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A Bipolar Pythagorean Neutrosophic Algebraic Lattice Model with Hybrid Entropy Weighting for Credit Risk Evaluation for Agricultural SMEs Supply Chain Finance

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Abstract-In agricultural supply chain finance (SCF), small and medium-sized enterprises (SMEs) often face severe credit risk due to unstable production, market volatility, and limited access to formal financial records. Traditional risk assessment models fail to handle uncertainty, contradiction, and bipolarity in real-world data, especially for agriculture. To address this gap, we propose a new mathematical model based on bipolar Pythagorean neutrosophic theory. This model introduces an algebraic lattice structure to represent the complex relationships between risk criteria and integrates a novel hybrid entropy weighting method. The framework allows for the evaluation of both positive and negative aspects of credit risk using bipolar neutrosophic sets, while preserving mathematical rigor through lattice-based operations. Several new formulas for entropy, aggregation, and distance are developed to support accurate and flexible assessment. This model is especially suitable for agricultural SMEs, where data is often vague, contradictory, or incomplete. A numerical case study is provided to show the practical performance of the proposed model.

Keywords-Credit Risk, Agricultural SMEs, Supply Chain Finance, Bipolar Pythagorean Neutrosophic Set, Entropy, Algebraic Lattice, Uncertainty Modeling, Fuzziness, Neutrosophic Logic.

1. Introduction

Agriculture is a cornerstone of global economies, particularly in developing countries, where it ensures food security, supports livelihoods, and drives rural development [1]. Despite its significance, agricultural SMEs face persistent challenges in accessing finance, including seasonal income variability, exposure to weather-related risks, limited collateral, and unstable supply chains [2]. These barriers restrict their ability to invest in productivity-enhancing technologies, expand operations, or mitigate risks. SCF has emerged as an innovative solution to address these challenges by integrating suppliers, producers, and buyers into a coordinated financing framework [3]. By leveraging relationships within the supply chain, SCF provides liquidity and reduces financial barriers for agricultural SMEs. However, the effectiveness of SCF is hindered by credit risk—the likelihood that a borrower will default or fail to meet repayment obligations [4].

Evaluating credit risk in agricultural SMEs is inherently complex due to several factors. First, financial and operational data in rural markets are often incomplete, inconsistent, or unreliable, making traditional risk assessment methods difficult to apply [5]. Second, agricultural SMEs may exhibit conflicting characteristics, such as high production potential alongside poor repayment histories, which complicates their risk profiles [6]. Third, external factors such as climate variability, market volatility, and geopolitical disruptions introduce additional layers of uncertainty, ambiguity, and contradiction that conventional models struggle to address [7]. These challenges demand a robust and flexible approach to credit risk evaluation tailored to the unique dynamics of agricultural finance.

Traditional credit risk assessment methods, such as logistic regression, credit scoring models, and machine learning techniques, often fall short in this context [8]. These approaches typically rely on structured, complete data and assume binary or probabilistic outcomes, which do not adequately capture the nuances of agricultural markets. For instance, they fail to account for bipolarity (simultaneous positive and negative risk attributes), degrees of indeterminacy, or the subjective judgment of financial experts, all of which are critical in agricultural lending decisions [9]. To address these limitations, this study proposes a novel credit risk evaluation model based on Bipolar Pythagorean Neutrosophic Sets (BPNNS). BPNNS extends fuzzy and intuitionistic fuzzy set theories by incorporating truth, indeterminacy, and falsity in both positive and negative dimensions, enabling a more comprehensive representation of uncertain and conflicting financial data [10].

The proposed model leverages a neutrosophic algebraic lattice, a mathematical structure that facilitates the aggregation and classification of risk profiles through stable and closed operations. Unlike ranking-based methods, such as TOPSIS or SWARA, which may lead to information loss through forced normalization, the lattice-based approach preserves the richness of the data and avoids reliance on rigid ranking schemes [11]. To determine the importance of each risk factor, this study introduces a hybrid entropy weighting scheme that integrates bipolar entropy with Pythagorean neutrosophic measures. This method accurately captures the inherent uncertainty of each criterion and its relative influence in decision-making [12].

The contributions of this paper are as follows:

1. A novel credit risk evaluation model for agricultural SMEs based on Bipolar Pythagorean Neutrosophic Sets and a neutrosophic algebraic lattice.
2. A new hybrid entropy-based weighting formula designed specifically for bipolar neutrosophic criteria.

3. Original mathematical definitions for distance, similarity, and aggregation operators within the BPNNS framework.
4. A practical numerical example demonstrating the model's applicability in real-world agricultural lending scenarios.

This framework provides a mathematically rigorous, flexible, and interpretable tool for assessing credit risk in environments characterized by uncertainty and incomplete data. By addressing the unique challenges of agricultural SMEs, it offers significant potential to enhance the effectiveness of supply chain finance and support sustainable rural development.

2. Preliminaries

In this section, we define the mathematical foundations required for our model. These definitions extend classical fuzzy and neutrosophic theories and are adapted specifically to support bipolar Pythagorean neutrosophic logic within an algebraic lattice framework.

2.1 Pythagorean Neutrosophic Set

Let X be a universal set. A Pythagorean neutrosophic set (PNS) on X is defined as $A = \{\langle x, T(x), I(x), F(x)\rangle : x \in X\}$

Where:
$T(x) \in [0,1]$: degree of truth,

$I(x) \in [0,1]$: degree of indeterminacy,

$F(x) \in [0,1]$: degree of falsity,
subject to the constraint:

$$T^2(x) + I^2(x) + F^2(x) \leq 1$$

This condition extends the classic fuzzy and intuitionistic models by permitting a greater tolerance for indeterminacy, which is crucial in credit assessment contexts.

2.2 Bipolar Pythagorean Neutrosophic Set (BPNS)

Let X be a universal set. BPNS on X is defined as $B = \{\langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)\rangle : x \in X\}$

Where:
$T^+, I^+, F^+ : X \to [0,1]$ represent positive truth, indeterminacy, and falsity,

$T^-, I^-, F^- : X \to [0,1]$ represent negative truth, indeterminacy, and falsity,
subject to the conditions:

$$(T^{+}(x))^{2}+(I^{+}(x))^{2}+(F^{+}(x))^{2}\leq 1,(T^{-}(x))^{2}+(I^{-}(x))^{2}+(F^{-}(x))^{2}\leq 1$$

This structure enables the modeling of both favorable and unfavorable attributes of an entity simultaneously, a necessity when assessing agricultural SMEs that exhibit dual characteristics.

2.3 Neutrosophic Bipolar Aggregation Operator

Given a set of BPNS values $\{B_1, B_2, \ldots, B_n\}$, where each $B_j = \langle T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^- \rangle$. We define the Bipolar Neutrosophic Weighted Averaging (BNWA) operator as:

$$\mathrm{BNWA}(B_1, \ldots, B_n; w_j) = \langle T^+, I^+, F^+, T^-, I^-, F^- \rangle$$

with components:

$$T^+ = \sum_{j=1}^{n} w_j T_j^+, \quad I^+ = \sum_{j=1}^{n} w_j I_j^+, \quad F^+ = \sum_{j=1}^{n} w_j F_j^+$$

$$T^- = \sum_{j=1}^{n} w_j T_j, \quad I^- = \sum_{j=1}^{n} w_j I_j^-, \quad F^- = \sum_{j=1}^{n} w_j F_j^-$$

Where $w_j \geq 0$ and $\sum\ w_j = 1$. This operator aggregates expert judgments or source assessments under uncertainty, preserving bipolar structure.

2.4 Bipolar Entropy Measure

Let $B = \langle T^+, I^+, F^+, T^-, I^-, F^- \rangle$. We define the Bipolar Pythagorean Neutrosophic Entropy as:

$$E(B) = 1 - \frac{1}{6}(T^+\log\ T^+ + I^+\log\ I^+ + F^+\log\ F^+ + T^-\log\ T^- + I^-\log\ I^- + F^-\log\ F^-)$$

Where zero-log-zero terms are interpreted as zero (i.e., $0\log\ 0 = 0$). This entropy reflects both the uncertainty and the divergence of a risk factor in terms of its bipolar neutrosophic composition.

2.5 Algebraic Lattice Structure

Let $\mathcal{L}$ be a set of bipolar Pythagorean neutrosophic evaluations. Define two binary operations:

1. Join (supremum): $B_i \sqcup B_j$: takes component-wise maximum for truth, and minimum for falsity.
2. Meet (infimum): $B_i \sqcap B_j$: takes component-wise minimum for truth, and maximum for falsity.

For any $B_i, B_j \in \mathcal{L}$, these operations satisfy:

Commutativity: $B_i \sqcup B_j = B_j \sqcup B_i$

Associativity: $(B_i \sqcup B_j) \sqcup B_k = B_i \sqcup (B_j \sqcup B_k)$

Absorption: $B_i \sqcup (B_i \sqcap B_j) = B_i$, and $B_i \sqcap (B_i \sqcup B_j) = B_i$

Thus, $(\mathcal{L},\sqcup,\sqcap)$ forms a complete distributive lattice, which allows for classification and comparison of credit risk profiles.

3. The Proposed Model

In this section, we construct the full Bipolar Pythagorean Neutrosophic Algebraic Lattice Model for credit risk evaluation in agricultural supply chain finance. The model operates in seven well-defined mathematical stages, ensuring logical completeness, stability under uncertainty, and algebraic consistency.

3.1 Problem Description

Let:
$\mathcal{A} = \{A_1, A_2, \dots, A_m\}$: the set of agricultural SMEs (alternatives).

$\mathcal{C} = \{C_1, C_2, \dots, C_n\}$: the set of credit risk criteria (e.g., repayment history, collateral quality, seasonal exposure, buyer dependency).

$\mathcal{E} = \{E_1, E_2, \dots, E_k\}$: the group of credit officers or domain experts.

Each expert E_l provides an evaluation of each SME A_i under criterion C_j, using a Bipolar Pythagorean

Neutrosophic Number (BPNN):

$$x_{ijl}^{(B)} = \langle T_{ijl}^{+}, I_{ijl}^{+}, F_{ijl}^{+}, T_{ijl}^{-}, I_{ijl}^{-}, F_{ijl}^{-} \rangle$$

Let each expert E_l be assigned a weight $w_l \in [0,1]$, with $\sum_{l=1}^{k} w_l = 1$.
The aggregated evaluation for alternative A_i on criterion C_j is computed using the BNWA operator:

$$\bar{x}_{ij}^{(B)} = \sum_{l=1}^{k} w_l \cdot x_{ijl}^{(B)}$$

This yields the aggregated decision matrix:

$$\mathbb{X} = \left[\bar{x}_{ij}^{(B)}\right]_{m \times n}$$

3.2 Hybrid Entropy-Based Weighting of Criteria

For each criterion C_j, we define its Bipolar Neutrosophic Entropy E_j using the average BPNNs across alternatives:

Let $\bar{x}_{\cdot j}^{(B)} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_{ij}^{(B)} = \langle \bar{T}_j^+, \bar{I}_j^+, \bar{F}_j^+, \bar{T}_j^-, \bar{I}_j^-, \bar{F}_j^- \rangle$

Then $E_j = 1 - \frac{1}{6}\sum_{\gamma\in\Theta} \gamma_j \log \gamma_j$ where $\Theta = \{\bar{T}_j^+, \bar{I}_j^+, \bar{F}_j^+, \bar{T}_j, \bar{I}_j^-, \bar{F}_j^-\}$

We define the objective weight for criterion C_j as:

$$w_j = \frac{1 - E_j}{\sum_{s=1}^{n} (1 - E_s)}$$

This ensures that criteria with less entropy (more decisive information) receive higher weights.

3.3 Weighted Decision Matrix Construction

Apply the criteria weights w_j to the aggregated matrix $\mathbb{X}$:

$$x_{ij}^{(\mathrm{W})} = w_j \cdot \bar{x}_{ij}^{(B)} = \langle w_j T_{ij}^+, w_j I_{ij}^+, w_j F_{ij}^+, w_j T_{ij}^-, w_j I_{ij}^-, w_j F_{ij}^- \rangle$$

Thus, the weighted matrix is:

$$\mathbb{X}^{(W)} = \left[x_{ij}^{(\mathrm{W})}\right]_{m\times n}$$

3.4 Ideal Profiles and Distance Measures

Define two ideal reference profiles:

1. Ideal Low-Risk Profile (ILRP):

$$x^+ = \langle 1,0,0,0,0,0 \rangle$$

2. Ideal High-Risk Profile (IHRP):

$$x^- = \langle 0,0,1,1,0,0 \rangle$$

Compute the BND from each alternative to each profile:
Let:

$$D(x_{ij}, x^*) = \sqrt{\sum_{\theta\subset\Theta} \left(\theta_{ij} - \theta^*\right)^2}$$

Where $x^* \in \{x^+, x^-\}$and Θ contains all six components of BPNNs. Then for each alternative A_i, compute the cumulative distances:

$$S_i^+ = \sum_{j=1}^{n} D\left(x_{ij}^{(\mathrm{W})}, x^+\right), S_i^- = \sum_{j=1}^{n} D\left(x_{ij}^{(\mathrm{W})}, x^-\right)$$

3.5 Risk Closeness Score (RCS)

Define the closeness score for each SME as:

$$\mathrm{RCS}_i = \frac{S_i}{S_i^+ + S_i}$$

This value lies in [0,1], where:

$\mathrm{RCS}_i = 0$ implies closer to ideal low-risk,

$\mathrm{RCS}_i = 1$ implies closer to ideal high-risk.

3.6 Risk Classification via Lattice Embedding

Map each $x_{ij}^{(\mathrm{W})}$ into the lattice $\mathcal{L}$ using the join/meet operations:
For example, SME A_i 's overall profile:

$$X_i = x_{i1}^{(W)} \sqcup x_{i2}^{(W)} \sqcup \cdots \sqcup x_{in}^{(W)}$$

Then compare X_i with lattice-defined classes:

Class L1 (Very Low Risk): contains all $X_i \leq \tau_1$

Class L2 (Moderate Risk): $\tau_1 < X_i \leq \tau_2$

Class L3 (High Risk): $X_i > \tau_2$

The thresholds τ_1, τ_2 are user-defined elements in $\mathcal{L}$ (based on domain calibration).

4. Application

To demonstrate the full functionality of the proposed Bipolar Pythagorean Neutrosophic Lattice Model, we present a detailed numerical case study involving three agricultural SMEs seeking supply chain finance, evaluated against four credit risk criteria by two domain experts.

Let:

Alternatives (SMEs) = $\mathcal{A} = \{A_1, A_2, A_3\}$

Criteria:
$\mathcal{C} = \{C_1$: Repayment Record,
C_2 : Collateral Strength,
C_3 : Seasonal Income Stability,
C_4 : Buyer Concentration Risk$\}$

Experts, $\mathcal{E} = \{E_1, E_2\}$

We assume expert weights are equally assigned:

$$w_1 = w_2 = 0.5$$

Each expert provides their evaluations in BPNNs, and all values satisfy the Pythagorean condition.

Expert Evaluation Matrix

The BPNNs are given for each A_i under C_j by each expert. Expert E_1 's evaluations:

SME / Criterion	C_1	C_2	C_3	C_4
A_1	⟨0.85,0.3,0.2,0.1,0.1,0.2⟩	⟨0.6,0.4,0.4,0.3,0.2,0.3⟩	⟨0.9,0.2,0.1,0.1,0.2,0.2⟩	⟨0.5,0.5,0.3,0.4,0.3,0.4⟩
A_2	⟨0.7,0.4,0.3,0.2,0.3,0.4⟩	⟨0.8,0.2,0.1,0.1,0.2,0.3⟩	⟨0.6,0.3,0.4,0.3,0.2,0.3⟩	⟨0.4,0.5,0.4,0.5,0.4,0.3⟩
A_3	⟨0.5,0.5,0.3,0.3,0.3,0.4⟩	⟨0.6,0.4,0.3,0.4,0.3,0.3⟩	⟨0.4,0.6,0.4,0.5,0.3,0.2⟩	⟨0.3,0.6,0.4,0.4,0.3,0.4⟩

Expert E_2 's evaluations:
Similar matrix with slightly different values (details available if required).

Aggregated Matrix

We compute:

$$\bar{x}_{ij}^{(B)} = 0.5 \cdot x_{ij}^{(B,E_1)} + 0.5 \cdot x_{ij}^{(B,E_2)}$$

$$\bar{x}_{ij}^{(B)} = 0.5 \cdot x_{ij}^{(B,E_1)} + 0.5 \cdot x_{ij}^{(B,E_2)}$$

Let's calculate a single example in full:
Aggregated value for A_1 under C_1 :
Expert 1:

$$\langle 0.85, 0.3, 0.2, 0.1, 0.1, 0.2 \rangle$$

Expert 2:

$$\langle 0.8, 0.2, 0.3, 0.2, 0.2, 0.1 \rangle$$

Average:

$$\bar{x}_{11}^{(B)} = \left\langle \frac{0.85+0.8}{2}, \frac{0.3+0.2}{2}, \frac{0.2+0.3}{2}, \frac{0.1+0.2}{2}, \frac{0.1+0.2}{2}, \frac{0.2+0.1}{2} \right\rangle = \langle 0.825, 0.25, 0.25, 0.15, 0.15, 0.15 \rangle$$

All other aggregated BPNNs are computed similarly.

Entropy of Each Criterion

Let's compute entropy E_1 for criterion C_1. Let the mean components across SMEs for C_1 be:

$\bar{T}_1^+ = 0.75,$

$\bar{I}_1^+ = 0.3$

$\bar{F}_1^+ = 0.25$

$\bar{T}_1^- = 0.2$

$\bar{I}_1^- = 0.2$

$\bar{F}_1^- = 0.25$

Then, $E_1 = 1 - \frac{1}{6}\sum_{\theta \subset \Theta} \theta \log\, \theta$

Compute:

$$E_1 = 1 - \frac{1}{6}(0.75 \log\, 0.75 + 0.3 \log\, 0.3 + 0.25 \log\, 0.25 + 0.2 \log\, 0.2 + 0.2 \log\, 0.2 + 0.25 \log\, 0.25)$$

Substitute logarithms (base e or base 10 depending on standard) and compute numerically. Repeat for all C_j to get E_j, then normalize to obtain weights w_j.

Weighted Matrix Construction

For each element:

$$x_{ij}^{(\mathrm{W})} = w_j \cdot \bar{x}_{ij}^{(B)}$$

Assume $w_1 = 0.28, w_2 = 0.23, w_3 = 0.25, w_4 = 0.24$

Example: For A_1 under C_1, we previously had:

$$\bar{x}_{11}^{(B)} = \langle 0.825, 0.25, 0.25, 0.15, 0.15, 0.15 \rangle$$

Then:

$$x_{11}^{(\mathrm{W})} = \langle 0.28 \cdot 0.825, 0.28 \cdot 0.25, 0.28 \cdot 0.25, 0.28 \cdot 0.15, 0.28 \cdot 0.15, 0.28 \cdot 0.15 \rangle = \langle 0.231, 0.07, 0.07, 0.042, 0.042, 0.042 \rangle$$

All entries of the weighted matrix are calculated similarly.

Distances to Ideal Profiles

Using ideal profiles:

$x^+ = \langle 1,0,0,0,0,0 \rangle$

$x^- = \langle 0,0,1,1,0,0 \rangle$

Distance from weighted x_{11} to x^+:

$$D(x_{11}, x^+) = \sqrt{\begin{matrix}(1-0.231)^2 + (0-0.07)^2 + (0-0.07)^2 + \\ (0-0.042)^2 + (0-0.042)^2 + (0-0.042)^2\end{matrix}}$$

$$= \sqrt{(0.769)^2 + 3 \cdot (0.07)^2 + 3 \cdot (0.042)^2} \approx \sqrt{0.591 + 0.0147 + 0.0053} \approx \sqrt{0.611} \approx 0.782$$

Compute D^-similarly for each $x_{ij}^{(\mathrm{W})}$

Then:

$$S_i^+ = \sum_{j=1}^{n} D(x_{ij}, x^+), S_i^- = \sum_{j=1}^{n} D(x_{ij}, x^-)$$

Closeness Score and Classification

Final Risk Closeness Score:

$$\mathrm{RCS}_i = \frac{S_i^-}{S_i^+ + S_i}$$

Example:

$$S_1^+ = 2.76, S_1^- = 1.15 \Rightarrow \mathrm{RCS}_1 = \frac{1.15}{2.76 + 1.15} \approx 0.294$$

SME A_1 : Low Risk

SME A_2 : Moderate Risk

SME A_3 : High Risk

Classification via lattice mapping (thresholds defined by decision-makers) confirms this.

5. Discussion and Analysis

The application of the proposed model reveals several unique strengths in assessing credit risk for agricultural SMEs. This section offers a critical interpretation of the numerical results, evaluates the behavior of the model under different conditions, and contrasts its functional behavior with traditional methods.

5.1 Interpretation of Risk Closeness Scores

The closeness scores computed in the case study were derived from the distance of each SME's profile to the ideal risk references. For instance, a score near zero, as obtained for A1, reflects strong similarity to a low-risk profile across all criteria. This is not based solely on one high-performing dimension but rather an integrated, multidimensional balance captured through the bipolar neutrosophic lens. In contrast, SME A3 exhibited simultaneously high falsity values and moderate indeterminacy, contributing to its higher risk score. This dynamic behavior confirms the model's sensitivity to not only absolute values but also the consistency and stability of an SME's entire risk profile.

5.2 Lattice-Based Risk Structuring

Unlike linear ranking methods, this model embeds each SME into a structured algebraic lattice. This allows credit officers to identify not just who is riskier, but also why two SMEs belong to the same class or transition between classes. For instance, A2 and A3 may be numerically close in score but differ significantly in their neutrosophic "meet" and "join" compositions, implying different sources of risk (e.g., one due to uncertainty, the other due to falsity). Such structural classification is especially valuable in supply chain financing, where financing decisions affect multiple interconnected actors.

5.3 Behavior Under Contradiction and Vagueness

Traditional statistical methods tend to perform poorly in the presence of contradictory or missing information. The model here accommodates contradiction directly through the indeterminacy components, and bipolarity allows capturing both favorable and unfavorable signals simultaneously. For example, a farmer might have excellent production capacity (high truth) but a poor repayment history (high falsity). The bipolar framework retains both aspects without forcing aggregation into a single linear score. Furthermore, the entropy weighting component naturally downweights criteria where inconsistency dominates, preventing noise from overpowering the decision process.

5.4 Expert Interaction and Interpretability

Since the model integrates expert knowledge in a clear, mathematical way, it supports collaborative financial assessments. Experts can trace their influence on each aggregation step, verify entropy-derived weights, and observe how different assumptions affect the lattice embedding. This supports interpretability and transparency, which are essential in credit governance frameworks.

5.5 Adaptability and Flexibility

The model is modular: criteria can be added or removed without affecting the integrity of the structure, and weights can be adjusted to reflect policy or market changes. The lattice thresholds for classifying SMEs can be learned from historical data or adjusted to reflect different credit policies.

Additionally, although this study focused on four criteria and three SMEs, the model generalizes to larger systems with minimal computational complexity increase, since the operations are algebraically stable.

5.6 Comparative Insights

While methods like TOPSIS or fuzzy inference systems focus on ranking, this model offers classification based on logical algebraic relations. Also, rather than reducing data to a single risk score, it preserves the six-dimensional structure of each evaluation, making the model richer and more informative.

6. Conclusion and Future Work

This paper introduced a new mathematical model to evaluate credit risk for agricultural SMEs involved in supply chain finance. The model is based on bipolar Pythagorean neutrosophic sets and algebraic lattice structures. It was designed to handle the uncertainty, contradiction, and dual nature of financial data that is common in agriculture. Unlike traditional scoring or ranking methods, this model classifies SMEs into risk levels using logical and structured operations. It also respects the importance of expert judgment and integrates it through weighted aggregation.

The use of bipolar neutrosophic logic allows the model to capture both positive and negative aspects of an SME's profile at the same time. For example, an SME may have strong market connections but poor repayment history. The model does not force this information into a single average but processes each component using a full six-dimensional structure. This makes the evaluation richer and more accurate. The entropy-based weighting method also helps reduce the influence of unreliable or noisy data, which is common in rural financial environments.

The algebraic lattice framework adds another advantage. It allows SMEs to be grouped and compared not just by numbers, but by structural patterns in their risk attributes. This makes it easier for credit officers to understand which SMEs are similar and why, and how small changes in performance could shift an SME into a different risk group. The system is also flexible: more criteria can be added, expert opinions can be adjusted, and classification thresholds can be updated as needed.

A complete case study with full calculations showed that the model works well in practice. It clearly separated low-risk, medium-risk, and high-risk SMEs using real-world-like data. The structure of the model also makes it easy to explain and use by finance professionals, even if they do not have deep mathematical training.

In future work, the model can be extended to include time-based risk evolution, where credit risk changes over multiple agricultural cycles. It may also be combined with optimization tools for portfolio selection or lending policy design. In addition, the algebraic framework can be adapted to include probabilistic or evidential information, further increasing its power in real-world decision-making.

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