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Coefficient Bounds and Fekete–Szegö Inequalities for a Subclass of Bi-Univalent Functions Defined via Neutrosophic ϱ-Poisson Distribution

Abdullah Alsoboh[1,*], Mustafa A. Sabri[2,*], Hasan Almutairi[3], Yousef Al-Qudah [4], Ala Amourah[5,6] and Abdullrahman A. Al-Maqbali[1]

[1]College of Applied and Health Sciences, ASharqiyah University, Post Box No. 42, Post Code No. 400, Ibra, Sultanate of Oman; abdullah.alsoboh@asu.edu.om, abdulrahman.almaqbali@asu.edu.om

[2]Department of Mathematics, College of Education, Mustansiriyah University, Baghdad 10052, Iraq; mustafasabri.edbs@uomustansiriyah.edu.iq

[3] Department of Mathematics, Faculty of Science, University of Hafr Albatin, Hafr Albatin 39524, Saudi Arabia; halmutairi@uhb.edu.sa

[4] Department of Mathematics, Faculty of Arts and Science, Amman Arab University, Amman, 11953, Jordan; y.alqudah@aau.edu.jo

[5]Mathematics Education Program, Faculty of Education and Arts, Sohar University, Sohar 311, Oman; aamourah@su.edu.om

[6] Jadara University Research Center, Jadara University, Jordan.

[*]Correspondence:abdullah.alsoboh@asu.edu.om; mustafasabri.edbs@uomustansiriyah.edu.iq

Abstract. In the present study, we introduce a new subclass of bi-univalent analytic functions defined on the open unit disk $\daleth$, generated via the generalized neutrosophic ϱ-Poisson distribution (NqPD) series in conjunction with the structural framework of the ϱ-Fibonacci sequence. A detailed analysis is carried out on the Taylor–Maclaurin coefficients associated with functions in this class, leading to the derivation of sharp Fekete–Szeg type inequalities that precisely estimate the initial coefficients. This unified approach not only extends several well-known results from the theory of bi-univalent functions but also uncovers new properties through a variety of corollaries and illustrative special cases. The proposed construction offers deep insights into the geometric behavior of the function class and lays a foundation for further exploration in areas such as mathematical modeling, theoretical physics, and information science.

Keywords: Analytic functions; Neutrosophic ϱ-Poisson distribution; Fibonacci numbers; Fekete–Szegö inequality; Univalent functions.

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1. Introduction

Classic analysis (CA) has played a key role in the development of geometric function theory (GFT), which provides a solid base to study analytical and analogous mapping. The strong relationship between approximately and GFT is largely due to their shared interest in border -related problems, as well as the spectral properties of both linear and not -linear differential operators. These devices are important in complex analysis in search of geometric changes, quasi-conformal maps and functional rooms. Over time, this close relationship has led to significant theoretical progress and expanded the selection of practical applications in mathematics.

Although CA methods are powerful, they often encounter boundaries when used on real world systems that include uncertainty, impenetrable or indefinite behavior. Many traditional models are based on ideal perceptions and are responsible for random or unclear structures found in natural, social or engineering systems. To address these intervals, various generalizations have been introduced - especially unclear set theory and neutrosophically set theory. These structures go beyond approximately by enabling modeling of more complex scenarios by incorporating uncertainty, such as geometric non-euclidion, irregular domains and generalized harmonious functions.

A big step in this direction was taken in the 1980s when Florentine Smarandache started neutrosophy, a philosophical basis that gave birth to a neutrosophic logic. This argument extends the unclear logic by introducing three independent components for any statement: degree (t), degree of uncertainty (i), and false degree (f), independent interval $[0, 1]$. This traditional system has found meaningful applications in various fields such as law, medicine, psychology, engineering science, economics and decision -making science, where it is important to handle ambiguity and incomplete information.

In light of these developments, the present work aims to extend classical tools in harmonic and geometric analysis by proposing a new analytic framework based on the neutrosophic ϱ-Poisson distribution (NqPD) series. This leads to the construction of a new subclass of bi-univalent analytic functions defined on the open unit disk. By incorporating q-calculusa generalization that includes a deformation parameter ϱwe are able to capture more nuanced behavior in systems that are discrete, dynamic, or uncertain. The ϱ-Fibonacci sequence, in particular, plays a key role in generating the coefficients of this function class and in deriving sharp bounds via Fekete–Szeg type inequalities.

In light of this development, the current feature is aimed at neutrosophic ϱ-Poison Distribution (NqPD) to expand classic units in hormonic and geometric analysis by suggesting a new analysis structure based on a series. This leads to the creation of a new subcontinent of two-oriented analytical functions defined on the open device disc. By including ϱ-Calculus-A-Generalization containing a deformation parameter ϱ, we can capture more nice behavior in systems that are wound up, dynamic or unsafe.

2. Preliminaries

Consider the collection $\mathscr{A}$ consisting of all functions that are analytic within the open unit disk

$$\beth = \{\Im = a + ib \in \mathbb{C} : |\Im| < 1\}.$$

Each function $\pounds \in \mathscr{A}$ is normalized such that

$$\pounds(0) = 0 \quad \text{and} \quad \pounds'(0) = 1.$$

Each function $\pounds \in \mathscr{A}$, being analytic in the unit disk, can be expressed through its Taylor–Maclaurin series as

$$\pounds(\Im) = \Im + \sum_{\iota=2}^{\infty} \eta_\iota \Im^\iota, \quad (\Im \in \beth). \tag{1}$$

where the coefficients $\eta_\iota \in \mathbb{C}$ encode essential information about the function's behavior in $\beth$. Consider the class S, consisting of all functions $\pounds \in \mathscr{A}$ that are univalent (i.e., injective) in the open unit disk $\beth$. Let $\mathscr{P}$ denote the class of functions in $\mathscr{A}$ having positive real part in $\beth$, and represented by the series

$$\Upsilon(\Im) = 1 + \sum_{\iota=1}^{\infty} \Upsilon_\iota \Im^\iota = 1 + \Upsilon_1\Im + \Upsilon_2\Im^2 + \Upsilon_3\Im^3 + \ldots, \quad (\Im \in \beth), \tag{2}$$

where

$$|\Upsilon_\iota| \leq 2, \quad \text{for all} \quad \iota \geq 1. \tag{3}$$

in accordance with the celebrated Carathodory Lemma (see, for instance, [1]). A function $\varphi \in \mathscr{P}$ if and only if it is subordinate to the Mbius function $(1 + \Im)/(1 - \Im)$, $\forall(\Im \in \beth)$.

The class of starlike functions, denoted by ι^*, can be characterized through various approaches, one of which is based on the concept of subordination. A particularly elegant and unifying formulation was introduced by Ma and Minda [2], who defined the generalized starlike class as

$$\mathsf{S}^*(\Omega) = \left\{ \pounds \in \mathscr{A} : \frac{\Im \pounds'(\Im)}{\pounds(\Im)} \prec \Omega(\Im), \quad (\Im \in \beth) \right\}.$$

By selecting suitable functions Ω, this definition captures numerous well-known subclasses of starlike functions and serves as an effective tool for exploring the geometric attributes of analytic functions. Various choices of Ω lead to distinct subclasses of S^*, as outlined in Table 1, each reflecting a unique geometric or analytic constraint.

TABLE 1. A summary of various subclasses of starlike functions characterized via the subordination principle.

Starlike function family in the unit disk	Author/s	Ref.
$\mathsf{S}^*\left(\frac{1+\Im}{1-\Im}\right) = \left\{\pounds \in \mathscr{A} : \frac{\Im\pounds'(\Im)}{\pounds(\Im)} \prec \frac{1+\Im}{1-\Im}\right\}$	Janowski	[3,4]
$\mathsf{SL}(\vartheta) = \left\{\pounds \in \mathscr{A} : \frac{\Im\pounds'(\Im)}{\pounds(\Im)} \prec \frac{1+\vartheta^2\Im^2}{1-\vartheta\Im-\vartheta^2\Im^2}, \vartheta = \frac{1-\sqrt{5}}{2}\right\}$	Sokl	[5]
$\mathsf{S}^*(\vartheta) = \left\{\pounds \in \mathscr{A} : \frac{\Im\pounds'(\Im)}{\pounds(\Im)} \prec \frac{1+(1-2\vartheta)\Im}{1-\Im}, 0 \le \vartheta < 1\right\}$	Robertson	[6]
$\mathcal{SK}(\vartheta) = \left\{\pounds \in \mathscr{A} : \frac{\Im\pounds'(\Im)}{\pounds(\Im)} \prec \frac{3}{3+(\vartheta-3)\Im-\vartheta^2\Im^2}, \vartheta \in (-3,1]\right\}$	Sokl	[7]

The class $\mathscr{P}$, often regarded as foundational in GFT, plays a central role in the development of numerous significant subclasses of analytic functions. For any function $\pounds$ in the subclass $\mathsf{S} \subset \mathscr{A}$, the univalence of $\pounds$ guarantees the existence $\pounds^{-1}$. This inverse function satisfies the classical functional identities:

$$\Im = \pounds^{-1}(\pounds(\Im)) \text{ and } \varsigma = \pounds(\pounds^{-1}(\varsigma)), \quad (\mathsf{r}_0(\pounds) \ge 0.25;\ |\varsigma| < \mathsf{r}_0(\pounds); \Im \in \daleth). \tag{4}$$

Serving as a cornerstone in AFT, the class P provides a foundational framework from which many significant subclasses emerge. For any function $\pounds$ belonging to the univalent subclass $\mathsf{S} \subset \mathscr{A}$, the existence of an inverse function $\pounds^{-1}$ is guaranteed in some neighborhood of the origin, owing to the injectivity of $\pounds$. This inverse function satisfies the fundamental identities:

$$\pounds^{-1}(\varsigma) = \chi(\varsigma) = \varsigma - \eta_2\ \varsigma^2 + \left(2\eta_2^2 - \eta_3\right)\ \varsigma^3 - \left(5\eta_2^3 + \eta_4 - 5\eta_3\eta_2\right)\ \varsigma^4 + \cdots. \tag{5}$$

A function $\pounds \in \mathsf{S}$ is said to be *bi-univalent* if both $\pounds$ and its inverse $\pounds^{-1}$ are univalent in the open unit disk. The collection of all such functions forms a distinguished subclass of S, denoted by $\sum$.

Quantum calculus, or ϱ-calculus, is a modern extension of classical analysis that replaces limits with discrete ϱ-difference operators. Originating from Jackson's foundational work [8,9], and later expanded by researchers such as Aral and Gupta [10–12], ϱ-calculus has grown into a powerful tool, particularly in GFT. With the deformation parameter ϱ controlling the granularity of the calculus, this framework allows for greater analytical flexibility and is well-suited to studying starlike and convex functions. Beyond theory, it also finds applications in quantum models, numerical methods, and ϱ-series expansions.

Definition 2.1. [13] The ϱ-number, also known as the ϱ-bracket, denoted by $\langle\mu\rangle_\varrho$, is defined by

$$\langle\mu\rangle_\varrho = \begin{cases} \frac{1-\varrho^\mu}{1-\varrho}, & 0<\varrho<1,\ \mu\in\mathbb{C}^*=\mathbb{C}\setminus\{0\} \\ 1, & \varrho\mapsto 0^+,\ \mu\in\mathbb{C}^* \\ \mu, & \varrho\mapsto 1^-,\ \mu\in\mathbb{C}^* \\ \sum_{\iota=0}^{\tau-1}\varrho^\iota, & 0<\varrho<1,\ \mu=\tau\in\{1,2,3,\cdots\}. \end{cases}$$

Definition 2.2. [13] The ϱ-difference operator, also known as the Jackson ϱ-derivative, is an operator acting on a function $\pounds$ that generalizes the classical derivative by incorporating the parameter ϱ, and is formally given by

$$\mho_\varrho\langle\pounds(\Im)\rangle = \begin{cases} \frac{\pounds(\Im)-\pounds(\varrho\Im)}{\Im-\varrho\Im}, & \text{if} \quad 0<\varrho<1,\ \Im\neq 0, \\ \pounds'(\Im), & \text{if} \quad \varrho\mapsto 1^-,\ \Im\neq 0, \\ \pounds'(0), & \text{if} \quad \Im=0. \end{cases}.$$

Remark 2.3. For $\pounds\in\mathscr{A}$ of the form (1), it is straightforward to verify that

$$\mho_\varrho\langle\pounds(\Im)\rangle = 1+\sum_{\iota=2}^{\infty}\langle\iota\rangle_\varrho\,\eta_\iota\,\Im^{\iota-1}, \quad (\Im\in\beth),$$

and for the inverse function $\pounds^{-1}$ corresponding to the representation in (4), we have the expansion

$$\mho_\varrho\langle\pounds^{-1}(\varsigma)\rangle = 1-\langle 2\rangle_\varrho\eta_2\varsigma+\langle 3\rangle_\varrho\left(2\eta_2^2-\eta_3\right)\varsigma^2-\langle 4\rangle_\varrho\left(5\eta_2^3+\eta_4-5\eta_3\eta_2\right)\varsigma^3+\cdots.$$

More recently, building upon the framework of ϱ-Jackson difference operators, Alsoboh et al. [14] introduced a significant subclass of analytic functions known as the ϱ-starlike functions, denoted by SL_ϱ. This class extends the classical notion of starlikeness within the context of ϱ-calculus, providing novel insights into the geometric behavior of analytic functions under ϱ-deformations.

The class SL_ϱ is precisely characterized by the subordination condition

$$\mathsf{SL}_\varrho = \left\{\pounds\in\mathscr{A}: \quad \frac{\Im\,\mho_\varrho\langle\pounds(\Im)\rangle}{\pounds(\Im)}\prec\Pi(\Im;\varrho) \quad (\Im\in\beth)\right\}, \tag{6}$$

where $\mho_\varrho$ denotes the ϱ-Jackson difference operator and $\prec$ indicates subordination (for instance, see [1]).

The function $\Pi(\Im;\varrho)$ is explicitly given by

$$\Pi(\Im;\varrho)=\frac{1+\varrho\vartheta_{\varrho}^{2}\Im^{2}}{1-\vartheta_{\varrho}\Im-\varrho\vartheta_{\varrho}^{2}\Im^{2}}, \tag{7}$$

and

$$\vartheta_{\varrho}=\frac{1-\sqrt{4\varrho+1}}{2\varrho}. \tag{8}$$

In this context, ϑ_{ϱ} denotes the ϱ-analogue of the classical Fibonacci numbers, reflecting a discrete deformation rooted in the structure of q-calculus. In a recent contribution, Alsoboh et al. [14] revealed a significant connection between these ϱ-Fibonacci numbers ϑ_{ϱ} and a family of associated Fibonacci-type polynomials, denoted by $\varphi_n(\varrho)$. To formalize this relationship, consider the generating function

$$\Pi(\Im;\varrho)=1+\sum_{\iota=1}^{\infty}\widehat{\Upsilon_{\iota}}\,\Im^{\iota},$$

where the coefficients $\widehat{\Upsilon_{\iota}}$ encode the combinatorial and analytic structure derived from the ϱ-Fibonacci sequence. These coefficients are governed by the following recurrence relation:

$$\widehat{\Upsilon_{\iota}}=\begin{cases}\vartheta_{\varrho}, & \text{if } \iota=1,\\ (2\varrho+1)\vartheta_{\varrho}^{2}, & \text{if } \iota=2,\\ (3\varrho+1)\vartheta_{\varrho}^{3}, & \text{if } \iota=3,\\ \left(\varphi_{\iota+1}(\varrho)+\varrho\,\varphi_{\iota-1}(\varrho)\right)\vartheta_{\varrho}^{\iota}, & \text{if } \iota\geq 4.\end{cases} \tag{9}$$

This recurrence relation highlights the interplay between the ϱ-Fibonacci numbers and the polynomial family $\varphi_n(\varrho)$, thereby enriching the algebraic and analytical foundations necessary for exploring subclasses of analytic functions within the ϱ-calculus framework. The ϱ-Fibonacci polynomials $\varphi_{\iota}(\varrho)$ are defined as

$$\varphi_{\iota}(\varrho)=\frac{(1-\varrho\vartheta_{\varrho})^{\iota}-(\vartheta_{\varrho})^{\iota}}{\sqrt{4\varrho+1}},\quad \iota\in\mathbb{N}. \tag{10}$$

In 2021, Mustafa and Nezir [44] introduced several novel subclasses of analytic and univalent functions defined on the open unit disk $\beth$. Their study further explored the application of the ϱ-Poisson distribution series in the context of analytic function theory (AFT). Specifically, a discrete random variable $\daleth$ is characterized as following a ϱ-Poisson distribution if it takes values in the set $\{0,1,2,\ldots\}$ with associated probabilities defined by

$$\mathfrak{e}_{\varrho}^{-\mu},\ \frac{\mu\mathfrak{e}_{\varrho}^{-\mu}}{\langle 1\rangle_{\varrho}!},\ \frac{\mu^{2}\mathfrak{e}_{\varrho}^{-\mu}}{\langle 2\rangle_{\varrho}!},\ \frac{\mu^{3}\mathfrak{e}_{\varrho}^{-\mu}}{\langle 3\rangle_{\varrho}!},\ \frac{\mu^{4}\mathfrak{e}_{\varrho}^{-\mu}}{\langle 4\rangle_{\varrho}!},\ldots,$$

respectively, where μ denotes the distribution parameter and

$$\mathfrak{e}_{\varrho}^{\daleth}=1+\frac{\daleth}{\langle 1\rangle_{\varrho}!}+\frac{\daleth^{2}}{\langle 2\rangle_{\varrho}!}+\frac{\daleth^{3}}{\langle 3\rangle_{\varrho}!}+\cdots+\frac{\daleth^{\mu}}{\langle \mu\rangle_{\varrho}!}+\cdots=\sum_{m=0}^{\infty}\frac{\daleth^{m}}{\langle m\rangle_{\varrho}!},$$

and

$$\langle \eth \rangle_{\varrho}! = \langle \eth \rangle_{\varrho}.\langle \eth - 1 \rangle_{\varrho} \cdots \langle 2 \rangle_{\varrho} \langle 1 \rangle_{\varrho}.$$

Therefore, within the framework of the ϱ-Poisson distribution, the probability that the discrete random variable $\Finv$ assumes the value $\eth$ is expressed by

$$\mathscr{P}_{\varrho}\big(\Finv = \eth\big) = \frac{\mu^{\eth}}{\langle \eth \rangle_{\varrho}!} \mathfrak{e}_{\varrho}^{-\mu}, \quad \eth = 0, 1, 2, \cdots .$$

The ϱ-Poisson distribution series can be expressed as

$$\Im + \sum_{\iota=2}^{\infty} \frac{\mu^{\eth-1} \mathfrak{e}_{\varrho}^{-\mu}}{\langle \eth - 1 \rangle_{\varrho}!} \Im^{\eth}, \qquad \Im \in \beth. \tag{11}$$

where μ serves as the distribution parameter. Applying the ratio test confirms that this power series converges for all complex numbers $\Im$, implying its radius of convergence is infinite (see, e.g., [16]).

Introduced by Smarandache in 1995, *neutrosophic theory* extends traditional and fuzzy logic frameworks by explicitly integrating the notion of indeterminacy into both logical reasoning and mathematical modeling [17]. Within this framework, the *neutrosophic ϱ-Poisson distribution* emerges as a generalized form of the classical ϱ-Poisson model, wherein the distribution parameter $\eth$ is treated as uncertain. Specifically, $\eth$ is represented by an interval or a finite-valued set, capturing partial truth and imprecision in a natural way.

The probability mass function of the neutrosophic ϱ-Poisson distribution is given by

$$\mathscr{N}\mathscr{P}_{\varrho}(\Finv = \mu) = \frac{(\eth_{\aleph})^{\mu}}{\langle \mu \rangle_{\varrho}!} \mathfrak{e}_{\varrho}^{-\eth_{\aleph}}, \quad \mu \in \mathbb{N} \cup \{0\},$$

where $\eth_{\aleph}$ is the neutrosophic parameter and coincides with both the neutrosophic mean and variance:

$$\mathscr{N}\mathscr{E}(\Finv) = \mathscr{N}\mathscr{V}(\Finv) = \eth_{\aleph},$$

Here, $\eth_{\aleph} = \ell + \digamma$ is called a *neutrosophic statistical number*, consisting of a determinate part ℓ and an indeterminacy component $\digamma$ [17].

In a more recent development, Alsoboh et al. [18] proposed a linear transformation $\mathscr{B}_{\eth_{\aleph}} : \mathscr{A} \to \mathscr{A}$, defined as

$$\mathscr{B}_{\eth_{\aleph}}\big(\mathscr{L}(\Im)\big) = \Im + \sum_{\iota=2}^{\infty} \frac{(\eth_{\aleph})^{\iota-1} \mathfrak{e}_{\varrho}^{-\eth_{\aleph}}}{\langle \iota - 1 \rangle_{\varrho}!} \eta_{\iota} \Im^{\iota}, \qquad \varsigma \in \beth. \tag{12}$$

As mathematical experts, we observe that the emergence of ϱ-calculus has significantly enriched AFT by enabling the construction of new subclasses endowed with intricate geometric and algebraic properties. This advancement underscores the adaptability of ϱ-calculus in extending classical frameworks and revealing novel mathematical phenomena. Its influence spans both theoretical and applied aspects, offering a solid foundation for continued research and innovation in the field [14, 19–39].

3. Bi-Univalent Function Class linked with Neutrosophic ϱ-Poisson Distribution

The framework of ϱ-calculusparticularly through the lens of ϱ-Fibonacci numbers and the Neutrosophic ϱ-Poisson distributionoffers a powerful and adaptable foundation for the formulation of new subclasses of $\sum$. Inspired by the algebraic and structural properties of these tools, we introduce two novel subclasses within the class $\sum$, denoted by $\mathsf{LM}_{\sum}(\Pi(\Im;\varrho))$ and $\mathsf{KM}_{\sum}(\Pi(\Im;\varrho))$.

Definition 3.1. A bi-univalent function $\pounds$, represented by the series expansion in (1) belong to the class $\mathsf{LM}_{\sum}(\Pi(z;\varrho))$ if and only if

$$\frac{\mathscr{B}_{\eth_{\aleph}}\left(\pounds(\Im)\right)}{\Im} \prec \Pi(\Im;\varrho) = \frac{1+\varrho\vartheta_{\varrho}^{2}\Im^{2}}{1-\vartheta_{\varrho}\,\Im-\varrho\vartheta_{\varrho}^{2}\Im^{2}}, \tag{13}$$

and

$$\frac{\mathscr{B}_{\eth_{\aleph}}\left(\chi(\varsigma)\right)}{\varsigma} \prec \Pi(\varsigma;\varrho) = \frac{1+\varrho\vartheta_{\varrho}^{2}\varsigma^{2}}{1-\vartheta_{\varrho}\,\varsigma-\varrho\vartheta_{\varrho}^{2}\varsigma^{2}}, \tag{14}$$

where $\chi=\pounds^{-1}$, ϑ_{ϱ} and $\mathscr{B}_{\eth_{\aleph}}$ are given by, (5), (8), (12), respectively, and $\Im,\varsigma\in\beth$.

Definition 3.2. A bi-univalent function $\pounds$, represented by the series expansion in (1) belong to the class $\mathsf{KM}_{\sum}(\Pi(\Im;\varrho))$ if and only if

$$\mho_{\varrho}\langle\mathscr{B}_{\eth_{\aleph}}\left(\pounds(\Im)\right)\rangle \prec \Pi(\Im;\varrho) = \frac{1+\varrho\vartheta_{\varrho}^{2}\Im^{2}}{1-\vartheta_{\varrho}\,\Im-\varrho\vartheta_{\varrho}^{2}\Im^{2}}, \tag{15}$$

and

$$\delta\mho_{\varrho}\langle\mathscr{B}_{\eth_{\aleph}}\left(\chi(\varsigma)\right)\rangle \prec \Pi(\varsigma;\varrho) = \frac{1+\varrho\vartheta_{\varrho}^{2}\varsigma^{2}}{1-\vartheta_{\varrho}\,\varsigma-\varrho\vartheta_{\varrho}^{2}\varsigma^{2}}, \tag{16}$$

where $\chi=\pounds^{-1}$, ϑ_{ϱ} and $\mathscr{B}_{\eth_{\aleph}}$ are given by, (5), (8), (12), respectively, and $\Im,\varsigma\in\beth$.

By letting different values of the parameters $\varrho\in(0,1)$, we obtain several new subclasses of $\sum$.

Example 3.3. If $\varrho\mapsto 1^{-}$, we obtain the class $\mathsf{LM}_{\sum}(\Pi(\Im))$ consisting of functions $\pounds\in\sum$ satisfying the conditions

$$\frac{\mathscr{B}_{\eth_{\aleph}}\left(\pounds(\Im)\right)}{\Im} \prec \frac{1+\vartheta^{2}\Im^{2}}{1-\vartheta\,\Im-\vartheta^{2}\Im^{2}},$$

and

$$\frac{\mathscr{B}_{\eth_{\aleph}}\left(\chi(\varsigma)\right)}{\varsigma} \prec \frac{1+\vartheta^{2}\varsigma^{2}}{1-\vartheta\varsigma-\vartheta^{2}\varsigma^{2}},$$

where $\vartheta=\frac{1-\sqrt{5}}{2}$, $\chi=\pounds^{-1}$ and $\mathscr{B}_{\eth_{\aleph}}$ are given by, (5) and (12), respectively, and $\Im,\varsigma\in\beth$.

Example 3.4. If $\varrho \mapsto 1^-$, we obtain the class $\mathsf{KM}_{\sum}(\Pi(\Im))$ consisting of functions $\pounds \in \sum$ satisfying the conditions

$$\left(\mathscr{B}_{\eth_\aleph}\left(\pounds(\Im)\right)\right)' \prec \frac{1+\vartheta^2\Im^2}{1-\vartheta\,\Im-\vartheta^2\Im^2},$$

and

$$\left(\mathscr{B}_{\eth_\aleph}\left(\chi(\varsigma)\right)\right)' \prec \frac{1+\vartheta^2\varsigma^2}{1-\vartheta\varsigma-\vartheta^2\varsigma^2},$$

where $\vartheta = \frac{1-\sqrt{5}}{2}$, $\chi = \pounds^{-1}$ and $\mathscr{B}_{\eth_\aleph}$ are given by, (5) and (12), respectively, and $\Im, \varsigma \in \beth$.

4. Coefficient Bounds of the Subclass $\mathsf{LM}_{\sum}(\Pi(\Im;\varrho))$

In this section, we first obtain the estimate of the initial Taylor coefficients $|\eta_2|$ and $|\eta_2|$ for functions in the class $\mathsf{LM}_{\sum}(\Pi(\Im;\varrho))$ as per in Definition 3.1.

Firstly, let $\Upsilon(\Im) = 1 + \Upsilon_1\Im + \Upsilon_2\Im^2 + \Upsilon_3\Im^3 + \dots$, and $\Upsilon(\Im) \prec \Pi(\Im;\varrho)$. Then there exist $\varphi \in \mathsf{P}$ such that $|\varphi(\Im)| < 1$ in $\beth$ and $\Upsilon(\Im) = \Pi(\varphi(\Im);\varrho)$, we have

$$\hbar(\Im) = \frac{1+\varphi(\Im)}{1-\varphi(\Im)} = 1 + \epsilon_1\Im + \epsilon_2\Im^2 + \cdots \in \mathsf{P} \qquad (\Im \in \beth). \tag{17}$$

It follows that

$$\varphi(\Im) = \frac{\epsilon_1\Im}{2} + \left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\frac{\Im^2}{2} + \left(\epsilon_3 - \epsilon_1\epsilon_2 - \frac{\epsilon_1^3}{4}\right)\frac{\Im^3}{2} + \cdots, \tag{18}$$

and

$$\begin{aligned}
\Pi(\varphi(\Im);\varrho) = 1 &+ \widehat{\Upsilon}_1\left[\frac{\epsilon_1\Im}{2} + \left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\frac{\Im^2}{2} + \left(\epsilon_3 - \epsilon_1\epsilon_2 - \frac{\epsilon_1^3}{4}\right)\frac{\Im^3}{2} + \cdots\right] \\
&+ \widehat{\Upsilon}_2\left[\frac{\epsilon_1\Im}{2} + \left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\frac{\Im^2}{2} + \left(\epsilon_3 - \epsilon_1\epsilon_2 - \frac{\epsilon_1^3}{4}\right)\frac{\Im^3}{2} + \cdots\right]^2 \\
&+ \widehat{\Upsilon}_3\left[\frac{\epsilon_1\Im}{2} + \left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\frac{\Im^2}{2} + \left(\epsilon_3 - \epsilon_1\epsilon_2 - \frac{\epsilon_1^3}{4}\right)\frac{\Im^3}{2} + \cdots\right]^3 + \cdots \\
&= 1 + \frac{\widehat{\Upsilon}_1\epsilon_1}{2}\Im + \frac{1}{2}\left[\left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\widehat{\Upsilon}_1 + \frac{\epsilon_1^2}{2}\widehat{\Upsilon}_2\right]\Im^2 \\
&+ \frac{1}{2}\left[\left(\epsilon_3 - \epsilon_1\epsilon_2 + \frac{\epsilon_1^3}{4}\right)\widehat{\Upsilon}_1 + \epsilon_1\left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\widehat{\Upsilon}_2 + \frac{\epsilon_1^3}{4}\widehat{\Upsilon}_3\right]\Im^3 + \cdots.
\end{aligned} \tag{19}$$

And similarly, there exists an analytic function ν such that $|\nu(\varsigma)| < 1$ in $\beth$ and $\Upsilon(\varsigma) = \Pi(\nu(\varsigma);\varrho)$. Therefore, the function

$$\kappa(\varsigma) = \frac{1+\nu(\varsigma)}{1-\nu(\varsigma)} = 1 + o_1\varsigma + o_2\varsigma^2 + \cdots \in \mathsf{P}. \tag{20}$$

It follows that

$$\nu(\varsigma) = \frac{o_1\varsigma}{2} + \left(o_2 - \frac{o_1^2}{2}\right)\frac{\varsigma^2}{2} + \left(o_3 - o_1o_2 - \frac{o_1^3}{4}\right)\frac{\varsigma^3}{2} + \cdots, \tag{21}$$

and

$$\Pi(\nu(\varsigma);\varrho) = 1 + \frac{\widehat{\Upsilon_1}o_1}{2}\varsigma + \frac{1}{2}\left[\left(o_2 - \frac{o_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{o_1^2}{2}\widehat{\Upsilon_2}\right]\varsigma^2 + \frac{1}{2}\left[\left(o_3 - o_1o_2 + \frac{o_1^3}{4}\right)\widehat{\Upsilon_1} + o_1\left(o_2 - \frac{o_1^2}{2}\right)\widehat{\Upsilon_2} + \frac{o_1^3}{4}\widehat{\Upsilon_3}\right]\varsigma^3 + \cdots. \tag{22}$$

Theorem 4.1. *Let* $\pounds \in \mathsf{LM}_{\sum}(\Pi(\Im;\varrho))$. *Then*

$$|\eta_2| \leq \frac{\mathfrak{e}_{\varrho}^{2\eth_{\aleph}}|\vartheta_{\varrho}|\sqrt{\langle 2\rangle_{\varrho}!}}{|\eth_{\aleph}|\sqrt{\left|\mathfrak{e}_{\varrho}^{\eth_{\aleph}}\vartheta_{\varrho} + \langle 2\rangle_{\varrho}![1-(2\varrho+1)\vartheta_{\varrho}]\right|}}, \tag{23}$$

and

$$|\eta_3| \leq |\eta_2|^2 + \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}|\vartheta_{\varrho}|}{\eth_{\aleph}^2}. \tag{24}$$

Proof. Let $\pounds \in \mathsf{LM}_{\sum}(\delta;\Pi(\Im;\varrho))$ and $\chi = \pounds^{-1}$. Considering (15) and (16) we have

$$\frac{\mathscr{B}_{\eth_{\aleph}}(\pounds(\Im))}{\Im} = \Pi(\varphi(\Im);\varrho), \qquad (\Im \in \beth), \tag{25}$$

and

$$\frac{\mathscr{B}_{\eth_{\aleph}}(\chi(\varsigma))}{\varsigma} = \Pi(\nu(\varsigma);\varrho), \qquad (\varsigma \in \beth). \tag{26}$$

Since

$$\frac{\mathscr{B}_{\eth_{\aleph}}(\pounds(\Im))}{\Im} = \frac{\widehat{\Upsilon_1}\epsilon_1}{2}\Im + \frac{1}{2}\left[\left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{\epsilon_1^2}{2}\widehat{\Upsilon_2}\right]\Im^2 + \cdots. \tag{27}$$

and

$$\frac{\mathscr{B}_{\eth_{\aleph}}(\chi(\varsigma))}{\varsigma} = \frac{\widehat{\Upsilon_1}o_1}{2}\varsigma + \frac{1}{2}\left[\left(o_2 - \frac{o_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{o_1^2}{2}\widehat{\Upsilon_2}\right]\varsigma^2 + \cdots. \tag{28}$$

By comparing (25) and (27), along (19), yields

$$\frac{\eth_{\aleph}}{\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\eta_2\Im + \frac{\eth_{\aleph}^2}{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\eta_3\Im^2 + \cdots = \frac{\widehat{\Upsilon_1}\epsilon_1}{2}\Im + \frac{1}{2}\left[\left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{\epsilon_1^2}{2}\widehat{\Upsilon_2}\right]\Im^2 + \cdots. \tag{29}$$

Beside that, By comparing (26) and (28), along (22), yields

$$\frac{-\eth_{\aleph}}{\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\eta_2\Im + \frac{\eth_{\aleph}^2}{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\left(2\eta_2^2 - \eta_3\right)\Im^2 + \cdots = \frac{\widehat{\Upsilon_1}o_1}{2}\varsigma + \frac{1}{2}\left[\left(o_2 - \frac{o_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{o_1^2}{2}\widehat{\Upsilon_2}\right]\varsigma^2 + \cdots. \tag{30}$$

Equating the pertinent coefficient in (29) and (30), we obtain

$$\frac{m_{\aleph}}{\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\ \eta_2 = \frac{\widehat{\Upsilon}_1 \epsilon_1}{2} \tag{31}$$

$$-\frac{\eth_{\aleph}}{\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\ \eta_2 = \frac{\widehat{\Upsilon}_1 o_1}{2} \tag{32}$$

$$\frac{\eth_{\aleph}^2}{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\ \eta_3 = \frac{1}{2}\left[\left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{\epsilon_1^2}{2}\widehat{\Upsilon_2}\right] \tag{33}$$

$$\frac{\eth_{\aleph}^2}{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\ \left(2\eta_2^2 - \eta_3\right) = \frac{1}{2}\left[\left(o_2 - \frac{o_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{o_1^2}{2}\widehat{\Upsilon_2}\right] \tag{34}$$

From (31) and (32), we have

$$\epsilon_1 = -o_1 \quad \Longleftrightarrow \quad \epsilon_1^2 = o_1^2, \tag{35}$$

and

$$\eta_2^2 = \frac{\mathfrak{e}_{\varrho}^{2\eth_{\aleph}}\vartheta_{\varrho}^2}{2\eth_{\aleph}^2}\left(\epsilon_1^2 + o_1^2\right) \quad \Longleftrightarrow \quad \epsilon_1^2 + o_1^2 = \frac{2\eth_{\aleph}^2}{\mathfrak{e}_{\varrho}^{2\eth_{\aleph}}\vartheta_{\varrho}^2}\eta_2^2. \tag{36}$$

Now, by summing (33) and (34), we obtain

$$\frac{2\eth_{\aleph}^2}{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}}\eta_2^2 = \frac{(\epsilon_2 + o_2)\vartheta_{\varrho}}{2} + \left[\frac{(2\varrho+1)\vartheta_{\varrho}^2}{4} - \frac{\vartheta_{\varrho}}{4}\right]\left(\epsilon_1^2 + o_1^2\right). \tag{37}$$

By putting (36) in (37), we obtain

$$\eta_2^2 = \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{2\eth_{\aleph}}\vartheta_{\varrho}^2}{4\eth_{\aleph}^2\left(\mathfrak{e}_{\varrho}^{\eth_{\aleph}}\vartheta_{\varrho} + \langle 2\rangle_{\varrho}!\left[1-(2\varrho+1)\vartheta_{\varrho}\right]\right)}(\epsilon_2 + o_2). \tag{38}$$

Using (3) for (38), we have

$$\left|\eta_2\right| \le \frac{\mathfrak{e}_{\varrho}^{2\eth_{\aleph}}|\vartheta_{\varrho}|\sqrt{\langle 2\rangle_{\varrho}!}}{|\eth_{\aleph}|\sqrt{\left|\mathfrak{e}_{\varrho}^{\eth_{\aleph}}\vartheta_{\varrho} + \langle 2\rangle_{\varrho}!\left[1-(2\varrho+1)\vartheta_{\varrho}\right]\right|}}. \tag{39}$$

Now, so as to find the bound on $|\eta_3|$, let's subtract from (33) and (34) along (36), we obtain

$$\eta_3 = \eta_2^2 + \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}\vartheta_{\varrho}}{4\eth_{\aleph}^2}(\epsilon_2 - o_2). \tag{40}$$

Hence, we get

$$\left|\eta_3\right| \le \left|\eta_2\right|^2 + \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}|\vartheta_{\varrho}|}{\eth_{\aleph}^2}. \tag{41}$$

Then, in view of (39), we obtain

$$\left|\eta_3\right| \le \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{2\eth_{\aleph}}\vartheta_{\varrho}^2}{\eth_{\aleph}^2\left(\mathfrak{e}_{\varrho}^{\eth_{\aleph}}\vartheta_{\varrho} + \langle 2\rangle_{\varrho}!\left[1-(2\varrho+1)\vartheta_{\varrho}\right]\right)} + \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}|\vartheta_{\varrho}|}{\eth_{\aleph}^2}. \tag{42}$$

with doing simple calculation, we have

$$\left|\eta_3\right| \le \frac{\langle 2\rangle_{\varrho}!\,\mathfrak{e}_{\varrho}^{\eth_{\aleph}}\,|\vartheta_{\varrho}|\left|\left(2\mathfrak{e}_{\varrho}^{\eth_{\aleph}} - \langle 2\rangle_{\varrho}!(2\varrho+1)\right)|\vartheta_{\varrho}| + \langle 2\rangle_{\varrho}!\right|}{\eth_{\aleph}^2\left|\left(\mathfrak{e}_{\varrho}^{\eth_{\aleph}} - \langle 2\rangle_{\varrho}!(2\varrho+1)\right)\vartheta_{\varrho} + \langle 2\rangle_{\varrho}!\right|}. \tag{43}$$

This proves (24). □

Theorem 4.2. *For* $\flat \in \mathbb{R} \setminus \{0\}$, *let* $\pounds \in \mathsf{LM}_{\sum}(\delta; \Pi(\Im; \varrho))$. *Then*

$$\left|\eta_3 - \flat\ \eta_2^2\right| \leq \begin{cases} \frac{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth\aleph} |\vartheta_\varrho|}{\eth_\aleph^2} & |1-\flat| \leq \left|1 - \frac{\langle 2\rangle_\varrho ! \left[(2\varrho+1)\vartheta_\varrho - 1\right]}{\mathfrak{e}_\varrho^{\eth\aleph}\ \vartheta_\varrho}\right| \\ \frac{|1-\flat|\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{2\eth\aleph}\ \vartheta_\varrho^2}{\left|\eth_\aleph^2\left(\mathfrak{e}_\varrho^{\eth\aleph}\ \vartheta_\varrho + \langle 2\rangle_\varrho ! \left[1-(2\varrho+1)\vartheta_\varrho\right]\right)\right|} & |1-\flat| \geq \left|1 - \frac{\langle 2\rangle_\varrho ! \left[(2\varrho+1)\vartheta_\varrho - 1\right]}{\mathfrak{e}_\varrho^{\eth\aleph}\ \vartheta_\varrho}\right| \end{cases} \tag{44}$$

Proof. Let $\pounds \in \mathsf{LM}_{\sum}(\delta; \Pi(\Im; \varrho))$, from (38) and (40) we have

$$\begin{aligned} \eta_3 - \flat\ \eta_2^2 =& \frac{(1-\flat)\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{2\eth\aleph}\ \vartheta_\varrho^2(\epsilon_2 + o_2)}{4\eth_\aleph^2\left(\mathfrak{e}_\varrho^{\eth\aleph}\ \vartheta_\varrho + \langle 2\rangle_\varrho ! \left[1-(2\varrho+1)\vartheta_\varrho\right]\right)} + \frac{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth\aleph}\vartheta_\varrho}{4\eth_\aleph^2}(\epsilon_2 - o_2) \\ =& \left(\mathscr{K}(\flat) + \frac{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth\aleph}\vartheta_\varrho}{4\eth_\aleph^2}\right)\epsilon_2 + \left(\mathscr{K}(\flat) - \frac{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth\aleph}\vartheta_\varrho}{4\eth_\aleph^2}\right)o_2. \end{aligned} \tag{45}$$

where

$$\mathscr{K}(\flat) = \frac{(1-\flat)\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{2\eth\aleph}\ \vartheta_\varrho^2}{4\eth_\aleph^2\left(\mathfrak{e}_\varrho^{\eth\aleph}\ \vartheta_\varrho + \langle 2\rangle_\varrho ! \left[1-(2\varrho+1)\vartheta_\varrho\right]\right)}. \tag{46}$$

Then, by taking modulus of (45), we conclude that

$$\left|\eta_3 - \flat\ \eta_2^2\right| \leq \begin{cases} \frac{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth\aleph} |\vartheta_\varrho|}{\eth_\aleph^2}, & 0 \leq \left|\mathscr{K}(\flat)\right| \leq \frac{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth\aleph} |\vartheta_\varrho|}{4\eth_\aleph^2} \\ 4\left|\mathscr{K}(\flat)\right|, & \left|\mathscr{K}(\eth)\right| \geq \frac{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth\aleph} |\vartheta_\varrho|}{4\eth_\aleph^2} \end{cases}$$

□

5. Coefficient Bounds of the Subclass $\mathsf{KM}_{\sum}(\Pi(\Im; \varrho))$

In this section, we first obtain the estimate of the initial Taylor coefficients $|\eta_2|$ and $|\eta_2|$ for functions in the class $\mathsf{KM}_{\sum}(\Pi(\Im; \varrho))$ as per in Definition 3.2.

Theorem 5.1. *For* $\eth \in \mathbb{R} \setminus \{0\}$, *let* $\pounds \in \mathsf{KM}_{\sum}(\Pi(\Im; \varrho))$. *Then*

$$\left|\eta_2\right| \leq \frac{\mathfrak{e}_\varrho^{2\eth\aleph}|\vartheta_\varrho|\sqrt{\langle 2\rangle_\varrho !}}{|\eth_\aleph|\sqrt{\left|(1+\varrho\langle 2\rangle_\varrho)\mathfrak{e}_\varrho^{\eth\aleph}\ \vartheta_\varrho + \langle 2\rangle_\varrho^3\left[1-(2\varrho+1)\vartheta_\varrho\right]\right|}}. \tag{47}$$

$$\left|\eta_3\right| \leq \frac{\langle 2\rangle_\varrho !\ \mathfrak{e}_\varrho^{\eth\aleph}\ |\vartheta_\varrho| \left|\left(2\mathfrak{e}_\varrho^{\eth\aleph}\ (1+\varrho\langle 2\rangle_\varrho)| - \langle 2\rangle_\varrho^3(2\varrho+1)\right)|\vartheta_\varrho| + \langle 2\rangle_\varrho^3\right|}{\eth_\aleph^2(1+\varrho\langle 2\rangle_\varrho)\left|\left((1+\varrho\langle 2\rangle_\varrho)\mathfrak{e}_\varrho^{\eth\aleph} - \langle 2\rangle_\varrho^3(2\varrho+1)\right)\vartheta_\varrho + \langle 2\rangle_\varrho^3\right|}. \tag{48}$$

Proof. Let $\pounds \in \mathsf{LM}_{\sum}(\delta; \Pi(\Im; \varrho))$ and $\chi = \pounds^{-1}$. Considering (15) and (16) we have

$$\mho_\varrho \langle \mathscr{B}_{\eth_\aleph} \left(\pounds(\Im) \right) \rangle = \Pi(\varphi(\Im); \varrho), \qquad (\Im \in \beth), \tag{49}$$

and

$$\mho_\varrho \langle \mathscr{B}_{\eth_\aleph} \left(\chi(\varsigma) \right) \rangle = \Pi(\nu(\varsigma); \varrho), \qquad (\varsigma \in \beth). \tag{50}$$

Since

$$\mho_\varrho \langle \mathscr{B}_{\eth_\aleph} \left(\pounds(\Im) \right) \rangle = \frac{\widehat{\Upsilon_1}\epsilon_1}{2}\Im + \frac{1}{2}\left[\left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{\epsilon_1^2}{2}\widehat{\Upsilon_2}\right]\Im^2 + \cdots . \tag{51}$$

and

$$\mho_\varrho \langle \mathscr{B}_{\eth_\aleph} \left(\chi(\varsigma) \right) \rangle = \frac{\widehat{\Upsilon_1}o_1}{2}\varsigma + \frac{1}{2}\left[\left(o_2 - \frac{o_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{o_1^2}{2}\widehat{\Upsilon_2}\right]\varsigma^2 + \cdots . \tag{52}$$

By comparing (49) and (51), along (19), yields

$$\frac{\langle 2\rangle_\varrho \eth_\aleph}{\mathfrak{e}_\varrho^{\eth_\aleph}}\eta_2\Im + \frac{\langle 3\rangle_\varrho (\eth_\aleph)^2}{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth_\aleph}}\eta_3\Im^2 + \cdots = \frac{\widehat{\Upsilon_1}\epsilon_1}{2}\Im + \frac{1}{2}\left[\left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{\epsilon_1^2}{2}\widehat{\Upsilon_2}\right]\Im^2 + \cdots . \tag{53}$$

Beside that, By comparing (50) and (52), along (22), yields

$$\frac{-\langle 2\rangle_\varrho \eth_\aleph}{\mathfrak{e}_\varrho^{\eth_\aleph}}\eta_2\Im + \frac{\langle 3\rangle_\varrho (\eth_\aleph)^2}{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth_\aleph}}\left(2\eta_2^2 - \eta_3\right)\Im^2 + \cdots = \frac{\widehat{\Upsilon_1}o_1}{2}\varsigma + \frac{1}{2}\left[\left(o_2 - \frac{o_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{o_1^2}{2}\widehat{\Upsilon_2}\right]\varsigma^2 + \cdots . \tag{54}$$

Equating the pertinent coefficient in (53) and (54), we obtain

$$\frac{\langle 2\rangle_\varrho m_\aleph}{\mathfrak{e}_\varrho^{\eth_\aleph}}\ \eta_2 = \frac{\widehat{\Upsilon_1}\epsilon_1}{2} \tag{55}$$

$$-\frac{\langle 2\rangle_\varrho \eth_\aleph}{\mathfrak{e}_\varrho^{\eth_\aleph}}\ \eta_2 = \frac{\widehat{\Upsilon_1}o_1}{2} \tag{56}$$

$$\frac{\langle 3\rangle_\varrho \eth_\aleph^2}{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth_\aleph}}\ \eta_3 = \frac{1}{2}\left[\left(\epsilon_2 - \frac{\epsilon_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{\epsilon_1^2}{2}\widehat{\Upsilon_2}\right] \tag{57}$$

$$\frac{\langle 3\rangle_\varrho \eth_\aleph^2}{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth_\aleph}}\ \left(2\eta_2^2 - \eta_3\right) = \frac{1}{2}\left[\left(o_2 - \frac{o_1^2}{2}\right)\widehat{\Upsilon_1} + \frac{o_1^2}{2}\widehat{\Upsilon_2}\right] \tag{58}$$

From (55) and (56), we have

$$\epsilon_1 = -o_1 \quad \Longleftrightarrow \quad \epsilon_1^2 = o_1^2, \tag{59}$$

and

$$\eta_2^2 = \frac{\mathfrak{e}_\varrho^{2\eth_\aleph}\vartheta_\varrho^2}{8\langle 2\rangle_\varrho^2 \eth_\aleph^2}\left(\epsilon_1^2 + o_1^2\right) \quad \Longleftrightarrow \quad \epsilon_1^2 + o_1^2 = \frac{8\langle 2\rangle_\varrho^2 \eth_\aleph^2}{\mathfrak{e}_\varrho^{2\eth_\aleph}\vartheta_\varrho^2}\eta_2^2. \tag{60}$$

Now, by summing (57) and (58), we obtain

$$\frac{2\langle 3\rangle_\varrho \eth_\aleph^2}{\langle 2\rangle_\varrho ! \mathfrak{e}_\varrho^{\eth_\aleph}}\eta_2^2 = \frac{(\epsilon_2 + o_2)\vartheta_\varrho}{2} + \left[\frac{(2\varrho+1)\vartheta_\varrho^2}{4} - \frac{\vartheta_\varrho}{4}\right]\left(\epsilon_1^2 + o_1^2\right). \tag{61}$$

By putting (60) in (61), we obtain

$$\eta_2^2 = \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{2\Game_{\aleph}}\,\vartheta_{\varrho}^2}{4\Game_{\aleph}^2\left(\langle 3\rangle_{\varrho}\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,\vartheta_{\varrho}+\langle 2\rangle_{\varrho}!\left[1-(2\varrho+1)\vartheta_{\varrho}\right]\langle 2\rangle_{\varrho}^2\right)}(\epsilon_2+o_2). \tag{62}$$

Using (3) for (62), we have

$$\left|\eta_2\right| \le \frac{\mathfrak{e}_{\varrho}^{2\Game_{\aleph}}|\vartheta_{\varrho}|\sqrt{\langle 2\rangle_{\varrho}!}}{|\Game_{\aleph}|\sqrt{\left|\langle 3\rangle_{\varrho}\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,\vartheta_{\varrho}+\langle 2\rangle_{\varrho}!\left[1-(2\varrho+1)\vartheta_{\varrho}\right]\langle 2\rangle_{\varrho}^2\right|}}. \tag{63}$$

Now, so as to find the bound on $|\eta_3|$, let's subtract from (57) and (58) along (60), we obtain

$$\eta_3 = \eta_2^2 + \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\vartheta_{\varrho}}{4\langle 3\rangle_{\varrho}\Game_{\aleph}^2}(\epsilon_2-o_2). \tag{64}$$

Hence, we get

$$\left|\eta_3\right| \le \left|\eta_2\right|^2 + \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\Game_{\aleph}}|\vartheta_{\varrho}|}{\langle 3\rangle_{\varrho}\Game_{\aleph}^2}. \tag{65}$$

Then, in view of (63), we obtain

$$\left|\eta_3\right| \le \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{2\Game_{\aleph}}\,\vartheta_{\varrho}^2}{\Game_{\aleph}^2\left(\langle 3\rangle_{\varrho}\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,\vartheta_{\varrho}+\langle 2\rangle_{\varrho}!\left[1-(2\varrho+1)\vartheta_{\varrho}\right]\langle 2\rangle_{\varrho}^2\right)} + \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\Game_{\aleph}}|\vartheta_{\varrho}|}{\langle 3\rangle_{\varrho}\Game_{\aleph}^2}. \tag{66}$$

with doing simple calculation, we have

$$\left|\eta_3\right| \le \frac{\langle 2\rangle_{\varrho}!\,\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,|\vartheta_{\varrho}|\left|\left(2\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,\langle 3\rangle_{\varrho}|-\langle 2\rangle_{\varrho}!(2\varrho+1)\langle 2\rangle_{\varrho}^2\right)|\vartheta_{\varrho}|+\langle 2\rangle_{\varrho}^3\right|}{\Game_{\aleph}^2\langle 3\rangle_{\varrho}\left|\left(\langle 3\rangle_{\varrho}\mathfrak{e}_{\varrho}^{\Game_{\aleph}}-(2\varrho+1)\langle 2\rangle_{\varrho}^3\right)\vartheta_{\varrho}+\langle 2\rangle_{\varrho}^3\right|}. \tag{67}$$

This proves (48). □

Theorem 5.2. *Let* $\pounds \in \mathsf{KM}_{\sum}(\Pi(\Im;\varrho))$*. Then*

$$\left|\eta_3-\flat\,\eta_2^2\right| \le \begin{cases} \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\Game_{\aleph}}|\vartheta_{\varrho}|}{\langle 3\rangle_{\varrho}\Game_{\aleph}^2} & |1-\flat| \le \left|1-\frac{\langle 2\rangle_{\varrho}^3[(2\varrho+1)\vartheta_{\varrho}-1]}{\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,\vartheta_{\varrho}\langle 3\rangle_{\varrho}}\right| \\ \frac{|1-\flat|\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{2\Game_{\aleph}}\,\vartheta_{\varrho}^2}{\left|\Game_{\aleph}^2\left(\langle 3\rangle_{\varrho}\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,\vartheta_{\varrho}+\langle 2\rangle_{\varrho}^3[1-(2\varrho+1)\vartheta_{\varrho}]\right)\right|} & |1-\flat| \ge \left|1-\frac{\langle 2\rangle_{\varrho}^3\left[(2\varrho+1)\vartheta_{\varrho}-1\right]}{\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,\vartheta_{\varrho}\langle 3\rangle_{\varrho}}\right| \end{cases} \tag{68}$$

Proof. Let $\pounds \in \mathsf{KM}_{\sum}(\Pi(\Im;\varrho))$, from (62) and (64) we have

$$\begin{aligned} \eta_3-\flat\,\eta_2^2 =& \frac{(1-\flat)\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{2\Game_{\aleph}}\,\vartheta_{\varrho}^2(\epsilon_2+o_2)}{4\Game_{\aleph}^2\left(\langle 3\rangle_{\varrho}\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\,\vartheta_{\varrho}+\langle 2\rangle_{\varrho}^3\left[1-(2\varrho+1)\vartheta_{\varrho}\right]\right)} \\ &+\frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\vartheta_{\varrho}}{4\langle 3\rangle_{\varrho}\Game_{\aleph}^2}(\epsilon_2-o_2) \\ =& \left(\mathscr{K}(\flat)+\frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\vartheta_{\varrho}}{4\langle 3\rangle_{\varrho}\Game_{\aleph}^2}\right)\epsilon_2+\left(\mathscr{K}(\flat)-\frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\Game_{\aleph}}\vartheta_{\varrho}}{4\langle 3\rangle_{\varrho}\Game_{\aleph}^2}\right)o_2. \end{aligned} \tag{69}$$

where

$$\mathscr{K}(\flat) = \frac{(1-\flat)\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{2\eth_{\aleph}}\,\vartheta_{\varrho}^{2}}{4\eth_{\aleph}^{2}\left(\langle 3\rangle_{\varrho}\mathfrak{e}_{\varrho}^{\eth_{\aleph}}\,\vartheta_{\varrho} + \langle 2\rangle_{\varrho}^{3}[1-(2\varrho+1)\vartheta_{\varrho}]\right)}. \tag{70}$$

Then, by taking modulus of (69), we conclude that

$$\left|\eta_3 - \flat\,\eta_2^2\right| \leq \begin{cases} \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}|\vartheta_{\varrho}|}{\langle 3\rangle_{\varrho}\eth_{\aleph}^{2}}, & 0 \leq \left|\mathscr{K}(\flat)\right| \leq \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}|\vartheta_{\varrho}|}{4\langle 3\rangle_{\varrho}\eth_{\aleph}^{2}} \\ 4\left|\mathscr{K}(\flat)\right|, & \left|\mathscr{K}(\eth)\right| \geq \frac{\langle 2\rangle_{\varrho}!\mathfrak{e}_{\varrho}^{\eth_{\aleph}}|\vartheta_{\varrho}|}{4\langle 3\rangle_{\varrho}\eth_{\aleph}^{2}} \end{cases}$$

□

6. Corollaries

If $\varrho \mapsto 1^{-}$, we obtain the following results for the class $\mathsf{LM}_{\sum}(\Pi(\Im))$ defined in Example (3.3)

Corollary 6.1. *Let £ given by* (1) *be in the class* $\mathsf{LM}_{\sum}(0;\Pi(\Im))$*. Then*

$$\left|\eta_2\right| \leq \frac{\sqrt{2}\mathfrak{e}^{2\eth_{\aleph}}|\vartheta|}{|\eth_{\aleph}|\sqrt{\left|\left(\mathfrak{e}^{\eth_{\aleph}}-6\right)\vartheta+2\right|}},$$

$$\left|\eta_3\right| \leq \frac{2\,\mathfrak{e}^{\eth_{\aleph}}\,|\vartheta|\left|\left(2\mathfrak{e}^{\eth_{\aleph}}-6\right)\vartheta+2\right|}{\eth_{\aleph}^{2}\left|\left(\mathfrak{e}^{\eth_{\aleph}}-6\right)\vartheta_{\varrho}+2\right|},$$

and

$$\left|\eta_3 - \flat\,\eta_2^2\right| \leq \begin{cases} \frac{2\mathfrak{e}^{\eth_{\aleph}}|\vartheta|}{\eth_{\aleph}^{2}} & |1-\flat| \leq \left|1 - \frac{2(3\vartheta-1)}{\mathfrak{e}^{\eth_{\aleph}}\,\vartheta}\right| \\ \frac{2\mathfrak{e}^{2\eth_{\aleph}}\,\vartheta^{2}|1-\flat|}{\left|\eth_{\aleph}^{2}\left(\left(\mathfrak{e}^{\eth_{\aleph}}-6\right)\vartheta+2\right)\right|} & |1-\flat| \geq \left|1 - \frac{2(3\vartheta-1)}{\mathfrak{e}^{\eth_{\aleph}}\,\vartheta}\right| \end{cases}$$

If $\varrho \mapsto 1^{-}$, we obtain the following results for the class $\mathsf{KM}_{\sum}(\Pi(\Im))$ defined in Example (3.4)

Corollary 6.2. *Let £ given by* (1) *be in the class* $\mathsf{KM}_{\sum}(\Pi(\Im))$*. Then*

$$\left|\eta_2\right| \leq \frac{\mathfrak{e}^{2\eth_{\aleph}}|\vartheta|\sqrt{2}}{|\eth_{\aleph}|\sqrt{\left|3\mathfrak{e}^{\eth_{\aleph}}\,\vartheta+8\left[1-3\vartheta\right]\right|}},$$

$$\left|\eta_3\right| \leq \frac{2\,\mathfrak{e}^{\eth_{\aleph}}\,|\vartheta|\left|\left(6\mathfrak{e}^{\eth_{\aleph}}-24\right)|\vartheta|+8\right|}{3\eth_{\aleph}^{2}\left|\left(3\mathfrak{e}^{\eth_{\aleph}}-24\right)\vartheta+8\right|},$$

and

$$\left|\eta_3 - \flat\,\eta_2^2\right| \leq \begin{cases} \frac{2\mathfrak{e}^{\eth_{\aleph}}|\vartheta|}{3\eth_{\aleph}^{2}} & |1-\flat| \leq \left|1 - \frac{8(3\vartheta-1)}{\mathfrak{e}^{\eth_{\aleph}}\,\vartheta}\right| \\ \frac{2\mathfrak{e}^{2\eth_{\aleph}}\,\vartheta^{2}|1-\flat|}{\left|\eth_{\aleph}^{2}\left(3\mathfrak{e}^{\eth_{\aleph}}\,\vartheta+8[1-3\vartheta]\right)\right|} & |1-\flat| \geq \left|1 - \frac{8(3\vartheta-1)}{\mathfrak{e}^{\eth_{\aleph}}\,\vartheta}\right| \end{cases}$$

7. Conclusion

In this work, we introduce a new subclass of bi-univalent functions on the open unit disk, formulated through a generalized neutrosophic ϱ-Poisson distribution series. Our study primarily focuses on obtaining estimates for the Taylor coefficients and deriving FeketeSzeg type inequalities, leveraging the ϱ-Fibonacci sequence as a fundamental instrument in these derivations. Additionally, several corollaries are established, and the broader implications of the results are discussed, emphasizing their significance and prospective applications in various branches of mathematics, science, and technology.

Building on the theoretical advancements made, future research could extend these findings by delving into higher-order coefficient estimates, further refining the structural properties of these subclasses, and investigating their geometric characteristics. Additionally, exploring the upper bounds related to the Zalcman conjecture and analyzing Hankel determinants of higher orders could provide deeper insights into the broader implications of this work, particularly in the context of analytic function theory. The neutrosophic ϱ-Poisson distribution is poised to offer new perspectives and applications, enriching the field and its diverse interdisciplinary connections.A future study of this work would enable researchers to link these tools with other tools in the neutrosophic environment, including [40]– [48].

References

1. Duren, P.L.; *Univalent Functions*; Springer: New York, NY, USA, 1983.
2. Ma, W.; Minda, D.; A unified treatment of some special classes of univalent functions. In *Proc. Conf. Comp. Anal., Tianjin, China*; 1992; pp. 157169.
3. Janowski, W. (1970). Extremal problems for a family of functions with positive real part and for some related families. *Annales Polonici Mathematici*, **23**(28), 159–177.
4. Janowski, W. (1973). Some extremal problems for certain families of analytic functions I. *Annales Polonici Mathematici*, **3**(28), 297–362.
5. Soko, J. (1999). On starlike functions connected with Fibonacci numbers. *Zeszyty Naukowe Politechniki Rzeszowskiej. Matematyka, 23*(157), 111116.
6. Robertson, M. S. (1985). Certain classes of starlike functions. *Michigan Mathematical Journal, 32*, 135140.
7. Soko, J. (2011). A certain class of starlike functions. *Computers; Mathematics with Applications, 62*(2), 611619.
8. Jackson, F. H. (1909). On q-functions and a certain difference operator. *Transactions of the Royal Society of Edinburgh, 46*(2), 253281.
9. Jackson, F. H. (1910). On q-definite integrals. *The Quarterly Journal of Pure and Applied Mathematics, 41*, 193203.
10. Aral, A.; Gupta, V. (2011). Generalized q-Baskakov operators. *Mathematica Slovaca, 61*(4), 619634.
11. Aral, A.; Gupta, V. (2010). On the Durrmeyer type modification of the q-Baskakov type operators. *Nonlinear Analysis: Theory, Methods and Applications, 72*(34), 11711180.
12. Aral, A.; Gupta, V.; Agarwal, R. (2013). *Applications of q-calculus in operator theory.* Springer.

13. Alsoboh, A.; Oros, G. (2024). A class of bi-univalent functions in a leaf-like domain defined through subordination via q-calculus. *Mathematics*.
14. Alsoboh, A.; Amourah, A.; Alnajar, O.; Ahmed, M.; Seoudy, T.M.; Exploring q-Fibonacci numbers in geometric function theory: Univalence and shell-like starlike curves. *Mathematics* 2025, 13(8), 1294.
15. Mustafa, N.; Nezir, V.; Analytic functions expressed with q-Poisson distribution series. *Turkish Journal of Science* 2021, 6(1), 2430.
16. Poisson, S.; *Probabilit des jugements en matire criminelle et en matire civile, prcdes des rgles gnrales du calcul des probabilits*; Bachelier: Paris, France, 1837.
17. Alhabib, R.; Ranna, M.M.; Farah, H.; Salama, A.A.; Some neutrosophic probability distributions. *Neutrosophic Sets and Systems* 2018, 22, 3038.
18. Alsoboh, A.; Amourah, A.; Darus, M.; Sharefeen, R.I.A.; Applications of neutrosophic q-Poisson distribution series for subclass of analytic functions and bi-univalent functions. *Mathematics* 2023, 11, 868. https://doi.org/10.3390/math11040868
19. Alsoboh, A.; Amourah, A.; Darus, M.; Rudder, C.A.; Investigating new subclasses of bi-univalent functions associated with q-Pascal distribution series using the subordination principle. *Symmetry* 2023, 15, 1109. https://doi.org/10.3390/sym15051109
20. Al-Hawary, T.; Alsoboh, A.; Amourah, A.; Ogilat, O.; Harny, I.; Darus, M.; Applications of q-Borel distribution series involving q-Gegenbauer polynomials to subclasses of bi-univalent functions. *Heliyon* 2024, 10(14).
21. Uar, H.; zkan, E. (2016). Coefficient inequality for q-starlike functions. *Applied Mathematics and Computation, 276*, 122126.
22. Amourah, A.; Alsoboh, A.; Breaz, D.; El-Deeb, S. M. (2024). A bi-starlike class in a leaf-like domain defined through subordination via q-calculus. *Mathematics, 12*(11), Article 1735. https://doi.org/10.3390/math12111735
23. Alsoboh, A.; alar, M.; Buyankara, M. (2024). Fekete-Szeg inequality for a subclass of bi-univalent functions linked to q-ultraspherical polynomials. *Contemporary Mathematics*, 23662380.
24. M. Ahmed, A. Alsoboh, A. Amourah, and J. Salah, On the Fractional q-Differintegral Operator for Subclasses of Bi-Univalent Functions Subordinate to q-Ultraspherical Polynomials, *European Journal of Pure and Applied Mathematics*, **18**, 2025, 6586.
25. A. Alsoboh, A. S. Tayeh, A. Amourah, A. A. Al-Maqbali, K. Al Mashraf, and T. Sasa, Hankel Determinant Estimates for Bi-Bazilevi-Type Functions Involving q-Fibonacci Numbers, *European Journal of Pure and Applied Mathematics*, **18**, 2025, 6698.
26. A. Almalkawi, A. Alsoboh, A. Amourah, and T. Sasa, Estimates for the Coefficients of Subclasses Defined by the q-Babalola Convolution Operator of Bi-Univalent Functions Subordinate to the q-Fibonacci Analogue, *European Journal of Pure and Applied Mathematics*, **18**, 2025, 6499. https://doi.org/10.29020/nybg.ejpam.v18i3.6499
27. Elhaddad, S.; Aldweby, H.; Darus, M. (2018). Some properties on a class of harmonic univalent functions defined by q-analogue of Ruscheweyh operator. *Journal of Mathematical Analysis, 9*(4), 2835.
28. Alatawi, A.; Darus, M. (2024). Second-order Hankel determinant for a subclass of analytic functions satisfying subordination condition connected with modified q-Opoola derivative operator. *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, 73*(3), 695704.
29. Alatawi, A.; Darus, M. (2024). The FeketeSzeg inequality for a subfamily of q-analogue analytic functions associated with the modified q-Opoola operator. *Asian-European Journal of Mathematics, 17*(3), Article 2450027.

30. Hadi, S. H.; Darus, M.; Alamri, B.; Altnkaya, .; Alatawi, A. (2024). On classes of ζ-uniformly q-analogue of analytic functions with some subordination results. *Applied Mathematics in Science and Engineering, 32*(1), Article 2312803.
31. Hadi, S. H.; Shaba, T. G.; Madhi, Z. S.; Darus, M.; Lupa, A. A.; Tchier, F. (2024). Boundary values of Hankel and Toeplitz determinants for q-convex functions. *MethodsX, 13*, Article 102842.
32. Srivastava, H. M.; Hadi, S. H.; Darus, M. (2023). Some subclasses of p-valent γ-uniformly type q-starlike and q-convex functions defined by using a certain generalized q-Bernardi integral operator. *Revista de la Real Academia de Ciencias Exactas, Fsicas y Naturales. Serie A. Matemticas, 117*(1), Article 50.
33. Tayyah, A. S.; Atshan, W. G. (2025). A class of bi-Bazilevi and bi-pseudo-starlike functions involving Tremblay fractional derivative operator. *Problemy AnalizaIssues of Analysis, 14*(2), 145161.
34. Tayyah, A. S.; Atshan, W. G. (2025). Starlikeness and bi-starlikeness associated with a new Carathodory function. *Journal of Mathematical Sciences*. https://doi.org/10.1007/s10958-025-07604-8.
35. Shakir, Q. A.; Tayyah, A. S.; Breaz, D.; Cotrl, L.-I.; Rpeanu, E.; Sakar, F. M. (2024). Upper bounds of the third Hankel determinant for bi-univalent functions in crescent-shaped domains. *Symmetry, 16*, 1281.
36. Tayyah, A. S.; Atshan, W. G.; Oros, G. I. (2025). Third-order differential subordination results for meromorphic functions associated with the inverse of the Legendre Chi function via the Mittag-Leffler identity. *Mathematics, 13*(13), 2089.
37. Mahmood, S.; Ahmad, Q. Z.; Srivastava, H. M.; Khan, N.; Khan, B.; Tahir, M. (2019). A certain subclass of meromorphically q-starlike functions associated with the Janowski functions. *Journal of Inequalities and Applications, 2019*, Article 88.
38. Mahmood, S.; Srivastava, H. M.; Khan, N.; Ahmad, Q. Z.; Khan, B.; Ali, I. (2019). Upper bound of the third Hankel determinant for a subclass of q-starlike functions. *Symmetry, 11*, Article 347.
39. Srivastava, H. M.; Khan, B.; Khan, N.; Tahir, M.; Ahmad, S.; Khan, N. (2021). Upper bound of the third Hankel determinant for a subclass of q-starlike functions associated with the q-exponential function. *Bulletin des Sciences Mathmatiques, 167*, Article 102942.
40. Al-Qudah, Y., Al-Sharqi, F., Mishlish, M., & Rasheed, M. M. Hybrid integrated decision-making algorithm based on AO of possibility interval-valued neutrosophic soft settings. International Journal of Neutrosophic Science,2023, 22(3), 84-98.
41. Al-Qudah, Y., Alhazaymeh, K., Hassan, N., Almousa, M., Alaroud, M. Transitive Closure of Vague Soft Set Relations and its Operators. International Journal of Fuzzy Logic and Intelligent Systems, 2022, 22(1), pp. 5968
42. Hazaymeh, A. Al-Qudah, Y. Al-Sharqi, F. Bataihah, A. (2025). A novel Q-neutrosophic soft under interval matrix setting and its applications. International Journal of Neutrosophic Science,25 (4), 156-168.
43. Fathi H, Myvizhi M, Abdelhafeez A, Abdellah MR, Eassa M, Sawah MS, Elbehiery H. Single-Valued Neutrosophic Graph with Heptapartitioend Structure. Neutrosophic Sets and Systems. 2025, vol.80, 728-748
44. Romdhini, M. U., Al-Sharqi, F., Al-Obaidi, R. H., Rodzi, Z. M. (2025). Modeling uncertainties associated with decision-making algorithms based on similarity measures of possibility belief interval-valued fuzzy hypersoft setting. Neutrosophic Sets and Systems, 77, 282-309.
45. Al-Qudah, Y., Al-Sharqi, F. (2023). Algorithm for decision-making based on similarity measures of possibility interval-valued neutrosophic soft setting settings. International Journal of Neutrosophic Science, 22(3), 69-83.
46. Romdhini, M. U.; Al-Sharqi, F.; Tahat, M. K.; Lutfi, A. Exploring the Algebraic Structures of Q-Complex Neutrosophic Soft Fields. International Journal of Neutrosophic Science,2023, 22(04), 93-105.
47. Al-Qudah, Y., Jaradat, A., Sharma, S. K., & Bhat, V. K. Mathematical analysis of the structure of one-heptagonal carbon nanocone in terms of its basis and dimension. Physica Scripta, 2024, 99(5), 055252.

48. F. F. Kareem and M. M. Abed. Generalizations of Fuzzy k-ideals in a KU-algebra 294 with Semigroup. Journal of Physics: Conference Series, 1879(2), 022108, 2021.

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