0.2,-0.13],-0.15)
A5	([0.6,1.0],0.9),([0.44,0.48],0.46),([0.09,0.37],0.19),([-0.97,-0.95],-0.96),([-0.4,-0.21],-0.21),([-0.17,-0.04],-0.16)	([0.6,0.87],0.75),([0.18,0.39],0.34),([0.05,0.35],0.31),([-0.55,-0.54],-0.55),([-0.13,-0.07],-0.11),([-0.17,-0.03],-0.08)	([0.94,0.96],0.95),([0.31,0.39],0.32),([0.19,0.42],0.21),([-0.69,-0.5],-0.52),([-0.45,-0.39],-0.44),([-0.22,-0.09],-0.18)	([0.57,0.61],0.58),([0.35,0.42],0.36),([0.03,0.5],0.36),([-0.97,-0.64],-0.95),([-0.13,-0.09],-0.11),([-0.45,-0.39],-0.39)	([0.56,0.72],0.7),([0.42,0.45],0.44),([0.17,0.32],0.21),([-0.8,-0.61],-0.64),([-0.14,-0.1],-0.11),([-0.45,-0.35],-0.37)
A6	([0.89,0.96],0.93),([0.2,0.42],0.34),([0.22,0.47],0.4),([-0.59,-0.55],-0.58),([-0.39,-0.15],-0.22),([-0.15,-0.11],-0.11)	([0.86,0.97],0.9),([0.36,0.43],0.39),([0.37,0.39],0.37),([-0.75,-0.74],-0.75),([-0.38,-0.23],-0.35),([-0.35,-0.1],-0.33)	([0.6,0.75],0.65),([0.03,0.28],0.2),([0.03,0.41],0.41),([-0.89,-0.67],-0.74),([-0.17,-0.06],-0.11),([-0.3,-0.2],-0.23)	([0.87,0.95],0.94),([0.14,0.37],0.31),([0.24,0.36],0.33),([-0.94,-0.55],-0.94),([-0.33,-0.26],-0.3),([-0.36,-0.21],-0.29)	([0.91,0.94],0.91),([0.34,0.42],0.35),([0.38,0.47],0.39),([-0.65,-0.55],-0.58),([-0.43,-0.19],-0.23),([-0.13,-0.01],-0.1)
A7	([0.58,0.69],0.6),([0.44,0.46],0.45),([0.38,0.44],0.44),([-0.89,-0.55],-0.8),([-0.33,-0.16],-0.22),([-0.36,-0.14],-0.29)	([0.77,0.93],0.86),([0.28,0.39],0.34),([0.17,0.46],0.35),([-0.77,-0.64],-0.67),([-0.49,-0.32],-0.35),([-0.36,-0.21],-0.25)	([0.71,0.74],0.73),([0.25,0.49],0.44),([0.08,0.37],0.35),([-0.91,-0.69],-0.86),([-0.46,-0.25],-0.38),([-0.21,-0.13],-0.2)	([0.51,0.64],0.52),([0.34,0.37],0.36),([0.24,0.3],0.27),([-0.99,-0.86],-0.88),([-0.13,-0.02],-0.12),([-0.14,-0.07],-0.11)	([0.73,0.96],0.77),([0.33,0.43],0.38),([0.27,0.39],0.27),([-0.87,-0.66],-0.82),([-0.24,-0.03],-0.09),([-0.26,-0.06],-0.06)

Table 4: CBNSS Matrix for Expert 4's Evaluation of Multimedia-Aided Piano Instruction Techniques.

	C1	C2	C3	C4	C5
A1	([0.54,0.73],0.68),([0.36,0.4],0.36),([0.25,0.44],0.39),([-0.72,-	([0.66,0.97],0.74),([0.01,0.34],0.23),([0.43,0.47],0.44),([-	([0.82,0.88],0.83),([0.21,0.45],0.27),([0.21,0.32],0.23),([-	([0.61,0.81],0.66),([0.05,0.23],0.12),([0.22,0.41],0.31),([-	([0.9,0.94],0.91),([0.07,0.09],0.07),([0.29,0.43],0.43),([-0.85,-

	0.67],-0.71),([-0.36,-0.08],-0.09),([-0.29,-0.28],-0.29)	0.58,-0.57],-0.57),([-0.34,-0.24],-0.29),([-0.22,-0.0],-0.01)	0.78,-0.55],-0.7),([-0.37,-0.3],-0.34),([-0.23,-0.04],-0.09)	0.57,-0.52],-0.57),([-0.22,-0.1],-0.2),([-0.26,-0.23],-0.25)	0.82],-0.84),([-0.34,-0.23],-0.26),([-0.24,-0.09],-0.22)
A2	([0.65,0.71],0.69),([0.14,0.46],0.14),([0.45,0.47],0.46),([-0.8,-0.58],-0.72),([-0.16,-0.06],-0.08),([-0.35,-0.2],-0.34)	([0.77,0.86],0.78),([0.05,0.13],0.11),([0.1,0.21],0.16),([-0.62,-0.55],-0.61),([-0.3,-0.26],-0.28),([-0.17,-0.04],-0.11)	([0.79,0.92],0.81),([0.16,0.45],0.41),([0.21,0.49],0.27),([-0.71,-0.67],-0.68),([-0.24,-0.16],-0.22),([-0.24,-0.21],-0.22)	([0.94,0.97],0.94),([0.22,0.28],0.24),([0.27,0.34],0.33),([-0.91,-0.69],-0.78),([-0.5,-0.49],-0.49),([-0.42,-0.39],-0.41)	([0.72,0.74],0.73),([0.23,0.49],0.46),([0.17,0.19],0.17),([-0.99,-0.79],-0.8),([-0.44,-0.42],-0.43),([-0.21,-0.11],-0.12)
A3	([0.76,0.88],0.82),([0.29,0.31],0.29),([0.26,0.4],0.27),([-0.75,-0.6],-0.7),([-0.24,-0.04],-0.08),([-0.4,-0.07],-0.17)	([0.57,0.98],0.59),([0.38,0.46],0.44),([0.21,0.34],0.24),([-0.71,-0.59],-0.68),([-0.48,-0.23],-0.44),([-0.19,-0.1],-0.16)	([0.81,0.91],0.84),([0.14,0.22],0.21),([0.01,0.17],0.12),([-0.71,-0.51],-0.6),([-0.17,-0.04],-0.16),([-0.2,-0.06],-0.14)	([0.92,0.96],0.93),([0.04,0.33],0.15),([0.05,0.21],0.12),([-0.77,-0.53],-0.67),([-0.31,-0.09],-0.11),([-0.12,-0.1],-0.11)	([0.77,0.96],0.91),([0.44,0.46],0.46),([0.43,0.46],0.44),([-0.83,-0.71],-0.79),([-0.29,-0.24],-0.28),([-0.29,-0.13],-0.26)
A4	([0.85,0.93],0.9),([0.2,0.25],0.22),([0.15,0.31],0.25),([-0.96,-0.85],-0.87),([-0.27,-0.07],-0.25),([-0.12,-0.01],-0.07)	([0.84,0.93],0.88),([0.09,0.45],0.42),([0.4,0.49],0.45),([-0.89,-0.71],-0.8),([-0.13,-0.03],-0.12),([-0.49,-0.45],-0.46)	([0.71,0.96],0.78),([0.25,0.35],0.34),([0.44,0.46],0.46),([-0.81,-0.79],-0.79),([-0.22,-0.02],-0.22),([-0.39,-0.05],-0.06)	([0.77,0.85],0.78),([0.37,0.49],0.42),([0.37,0.38],0.37),([-0.71,-0.69],-0.7),([-0.4,-0.37],-0.4),([-0.27,-0.08],-0.19)	([0.53,0.7],0.7),([0.12,0.49],0.45),([0.25,0.34],0.31),([-0.74,-0.7],-0.72),([-0.33,-0.12],-0.2),([-0.31,-0.18],-0.28)
A5	([0.88,0.93],0.89),([0.26,0.41],0.4),([0.08,0.24],0.12),([-0.85,-0.83],-0.84),([-0.49,-0.08],-0.13),([-0.4,-0.19],-0.22)	([0.87,0.93],0.92),([0.35,0.5],0.43),([0.09,0.31],0.12),([-0.87,-0.57],-0.66),([-0.28,-0.1],-0.14),([-0.2,-0.11],-0.17)	([0.83,0.89],0.85),([0.37,0.41],0.39),([0.39,0.41],0.4),([-0.68,-0.58],-0.65),([-0.46,-0.34],-0.37),([-0.5,-0.26],-0.49)	([0.84,0.86],0.85),([0.04,0.21],0.19),([0.42,0.48],0.46),([-0.69,-0.61],-0.68),([-0.27,-0.26],-0.26),([-0.26,-0.02],-0.24)	([0.68,0.7],0.68),([0.21,0.37],0.34),([0.1,0.49],0.21),([-0.59,-0.52],-0.53),([-0.3,-0.04],-0.05),([-0.45,-0.09],-0.4)
A6	([0.58,0.92],0.79),([0.21,0.36],0.23),([0.06,0.1],0.09),([-0.67,-0.58],-0.61),([-0.33,-0.09],-0.1),([-0.21,-0.17],-0.21)	([0.89,0.99],0.9),([0.12,0.49],0.45),([0.06,0.16],0.14),([-0.92,-0.54],-0.73),([-0.25,-0.2],-0.2),([-0.33,-0.12],-0.24)	([0.91,0.99],0.98),([0.3,0.42],0.36),([0.4,0.43],0.4),([-0.96,-0.84],-0.86),([-0.2,-0.18],-0.19),([-0.28,-0.0],-0.25)	([0.74,0.79],0.77),([0.01,0.22],0.19),([0.13,0.31],0.25),([-0.74,-0.72],-0.73),([-0.45,-0.19],-0.28),([-0.37,-0.1],-0.3)	([0.71,0.99],0.87),([0.22,0.46],0.43),([0.26,0.36],0.36),([-0.87,-0.79],-0.84),([-0.21,-0.12],-0.14),([-0.31,-0.15],-0.19)
A7	([0.53,0.67],0.53),([0.12,0.22],0.13),([0.41,0.44],0.44),([-0.88,-0.59],-0.84),([-0.33,-0.07],-0.19),([-0.48,-0.24],-0.38)	([0.94,0.98],0.94),([0.29,0.33],0.32),([0.4,0.45],0.41),([-0.93,-0.91],-0.91),([-0.29,-0.13],-0.16),([-0.23,-0.02],-0.12)	([0.54,0.96],0.91),([0.3,0.47],0.38),([0.4,0.46],0.44),([-0.77,-0.65],-0.69),([-0.38,-0.21],-0.28),([-0.21,-0.08],-0.12)	([0.64,0.93],0.84),([0.2,0.44],0.37),([0.37,0.48],0.41),([-0.59,-0.55],-0.58),([-0.38,-0.15],-0.28),([-0.36,-0.14],-0.33)	([0.76,0.99],0.9),([0.09,0.14],0.14),([0.38,0.44],0.38),([-0.62,-0.51],-0.57),([-0.48,-0.05],-0.18),([-0.33,-0.16],-0.21)

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