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δ-Open Maps and δ-Closed Maps in Interval-Valued Neutrosophic Soft Topological Spaces

Vijayalakshmi B[1], Madhunika S[2]
[1]Assistant Professor, Department of Mathematics, Annamalai University, Annamalai Nagar - 608002, Tamilnadu, India; mathvijaya2006au@gmail.com
[2]Research Scholar, Department of Mathematics, Annamalai University, Annamalai Nagar - 608002, Tamilnadu, India; madhunika2020@gmail.com
*Correspondence: mathvijaya2006au@gmail.com

Abstract. This paper introduces and investigates interval-valued neutrosophic soft δ-open and δ-closed maps within the framework of interval-valued neutrosophic soft topological spaces, offering a robust generalization of classical topological mappings under uncertainty. Furthermore, the concept of interval-valued neutrosophic soft $\delta\beta$-homeomorphism is formulated to extend the notion of topological equivalence in soft and uncertain environments. Several fundamental properties, structural characterizations, and illustrative examples are established to substantiate the theoretical development and to demonstrate its potential applicability in complex decision-making systems governed by indeterminacy and imprecision.

Keywords: interval - valued neutrosophic soft sets, interval - valued neutrosophic soft topological spaces, interval - valued neutrosophic soft δ-open(δ-closed) Maps, interval - valued neutrosophic soft $\delta\beta$ - homeomorphism

1. **Introduction**

The seminal contribution of Zadeh [16] in 1965, through the formulation of fuzzy sets, established a robust mathematical foundation for handling imprecision and vagueness inherent in real-world phenomena. Building upon this, Atanassov [4] introduced intuitionistic fuzzy sets in 1986 by incorporating non-membership values, thereby enhancing the expressive capability of fuzzy set theory. In response to the need for modeling indeterminacy and inconsistency, Smarandache [11] developed the theory of neutrosophic sets, which further broadened the scope of uncertainty modeling. Subsequently, Smarandache et al. [12] investigated neutrosophic topological spaces based on these enriched set structures. In a parallel stream, Molodtsov [8] introduced soft set theory as a parameterized approach to managing uncertainties, which was later expanded by Maji [7] through the formulation of neutrosophic soft sets, combining the strengths of both theories.

To better capture interval-based uncertainty, Wang et al. [15] introduced interval-valued neutrosophic sets, which were extended into the soft set framework by Deli [5]. Mukherjee et al. [3] advanced these notions by establishing their topological foundations. In the context of generalized open sets, Vadivel et al. [13] proposed δ-open sets in neutrosophic topology, while Acikgoz et al. [1] examined their properties within neutrosophic soft topological spaces. More recently, Jayasudha and Raghavi [2] introduced interval-valued neutrosophic hypersoft topological spaces, providing a novel structure for modeling multi-parameter uncertainties. Extending this direction, Saeed et al. [9] investigated interval-valued complex neutrosophic sets and their associated soft topologies.

Motivated by these significant developments, the present study introduces and examines the concepts of interval-valued neutrosophic soft δ-open and δ-closed maps. In addition, we define and analyze interval-valued neutrosophic soft $\delta\beta$-homeomorphisms and establish several characterizations and illustrative examples. These contributions aim to enrich the theoretical landscape of interval-valued neutrosophic soft topological spaces and provide a foundation for further exploration in uncertain and imprecise environments.

2. **Preliminaries**

This section offers a summary of essential definitions refers to *neutrosophic soft and interval valued neutrosophic soft sets* to ensure thorough understanding.

Definition 2.1. [6]

Assume that $\mathbb{W}$ is the underlying universe and let ϱ be a set of parameters. The collection of all neutrosophic sets within $\mathbb{W}$ is represented by $\mathcal{P}(\mathbb{W})$. A neutrosophic soft set, denoted as (S, ϱ), over $\mathbb{W}$ (abbreviated as NSS) is defined by

$$(S, \varrho) = \left\{(\varphi, \langle \varepsilon, \mu_{S(\varphi)}(\varepsilon), \sigma_{S(\varphi)}(\varepsilon), \nu_{S(\varphi)}(\varepsilon)\rangle) : \varepsilon \in \mathbb{W}, \varphi \in \varrho\right\},$$

where $\mu_{S(\varphi)}(\varepsilon), \sigma_{S(\varphi)}(\varepsilon), \nu_{S(\varphi)}(\varepsilon) \in [0, 1]$ are called the degree of membership, degree of indeterminacy, and degree of non-membership functions of $S(\varphi)$, respectively. The maximum value for each of μ, σ, and ν is 1.

The inequality $0 \leq \mu_{S(\varphi)}(\varepsilon) + \sigma_{S(\varphi)}(\varepsilon) + \nu_{S(\varphi)}(\varepsilon) \leq 3$ naturally holds.

Definition 2.2. [5]

Let $\mathbb{W}$ be a universal set, and ϱ be a collection of parameters. The set of all interval-valued neutrosophic soft sets on $\mathbb{W}$ is denoted by I_vNSS ($\mathbb{W}$). An interval-valued neutrosophic soft set (abbreviated as I_vNSS) on $\mathbb{W}$ is represented by the pair (A, ϱ), where A is a mapping defined as $A : \varrho \to \mathrm{I}_v\mathrm{NSS}(\mathbb{W})$. The collection of all such interval-valued neutrosophic soft sets on $\mathbb{W}$ is represented as $\mathrm{I}_v\mathrm{NSS}(\mathbb{W})$.

Definition 2.3. [5]

An $\mathrm{I}_v\mathrm{NSS}$ (A, ϱ) over the universe $\mathbb{W}$ is said to be null $\mathrm{I}_v\mathrm{NSS}$ with respect to ϱ if $\mu_{D(e_1)}(w) = [0,0], \sigma_{D(e_1)}(w) = [0,0], \nu_{D(e_1)}(w) = [1,1]$ $\forall w \in \mathbb{W}$, $\forall e \in \varrho$ It is denoted by $0_{(\mathbb{W},\varrho)}$.

Definition 2.4. [5]

An $\mathrm{I}_v\mathrm{NSS}$ (A, ϱ) over the universe $\mathbb{W}$ is said to be universe $\mathrm{I}_v\mathrm{NSS}$ with respect to ϱ if $\mu_{D(e_1)}(w) = [1,1], \sigma_{D(e_1)}(w) = [1,1], \nu_{D(e_1)}(w) = [0,0]$ $\forall w \in \mathbb{W}$, $\forall e \in \varrho$ It is denoted by $1_{(\mathbb{W},\varrho)}$.

Definition 2.5. [5]

Let $\mathbb{W}$ be a universe set and ϱ be a set of parameters. Let $(A, \varrho_1), (D, \varrho_2) \in \mathrm{I}_vNSS(\mathbb{W})$, where $A : \varrho_1 \to \mathrm{I}_vNSS(\mathbb{W})$ is defined by $A(e_1) = \{(w, \mu_{A(e_1)}(w), \sigma_{A(e_1)}(w), \nu_{A(e_1)}(w)) : w \in \mathbb{W}\}$ and $D : \varrho_2 \to \mathrm{I}_vNSS(\mathbb{W})$ is defined by $D(e_1) = \{(w, \mu_{D(e_1)}(w), \sigma_{D(e_1)}(w), \nu_{D(e_1)}(w)) : w \in \mathbb{W}\}$ where

$\mu_{A(e_1)}(w), \sigma_{A(e_1)}(w), \nu_{A(e_1)}(w), \mu_{D(e_1)}(w), \sigma_{D(e_1)}(w), \nu_{D(e_1)}(w), \in Int([0,1])$ for $w \in \mathbb{W}$. Then

(i) (A, ϱ_1) is called $\mathrm{I}_v\mathrm{NS}$ subset of (D, ϱ_2) (denoted by $(A, \varrho_1) \subseteq (D, \varrho_2)$ if $\varrho_1 \subseteq \varrho_2$ and $\mu_{A(e_1)}(w) \leq \mu_{D(e_1)}(w), \sigma_{A(e_1)}(w) \leq \sigma_{D(e_1)}(w), \nu_{A(e_1)}(w) \geq \nu_{D(e_1)}(w) \forall w \in \mathbb{W}$.
Where $\mu_{A(e_1)}(w) \leq \mu_{D(e_1)}(w)$ iff $\inf \mu_{A(e_1)} \leq \inf \mu_{D(e_1)}$ and $\sup \mu_{A(e_1)} \leq \sup \mu_{D(e_1)}$
$\sigma_{A(e_1)}(w) \leq \sigma_{D(e_1)}(w)$ iff $\inf \sigma_{A(e_1)} \leq \inf \sigma_{D(e_1)}$ and $\sup \sigma_{A(e_1)} \leq \sup \sigma_{D(e_1)}$
$\nu_{A(e_1)}(w) \geq \nu_{D(e_1)}(w)$ iff $\inf \nu_{A(e_1)} \geq \inf \nu_{D(e_1)}$ and $\sup \nu_{A(e_1)} \geq \sup \nu_{D(e_1)}$

(ii) their union, represented by $(A, \varrho_1) \cup (D, \varrho_2) = (S, \varrho_3)$, is an $\mathrm{I}_v\mathrm{NSS}$ over $\mathbb{W}$, where $\varrho_3 = \varrho_1 \cup \varrho_2$ and $e \in \varrho_3, S : \varrho_3 \to I_vNSS(\mathbb{W})$, where
$S(e_1) = \{(w, \mu_{S(e_1)}(w), \sigma_{S(e_1)}(w), \nu_{S(e_1)}(w)) : w \in \mathbb{W}\}$, where for $x \in \mathbb{W}$

$$\mu_{S(e_1)}(w) = \begin{cases} \mu_{A(e_1)}(w) & \text{if} \quad e \in \varrho_1 - \varrho_2 \\ \mu_{D(e_1)}(w) & \text{if} \quad e \in \varrho_2 - \varrho_1 \\ \mu_{A(e_1)}(w) \cup \mu_{D(e_1)}(w) & \text{if} \quad e \in \varrho_2 \cap \varrho_1 \end{cases}$$

$$\sigma_{S(e_1)}(w) = \begin{cases} \sigma_{A(e_1)}(w) & \text{if} \quad e \in \varrho_1 - \varrho_2 \\ \sigma_{D(e_1)}(w) & \text{if} \quad e \in \varrho_2 - \varrho_1 \\ \sigma_{A(e_1)}(w) \cup \sigma_{D(e_1)}(w) & \text{if} \quad e \in \varrho_2 \cap \varrho_1 \end{cases}$$

$$\nu_{S(e_1)}(w) = \begin{cases} \nu_{A(e_1)}(w) & \text{if} \quad e \in \varrho_1 - \varrho_2 \\ \nu_{D(e_1)}(w) & \text{if} \quad e \in \varrho_2 - \varrho_1 \\ \nu_{A(e_1)}(w) \cap \nu_{D(e_1)}(w) & \text{if} \quad e \in \varrho_2 \cap \varrho_1 \end{cases}$$

(iii) Their intersection, denoted by $(A, \varrho_1) \cap (D, \varrho_2) = (S, \varrho_3)$, is an I_vNSS over $\mathbb{W}$ where $\varrho_3 = \varrho_1 \cap \varrho_2$ and for $e \in \varrho_3, S : \varrho_3 \to IVNS(\mathbb{W})$ is denoted by

$S(e_1) = \{(w, \mu_{S(e_1)}(w), \sigma_{S(e_1)}(w), \nu_{S(e_1)}(w)) : w \in \mathbb{W}\}$, where for $w \in \mathbb{W}$ and $e \in \varrho_3$ inf $\mu_{S(e_1)}(w) = \mu_{A(e_1)}(w) \cap \mu_{D(e_1)}(w), \sigma_{S(e_1)}(w) = \sigma_{A(e_1)}(w) \cap \sigma_{D(e_1)}(w)$ and $\nu_{S(e_1)} = \nu_{A(e_1)}(w) \cup \nu_{D(e_1)}(w)$.

(iv) The complement of (A, ϱ_1), denoted by $(A, \varrho_1)^c$ is an I_vNSS over $\mathbb{W}$ and is defined as $(A, \varrho)^c = (A^c, \urcorner\varrho_1)$, where $A^c : \urcorner\varrho_1 \to \mathrm{I}_vNSS(\mathbb{W})$ is denoted by

$A^c(e_1) = (w, \nu_{A(e_1)}(w), [1 - sup\sigma_{A(e}(w), 1 - inf\sigma_{A(e_1)}(w)], \mu_{A(e_1)}(w)) : w \in \mathbb{W}$ for $e \in \varrho_1$

Definition 2.6. [3]

An interval valued neutrosophic soft topology (shortly, I_vNSt) on an underlying universe $\mathbb{W}$ is a collection of τ of I_vNS subsets (S, ϱ) of $\mathbb{W}$ where ϱ be the parameters set, satisfying

(1) $0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)} \in \tau$.

(2) $[(S, \varrho) \cap (D, \varrho)] \in \tau$ for any $(S, \varrho), (D, \varrho) \in \tau$.

(3) $\bigcup_{k \in K} (S, \varrho)_k \in \tau$, for every $(S, \varrho_k) : k \in K \subseteq \tau$.

Then $(\mathbb{W}, \tau, \varrho)$ is known as *interval valued neutrosophic soft topological space* (shortly, I_vNSts) and the elements of τ are known as *interval valued neutrosophic soft open sets* (shortly, I_vNSOS) in $\mathbb{W}$. A I_vNSS (S, ϱ) is known as *interval valued neutrosophic soft closed set* (shortly, I_vNSCS) if its complement $(S, \varrho)^c$ is I_vNSOS.

Definition 2.7. [3]

Let $(\mathbb{W}, \tau, \varrho)$ be a I_vNSts & let (S, ϱ) is a I_vNSS on $\mathbb{W}$. *The interval valued neutrosophic soft interior* of (S, ϱ) (in brief, I_vNSint(S, ϱ)) and *the interval valued neutrosophic soft closure* of (S, ϱ) (in brief, I_vNScl(S, ϱ)) are represented as

$\mathrm{I}_v\mathrm{NSint}(S, \varrho) = \bigcup\{(D, \varrho) : (D, \varrho) \subseteq (S, \varrho)$ and (D, ϱ) is a I_vNSOS in $\mathbb{W}\}$.

$\mathrm{I}_v\mathrm{NScl}(S, \varrho) = \bigcap\{(D, \varrho) : (D, \varrho) \supseteq (S, \varrho)$ and (D, ϱ) is a I_vNSCS in $\mathbb{W}\}$.

Definition 2.8. [14]

Let $(\mathbb{W}, \tau, \varrho)$ be an I_vNSts on $\mathbb{W}$ and let (A, ϱ) is called the I_vNS

(i) regular-open set(shortly, I_vNSROS) if $(A, \varrho) = \mathrm{I}_v\mathrm{NSint}(\mathrm{I}_v\mathrm{NScl}(A, \varrho))$.

(ii) pre-open set(shortly, I_vNSPOS) if $(A, \varrho) \subseteq \mathrm{I}_v\mathrm{NSint}(\mathrm{I}_v\mathrm{NScl}(A, \varrho))$.

(iii) semi-open set(shortly, I_vNSSOS) if $(A, \varrho) \subseteq \mathrm{I}_v\mathrm{NScl}(\mathrm{I}_v\mathrm{NSint}(A, \varrho))$.

(iv) α-open set(shortly, I_vNSαOS) if $(A, \varrho) \subseteq \mathrm{I}_v\mathrm{NSint}(\mathrm{I}_v\mathrm{NScl}(\mathrm{I}_v\mathrm{NSint}(A, \varrho)))$.

(v) β-open set(shortly, I_vNSβOS) if $(A, \varrho) \subseteq \mathrm{I}_v\mathrm{NScl}(\mathrm{I}_v\mathrm{NSint}(\mathrm{I}_v\mathrm{NScl}(A, \varrho)))$.

The complement of a I_vNSROS (resp. I_vNSPOS, I_vNSSOS, I_vNSαOS and I_vNSβOS) is called the interval valued neutrosophic soft regular (resp. pre, semi, α and β) closed set (shortly, I_vNSRCS (resp. I_vNSPCS, I_vNSSCS, I_vNSαCS and I_vNSβCS)) in $\mathbb{W}$

The family of all I_vNSROS(resp. I_vNSRCS, I_vNSPOS, I_vNSPCS, I_vNSSOS, I_vNSSCS, I_vNSαOS, I_vNSαCS, I_vNSβOS and I_vNSβCS) of $\mathbb{W}$ is represented by I_vNSROS($\mathbb{W}$) (resp. I_vNSRCS($\mathbb{W}$), I_vNSPOS($\mathbb{W}$) I_vNSPCS($\mathbb{W}$), I_vNSSOS($\mathbb{W}$), I_vNSSCS($\mathbb{W}$), I_vNSαOS($\mathbb{W}$), I_vNSαCS($\mathbb{W}$), I_vNSβOS($\mathbb{W}$) and I_vNSβCS($\mathbb{W}$)).

Definition 2.9. [14]

Let (A, ϱ) be a I_vNSts. Then

(i) interval valued neutrosophic soft δ-interior of (A, ϱ) (in short, I_vNSδint(A, ϱ)) is defined by

$$I_vNS\delta int(A, \varrho) = \bigcup\{(D, \varrho) : (D, \varrho) \subseteq (A, \varrho) \ \& \ (D, \varrho) \text{ is a } I_vNSROS \text{ in } \mathbb{W}\}$$

(ii) interval valued neutrosophic soft δ-closure of (A, ϱ) (in short, I_vNSδcl(A, ϱ)) is defined by

$$I_vNS\delta cl(A, \varrho) = \bigcap\{(D, \varrho) : (D, \varrho) \supseteq (A, \varrho) \ \& \ (D, \varrho) \text{ is a } I_vNSRCS \text{ in } \mathbb{W}\}$$

Definition 2.10. [14]

An I_vNSS (A, ϱ) is known as I_vNSS

(1) δ-open set (shortly, I_vNSδOS) if $(A, \varrho) = I_v$NSδint(A, ϱ).

(2) δ-pre open set (in short, I_vNSδPOS) if $(A, \varrho) \subseteq I_v$NSint($I_vNS\delta$cl$(A, \varrho)$).

(3) δ-semi open set (in short, I_vNSδSOS) if $(A, \varrho) \subseteq I_v$NScl($I_vNS\delta$int$(A, \varrho)$).

(4) $\delta\alpha$ open or a-open set (in short, I_vNS$\delta\alpha$OS or I_vNSaOS) if $(A, \varrho) \subseteq I_v$NSint($I_v$NScl($I_vNS\delta$int (A, ϱ))).

(5) $\delta\beta$ open or e*-open set (in short, I_vNS$\delta\beta$OS or I_vNSe*OS) if $(A, \varrho) \subseteq I_v$NScl($I_v$NSint($I_vNS\delta$cl (A, ϱ))).

The complement of an I_vNSδOS(resp. I_vNSδPOS, I_vNSδSOS, I_vNS$\delta\alpha$OS and I_vNS$\delta\beta$OS) is called the interval valued neutrosophic soft δ(resp. δ-pre, δ-semi, δ-α and δ-β) closed set (shortly, I_vNSδCS(resp. I_vNSδPCS, I_vNSδSCS, I_vNS$\delta\alpha$CS and I_vNS$\delta\beta$CS)) in $\mathbb{W}$.

The family of all I_vNSδPOS(resp. I_vNSδPCS, I_vNSδSOS, I_vNSδSCS, I_vNS$\delta\alpha$OS, I_vNS$\delta\alpha$CS, I_vNS$\delta\beta$OS and I_vNS$\delta\beta$CS) of $\mathbb{W}$ is represented by I_vNSδPOS($\mathbb{W}$) (resp. I_vNSδPCS($\mathbb{W}$), I_vNSδSOS($\mathbb{W}$), I_vNSδSCS($\mathbb{W}$), I_vNS$\delta\alpha$OS($\mathbb{W}$), I_vNS$\delta\alpha$CS($\mathbb{W}$), I_vNS$\delta\beta$OS($\mathbb{W}$) and I_vNS$\delta\beta$CS($\mathbb{W}$)).

Definition 2.11. [14]

An I_vNSS (A, ϱ) is known as I_vNSδ-pre(resp. I_vNSδ-semi, I_vNSδ-α and I_vNSδ-β) interior of (A, ϱ) (shortly, I_vNSδPint(A, ϱ)(resp. I_vNSδSint(A, ϱ), I_vNS$\delta\alpha$int(A, ϱ) and I_vNS$\delta\beta$int(A, ϱ))) is the union of all I_vNSδPOS(resp. I_vNSδSOS, I_vNS$\delta\alpha$OS and I_vNS$\delta\beta$OS) contained in (A, ϱ).

Definition 2.12. [14]

An I_vNSS (A, ϱ) is known as I_vNSδ-pre(resp. I_vNSδ-semi, I_vNSδ-α and I_vNSδ-β) closure of (A, ϱ) (shortly, I_vNSδPcl(A, ϱ)(resp. I_vNSδScl(A, ϱ), I_vNS$\delta\alpha$cl(A, ϱ) and I_vNS$\delta\beta$cl(A, ϱ)))

is the intersection of all I_vNSδPCS(resp. I_vNSδSCS, I_vNS$\delta\alpha$CS and I_vNS$\delta\beta$CS) contained in (A, ϱ).

3. Interval - valued neutrosophic soft δ - open maps in I_vNSts

In Sections 3, 4 & 5, let $(\mathbb{W}, \tau, \varrho)$ and $(\mathbb{T}, \sigma, \varrho)$ be any two I_vNSts. Let $\mathcal{G}$: $(\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a function. Let (B, ϱ) and (P, ϱ) be an I_vNS sets in I_vNSts.

Definition 3.1. Let $\mathcal{G}$: $(\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be any function. Then, $\mathcal{G}$ is said to be an interval valued neutrosophic soft (resp. δ, $\delta\alpha$, δS, δP & $\delta\beta$ or $e*$) open map (in brief, I_vNSO (resp. I_vNSδO, I_vNS$\delta\alpha$O I_vNSδSO, I_vNSδPO & I_vNS$\delta\beta$O or I_vNSe*O)) if the image of every I_vNSOS in $(\mathbb{W}, \tau, \varrho)$ is a I_vNSOS (resp. I_vNSδOS, I_vNS$\delta\alpha$OS I_vNSδSOS, I_vNSδPOS & I_vNS$\delta\beta$OS or I_vNSe*OS)) in $(\mathbb{T}, \sigma, \varrho)$.

Theorem 3.1. The statements are hold for I_vNSO Mapping.

(i) Every $I_vNS\delta$O map is an I_vNSO map.
(ii) Every I_vNSO map is an $I_vNS\delta S$O map.
(iii) Every I_vNSO map is an $I_vNS\delta P$O map.
(iv) Every $I_vNS\delta S$O map is an I_vNSe^*O map.
(v) Every $I_vNS\delta P$O map is an I_vNSe^*O map.
(vi) Every $I_vNS\delta\alpha$O map is an $I_vNS\delta S$O map.
(vii) Every $I_vNS\delta\alpha$O map is an $I_vNS\delta$PO map.

But not conversely.

Proof.

(i) Let (P, ϱ) be an $I_vNS\delta$OS in $\mathbb{W}$. Since $\mathcal{G}$ is $I_vNS\delta$O map, $\mathcal{G}(P, \varrho)$ is an $I_vNS\delta$OS in $\mathbb{T}$. Since every I_vNSδOS is an I_vNSOS, $\mathcal{G}(P, \varrho)$ is an I_vNSOS in $\mathbb{T}$. Hence $\mathcal{G}$ is an I_vNSO map.

The proofs of other cases are similar. ■

Example 3.1. Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}, \varrho = \{e_1, e_2\}$ and I_vNSS (V_1, ϱ) in $\mathbb{W}$ and (S_1, ϱ), (S_2, ϱ) and (S_3, ϱ) in $\mathbb{T}$ are defined as

$$(V_1, e_1) = \langle(\frac{\mu_{w_1}}{[0.1,0.3]}, \frac{\sigma_{w_1}}{[0.2,0.4]}, \frac{\nu_{w_1}}{[0.6,0.9]}), (\frac{\mu_{w_2}}{[0.2,0.4]}, \frac{\sigma_{w_2}}{[0.2,0.2]}, \frac{\nu_{w_2}}{[0.7,0.8]}), (\frac{\mu_{w_3}}{[0.1,0.3]}, \frac{\sigma_{w_3}}{[0.1,0.4]}, \frac{\nu_{w_3}}{[0.7,0.9]})\rangle$$

$$(V_1, e_2) = \langle(\frac{\mu_{w_1}}{[0.1,0.1]}, \frac{\sigma_{w_1}}{[0.1,0.3]}, \frac{\nu_{w_1}}{[0.7,0.8]}), (\frac{\mu_{w_2}}{[0.1,0.1]}, \frac{\sigma_{w_2}}{[0.1,0.4]}, \frac{\nu_{w_2}}{[0.8,0.9]}), (\frac{\mu_{w_3}}{[0.1,0.1]}, \frac{\sigma_{w_3}}{[0.2,0.3]}, \frac{\nu_{w_3}}{[0.8,0.9]})\rangle$$

$$(S_1, e_1) = \langle(\frac{\mu_{t_1}}{[0.1,0.3]}, \frac{\sigma_{t_1}}{[0.2,0.4]}, \frac{\nu_{t_1}}{[0.6,0.9]}), (\frac{\mu_{t_2}}{[0.2,0.4]}, \frac{\sigma_{t_2}}{[0.2,0.2]}, \frac{\nu_{t_2}}{[0.7,0.8]}), (\frac{\mu_{t_3}}{[0.1,0.3]}, \frac{\sigma_{t_3}}{[0.1,0.4]}, \frac{\nu_{t_3}}{[0.7,0.9]})\rangle$$

$$(S_1,e_2) = \langle(\frac{\mu_{t_1}}{[0.1,0.1]},\frac{\sigma_{t_1}}{[0.1,0.3]},\frac{\nu_{t_1}}{[0.7,0.8]}),(\frac{\mu_{t_2}}{[0.1,0.1]},\frac{\sigma_{t_2}}{[0.1,0.4]},\frac{\nu_{t_2}}{[0.8,0.9]}),(\frac{\mu_{t_3}}{[0.1,0.1]},\frac{\sigma_{t_3}}{[0.2,0.3]},\frac{\nu_{t_3}}{[0.8,0.9]})\rangle$$

$$(S_2,e_1) = \langle(\frac{\mu_{t_1}}{[0.2,0.5]},\frac{\sigma_{t_1}}{[0.6,0.7]},\frac{\nu_{t_1}}{[0.4,0.6]}),(\frac{\mu_{t_2}}{[0.3,0.5]},\frac{\sigma_{t_2}}{[0.4,0.5]},\frac{\nu_{t_2}}{[0.5,0.7]}),(\frac{\mu_{t_3}}{[0.4,0.5]},\frac{\sigma_{t_3}}{[0.6,0.8]},\frac{\nu_{t_3}}{[0.4,0.5]})\rangle$$

$$(S_2,e_2) = \langle(\frac{\mu_{t_1}}{[0.3,0.5]},\frac{\sigma_{t_1}}{[0.2,0.5]},\frac{\nu_{t_1}}{[0.3,0.5]}),(\frac{\mu_{t_2}}{[0.4,0.5]},\frac{\sigma_{t_2}}{[0.7,0.8]},\frac{\nu_{t_2}}{[0.4,0.6]}),(\frac{\mu_{t_3}}{[0.3,0.4]},\frac{\sigma_{t_3}}{[0.3,0.5]},\frac{\nu_{t_3}}{[0.4,0.6]})\rangle$$

$$(S_3,e_1) = \langle(\frac{\mu_{t_1}}{[0.2,0.4]},\frac{\sigma_{t_1}}{[0.2,0.5]},\frac{\nu_{t_1}}{[0.5,0.7]}),(\frac{\mu_{t_2}}{[0.2,0.5]},\frac{\sigma_{t_2}}{[0.3,0.5]},\frac{\nu_{t_2}}{[0.6,0.8]}),(\frac{\mu_{t_3}}{[0.2,0.5]},\frac{\sigma_{t_3}}{[0.1,0.5]},\frac{\nu_{t_3}}{[0.5,0.7]})\rangle$$

$$(S_3,e_2) = \langle(\frac{\mu_{t_1}}{[0.2,0.4]},\frac{\sigma_{t_1}}{[0.1,0.5]},\frac{\nu_{t_1}}{[0.6,0.8]}),(\frac{\mu_{t_2}}{[0.2,0.4]},\frac{\sigma_{t_2}}{[0.2,0.5]},\frac{\nu_{t_2}}{[0.6,0.9]}),(\frac{\mu_{t_3}}{[0.2,0.3]},\frac{\sigma_{t_3}}{[0.2,0.4]},\frac{\nu_{t_3}}{[0.7,0.8]})\rangle$$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1,\varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1,\varrho), (S_2,\varrho), (S_3,\varrho)\}$. Let $\mathcal{G}$: $(\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ be an identity mapping, then $\mathcal{G}$ is $\mathrm{I}_v N$SO map but not $\mathrm{I}_v NS\delta$O map because $\mathcal{G}(V_1,\varrho) = (S_1,\varrho)$ is an I_vNSOS in $\mathbb{T}$ but not $\mathrm{I}_v NS\delta$OS in $\mathbb{T}$.

The following Figure: 1 illustarte I_vNSδO sets in interval valued neutrosophic soft topological space.

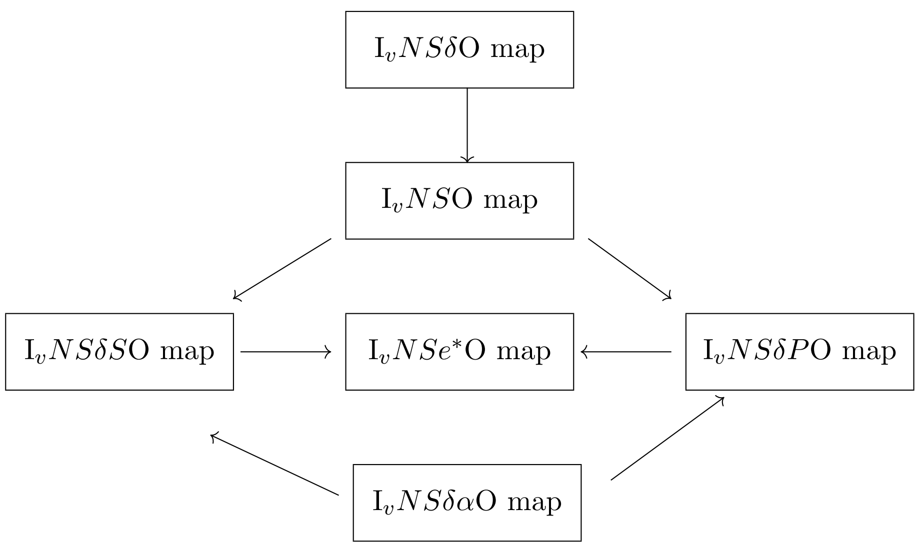

FIGURE 1. I_vNSδO maps in I_vNSts.

Example 3.2. Let $\mathbb{W} = \{w_1,w_2,w_3\} = \{t_1,t_2,t_3\} = \mathbb{T}, \varrho = \{e_1,e_2\}$ and I_vNSS (V_1,ϱ) in $\mathbb{W}$ and (S_1,ϱ), (S_2,ϱ), (S_3,ϱ) and (S_4,ϱ) in $\mathbb{T}$ are defined as

$$(V_1,e_1) = \langle(\frac{\mu_{w_1}}{[0.6,0.9]},\frac{\sigma_{w_1}}{[0.6,0.8]},\frac{\nu_{w_1}}{[0.1,0.3]}),(\frac{\mu_{w_2}}{[0.7,0.8]},\frac{\sigma_{w_2}}{[0.8,0.8]},\frac{\nu_{w_2}}{[0.2,0.4]}),(\frac{\mu_{w_3}}{[0.7,0.9]},\frac{\sigma_{w_3}}{[0.6,0.9]},\frac{\nu_{w_3}}{[0.1,0.3]})\rangle$$

$$(V_1,e_2) = \langle(\frac{\mu_{w_1}}{[0.7,0.8]},\frac{\sigma_{w_1}}{[0.7,0.9]},\frac{\nu_{w_1}}{[0.1,0.1]}),(\frac{\mu_{w_2}}{[0.8,0.9]},\frac{\sigma_{w_2}}{[0.6,0.9]},\frac{\nu_{w_2}}{[0.1,0.1]}),(\frac{\mu_{w_3}}{[0.8,0.9]},\frac{\sigma_{w_3}}{[0.7,0.8]},\frac{\nu_{w_3}}{[0.1,0.1]})\rangle$$

$(S_1, e_1) = \langle(\frac{\mu_{t_1}}{[0.1,0.3]}, \frac{\sigma_{t_1}}{[0.2,0.4]}, \frac{\nu_{t_1}}{[0.6,0.9]}), (\frac{\mu_{t_2}}{[0.2,0.4]}, \frac{\sigma_{t_2}}{[0.2,0.2]}, \frac{\nu_{t_2}}{[0.7,0.8]}), (\frac{\mu_{t_3}}{[0.1,0.3]}, \frac{\sigma_{t_3}}{[0.1,0.4]}, \frac{\nu_{t_3}}{[0.7,0.9]})\rangle$

$(S_1, e_2) = \langle(\frac{\mu_{t_1}}{[0.1,0.1]}, \frac{\sigma_{t_1}}{[0.1,0.3]}, \frac{\nu_{t_1}}{[0.7,0.8]}), (\frac{\mu_{t_2}}{[0.1,0.1]}, \frac{\sigma_{t_2}}{[0.1,0.4]}, \frac{\nu_{t_2}}{[0.8,0.9]}), (\frac{\mu_{t_3}}{[0.1,0.1]}, \frac{\sigma_{t_3}}{[0.2,0.3]}, \frac{\nu_{t_3}}{[0.8,0.9]})\rangle$

$(S_2, e_1) = \langle(\frac{\mu_{t_1}}{[0.2,0.5]}, \frac{\sigma_{t_1}}{[0.6,0.7]}, \frac{\nu_{t_1}}{[0.4,0.6]}), (\frac{\mu_{t_2}}{[0.3,0.5]}, \frac{\sigma_{t_2}}{[0.4,0.5]}, \frac{\nu_{t_2}}{[0.5,0.7]}), (\frac{\mu_{t_3}}{[0.4,0.5]}, \frac{\sigma_{t_3}}{[0.6,0.8]}, \frac{\nu_{t_3}}{[0.4,0.5]})\rangle$

$(S_2, e_2) = \langle(\frac{\mu_{t_1}}{[0.3,0.5]}, \frac{\sigma_{t_1}}{[0.2,0.5]}, \frac{\nu_{t_1}}{[0.3,0.5]}), (\frac{\mu_{t_2}}{[0.4,0.5]}, \frac{\sigma_{t_2}}{[0.7,0.8]}, \frac{\nu_{t_2}}{[0.4,0.6]}), (\frac{\mu_{t_3}}{[0.3,0.4]}, \frac{\sigma_{t_3}}{[0.3,0.5]}, \frac{\nu_{t_3}}{[0.4,0.6]})\rangle$

$(S_3, e_1) = \langle(\frac{\mu_{t_1}}{[0.2,0.4]}, \frac{\sigma_{t_1}}{[0.2,0.5]}, \frac{\nu_{t_1}}{[0.5,0.7]}), (\frac{\mu_{t_2}}{[0.2,0.5]}, \frac{\sigma_{t_2}}{[0.3,0.5]}, \frac{\nu_{t_2}}{[0.6,0.8]}), (\frac{\mu_{t_3}}{[0.2,0.5]}, \frac{\sigma_{t_3}}{[0.1,0.5]}, \frac{\nu_{t_3}}{[0.5,0.7]})\rangle$

$(S_3, e_2) = \langle(\frac{\mu_{t_1}}{[0.2,0.4]}, \frac{\sigma_{t_1}}{[0.1,0.5]}, \frac{\nu_{t_1}}{[0.6,0.8]}), (\frac{\mu_{t_2}}{[0.2,0.4]}, \frac{\sigma_{t_2}}{[0.2,0.5]}, \frac{\nu_{t_2}}{[0.6,0.9]}), (\frac{\mu_{t_3}}{[0.2,0.3]}, \frac{\sigma_{t_3}}{[0.2,0.4]}, \frac{\nu_{t_3}}{[0.7,0.8]})\rangle$

$(S_4, e_1) = \langle(\frac{\mu_{t_1}}{[0.6,0.9]}, \frac{\sigma_{t_1}}{[0.6,0.8]}, \frac{\nu_{t_1}}{[0.1,0.3]}), (\frac{\mu_{t_2}}{[0.7,0.8]}, \frac{\sigma_{t_2}}{[0.8,0.8]}, \frac{\nu_{t_2}}{[0.2,0.4]}), (\frac{\mu_{t_3}}{[0.7,0.9]}, \frac{\sigma_{t_3}}{[0.6,0.9]}, \frac{\nu_{t_3}}{[0.1,0.3]})\rangle$

$(S_4, e_2) = \langle(\frac{\mu_{t_1}}{[0.7,0.8]}, \frac{\sigma_{t_1}}{[0.7,0.9]}, \frac{\nu_{t_1}}{[0.1,0.1]}), (\frac{\mu_{t_2}}{[0.8,0.9]}, \frac{\sigma_{t_2}}{[0.6,0.9]}, \frac{\nu_{t_2}}{[0.1,0.1]}), (\frac{\mu_{t_3}}{[0.8,0.9]}, \frac{\sigma_{t_3}}{[0.7,0.8]}, \frac{\nu_{t_3}}{[0.1,0.1]})\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$. Let $\mathcal{G}$: $(\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then

(i) $\mathcal{G}$ is $\mathrm{I}_v NS\delta$PO map but not $\mathrm{I}_v NS$O map because $\mathcal{G}(V_1, \varrho) = (S_4, \varrho)$ is an $\mathrm{I}_v NS\delta$POS in $\mathbb{T}$ but not I_vNSOS in $\mathbb{T}$.

(ii) $\mathcal{G}$ is $\mathrm{I}_v NS\delta$PO map but not $\mathrm{I}_v NS\delta$PO map because $\mathcal{G}(V_1, \varrho) = (S_4, \varrho)$ is an $\mathrm{I}_v NS\delta$POS in $\mathbb{T}$ but not $\mathrm{I}_v NS\delta\alpha$OS in $\mathbb{T}$.

Example 3.3. Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}, \varrho = \{e_1, e_2\}$ and I_vNSS (V_1, ϱ) in $\mathbb{W}$ and (S_1, ϱ), (S_2, ϱ), (S_3, ϱ) and (S_4, ϱ) in $\mathbb{T}$ are defined as

$(V_1, e_1) = \langle(\frac{\mu_{w_1}}{[0.4,0.6]}, \frac{\sigma_{w_1}}{[0.3,0.4]}, \frac{\nu_{w_1}}{[0.2,0.5]}), (\frac{\mu_{w_2}}{[0.5,0.7]}, \frac{\sigma_{w_2}}{[0.5,0.6]}, \frac{\nu_{w_2}}{[0.3,0.5]}), (\frac{\mu_{w_3}}{[0.4,0.5]}, \frac{\sigma_{w_3}}{[0.2,0.4]}, \frac{\nu_{w_3}}{[0.4,0.5]})\rangle$

$(V_1, e_2) = \langle(\frac{\mu_{w_1}}{[0.3,0.5]}, \frac{\sigma_{w_1}}{[0.5,0.8]}, \frac{\nu_{w_1}}{[0.3,0.5]}), (\frac{\mu_{w_2}}{[0.4,0.6]}, \frac{\sigma_{w_2}}{[0.2,0.3]}, \frac{\nu_{w_2}}{[0.4,0.5]}), (\frac{\mu_{w_3}}{[0.4,0.6]}, \frac{\sigma_{w_3}}{[0.5,0.7]}, \frac{\nu_{w_3}}{[0.3,0.4]})\rangle$

$(S_1, e_1) = \langle(\frac{\mu_{t_1}}{[0.1,0.3]}, \frac{\sigma_{t_1}}{[0.2,0.4]}, \frac{\nu_{t_1}}{[0.6,0.9]}), (\frac{\mu_{t_2}}{[0.2,0.4]}, \frac{\sigma_{t_2}}{[0.2,0.2]}, \frac{\nu_{t_2}}{[0.7,0.8]}), (\frac{\mu_{t_3}}{[0.1,0.3]}, \frac{\sigma_{t_3}}{[0.1,0.4]}, \frac{\nu_{t_3}}{[0.7,0.9]})\rangle$

$(S_1, e_2) = \langle(\frac{\mu_{t_1}}{[0.1,0.1]}, \frac{\sigma_{t_1}}{[0.1,0.3]}, \frac{\nu_{t_1}}{[0.7,0.8]}), (\frac{\mu_{t_2}}{[0.1,0.1]}, \frac{\sigma_{t_2}}{[0.1,0.4]}, \frac{\nu_{t_2}}{[0.8,0.9]}), (\frac{\mu_{t_3}}{[0.1,0.1]}, \frac{\sigma_{t_3}}{[0.2,0.3]}, \frac{\nu_{t_3}}{[0.8,0.9]})\rangle$

$(S_2, e_1) = \langle(\frac{\mu_{t_1}}{[0.2,0.5]}, \frac{\sigma_{t_1}}{[0.6,0.7]}, \frac{\nu_{t_1}}{[0.4,0.6]}), (\frac{\mu_{t_2}}{[0.3,0.5]}, \frac{\sigma_{t_2}}{[0.4,0.5]}, \frac{\nu_{t_2}}{[0.5,0.7]}), (\frac{\mu_{t_3}}{[0.4,0.5]}, \frac{\sigma_{t_3}}{[0.6,0.8]}, \frac{\nu_{t_3}}{[0.4,0.5]})\rangle$

$(S_2, e_2) = \langle(\frac{\mu_{t_1}}{[0.3,0.5]}, \frac{\sigma_{t_1}}{[0.2,0.5]}, \frac{\nu_{t_1}}{[0.3,0.5]}), (\frac{\mu_{t_2}}{[0.4,0.5]}, \frac{\sigma_{t_2}}{[0.7,0.8]}, \frac{\nu_{t_2}}{[0.4,0.6]}), (\frac{\mu_{t_3}}{[0.3,0.4]}, \frac{\sigma_{t_3}}{[0.3,0.5]}, \frac{\nu_{t_3}}{[0.4,0.6]})\rangle$

$(S_3, e_1) = \langle(\frac{\mu_{t_1}}{[0.2,0.4]}, \frac{\sigma_{t_1}}{[0.2,0.5]}, \frac{\nu_{t_1}}{[0.5,0.7]}), (\frac{\mu_{t_2}}{[0.2,0.5]}, \frac{\sigma_{t_2}}{[0.3,0.5]}, \frac{\nu_{t_2}}{[0.6,0.8]}), (\frac{\mu_{t_3}}{[0.2,0.5]}, \frac{\sigma_{t_3}}{[0.1,0.5]}, \frac{\nu_{t_3}}{[0.5,0.7]})\rangle$

$(S_3, e_2) = \langle(\frac{\mu_{t_1}}{[0.2,0.4]}, \frac{\sigma_{t_1}}{[0.1,0.5]}, \frac{\nu_{t_1}}{[0.6,0.8]}), (\frac{\mu_{t_2}}{[0.2,0.4]}, \frac{\sigma_{t_2}}{[0.2,0.5]}, \frac{\nu_{t_2}}{[0.6,0.9]}), (\frac{\mu_{t_3}}{[0.2,0.3]}, \frac{\sigma_{t_3}}{[0.2,0.4]}, \frac{\nu_{t_3}}{[0.7,0.8]})\rangle$

$(S_4, e_1) = \langle(\frac{\mu_{t_1}}{[0.4,0.6]}, \frac{\sigma_{t_1}}{[0.3,0.4]}, \frac{\nu_{t_1}}{[0.2,0.5]}), (\frac{\mu_{t_2}}{[0.5,0.7]}, \frac{\sigma_{t_2}}{[0.5,0.6]}, \frac{\nu_{t_2}}{[0.3,0.5]}), (\frac{\mu_{t_3}}{[0.4,0.5]}, \frac{\sigma_{t_3}}{[0.2,0.4]}, \frac{\nu_{t_3}}{[0.4,0.5]})\rangle$

$(S_4, e_2) = \langle(\frac{\mu_{t_1}}{[0.3,0.5]}, \frac{\sigma_{t_1}}{[0.5,0.8]}, \frac{\nu_{t_1}}{[0.3,0.5]}), (\frac{\mu_{t_2}}{[0.4,0.6]}, \frac{\sigma_{t_2}}{[0.2,0.3]}, \frac{\nu_{t_2}}{[0.4,0.5]}), (\frac{\mu_{t_3}}{[0.4,0.6]}, \frac{\sigma_{t_3}}{[0.5,0.7]}, \frac{\nu_{t_3}}{[0.3,0.4]})\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$. Let $\mathcal{G}$: $(\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then

(i) $\mathcal{G}$ is $\mathrm{I}_v NS\delta\beta$O map but not $\mathrm{I}_v NS\delta$SO map because $\mathcal{G}(V_1, \varrho) = (S_4, \varrho)$ is an $\mathrm{I}_v NS\delta\beta$OS in $\mathbb{T}$ but not $\mathrm{I}_v NS\delta$SOS in $\mathbb{T}$.

(ii) $\mathcal{G}$ is $\mathrm{I}_v NS\delta\beta$O map but not $\mathrm{I}_v NS\delta$PO map because $\mathcal{G}(V_1, \varrho) = (S_4, \varrho)$ is an $\mathrm{I}_v NS\delta\beta$OS in $\mathbb{T}$ but not $\mathrm{I}_v NS\delta$POS in $\mathbb{T}$.

Example 3.4. Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{t_1, t_2, t_3\} = \mathbb{T}, \varrho = \{e_1, e_2\}$ and I_vNSS (V_1, ϱ) in $\mathbb{W}$ and (S_1, ϱ), (S_2, ϱ), (S_3, ϱ) and (S_4, ϱ) in $\mathbb{T}$ are defined as

$(V_1, e_1) = \langle(\frac{\mu_{w_1}}{[0.5,0.7]}, \frac{\sigma_{w_1}}{[0.5,0.8]}, \frac{\nu_{w_1}}{[0.2,0.4]}), (\frac{\mu_{w_2}}{[0.6,0.8]}, \frac{\sigma_{w_2}}{[0.5,0.7]}, \frac{\nu_{w_2}}{[0.2,0.5]}), (\frac{\mu_{w_3}}{[0.5,0.7]}, \frac{\sigma_{w_3}}{[0.5,0.9]}, \frac{\nu_{w_3}}{[0.2,0.5]})\rangle$

$(V_1, e_2) = \langle(\frac{\mu_{w_1}}{[0.6,0.8]}, \frac{\sigma_{w_1}}{[0.5,0.9]}, \frac{\nu_{w_1}}{[0.2,0.4]}), (\frac{\mu_{w_2}}{[0.6,0.9]}, \frac{\sigma_{w_2}}{[0.5,0.8]}, \frac{\nu_{w_2}}{[0.2,0.4]}), (\frac{\mu_{w_3}}{[0.7,0.8]}, \frac{\sigma_{w_3}}{[0.6,0.8]}, \frac{\nu_{w_3}}{[0.2,0.3]})\rangle$

$(S_1, e_1) = \langle(\frac{\mu_{t_1}}{[0.1,0.3]}, \frac{\sigma_{t_1}}{[0.2,0.4]}, \frac{\nu_{t_1}}{[0.6,0.9]}), (\frac{\mu_{t_2}}{[0.2,0.4]}, \frac{\sigma_{t_2}}{[0.2,0.2]}, \frac{\nu_{t_2}}{[0.7,0.8]}), (\frac{\mu_{t_3}}{[0.1,0.3]}, \frac{\sigma_{t_3}}{[0.1,0.4]}, \frac{\nu_{t_3}}{[0.7,0.9]})\rangle$

$(S_1, e_2) = \langle(\frac{\mu_{t_1}}{[0.1,0.1]}, \frac{\sigma_{t_1}}{[0.1,0.3]}, \frac{\nu_{t_1}}{[0.7,0.8]}), (\frac{\mu_{t_2}}{[0.1,0.1]}, \frac{\sigma_{t_2}}{[0.1,0.4]}, \frac{\nu_{t_2}}{[0.8,0.9]}), (\frac{\mu_{t_3}}{[0.1,0.1]}, \frac{\sigma_{t_3}}{[0.2,0.3]}, \frac{\nu_{t_3}}{[0.8,0.9]})\rangle$

$(S_2, e_1) = \langle(\frac{\mu_{t_1}}{[0.2,0.5]}, \frac{\sigma_{t_1}}{[0.6,0.7]}, \frac{\nu_{t_1}}{[0.4,0.6]}), (\frac{\mu_{t_2}}{[0.3,0.5]}, \frac{\sigma_{t_2}}{[0.4,0.5]}, \frac{\nu_{t_2}}{[0.5,0.7]}), (\frac{\mu_{t_3}}{[0.4,0.5]}, \frac{\sigma_{t_3}}{[0.6,0.8]}, \frac{\nu_{t_3}}{[0.4,0.5]})\rangle$

$(S_2, e_2) = \langle(\frac{\mu_{t_1}}{[0.3,0.5]}, \frac{\sigma_{t_1}}{[0.2,0.5]}, \frac{\nu_{t_1}}{[0.3,0.5]}), (\frac{\mu_{t_2}}{[0.4,0.5]}, \frac{\sigma_{t_2}}{[0.7,0.8]}, \frac{\nu_{t_2}}{[0.4,0.6]}), (\frac{\mu_{t_3}}{[0.3,0.4]}, \frac{\sigma_{t_3}}{[0.3,0.5]}, \frac{\nu_{t_3}}{[0.4,0.6]})\rangle$

$(S_3, e_1) = \langle(\frac{\mu_{t_1}}{[0.2,0.4]}, \frac{\sigma_{t_1}}{[0.2,0.5]}, \frac{\nu_{t_1}}{[0.5,0.7]}), (\frac{\mu_{t_2}}{[0.2,0.5]}, \frac{\sigma_{t_2}}{[0.3,0.5]}, \frac{\nu_{t_2}}{[0.6,0.8]}), (\frac{\mu_{t_3}}{[0.2,0.5]}, \frac{\sigma_{t_3}}{[0.1,0.5]}, \frac{\nu_{t_3}}{[0.5,0.7]})\rangle$

$(S_3, e_2) = \langle(\frac{\mu_{t_1}}{[0.2,0.4]}, \frac{\sigma_{t_1}}{[0.1,0.5]}, \frac{\nu_{t_1}}{[0.6,0.8]}), (\frac{\mu_{t_2}}{[0.2,0.4]}, \frac{\sigma_{t_2}}{[0.2,0.5]}, \frac{\nu_{t_2}}{[0.6,0.9]}), (\frac{\mu_{t_3}}{[0.2,0.3]}, \frac{\sigma_{t_3}}{[0.2,0.4]}, \frac{\nu_{t_3}}{[0.7,0.8]})\rangle$

$(S_4, e_1) = \langle(\frac{\mu_{t_1}}{[0.5,0.7]}, \frac{\sigma_{t_1}}{[0.5,0.8]}, \frac{\nu_{t_1}}{[0.2,0.4]}), (\frac{\mu_{t_2}}{[0.6,0.8]}, \frac{\sigma_{t_2}}{[0.5,0.7]}, \frac{\nu_{t_2}}{[0.2,0.5]}), (\frac{\mu_{t_3}}{[0.5,0.7]}, \frac{\sigma_{t_3}}{[0.5,0.9]}, \frac{\nu_{t_3}}{[0.2,0.5]})\rangle$

$(S_4, e_2) = \langle(\frac{\mu_{t_1}}{[0.6,0.8]}, \frac{\sigma_{t_1}}{[0.5,0.9]}, \frac{\nu_{t_1}}{[0.2,0.4]}), (\frac{\mu_{t_2}}{[0.6,0.9]}, \frac{\sigma_{t_2}}{[0.5,0.8]}, \frac{\nu_{t_2}}{[0.2,0.4]}), (\frac{\mu_{t_3}}{[0.7,0.8]}, \frac{\sigma_{t_3}}{[0.6,0.8]}, \frac{\nu_{t_3}}{[0.2,0.3]})\rangle$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$. Let $\mathcal{G}$: $(\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then

(i) $\mathcal{G}$ is $\mathrm{I}_v NS\delta$SO map but not I_vNSO map because $\mathcal{G}(V_1, \varrho) = (S_4, \varrho)$ is an $\mathrm{I}_v NS\delta$SOS in $\mathbb{T}$ but not I_vNSOS in $\mathbb{T}$.

(ii) $\mathcal{G}$ is $\mathrm{I}_v NS\delta$SO map but not $\mathrm{I}_v NS\delta\alpha$O map because $\mathcal{G}(V_1, \varrho) = (S_4, \varrho)$ is an $\mathrm{I}_v NS\delta$SOS in $\mathbb{T}$ but not $\mathrm{I}_v NS\delta\alpha$OS in $\mathbb{T}$.

Theorem 3.2. A map $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is an $\mathrm{I}_v NSe^*O$ iff for every I_vNSS (P, ϱ) of $(\mathbb{W}, \tau, \varrho)$, $\mathcal{G}(\mathrm{I}_v NSint(P, \varrho)) \subseteq \mathrm{I}_v NSe^*int(\mathcal{G}(P, \varrho))$.

Proof. Necessity: Let $\mathcal{G}$ be a $\mathrm{I}_v NSe^*O$ map and (P, ϱ) be a $\mathrm{I}_v NSOS$ in $(\mathbb{W}, \tau, \varrho)$. Now, $\mathrm{I}_v NSint(P, \varrho) \subseteq (P, \varrho)$ implies $\mathcal{G}(\mathrm{I}_v NSint(P, \varrho)) \subseteq \mathcal{G}(P, \varrho)$. Since $\mathcal{G}$ is a $\mathrm{I}_v NSe^*OS$ map, $\mathcal{G}(\mathrm{I}_v NSint(P, \varrho))$ is a $\mathrm{I}_v NSe^*O$ in $(\mathbb{T}, \sigma, \varrho)$ such that $\mathcal{G}(\mathrm{I}_v NSint(P, \varrho)) \subseteq \mathcal{G}(P, \varrho)$ therefore $\mathcal{G}(\mathrm{I}_v NSint(P, \varrho)) \subseteq \mathrm{I}_v NSe^*int(\mathcal{G}(P, \varrho))$.

Sufficiency: Assume (P, ϱ) is an $\mathrm{I}_v NSOS$ of $(\mathbb{W}, \tau, \varrho)$. Then $\mathcal{G}(P, \varrho) = \mathcal{G}(\mathrm{I}_v NSint(P, \varrho)) \subseteq \mathrm{I}_v NSe^*int$ $(\mathcal{G}(P, \varrho))$. But $\mathrm{I}_v NSe^*int(\mathcal{G}(P, \varrho)) \subseteq \mathcal{G}(P, \varrho)$. So $\mathcal{G}(P, \varrho) = \mathrm{I}_v NSe^*int(P, \varrho)$ which implies $\mathcal{G}(P, \varrho)$ is a $\mathrm{I}_v NSe^*OS$ of $(\mathbb{T}, \sigma, \varrho)$ and hence $\mathcal{G}$ is a $\mathrm{I}_v NSe^*O$. ■

Theorem 3.3. If $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is an $\mathrm{I}_v NSe^*O$ map then $\mathrm{I}_v\mathrm{NSint}(\mathcal{G}^{-1}(P, \varrho)) \subseteq \mathcal{G}^{-1}(\mathrm{I}_v NSe^*\mathrm{int}$ $(P, \varrho))$ for every I_vNSS (P, ϱ) of $(\mathbb{T}, \sigma, \varrho)$.

Proof. Let (P, ϱ) be a I_vNSS of $(\mathbb{T}, \sigma, \varrho)$. Then $\mathrm{I}_v\mathrm{NSint}(\mathcal{G}^{-1}(P, \varrho))$ is a I_vNSOS in $(\mathbb{W}, \tau, \varrho)$. Since $\mathcal{G}$ is $\mathrm{I}_v\mathrm{NSe}^*$O, $\mathcal{G}(\mathrm{I}_v\mathrm{NSint}(\mathcal{G}^{-1}(P, \varrho)))$ is $\mathrm{I}_v NSe^*O$ in $(\mathbb{T}, \sigma, \varrho)$ and hence $\mathcal{G}(\mathrm{I}_v\mathrm{NSint}(\mathcal{G}^{-1}(P, \varrho))) \subseteq \mathrm{I}_v NSe^*int(\mathcal{G}(\mathcal{G}^{-1}(P, \varrho))) \subseteq \mathrm{I}_v\mathrm{NSe}^*\mathrm{int}(P, \varrho)$. Thus $\mathrm{I}_v\mathrm{NSint}(\mathcal{G}^{-1}(P, \varrho)) \subseteq \mathcal{G}^{-1}(\mathrm{I}_v\mathrm{NSe}^*\mathrm{int}(P, \varrho))$. ■

Theorem 3.4. A map $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is $\mathrm{I}_v NSe^*O$ iff for each $\mathrm{I}_v NSS$ (S, ϱ) of $(\mathbb{T}, \sigma, \varrho)$ and for each I_vNSCS (P, ϱ) of $(\mathbb{W}, \tau, \varrho)$ containing $\mathcal{G}^{-1}(S, \varrho)$ there is an $\mathrm{I}_v NSe^*$CS (B, ϱ) of $(\mathbb{T}, \sigma, \varrho)$ such that $(S, \varrho) \subseteq (P, \varrho)$ and $\mathcal{G}^{-1}(B, \varrho) \subseteq (P, \varrho)$.

Proof. Necessity: Assume $\mathcal{G}$ be a $\mathrm{I}_v NSe^*O$ map. Let (S, ϱ) be a $\mathrm{I}_v NSCS$ in $(\mathbb{T}, \tau, \varrho)$ and (P, ϱ) is a $\mathrm{I}_v NSCS$ in $(\mathbb{W}, \tau, \varrho)$ such that $\mathcal{G}^{-1}(S, \varrho) \subseteq (P, \varrho)$. Then, $(B, \varrho) = \mathcal{G}^{-1}(P, \varrho)^c)^c$ is $\mathrm{I}_v NSe^*CS$ of $(\mathbb{T}, \tau, \varrho)$ such that $\mathcal{G}^{-1}(B, \varrho) \subseteq (P, \varrho)$.

Sufficiency: Assume (V, ϱ) is a $\mathrm{I}_v NSOS$ of $(\mathbb{W}, \tau, \varrho)$. Then $\mathcal{G}^{-1}((\mathcal{G}(V, \varrho))^c \subseteq (V, \varrho)^c$ and $(V, \varrho)^c$ is $\mathrm{I}_v NSCS$ in $(\mathbb{W}, \tau, \varrho)$. By hypothesis there is a $\mathrm{I}_v NSe^*CS(B, \varrho)$ of $(\mathbb{T}, \tau, \varrho)$ such that $((\mathcal{G}(V, \varrho))^c \subseteq (B, \varrho)$ and $\mathcal{G}^{-1}(B, \varrho) \subseteq (V, \varrho)^c$. Therefore $(V, \varrho) \subseteq (\mathcal{G}^{-1}(B, \varrho))^c$. Hence

$(B,\varrho)^c \subseteq \mathcal{G}(B,\varrho) \subseteq \mathcal{G}((\mathcal{G}^{-1}(B,\varrho))^c) \subseteq (B,\varrho)^c$ which implies $\mathcal{G}(V,\varrho) = (B,\varrho)^c$. Since $(B,\varrho)^c$ is I_vNSe^*OS of $(\mathbb{T},\sigma,\varrho)$. Hence $\mathcal{G}(V,\varrho)$ is a I_vNSe^*OS in $(\mathbb{T},\sigma,\varrho)$ and thus $\mathcal{G}$ is I_vNSe^*O map. ■

Theorem 3.5. A map $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ is I_vNSe^*O iff $\mathcal{G}^{-1}(\mathrm{I}_vNSe^*cl(P,\varrho) \subseteq \mathrm{I}_vNScl(\mathcal{G}^{-1}(P,\varrho))$ for every I_vNSS (P,ϱ) of $(\mathbb{T},\sigma,\varrho)$.

Proof. Necessity: Assume $\mathcal{G}$ is a I_vNSe^*O map. For any I_vNSS (P,ϱ) of $(\mathbb{T},\tau,\varrho), \mathcal{G}^{-1}(P,\varrho) \subseteq \mathrm{I}_vNScl(\mathcal{G}^{-1}(P,\varrho))$. Therefore by theorem 3.4 there exists a I_vNSe^*CS (S,ϱ) in $(\mathbb{T},\sigma,\varrho)$ such that $(P,\varrho) \subseteq (S,\varrho)$ and $\mathcal{G}^{-1}(S,\varrho) \subseteq \mathrm{I}_vNScl(\mathcal{G}^{-1}(P,\varrho))$. Therefore we obtain that $\mathcal{G}^{-1}(\mathrm{I}_vNSe^*cl(P,\varrho)) \subseteq \mathcal{G}^{-1}(S,\varrho) \subseteq \mathrm{I}_vNScl(\mathcal{G}^{-1}(P,\varrho))$.

Sufficiency:Assume (P,ϱ) is a I_vNSS of $(\mathbb{T},\sigma,\varrho)$ and (S,ϱ) is a I_vNSCS of $(\mathbb{W},\tau,\varrho)$ containing $\mathcal{G}^{-1}(P,\varrho)$. Put $(A,\varrho) = \mathrm{I}_vNScl(P,\varrho)$, then $(P,\varrho) \subseteq (A,\varrho)$ and (A,ϱ) is I_vNSe^*CS and $\mathcal{G}^{-1}(A,\varrho) \subseteq \mathrm{I}_vNScl(\mathcal{G}^{-1}(P,\varrho)) \subseteq (S,\varrho)$. Then by Theorem 3.4, $\mathcal{G}$ is I_vNSe^*O map. ■

Theorem 3.6. If $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ and $\mathcal{H} : (\mathbb{T},\sigma,\varrho) \to (\mathbb{U},\rho,\varrho)$ are two interval - valued neutrosophic soft maps and $\mathcal{H}\circ\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{U},\rho,\varrho)$ is I_vNSe^*O. If $\mathcal{H} : (\mathbb{T},\sigma,\varrho) \to (\mathbb{U},\rho,\varrho)$ is I_vNSe^*-irr then $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ is I_vNSe^*O map.

Proof. Let (B,ϱ) be a I_vNSOS in $(\mathbb{W},\tau,\varrho)$. Then $\mathcal{H}\circ\mathcal{G}(S,\varrho)$ is I_vNSe^*O of $(\mathbb{U},\rho,\varrho)$ because $\mathcal{H}\circ\mathcal{G}$ is I_vNSe^*O map. Since $\mathcal{H}$ is I_vNSe^* - irr and $\mathcal{H}\circ G(B,\varrho)$ is I_vNSe^*OS of $(\mathbb{U},\rho,\varrho), \mathcal{G}^{-1}(\mathcal{H}\circ\mathcal{G}(B,\varrho)) = \mathcal{G}(B,\varrho)$ is I_vNSe^*OS in $(\mathbb{T},\sigma,\varrho)$. Hence $\mathcal{G}$ is I_vNSe^*O map. ■

Theorem 3.7. If $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ is I_vNSO and $\mathcal{H} : (\mathbb{T},\sigma,\varrho) \to (\mathbb{U},\rho,\varrho)$ is I_vNSe^*O maps then $\mathcal{H}\circ\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{U},\rho,\varrho)$ is I_vNSe^*O.

Proof. Let (B,ϱ) be a I_vNSOS in $(\mathbb{W},\tau,\varrho)$. Then $\mathcal{G}(B,\varrho)$ is a I_vNSOS of $(\mathbb{T},\sigma,\varrho)$ because $\mathcal{G}$ is a I_vNSO map. Since $\mathcal{H}$ is I_vNSe^*O, $\mathcal{H}(\mathcal{G}(B,\varrho)) = \mathcal{H}\circ\mathcal{G}(B,\varrho)$ is I_vNSe^*OS of $(\mathbb{U},\rho,\varrho)$. Hence $\mathcal{H}\circ\mathcal{G}$ is I_vNSe^*O map. ■

4. Interval - valued neutrosophic soft δ - closed maps

Definition 4.1. Let $(\mathbb{W},\tau,\varrho)$ and $(\mathbb{T},\sigma,\varrho)$ be any two $\mathrm{I}_vNSts's$. A map $\mathcal{G} : (\mathbb{W},\tau,\varrho) \to (\mathbb{T},\sigma,\varrho)$ is said to be an interval - valued neutrosophic soft (resp. δ, $\delta\alpha$, δS, δP and $\delta\beta$ or e*) closed map (briefly, I_vNSC (resp. $\mathrm{I}_vNS\delta C$, $\mathrm{I}_vNS\delta\alpha C$, $\mathrm{I}_vNS\delta SC$, $\mathrm{I}_vNS\delta PC$ and $\mathrm{I}_vNS\delta\beta C$ or I_vNSe*C)) if the image of every I_vNSCS in $(\mathbb{W},\tau,\varrho)$ is a I_vNSCS (resp. $\mathrm{I}_vNS\delta CS$, $\mathrm{I}_vNS\delta\alpha CS$, $\mathrm{I}_vNS\delta SCS$, $\mathrm{I}_vNS\delta PCS$ and $\mathrm{I}_vNS\delta\beta CS$ or I_vNSe*CS) in $(\mathbb{T},\sigma,\varrho)$.

Theorem 4.1. The following statements are hold:

(i) Every $\mathrm{I}_vNS\delta$C map is an I_vNSC map.
(ii) Every I_vNSC map is an $\mathrm{I}_vNS\delta S$C map.

(iii) Every $\mathrm{I}_v NS$C map is an $\mathrm{I}_v NS\delta P$C map.
(iv) Every $\mathrm{I}_v NS\delta SC$ map is an $\mathrm{I}_v NSe^*$C map.
(v) Every $\mathrm{I}_v NS\delta P$C map is an $\mathrm{I}_v NSe^*$C map.
(vi) Every $\mathrm{I}_v NS\delta\alpha$C map is an $\mathrm{I}_v NS\delta S$C map.
(vii) Every $\mathrm{I}_v NS\delta\alpha$C map is an $\mathrm{I}_v NS\delta$PC map.

Proof.

(i) Let (P, ϱ) be an $\mathrm{I}_v NS\delta$CS in $\mathbb{W}$. Since $\mathcal{G}$ is $\mathrm{I}_v NS\delta$C map, $\mathcal{G}(P, \varrho)$ is an $\mathrm{I}_v NS\delta$CS in $\mathbb{T}$. Since every I_vNSδCS is an I_vNSCS, $\mathcal{G}(P, \varrho)$ is an I_vNSCS in $\mathbb{T}$. Hence $\mathcal{G}$ is an I_vNSC map.

The other cases are similar. ∎

The following Figure: 2 illustarte NSZO sets in neutrosophic soft topological space.

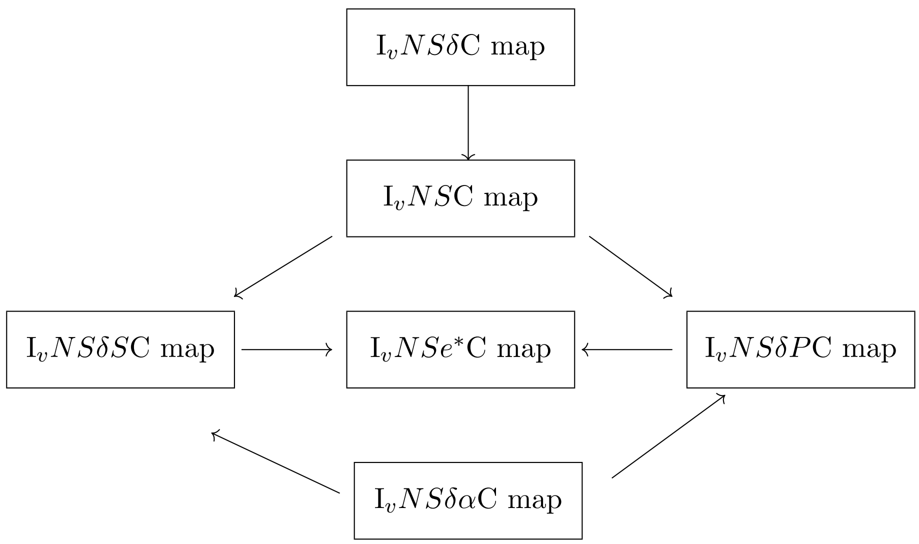

FIGURE 2. I_vNSδC maps in I_vNSts.

Example 4.1. In Example 3.1, $\mathcal{G}$ is I_vNSC mapping but not $\mathrm{I}_v NS\delta$C mapping because $(V_1, \varrho)^c$ is I_vNSCS in $\mathbb{T}$ but $\mathcal{G}(V_1, \varrho)^c = (S_1, \varrho)^c$ is not $\mathrm{I}_v NS\delta$CS in $\mathbb{T}$.

Example 4.2. In Example 3.2,

(i) $\mathcal{G}$ is $\mathrm{I}_v NS\delta$PC mapping but not I_vNSC mapping because $(V_1, \varrho)^c$ is $\mathrm{I}_v NS\delta$PCS in $\mathbb{T}$ but $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is not I_vNSCS in $\mathbb{T}$.
(ii) $\mathcal{G}$ is $\mathrm{I}_v NS\delta$PC mapping but not $\mathrm{I}_v NS\delta\alpha$C mapping because $(V_1, \varrho)^c$ is $\mathrm{I}_v NS\delta$PCS in $\mathbb{T}$ but $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is not $\mathrm{I}_v NS\delta\alpha$CS in $\mathbb{T}$.

Example 4.3. In Example 3.3,

(i) $\mathcal{G}$ is $\mathrm{I}_v NS\delta\beta$C mapping but not $\mathrm{I}_v NS\delta$SC mapping because $(V_1, \varrho)^c$ is $\mathrm{I}_v NS\delta\beta$CS in $\mathbb{T}$ but $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is not $\mathrm{I}_v NS\delta$SCS in $\mathbb{T}$.

(ii) $\mathcal{G}$ is $\mathrm{I}_v NS\delta\beta$C mapping but not $\mathrm{I}_v NS\delta$PC mapping because $(V_1, \varrho)^c$ is $\mathrm{I}_v NS\delta\beta$CS in $\mathbb{T}$ but $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is not $\mathrm{I}_v NS\delta$PCS in $\mathbb{T}$.

Example 4.4. In Example 3.4,

(i) $\mathcal{G}$ is $\mathrm{I}_v NS\delta$C mapping but not I_vNSC mapping because $(V_1, \varrho)^c$ is $\mathrm{I}_v NS\delta$SCS in $\mathbb{T}$ but $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is not I_vNSCS in $\mathbb{T}$.

(ii) $\mathcal{G}$ is $\mathrm{I}_v NS\delta$SC mapping but not $\mathrm{I}_v NS\delta\alpha$C mapping because $(V_1, \varrho)^c$ is $\mathrm{I}_v NS\delta$SCS in $\mathbb{T}$ but $\mathcal{G}(V_1, \varrho)^c = (S_4, \varrho)^c$ is not $\mathrm{I}_v NS\delta\alpha$CS in $\mathbb{T}$.

Theorem 4.2. A map $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is $\mathrm{I}_v NSe^*C$ iff for each $\mathrm{I}_v NSS(S, \varrho)$ of $(\mathbb{T}, \sigma, \varrho)$ and for each $\mathrm{I}_v NSOS(P, \varrho)$ of $(\mathbb{W}, \tau, \varrho)$ containing $\mathcal{G}^{-1}(S, \varrho)$ there is an $\mathrm{I}_v NSe^*OS(B, \varrho)$ of $(\mathbb{T}, \sigma, \varrho)$ such that $(S, \varrho) \subseteq (B, \varrho)$ and $\mathcal{G}^{-1}(B, \varrho) \subseteq (P, \varrho)$.

Proof. Necessity: Assume $\mathcal{G}$ is a $\mathrm{I}_v NSe^*C$ map. Let (S, ϱ) be the $\mathrm{I}_v NSCS$ of $(\mathbb{T}, \sigma, \varrho)$ and (P, ϱ) is a $\mathrm{I}_v NSOS$ of $(\mathbb{W}, \tau, \varrho)$ such that $\mathcal{G}^{-1}(S, \varrho) \subseteq (P, \varrho)$. Then $(B, \varrho) = \mathbb{T} - \mathcal{G}^{-1}(P, \varrho)^c$ is $\mathrm{I}_v NSe^*OS$ of $(\mathbb{T}, \sigma, \varrho)$ such that $\mathcal{G}^{-1}(B, \varrho) \subseteq (P, \varrho)$.

Sufficiency: Assume (B, ϱ) is a $\mathrm{I}_v NSCS$ of $(\mathbb{W}, \tau, \varrho)$. Then $(\mathcal{G}(B, \varrho))^c$ is a $\mathrm{I}_v NSS$ of $(\mathbb{T}, \sigma, \varrho)$ and $(B, \varrho)^c$ is $\mathrm{I}_v NSOS$ in $(\mathbb{W}, \tau, \varrho)$ such that $\mathcal{G}^{-1}((\mathcal{G}(B, \varrho))^c) \subseteq (B, \varrho)^c$. By hypothesis there is a $\mathrm{I}_v NSe^*O(B, \varrho)$ of $(\mathbb{T}, \sigma, \varrho)$ such that $(\mathcal{G}(B, \varrho))^c \subseteq (B, \varrho)$ and $\mathcal{G}^{-1}(B, \varrho) \subseteq (B, \varrho)^c$. Therefore $(B, \varrho) \subseteq (\mathcal{G}^{-1}(B, \varrho))^c$. Hence $(B, \varrho)^c \subseteq \mathcal{G}(B, \varrho) \subseteq \mathcal{G}((\mathcal{G}^{-1}(B, \varrho))^c) \subseteq (B, \varrho)^c$ which implies $\mathcal{G}(B, \varrho) = (B, \varrho)^c$. Since $(B, \varrho)^c$ is $\mathrm{I}_v NSe^*OS$ of $(\mathbb{T}, \sigma, \varrho)$. Hence $\mathcal{G}(B, \varrho)$ is $\mathrm{I}_v NSe^*C$ in $(\mathbb{T}, \sigma, \varrho)$ and thus $\mathcal{G}$ is $\mathrm{I}_v NSe^*C$ map. ■

Theorem 4.3. If $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is $\mathrm{I}_v NSC$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ is $\mathrm{I}_v NSe^*C$, then $\mathcal{H} \circ \mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{U}, \rho, \varrho)$ is $\mathrm{I}_v NSe^*C$.

Proof. Let (B, ϱ) be a $\mathrm{I}_v NSCS$ in $(\mathbb{W}, \tau, \varrho)$. Then $\mathcal{G}(B, \varrho)$ is $\mathrm{I}_v NSCS$ of $(\mathbb{T}, \sigma, \varrho)$ because $\mathcal{G}$ is $\mathrm{I}_v NSC$ map. Now $(\mathcal{H} \circ \mathcal{G})(B, \varrho) = \mathcal{H}(\mathcal{G}(B, \varrho))$ is $\mathrm{I}_v NSe^*C$ in $(\mathbb{U}, \rho, \varrho)$ because $\mathcal{H}$ is $\mathrm{I}_v NSe^*C$ map. Thus $\mathcal{H} \circ \mathcal{G}$ is $\mathrm{I}_v NSe^*C$ map. ■

Theorem 4.4. If $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ is $\mathrm{I}_v NSe^*C$ map, then $\mathrm{I}_v NSe^*cl(\mathcal{G}(B, \varrho) \subseteq \mathcal{G}(\mathrm{I}_v NScl(B, \varrho))$.

Proof. Obvious. ■

Theorem 4.5. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ be $\mathrm{I}_v NSe^*C$ maps. If every $\mathrm{I}_v NSe^*CS$ of $(\mathbb{T}, \sigma, \varrho)$ is $\mathrm{I}_v NSC$ then, $\mathcal{H} \circ \mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{U}, \rho, \varrho)$ is $\mathrm{I}_v NSe^*C$.

Proof. Let (B, ϱ) be a $\mathrm{I}_v NSCS$ in $(\mathbb{W}, \tau, \varrho)$. Then $\mathcal{G}(B, \varrho)$ is $\mathrm{I}_v NSe^*CS$ of $(\mathbb{T}, \sigma, \varrho)$ because $\mathcal{G}$ is $\mathrm{I}_v NSe^*C$ map. By hypothesis $\mathcal{G}(B, \varrho)$ is $\mathrm{I}_v NSCS$ of $(\mathbb{T}, \sigma, \varrho)$. Now $\mathcal{H}(\mathcal{G}(B, \varrho)) =$

$(\mathcal{H}\circ\mathcal{G})(B,\varrho)$ is I_vNSe*CS in $(\mathbb{U},\rho,\varrho)$ because $\mathcal{H}$ is I_vNSe^*C map. Thus $\mathcal{H}\circ\mathcal{G}$ is I_vNSe^*C map. ■

Theorem 4.6. Let $\mathcal{G}:(\mathbb{W},\tau,\varrho)\to(\mathbb{T},\sigma,\varrho)$ be an objective map, then the following statements are equivalent:

(i) $\mathcal{G}$ is a I_vNSe^*O map.

(ii) $\mathcal{G}$ is a I_vNSe^*C map.

(iii) $\mathcal{G}^{-1}$ is a I_vNSe^*Cts map.

Proof. (i) ⇒ (ii): Let us assume that $\mathcal{G}$ is a I_vNSe^*O map. By definition, (B,ϱ) is a I_vNSOS in $(\mathbb{W},\tau,\varrho)$, then $\mathcal{G}(B,\varrho)$ is a I_vNSe^*OS in $(\mathbb{T},\sigma,\varrho)$. Here, (B,ϱ) is I_vNSCS in $(\mathbb{W},\tau,\varrho)$,then $\mathbb{W}-(B,\varrho)$ is a I_vNSOS in $(\mathbb{W},\tau,\varrho)$. By assumption, $\mathcal{G}(\mathbb{W}-(B,\varrho))$ is a I_vNSe^*OS in $(\mathbb{T},\sigma,\varrho)$. Hence, $\mathbb{T}-\mathcal{G}(\mathbb{W}-(B,\varrho)$ is a I_vNSe^*CS in $(\mathbb{T},\sigma,\varrho)$. Therefore, $\mathcal{G}$ is a I_vNSe^*C map.

(ii) ⇒ (iii) : Let (B,ϱ) be a I_vNSCS in $(\mathbb{W},\tau,\varrho)$ By(ii), $\mathcal{G}(B,\varrho)$ is a I_vNSe^*CS in $(\mathbb{T},\sigma,\varrho)$. Hence, $\mathcal{G}(B,\varrho)=(\mathcal{G}^{-1})^{-1}(B,\varrho)$, so $\mathcal{G}^{-1}$ is a I_vNSe^*CS in $(\mathbb{T},\sigma,\varrho)$. Hence, $\mathcal{G}^{-1}$ is I_vNSe^*Cts.

(iii) ⇒ (i): Let (B,ϱ) be a I_vNSOS in $(\mathbb{W},\tau,\varrho)$ By(iii), $(\mathcal{G}^{-1})^{-1}(B,\varrho)=\mathcal{G}(B,\varrho)$ is a I_vNSe^*O map. ■

5. Interval - valued neutrosophic soft e^*- homeomorphism

Definition 5.1. A bijection $\mathcal{G}:(\mathbb{W},\tau,\varrho)\to(\mathbb{T},\sigma,\varrho)$ is called a I_vNSe^* -homeomorphism (briefly I_vNSe^* Hom)if $\mathcal{G}$ and $\mathcal{G}^{-1}$ are I_vNSe^*Cts.

Theorem 5.1. Each I_vNSHom is a I_vNSe^* Hom.

Proof. Let $\mathcal{G}$ be I_vNSHom, then $\mathcal{G}$ and $\mathcal{G}^{-1}$ are I_vNSCts. But every I_vNSCts function is I_vNSe^*Cts. Hence, $\mathcal{G}$ and $\mathcal{G}^{-1}$ are I_vNSe^*Cts. Therefore, $\mathcal{G}$ is a I_vNSe^* Hom. ■

Example 5.1. Let $\mathbb{W}=\{w_1,w_2,w_3\}=\{t_1,t_2,t_3\}=\mathbb{T},\varrho=\{e_1,e_2\}$ and I_vNSS (V_1,ϱ) and (V_2,ϱ) in $\mathbb{W}$ and (S_1,ϱ) in $\mathbb{T}$ are defined as

$$(V_1,e_1)=\langle(\tfrac{\mu_{w_1}}{[0.2,0.5]},\tfrac{\sigma_{w_1}}{[0.3,0.5]},\tfrac{\nu_{w_1}}{[0.6,0.7]}),(\tfrac{\mu_{w_2}}{[0.1,0.4]},\tfrac{\sigma_{w_2}}{[0.2,0.5]},\tfrac{\nu_{w_2}}{[0.5,0.8]}),(\tfrac{\mu_{w_3}}{[0.2,0.3]},\tfrac{\sigma_{w_3}}{[0.3,0.5]},\tfrac{\nu_{w_3}}{[0.6,0.7]})\rangle$$

$$(V_1,e_2)=\langle(\tfrac{\mu_{w_1}}{[0.1,0.4]},\tfrac{\sigma_{w_1}}{[0.2,0.3]},\tfrac{\nu_{w_1}}{[0.6,0.8]}),(\tfrac{\mu_{w_2}}{[0.2,0.3]},\tfrac{\sigma_{w_2}}{[0.2,0.4]},\tfrac{\nu_{w_2}}{[0.5,0.9]}),(\tfrac{\mu_{w_3}}{[0.1,0.4]},\tfrac{\sigma_{w_3}}{[0.4,0.5]},\tfrac{\nu_{w_3}}{[0.7,0.9]})\rangle$$

$$(V_2,e_1)=\langle(\tfrac{\mu_{w_1}}{[0.3,0.5]},\tfrac{\sigma_{w_1}}{[0.4,0.5]},\tfrac{\nu_{w_1}}{[0.5,0.6]}),(\tfrac{\mu_{w_2}}{[0.2,0.5]},\tfrac{\sigma_{w_2}}{[0.3,0.5]},\tfrac{\nu_{w_2}}{[0.4,0.7]}),(\tfrac{\mu_{w_3}}{[0.2,0.3]},\tfrac{\sigma_{w_3}}{[0.4,0.5]},\tfrac{\nu_{w_3}}{[0.5,0.7]})\rangle$$

$$(V_2,e_2)=\langle(\tfrac{\mu_{w_1}}{[0.3,0.4]},\tfrac{\sigma_{w_1}}{[0.3,0.5]},\tfrac{\nu_{w_1}}{[0.4,0.7]}),(\tfrac{\mu_{w_2}}{[0.3,0.5]},\tfrac{\sigma_{w_2}}{[0.3,0.5]},\tfrac{\nu_{w_2}}{[0.4,0.6]}),(\tfrac{\mu_{w_3}}{[0.3,0.5]},\tfrac{\sigma_{w_3}}{[0.5,0.5]},\tfrac{\nu_{w_3}}{[0.5,0.7]})\rangle$$

$$(S_1, e_1) = \langle(\frac{\mu_{t_1}}{[0.3,0.5]}, \frac{\sigma_{t_1}}{[0.4,0.5]}, \frac{\nu_{t_1}}{[0.4,0.6]}), (\frac{\mu_{t_2}}{[0.3,0.5]}, \frac{\sigma_{t_2}}{[0.4,0.5]}, \frac{\nu_{t_2}}{[0.3,0.6]}), (\frac{\mu_{t_3}}{[0.3,0.4]}, \frac{\sigma_{t_3}}{[0.4,0.5]}, \frac{\nu_{t_3}}{[0.4,0.6]})\rangle$$

$$(S_1, e_2) = \langle(\frac{\mu_{t_1}}{[0.3,0.5]}, \frac{\sigma_{t_1}}{[0.4,0.5]}, \frac{\nu_{t_1}}{[0.3,0.5]}), (\frac{\mu_{t_2}}{[0.4,0.5]}, \frac{\sigma_{t_2}}{[0.3,0.5]}, \frac{\nu_{t_2}}{[0.3,0.5]}), (\frac{\mu_{t_3}}{[0.4,0.5]}, \frac{\sigma_{t_3}}{[0.5,0.7]}, \frac{\nu_{t_3}}{[0.4,0.6]})\rangle$$

Then, we have $\tau = \{0_{(\mathbb{W},\varrho)}, 1_{(\mathbb{W},\varrho)}, (V_1, \varrho), (V_2, \varrho)\}$ and $\sigma = \{0_{(\mathbb{T},\varrho)}, 1_{(\mathbb{T},\varrho)}, (S_1, \varrho)\}$. Let $\mathcal{G}$: $(\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be an identity mapping, then $\mathcal{G}$ is I$_v$NSe*Hom but not I$_v$NSHom.

Theorem 5.2. $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a bijective map. If $\mathcal{G}$ is I_vNSe^*Cts, then the following statements are equivalent:

(i) $\mathcal{G}$ is a I_vNSe^*C map.

(ii) $\mathcal{G}$ is a I_vNSe^*O map.

(iii) $\mathcal{G}^{-1}$ is a I_vNSe^*Hom map.

Proof. (i) $\to$ (ii): Assume that $\mathcal{G}$ is a bijective map and a I_vNSe^*C map. Hence, $\mathcal{G}^{-1}$ is a I_vNSe^*Cts map. We know that each I_vNSOS in $(\mathbb{W}, \tau, \varrho)$ is a I_vNSe^*OS in $(\mathbb{T}, \sigma, \varrho)$. Hence, $\mathcal{G}$ is a I_vNSe^*OS map.

(ii) $\Rightarrow$ (iii) : Let $\mathcal{G}$ be a bijective and I_vNSO map. Further, $\mathcal{G}^{-1}$ is a I_vNSe^*Cts map. Hence, $\mathcal{G}$ and $\mathcal{G}^{-1}$ are I_vNSe^*Cts. Therefore, $\mathcal{G}$ is a I_vNSe^*Hom.

(iii) $\Rightarrow$ (i): Let $\mathcal{G}$ be a I_vNSe^*Hom, then $\mathcal{G}$ and $\mathcal{G}^{-1}$ are I_vNSe^*Cts. Since each I$_v$NSCS in $(\mathbb{W}, \tau, \varrho)$ is a I_vNSe^*CS in $(\mathbb{T}, \sigma, \varrho)$, $\mathcal{G}$ is a I_vNSe^*C map. ∎

Definition 5.2. A I_vNSts $(\mathbb{W}, \tau, \varrho)$ is said to be an interval valued neutrosophic soft $e^*T_{\frac{1}{2}}$ (briefly, $\mathrm{I}_vNSe^*T_{\frac{1}{2}}$)- space if every I_vNSe^*CS is I_vNSC in $(\mathbb{W}, \tau, \varrho)$.

Theorem 5.3. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a I_vNSe^*Hom, then $\mathcal{G}$ is a I_vNSHom if $(\mathbb{W}, \tau, \varrho)$ and $(\mathbb{T}, \sigma, \varrho)$ are $\mathrm{I}_vNSe^*T_{\frac{1}{2}}$)- space.

Proof. Assume that (B, ϱ) is a I_vNSCS in $(\mathbb{T}, \sigma, \varrho)$, then $\mathcal{G}^{-1}(B, \varrho)$ is a I_vNSe^*CS in $(\mathbb{W}, \tau, \varrho)$. Since $(\mathbb{W}, \tau, \varrho)$ is an $\mathrm{I}_vNSe^*T_{\frac{1}{2}}$ space, $\mathcal{G}^{-1}(B, \varrho)$ is a I_vNSCS in $(\mathbb{W}, \tau, \varrho)$. Therefore, $\mathcal{G}$ is I_vNSCts. By hypothesis, $\mathcal{G}^{-1}$ is I_vNSe^*Cts. Let (A, ϱ) be a I_vNSCS in $(\mathbb{W}, \tau, \varrho)$. Then, $(\mathcal{G}^{-1})^{-1}(A, \varrho) = \mathcal{G}(A, \varrho)$ is a I_vNSCS in $(\mathbb{T}, \sigma, \varrho)$, by presumption. Since $(\mathbb{T}, \sigma, \varrho)$ is a $\mathrm{I}_vNSe^*T_{\frac{1}{2}}$)- space, $\mathcal{G}(A, \varrho)$ is a I_vNSCS in $(\mathbb{T}, \sigma, \varrho)$. Hence, $\mathcal{G}^{-1}$ is I_vNSCts. Hence, $\mathcal{G}$ is a I_vNSHom. ∎

Theorem 5.4. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ be a I_vNSts then the following are equivalent if $(\mathbb{T}, \sigma, \varrho)$ is a $\mathrm{I}_vNSe^*T_{\frac{1}{2}}$)- space.

(i) $\mathcal{G}$ is a I_vNSe^*C map.

(ii) If (B, ϱ) is a I_vNSOS in $(\mathbb{W}, \tau, \varrho)$, then $\mathcal{G}(B, \varrho)$ is I_vNSe^*OS in $(\mathbb{T}, \sigma, \varrho)$.

(iii) $\mathcal{G}(\mathrm{I}_vNSint(B, \varrho)) \subseteq \mathrm{I}_vNScl(\mathrm{I}_vNSint(\mathcal{G}(B, \varrho)))$ for every I_vNSS (B, ϱ) in $(\mathbb{W}, \tau, \varrho)$.

Proof. (i) $\Rightarrow$ (ii): Obvious.

(ii) $\Rightarrow$ (iii): Let (B, ϱ) be a $\mathrm{I}_v NSS$ in $(\mathbb{W}, \tau, \varrho)$. Then, $\mathrm{I}_v NSint(B, \varrho)$ is a $\mathrm{I}_v NSOS$ in $(\mathbb{W}, \tau, \varrho)$. Then, $\mathcal{G}(\mathrm{I}_v NSint(B, \varrho))$ is a I_vNSe*OS in $(\mathbb{T}, \sigma, \varrho)$. Since $(\mathbb{T}, \sigma, \varrho)$ is a $\mathrm{I}_v NSe^* T_{\frac{1}{2}})$ - space, $\mathcal{G}(\mathrm{I}_v NSint(B, \varrho))$ is a $\mathrm{I}_v NSOS$ in $(\mathbb{T}, \sigma, \varrho)$.

Therefore, $\mathcal{G}(\mathrm{I}_v NSint(B, \varrho)) = \mathrm{I}_v NSint(\mathcal{G}(\mathrm{I}_v NSint(B, \varrho))) \subseteq \mathrm{I}_v NScl(\mathrm{I}_v NSint(\mathcal{G}(B, \varrho)))$.

(iii) $\Rightarrow$ (i): Let (B, ϱ) be a $\mathrm{I}_v NSCS$ in $(\mathbb{W}, \tau, \varrho)$. Then, $(B, \varrho)^c$ is a $\mathrm{I}_v NSOS$ in $(\mathbb{W}, \tau, \varrho)$. From, $\mathcal{G}(\mathrm{I}_v NSint((B, \varrho)^c)) \subseteq \mathrm{I}_v NScl(\mathrm{I}_v NSint(\mathcal{G}(B, \varrho)^c)))$. Hence, $\mathcal{G}(B, \varrho)^c \subseteq \mathrm{I}_v NScl(\mathrm{I}_v NSint(\mathcal{G}(B, \varrho)^c)))$. Therefore, $\mathcal{G}(B, \varrho)^c$ is $\mathrm{I}_v NSe^* OS$ in $(\mathbb{T}, \sigma, \varrho)$. Therefore, $\mathcal{G}(B, \varrho)$ is a $\mathrm{I}_v NSe^* CS$ in $(\mathbb{W}, \tau, \varrho)$. Hence, $\mathcal{G}$ is a $\mathrm{I}_v NSC$ map. ■

Theorem 5.5. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ be $\mathrm{I}_v NSe^* C$, where $(\mathbb{T}, \sigma, \varrho)$ and $(\mathbb{U}, \rho, \varrho)$ are two $\mathrm{I}_v NSts's$ and $(\mathbb{T}, \sigma, \varrho)$ a $\mathrm{I}_v NSe^* T_{\frac{1}{2}}$ - space, then the composition $\mathcal{H} \circ \mathcal{G}$ is $\mathrm{I}_v NSe^* C$ map.

Proof. Let (B, ϱ) be a $\mathrm{I}_v NSCS$ in $(\mathbb{W}, \tau, \varrho)$. Since $\mathcal{G}$ is $\mathrm{I}_v NSe^* C$ and $\mathcal{G}(B, \varrho)$ is a $\mathrm{I}_v NSe^* CS$ in $(\mathbb{T}, \sigma, \varrho)$, by assumption, $\mathcal{G}(B, \varrho)$ is a $\mathrm{I}_v NSCS$ in $(\mathbb{T}, \sigma, \varrho)$. Since $\mathcal{H}$ is $\mathrm{I}_v NSe^* C, \mathcal{H}(\mathcal{G}(B, \varrho))$, is $\mathrm{I}_v NSe^* C$ in $(\mathbb{U}, \rho, \varrho)$ and $\mathcal{H}(\mathcal{G}(B, \varrho)) = \mathcal{H} \circ \mathcal{G}(B, \varrho)$. Therefore, $\mathcal{H} \circ \mathcal{G}$ is $\mathrm{I}_v NSe^* C$ map. ■

Theorem 5.6. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \to (\mathbb{T}, \sigma, \varrho)$ and $\mathcal{H} : (\mathbb{T}, \sigma, \varrho) \to (\mathbb{U}, \rho, \varrho)$ be two $\mathrm{I}_v NSts's$ then the followinng hold:

(i) if $\mathcal{H} \circ \mathcal{G}$ is $\mathrm{I}_v NSe^* O$ and $\mathcal{G}$ is $\mathrm{I}_v NSCts$, then $\mathcal{H}$ is $\mathrm{I}_v NSe^* O$ map.

(ii) if $\mathcal{H} \circ \mathcal{G}$ is $\mathrm{I}_v NSO$ and $\mathcal{H}$ is $\mathrm{I}_v NSe^* Cts$, then $\mathcal{G}$ is $\mathrm{I}_v NSe^* O$ map.

Proof. Obvious. ■

6. **Conclusions**

The study presented a detailed analysis of interval-valued neutrosophic soft δ-open and δ-closed maps, along with the formulation of the $\delta\beta$-homeomorphism. The theoretical findings, supported by illustrative examples, enrich the structural understanding of mappings in interval-valued neutrosophic soft topological spaces. These contributions serve as a stepping stone for further mathematical investigation and practical advancements in the study of generalized topological structures under uncertainty.

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