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Neutrosophic Optimal Transport of Rural Cultural Development Quality under Rural Revolutionary Topological Constraints: A Strategic Evaluation Model

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Abstract: Rural cultural revitalization demands evaluation models capable of addressing incomplete, inconsistent, and structurally interconnected data. This paper introduces a novel framework Neutrosophic Optimal Transport for Rural Cultural Quality (NOT-RCQ) which models the transformation from an existing rural cultural profile to a targeted revitalized state as an optimal transport problem in a refined neutrosophic measure space. Each cultural indicator is represented by a triplet of truth, indeterminacy, and falsehood components, allowing simultaneous quantification of verified achievements, uncertain data, and misleading or failed initiatives. Revolutionary topological constraints OverTopology, UnderTopology, OffTopology, and AntiTopology are incorporated to represent policy expansions, resource limitations, activation/deactivation of cultural structures, and stress-test conditions respectively. The model computes a Neutrosophic Optimal Transport Distance between the current and desired states, reflecting the minimal "cost" of transforming cultural quality under given structural constraints. A higher-quality cultural profile corresponds to a shorter transport distance. The proposed method enables policymakers to identify priority interventions, assess resilience to disruptions, and compare revitalization strategies through rigorous quantitative analysis. The approach is validated on a realistic synthetic dataset simulating rural heritage preservation, community engagement, infrastructure, and innovation indicators.

Keywords: Rural cultural revitalization, neutrosophic optimal transport, revolutionary topology, cultural development evaluation, truth-indeterminacy-falsehood modeling, strategic policy planning.

1. Introduction

Rural areas are vital to the cultural and economic fabric of any nation, yet they often face challenges such as economic decline, population loss, and erosion of cultural heritage. Rural revitalization strategies aim to address these issues by fostering sustainable development while preserving and promoting cultural identity. Cultural development in rural settings involves a rich mix of tangible heritage such as historical sites and artifacts, intangible traditions like folklore and local customs, creative industries such as crafts and festivals, and community engagement practices. These elements work together to strengthen the identity, cohesion, and resilience of rural communities. However, measuring the quality of cultural development is a complex task due to several challenges: incomplete or unreliable data, inconsistent community participation metrics, and the intricate interplay of cultural indicators influenced by local geography, governance structures, and social dynamics [1].

Traditional evaluation methods, such as composite indices or statistical models, often fall short in capturing the nuances of rural cultural systems. These methods typically assume complete and consistent data and treat indicators as independent, ignoring their interconnected nature. For example, increased community participation in cultural events may enhance heritage preservation efforts, creating feedback loops that are difficult to quantify using conventional approaches. Moreover, rural cultural data often contains a mix of verified facts, uncertainties, and even misleading information, further complicating analysis [2]. Structural constraints, such as limited funding, policy restrictions, or inadequate infrastructure, also shape the feasibility of cultural initiatives. To address these challenges, advanced theoretical tools are needed to model uncertainty, structural limitations, and indicator interdependencies effectively.

This study introduces the NOT-RCQ model, a novel framework designed to evaluate and guide rural cultural revitalization efforts. The model integrates three key concepts: neutrosophic logic, revolutionary topologies, and optimal transport theory. Neutrosophic logic, which represents truth, indeterminacy, and falsehood, allows for precise modeling of uncertain and contradictory cultural data [1]. Revolutionary topologies extend classical topological spaces to account for dynamic structural changes, such as policy-driven expansions or budgetary constraints, that influence cultural development [2]. Optimal transport theory provides a method to measure the effort required to transform a current cultural profile into a desired state by calculating the minimal "cost" of redistributing cultural indicator values [3]. By combining these tools, the NOT-RCQ model offers a quantitative measure of how close a rural area is to achieving its cultural revitalization goals, identifies the scale of intervention needed, and highlights key areas for policy focus. This paper develops the theoretical foundation of the NOT-RCQ model, provides its mathematical formulation, and illustrates its application through practical examples of rural cultural systems.

2. Literature Review

2.1 Assessing Rural Cultural Development Quality

Evaluating the quality of cultural development in rural areas is a critical component of broader revitalization strategies. Existing approaches often rely on composite indices that combine economic, social, and cultural indicators into a single score. For instance, UNESCO’s cultural vitality framework measures aspects like cultural participation and heritage preservation, while region-specific sustainability indices incorporate local cultural metrics [4]. These methods provide standardized benchmarks but face limitations in rural contexts, where data is often incomplete, qualitative, or influenced by political factors. Additionally, most conventional models assume that indicators are independent, overlooking systemic interactions, such as how cultural events can boost local tourism or how heritage preservation strengthens community identity [5]. These interconnections require a more flexible and nuanced evaluation approach to capture the dynamic nature of rural cultural systems.

2.2 Neutrosophic Logic in Evaluation Models

Neutrosophic logic, introduced by Smarandache, extends traditional fuzzy logic by incorporating three components: truth (T), indeterminacy (I), and falsehood (F) [1]. This framework is particularly suited for modeling complex systems with uncertainty and contradictory information, such as rural cultural data. Neutrosophic measure theory further formalizes these components, enabling the quantification of uncertainty in measurable spaces [6]. While neutrosophic logic has been applied in fields like medical diagnosis and engineering, its use in cultural development evaluation is still emerging. Existing studies using neutrosophic logic in rural contexts tend to focus on static analyses, such as assessing current cultural vitality, without addressing how to transform a cultural system toward a desired state [7]. This gap highlights the need for a dynamic, transformation-focused approach.

2.3 Revolutionary Topologies and Structural Dynamics

Revolutionary topologies, as explored in NeutroTopology, AntiTopology, OverTopology, and UnderTopology, offer a framework for modeling dynamic structural changes in complex systems [2]. Unlike classical topological spaces, revolutionary topologies allow for the expansion, contraction, deactivation, or inversion of connections between elements. In the context of rural cultural development, these structures can represent real-world dynamics, such as the expansion of cultural programs through new policies (OverTopology), the reduction of initiatives due to budget cuts (UnderTopology), or the suspension of cultural networks during crises (OffTopology) [8]. While these concepts have been studied in abstract mathematics and computer science, their application to socio-cultural systems, particularly under policy constraints, remains underexplored. Integrating revolutionary topologies into cultural evaluation models can provide a more realistic representation of structural limitations and opportunities.

2.4 Optimal Transport in Socio-Cultural Analysis

Optimal transport theory, originally developed in mathematics, calculates the most efficient way to redistribute resources or "mass" from one distribution to another based on a cost function [3]. In social sciences, it has been used to analyze migration patterns, resource allocation, and network optimization [9]. However, its application to cultural development is limited, with most studies focusing on tangible resources like funding or population rather than intangible cultural indicators. Furthermore, optimal transport has not been combined with uncertainty models like neutrosophic logic or structural frameworks like revolutionary topologies in the context of rural cultural systems. This integration is essential for quantifying the effort needed to achieve cultural revitalization goals while accounting for uncertainty and structural constraints.

2.5 Research Gap and Proposed Contribution

The literature reveals robust individual tools for evaluating cultural systems: neutrosophic logic for handling uncertainty, revolutionary topologies for modeling structural dynamics, and optimal transport for analyzing transformations. However, no existing framework combines these approaches to address the unique challenges of rural cultural development. Specifically, there is no model that:

Represents cultural indicators as neutrosophic measures to account for uncertainty and contradiction.

Incorporates revolutionary topologies to reflect policy and structural constraints.

Uses optimal transport to quantify the effort required to transform a current cultural profile into a target state.

The proposed NOT-RCQ model fills this gap by integrating these tools into a unified framework. It provides a comprehensive, uncertainty-sensitive, and constraint-aware approach to evaluating and guiding rural cultural revitalization, offering both theoretical rigor and practical applicability.

3. Methodology

3.1 Conceptual Framework

The NOT-RCQ model evaluates the "distance" between a rural area's *current* cultural development profile and its *target* profile under a revitalization strategy. This distance is interpreted as the minimal "cost" required to shift the truth, indeterminacy, and falsehood components of each cultural indicator toward the desired configuration, subject to real-world structural constraints.

The model integrates three pillars:

1. Refined Neutrosophic Representation - Each cultural indicator is described by a vector of triplets (T, I, F), possibly refined into multiple subcomponents ($T_1, T_2, \ldots$) to capture nuanced aspects such as different types of heritage or forms of participation.
2. Revolutionary Topological Constraints - The set of indicators and their interrelations form a revolutionary topological space, where OverTopology, UnderTopology, OffTopology, and AntiTopology capture expansions, restrictions, suspensions, and inversions of connections between indicators, respectively.
3. Optimal Transport Mechanism - Movement of T, I, and F "mass" from the current to target profile is optimised with respect to a cost function that depends both on the type of transformation (e.g., $I \rightarrow T$ vs. $F \rightarrow T$) and the structural constraints imposed by the current topology.

3.2 Data Representation

Let $\mathcal{X}$ denote the set of cultural indicators, such as:

C_1 : Heritage site preservation

C_2 : Local festival participation
C_3 : Traditional craft activity
C_4 : Cultural infrastructure availability
C_5 : Innovation in cultural industries
For each $C_j \in \mathcal{X}$, we define:

$$\mu^{\mathrm{src}}(C_j) = (T_j^{\mathrm{src}}, I_j^{\mathrm{src}}, F_j^{\mathrm{src}}), \mu^{\mathrm{tgt}}(C_j) = \left(T_j^{\mathrm{tgt}}, I_j^{\mathrm{tgt}}, F_j^{\mathrm{tgt}}\right)$$

where the superscripts "src" and "tgt" denote the source (current) and target (goal) states, respectively. Values are in [0,1] and may sum to less than or equal to 3 if refined components are used.

3.3 Revolutionary Topology Layer

Indicators are connected in a topology τ that captures functional relationships e.g., improving C_4 infrastructure - enables growth in C_2 - participation. Revolutionary topologies allow:

a. OverTopology: Introduction of new open sets e.g., newly funded cultural programs.
b. UnderTopology: Removal or restriction of open sets e.g., budget cuts.
c. OffTopology: Deactivation of specific connections or indicators e.g., cancellation of events during crises.
d. AntiTopology: Structural inversion representing disruptive events e.g., legal bans or cultural conflicts.

These variations alter the admissible transport paths and their costs.

3.4 Optimal Transport Formulation

We interpret T, I, and F components as "mass" to be moved from the current to target profile. For each pair of components $a, b \in \{T, I, F\}$ and indicators $x, y \in \mathcal{X}$, a transport plan $\gamma_{ab}(x, y)$ specifies the amount moved.

A cost matrix $c_{ab}(x, y \mid \tau)$ assigns a cost to transporting one unit of component a from indicator x to component b at indicator y, factoring in:

a. Transformation type cost: e.g., $F \to T$ may be higher than $I \to T$.
b. Topological distance: minimal number of open-set transitions required in τ.
c. Topology mode modifiers: penalties or discounts depending on Over/Under/Off/Anti topology states.

The objective is:

$$\min_{\{\gamma_{ab}\}} \sum_{a,b} \sum_{x,y} c_{ab}(x, y \mid \tau) \gamma_{ab}(x, y)$$

subject to mass-balance constraints preserving the total T, I, F quantities for each indicator in source and target.

3.5 Output Metrics

From the optimal plan we compute:
Neutrosophic Optimal Transport Distance: total minimal cost of transformation.
Quality Index:

$$Q=\frac{1}{1+\text{ Distance }}$$

A value closer to 1 indicates a profile already near the target; lower values indicate more extensive intervention needed. Additionally, the optimal plan reveals:

a. Which indicators require the largest transformations.
b. How much indeterminacy must be resolved.
c. Where structural constraints most affect progress.

4. Proposed Model

4.1 Preliminaries and Definitions

Let:

$\mathcal{X}=\{C_1, C_2, \ldots, C_n\}$ be the set of n cultural indicators.

Each indicator C_j is assigned a refined neutrosophic measure in the current (source) and target states:

$$\mu^{\text{src}}(C_j)=\left(T_j^{\text{src}}, I_j^{\text{src}}, F_j^{\text{src}}\right), \mu^{\text{tgt}}(C_j)=\left(T_j^{\text{tgt}}, I_j^{\text{tgt}}, F_j^{\text{tgt}}\right)$$

with $T, I, F \in[0,1]$ and no restriction that $T+I+F=1$ (to allow refined components).

Let $\mathcal{C}=\{T, I, F\}$ denote the component set.

A mass unit is an abstract measure corresponding to a proportion of a cultural attribute.

4.2 Revolutionary Topological Space

We define a Revolutionary Neutrosophic Topological Space ($\mathcal{X}, \tau$), where τ is a family of "open sets" representing groups of related indicators.

The topology may be in one of four modes:

1. OverTopology (τ^{+})- additional open sets are introduced.
2. UnderTopology (τ^{-})- some open sets are removed.
3. OffTopology (τ^{0}) - certain open sets or links are deactivated.
4. AntiTopology (τ^{*}) - inversion of open-set relationships.

The topological distance $d_\tau(x, y)$ between indicators x, y is the minimal number of open-set transitions required to connect them under the current topology mode.

4.3 Transport Plan

We introduce a transport tensor:

$$\Gamma=\{\gamma_{ab}(x, y) \mid a, b \in \mathcal{C}, x, y \in \mathcal{X}\}$$

where $\gamma_{ab}(x, y) \geq 0$ is the amount of component a at indicator x to be moved to component b at indicator y.

Mass-Balance Constraints:

$$\sum_{b \subset \mathcal{C}} \sum_{y \subset \mathcal{X}} \gamma_{ab}(x, y)=m_a^{\text{src}}(x), \forall a \in \mathcal{C}, x \in \mathcal{X}$$

$$\sum_{a \subset \mathcal{C}} \sum_{x \subset \mathcal{X}} \gamma_{ab}(x, y)=m_b^{\text{tgt}}(y), \forall b \in \mathcal{C}, y \in \mathcal{X}$$

where $m_a^{\text{src}}(x)$ and $m_b^{\text{tgt}}(y)$ are the source and target component masses for each indicator.

4.4 Cost Function

The cost of moving one unit of a at x to b at y is:

$$c_{ab}(x, y \mid \tau) = \lambda_{ab} \cdot d_\tau(x, y) \cdot \omega_\tau(x, y)$$

where:

λ_{ab} - transformation cost coefficient, reflecting difficulty of changing component a into b ($\lambda_{TT} < \lambda_{FT}$, typically).

$d_\tau(x, y)$ - topological distance in the current topology mode.

$\omega_\tau(x, y)$-mode multiplier: < 1 in OverTopology, > 1 in Under/OffTopology, large penalty in AntiTopology.

4.5 Objective Function

The Neutrosophic Optimal Transport Problem is:

$$\text{Distance}_{\text{NOT}} = \min_{\Gamma} \sum_{a,b\subset\mathcal{C}} \sum_{x,y\subset\mathcal{X}} c_{ab}(x, y \mid \tau)\gamma_{ab}(x, y)$$

subject to the mass-balance constraints above.

Quality Index

We define:

$$Q = \frac{1}{1 + \text{Distance}_{\text{NOT}}}$$

where $Q \in (0,1]$, with higher values indicating greater closeness to the target cultural profile under the given topology.

4.7 Properties

Proposition 1 (Non-Negativity):

Distance $_{\text{NOT}} \geq 0$, with equality iff $\mu^{\text{src}} = \mu^{\text{tgt}}$ component-wise.

Proposition 2 (Topology Sensitivity):

For identical source/target measures, Distance $_{\text{NOT}}$ varies with topology mode via ω_τ, enabling sensitivity analysis to policy shifts.

Proposition 3 (Triangle Inequality in Topology Space):

If τ_1, τ_2, τ_3 are sequential topology states, then:

$$\text{Distance}_{\text{NOT}}(\tau_1, \tau_3) \leq \text{Distance}_{\text{NOT}}(\tau_1, \tau_2) + \text{Distance}_{\text{NOT}}(\tau_2, \tau_3)$$

This allows staged transition planning.

5. Mathematical Equations

This section gives complete, transparent calculations for the NOT-RCQ model. We first formalize the optimization, then solve two numerical examples under different revolutionary topology modes. Every number, constraint, and cost is shown, with nothing skipped.

5.1 Formal optimization model

5.1.1 Data and variables

Indicators: $\mathcal{X} = \{C_1, C_2, \dots, C_n\}$.

Components: $\mathcal{C} = \{T, I, F\}$ (truth, indeterminacy, falsehood).

Source and target neutrosophic measures per indicator:

$$\mu^{\mathrm{src}}(C_j) = (T_j^{\mathrm{src}}, I_j^{\mathrm{src}}, F_j^{\mathrm{src}}), \mu^{\mathrm{tgt}}(C_j) = \left(T_j^{\mathrm{tgt}}, I_j^{\mathrm{tgt}}, F_j^{\mathrm{tgt}}\right).$$

Transport plan (decision variables):

$$\gamma_{ab}(x, y) \geq 0 \ \text{ for } a, b \in \{T, I, F\}, x, y \in \mathcal{X}.$$

5.1.2 Mass balance constraints

For every component a at every indicator x,

$$\sum_{b \subset \mathcal{C}} \sum_{y \subset \mathcal{X}} \gamma_{ab}(x, y) = m_a^{\mathrm{src}}(x) = a_x^{\mathrm{src}}.$$

For every component b at every indicator y.

$$\sum_{a \subset \mathcal{C}} \sum_{x \subset \mathcal{X}} \gamma_{ab}(x, y) = m_b^{\mathrm{tgt}}(y) = b_y^{\mathrm{tgt}}.$$

(Feasibility requires $\sum_{x,a} \ a_x^{\mathrm{src}} = \sum_{y,b} \ b_y^{\mathrm{tgt}}$.)

5.1.3 Costs and objective

Let the revolutionary topology be τ. Define:

Topological distance $d_\tau(x, y) \in \{1,2,3, \ldots\}$ with $d_\tau(x, x) = 1$ (positive "local" effort).

Transformation coefficients $\lambda_{ab} > 0$ (e.g., $F \rightarrow T$ costlier than $I \rightarrow T$).

Mode multiplier $\omega_\tau(x, y) > 0$ (e.g., < 1 in OverTopology, > 1 in UnderTopology, large in AntiTopology).

Per-unit cost:

$$c_{ab}(x, y \mid \tau) = \lambda_{ab} \cdot d_\tau(x, y) \cdot \omega_\tau(x, y).$$

Minimize total transport cost (the NOT-distance):

$$\text{Distance}_{\text{ NOT}} = \min_{\gamma \geq 0} \sum_{a,b} \sum_{x,y} c_{ab}(x, y \mid \tau) \gamma_{ab}(x, y)$$

subject to mass balance.

Quality index:

$$Q = \frac{1}{1 + \text{ Distance}_{\text{ NOT}}} \in (0,1]$$

5.2 Example A: Local transformations only (no cross-indicator moves)

We study three indicators:

- C_1 : Heritage preservation
- C_2 : Festival participation
- C_3 : Cultural infrastructure

5.2.1 Source and target profiles

Each indicator sums to 1.0 (over $T + I + F$).

Indicator	T^{src}	I^{src}	F^{src}	T^{tgt}	I^{tgt}	F^{tgt}
C_1	0.50	0.30	0.20	0.70	0.20	0.10
C_2	0.40	0.40	0.20	0.60	0.30	0.10

C_3	0.30	0.50	0.20	0.50	0.40	0.10

Global totals:

$$\sum T^{\mathrm{src}} = 1.20 \to \sum T^{\mathrm{tgt}} = 1.80 \text{ (need +0.60)},$$
$$\sum I^{\mathrm{src}} = 1.20 \to \sum I^{\mathrm{tgt}} = 0.90 \text{ (need -0.30)},$$
$$\sum F^{\mathrm{src}} = 0.60 \to \sum F^{\mathrm{tgt}} = 0.30 \text{ (need -0.30)}.$$

Thus, the extra 0.60 of T must come from 0.30 of I and 0.30 of F.

5.2.2 Topology & cost parameters

Distances: $d_\tau(x,x) = 1$ (local), cross-indicator distances unused here.

Transformation coefficients:

$$\lambda_{TT} = \lambda_{II} = \lambda_{FF} = 0.2, \lambda_{I\to T} = 1.0, \lambda_{F\to T} = 2.0.$$

Mode multiplier:

OverTopology: $\omega = 0.8$

UnderTopology: $\omega = 1.2$

AntiTopology: $\omega = 3.0$

Per-unit local costs ($d = 1$):

Over: $TT = II = FF = 0.16, I \to T = 0.8, F \to T = 1.6$

Under: $TT = II = FF = 0.24, I \to T = 1.2, F \to T = 2.4$

Anti: $TT = II = FF = 0.6, I \to T = 3.0, F \to T = 6.0$

5.2.3 Feasible optimal plan (balanced per indicator)

Each indicator's T deficit is 0.2 . To meet global component totals, split each deficit as:

0.1 from $I \to T$ (so global I reduces by 0.3),

0.1 from $F \to T$ (so global F reduces by 0.3).

For each C_j :

$T \to T: \gamma_{TT}(C_j, C_j) = T_j^{\mathrm{src}}$.

$I \to T$: 0.1.

$F \to T$: 0.1.

Remaining I stays as I locally to match I^{tgt} .

Remaining F stays as F locally to match F^{tgt}.

This exactly matches every target component at every indicator with no cross-indicator movement.

5.2.4 Cost under OverTopology ($\boldsymbol{\omega = 0.8}$)

Compute per indicator; unit costs: $TT = II = FF = 0.16, I \to T = 0.8, F \to T = 1.6$.

C_1 :

TT: $0.5 \times 0.16 = 0.080$

$I \to T$: $0.1 \times 0.8 = 0.080$

$F \to T$: $0.1 \times 1.6 = 0.160$

II: $0.2 \times 0.16 = 0.032$ (leftover I to I)

FF: $0.1 \times 0.16 = 0.016$ (leftover F to F)

Total $C_1 = 0.368$.
$C_2 = 0.4 \times 0.16 + 0.1 \times 0.8 + 0.1 \times 1.6 + 0.3 \times 0.16 + 0.1 \times 0.16 = 0.368$.
$C_3 = 0.3 \times 0.16 + 0.1 \times 0.8 + 0.1 \times 1.6 + 0.4 \times 0.16 + 0.1 \times 0.16 = 0.368$.
Total Distance (OverTopology): 1.104.
Quality: $Q = \frac{1}{1+1.104} = \frac{1}{2.104} \approx 0.4756$.

5.2.5 Cost under UnderTopology ($\omega = 1.2$)

Unit costs: $TT = II = FF = 0.24, I \to T = 1.2, F \to T = 2.4$.
Each indicator's total becomes:

$$0.5 \times 0.24 + 0.1 \times 1.2 + 0.1 \times 2.4 + (\text{ matching } II, FF) = 0.552.$$

(Analogous arithmetic for C_2, C_3.)
Total Distance (UnderTopology): 1.656.
Quality: $Q = \frac{1}{1+1.656} = \frac{1}{2.656} \approx 0.3767$.

5.2.6 Cost under AntiTopology ($\omega = 3.0$)

Unit costs: $TT = II = FF = 0.6, I \to T = 3.0, F \to T = 6.0$.
Per indicator:

$$0.5 \times 0.6 + 0.1 \times 3.0 + 0.1 \times 6.0 + (\text{ matching } II, FF) = 1.38.$$

Total Distance (AntiTopology): 4.14.
Quality: $Q = \frac{1}{1+4.14} = \frac{1}{5.14} \approx 0.1946$.
Takeaway (Example A): The identical transport pattern evaluated under different topology modes shows how structural conditions alone alter the distance and the quality score.

5.3 Example B: Cross-indicator transport with explicit distances

Now we force non-local moves (cross-indicator flows) and compute the full cost.

5.3.1 Source and new target profiles

Source stays as in Example A. New targets:

Indicator	T^{tgt}	I^{tgt}	F^{tgt}
C_1	0.40	0.50	0.10
C_2	0.70	0.20	0.10
C_3	0.70	0.20	0.10

Global totals unchanged (feasible): $\sum T = 1.8, \sum I = 0.9, \sum F = 0.3$.
This configuration requires moving I from C_2 to C_1, and moving some T and F across indicators.

5.3.2 Topology distances and costs

Distances:

$d(C_j, C_j) = 1$ (local).
$d(C_1, C_2) = 2, d(C_2, C_3) = 2$.
$d(C_1, C_3) = 3$.

Use OverTopology again ($\omega=0.8$).
Thus, unit costs for cross moves:

TT across 2 steps: $0.2\times 2\times 0.8=0.32$

II across 2 steps: 0.32

$F\rightarrow T$ across 2 steps: $2.0\times 2\times 0.8=3.20$

Local costs as before (Example A OverTopology).

5.3.3 Construct a feasible optimal plan

Step 1. Keep local T up to min(src,tgt) :

$C_1: TT=0.40$ (leave 0.10 of T^{src} to be reallocated)

$C_2: TT=0.40$ (deficit 0.30 to target 0.70)

$C_3: TT=0.30$ (deficit 0.40 to target 0.70)

Step 2. Move surplus T from C_1 to C_2 :
$\gamma_{TT}(C_1\rightarrow C_2)=0.10$ (distance 2).
Now C_2 deficit in T becomes 0.20 .

Step 3. Rebalance I (cross $I\rightarrow I$):
Targets demand $I_1=0.50$ (from 0.30), $I_2=0.20$ (from 0.40), $I_3=0.20$ (from 0.50).
Move 0.20 of I from C_2 to C_1 : $\gamma_{II}(C_2\rightarrow C_1)=0.20$ (distance 2).
Then reduce I at C_3 locally by converting to T (next step).

Step 4. Convert $I\rightarrow T$ locally at C_3 :
$\gamma_{I\rightarrow T}(C_3)=0.30$ (to reduce I_3 from $0.50\rightarrow 0.20$ and raise T_3 from $0.30\rightarrow 0.60$).
Remaining T_3 deficit = 0.10.

Step 5. Convert $F\rightarrow T$ to fill residual T deficits:

C_2 : deficit $0.20\rightarrow$ do $F\rightarrow T$ locally 0.10 and accept cross $F\rightarrow T 0.10$ from C_1.

C_3 : deficit $0.10\rightarrow F\rightarrow T$ locally 0.10 .

Check all targets now match:

I : after Step 3+4, $(I_1, I_2, I_3)=(0.50,0.20,0.20)$

F : each goes from $0.20\rightarrow 0.10$ (using local or cross $F\rightarrow T$)

T :

C_1: 0.40 (local TT)

C_2: $0.40($ local $TT)+0.10($ TT from $C_1)+0.10($ local $F\rightarrow T)+0.10($ cross $F\rightarrow T$ from $C_1)=0.70$

C_3: $0.30($ local $TT)+0.30($ local $I\rightarrow T)+0.10($ local $F\rightarrow T)=0.70$

5.3.4 Compute total cost (OverTopology)

Unit costs (recall):
Local: $TT=II=FF=0.16; I\rightarrow T=0.8; F\rightarrow T=1.6$.
Cross (distance 2): $TT=0.32; II=0.32; F\rightarrow T=3.20$.

List all flows and costs:
T → T (self):
C_1: $0.40 \times 0.16 = 0.064$
C_2: $0.40 \times 0.16 = 0.064$
C_3: $0.30 \times 0.16 = 0.048$
Subtotal = 0.176
T → T (cross):
$C_1 \to C_2$: $0.10 \times 0.32 = 0.032$
I → I (cross):
$C_2 \to C_1$: $0.20 \times 0.32 = 0.064$
I → T (local):
C_3: $0.30 \times 0.8 = 0.240$
F → T (local):
C_2: $0.10 \times 1.6 = 0.160$
C_3: $0.10 \times 1.6 = 0.160$
Subtotal = 0.320
F → T (cross):
$C_1 \to C_2$: $0.10 \times 3.20 = 0.320$
$I \to I$ (self) to carry remainder:
C_1: $0.30 \times 0.16 = 0.048$
C_2: $0.20 \times 0.16 = 0.032$
C_3: $0.20 \times 0.16 = 0.032$
Subtotal = 0.112
F → F (self) to carry remainder:
C_1: $0.10 \times 0.16 = 0.016$
C_2: $0.10 \times 0.16 = 0.016$
C_3: $0.10 \times 0.16 = 0.016$

Subtotal = 0.048
Grand total (Distancenot):

$$0.176 + 0.032 + 0.064 + 0.240 + 0.320 + 0.320 + 0.112 + 0.048 = 1.312.$$

Quality:

$$Q = \frac{1}{1 + 1.312} = \frac{1}{2.312} \approx 0.4323.$$

Thought (Example B): Compared to Example A's OverTopology ($Q \approx 0.476$), the need for crossindicator TT, II, and especially cross $F \to T$ inflates the distance (+ path length × mode multiplier), lowering quality to ≈ 0.432. This quantifies how spatial/structural configuration and policy "wiring" affect cultural upgrade cost.

These calculations confirm several important mathematical properties of the NOT-RCQ model. First, the transport plan is fully feasible, as it satisfies all mass balance requirements: every component for every indicator reaches its target value exactly, with no surplus or deficit. This ensures that the computed flows represent valid and implementable transformations. Second, the results highlight the model's sensitivity to topological mode multipliers ω. When flows are held constant and only the topology mode changes from Over to Under to Anti, the total transport distance increases monotonically, as shown in Example A ($1.104 \rightarrow 1.656 \rightarrow 4.14$). This demonstrates how structural or policy conditions alone can substantially alter the effort required for cultural development. Third, the structure of the distance reflects explicit spatial penalties. Cross-indicator movements are directly scaled by the topological distance $d_\tau(x,y)$, meaning that longer paths incur higher costs; in Example B, transporting $T \rightarrow T$ across two steps costs 0.32 per unit, compared to just 0.16 for local transfers. Finally, the model identifies actionable policy levers. Reducing $\lambda_{F\rightarrow T}$ through targeted "credibility repair" programs or decreasing d_τ and ω via OverTopology interventions such as creating new funding channels or direct operational links will strictly reduce the NOT distance, thereby increasing the quality index Q.

6. Results & Analysis

6.1 Scenario set and notation

We evaluate the NOT-RCQ model on the two worked setups from §5 without re-stating them. Scenario A permits local transformations only; Scenario B requires cross-indicator moves with explicit topological distances. Revolutionary topology modes are: Over, Under, Anti (mode multipliers conceptually derived from revolutionary topologies: Over/Under/Off/Anti) [1-2] .

Cost primitives

Transformation coefficients λ_{ab} follow neutrosophic measurement logic (costing conversions among T, I, F) [1]. The total distance is the linear program's optimum cost; the quality index is $Q = 1/(1 + \text{Distance}_{\text{NOT}})$.

6.2 Aggregate outcomes

Table 1. NOT distance and quality across topology modes (Scenario A)

Mode	Distance $_{\text{NOT}}$	Quality Q
Over	1.104	0.4756
Under	1.656	0.3767
Anti	4.140	0.1946

Holding flows fixed and changing only the topological mode multiplier shifts the objective monotonically: friendlier structural conditions (Over) compress the distance; adversarial conditions (Anti) inflate it. This isolates policy structure as a lever distinct from data values [2] .

Table 2. Cost composition (Scenario A, Over mode)

Component flow	Unit cost	Volume	Cost
$T \rightarrow T$ (local)	0.16	1.20	0.192
$I \rightarrow T$ (local)	0.80	0.30	0.240
$F \rightarrow T$ (local)	1.60	0.30	0.480
$I \rightarrow I, F \rightarrow F$ (local carry)	0.16	1.20	0.192
Total			1.104

The $F \rightarrow T$ conversions dominate cost under identical volumes; with $\lambda_{F\rightarrow T} > \lambda_{I\rightarrow T'}$, the model signals that credibility repair / failure reversal is the most expensive strand of cultural upgrading.

Table 3. Cost composition (Scenario B, Over mode with cross-indicator moves)

Component flow	Distance	Unit cost	Volume	Cost
$T \rightarrow T$ (local)	1	0.16	1.10	0.176
$T \rightarrow T($ cross $C_1 \rightarrow C_2)$	2	0.32	0.10	0.032
$I \rightarrow I($ cross $C_2 \rightarrow C_1)$	2	0.32	0.20	0.064
$I \rightarrow T($ local $C_3)$	1	0.80	0.30	0.240
$F \rightarrow T($ local $C_2, C_3)$	1	1.60	0.20	0.320
$F \rightarrow T($ cross $C_1 \rightarrow C_2)$	2	3.20	0.10	0.320
$I \rightarrow I, F \rightarrow F$ (local carry)	1	0.16	0.80	0.160
Total				1.312

Cross-indicator $F \rightarrow T$ is particularly punitive (distance × multiplier × λ). Strategically, reducing topological distance (e.g., creating direct programmatic links between the relevant indicators) would immediately drop the 3.20 unit cost to 1.60 if made local.

6.3 Sensitivity to policy/design levers

We test marginal changes independently (Scenario B, Over mode):

Reducing $\lambda_{F\rightarrow T}$ by 25% (stronger remediation programs, e.g., standardized documentation & audit protocols) brings the three $F \rightarrow T$ items to unit costs 1.20 (local) and 2.40 (cross). New total:

$$0.176 + 0.032 + 0.064 + 0.240 + (0.2 \times 1.20 = 0.24) + (0.1 \times 2.40 = 0.24) + 0.160 = 1.184$$

Quality rises to $1/(1 + 1.184) \approx 0.4577$.

Converting one cross $F \rightarrow T$ to local by adding a direct operational link (i.e., reducing d_τ from $2 \rightarrow 1$ through an OverTopology expansion) halves that line: $0.10 \times 1.60 = 0.160$. New total:

$$1.312 - 0.320 + 0.160 = 1.152, Q \approx 0.4647$$

Takeaway. Either cheaper conversion or shorter paths improves quality; together, they compound. This aligns with neutrosophic measurement's accommodation of indeterminacy and error as distinct "masses" to be reallocated, not merely rescaled [1] .

7. Discussion

The NOT-RCQ framework functions as a structured planning instrument by interpreting the computed distance as a quantitative lower bound on the minimal effort required to transform the current rural cultural profile into the desired target configuration under existing structural constraints. Rather than being a passive performance score, this metric is inherently action-oriented; each cost component corresponds to a specific type of policy or operational measure, such as training programs, archival documentation, cultural curation, or festival redesign. By decomposing total cost into flows such as $I \to T, F \to T$, and local versus crossindicator transfers, the model identifies where interventions would produce the most significant cost reductions-most notably in repairing errors and in establishing direct operational connections between interdependent indicators to shorten topological distances.

Revolutionary topology plays a decisive role in shaping these outcomes. OverTopology represents enabling conditions-such as the establishment of new institutions, creation of shared venues, and implementation of integrated cultural programs-which reduce both path lengths and multiplicative penalties in the model. In contrast, UnderTopology and AntiTopology operate as stress-test conditions, simulating the impact of restrictive policies, funding reductions, administrative bottlenecks, or disruptive events like closures. A marked increase in distance under AntiTopology, as observed in the results, signals vulnerability, whereas systems that maintain proximity to their target profile despite these penalties demonstrate inherent structural resilience [2].

The choice to model cultural indicators with neutrosophic mass rather than a single composite score is deliberate and foundational. Rural cultural datasets often contain a mixture of confirmed successes, uncertain or incomplete records, and documented failures. Representing these as distinct masses (T, I, F) aligns with neutrosophic probability theory [1], preserving the ability to track and manage uncertainty explicitly. This separation enables planners to make informed strategic choices: for example, deciding whether to convert $I \to T$ through additional surveys and documentation, or $F \to T$ through remediation of failed initiatives. Single-number fuzzy scores cannot expose such distinctions, making them less actionable in a targeted improvement strategy.

From a computational perspective, the NOT-RCQ model is formulated as a linear program over the transport tensor γ, which can be solved efficiently even for larger indicator sets. When refined componentssubdivisions within T, I, F-are introduced, the tensor increases in dimensionality but retains its linear structure, preserving tractability. Several natural extensions can further enhance applicability: entropic regularization to accelerate solution times, multi-period staging to plan gradual transitions, and Monte Carlo-based stochastic sampling to generate diverse scenarios without altering the deterministic optimal transport core [3]. Together, these features ensure that the model is both theoretically rigorous and practically adaptable for guiding rural cultural revitalization efforts under varied structural conditions.

8. Conclusion

This paper presented NOT-RCQ, a novel framework that reinterprets rural cultural upgrading as optimal transport of neutrosophic mass subject to revolutionary topological constraints. The approach operationalizes three realities of rural cultural systems: (i) truth/indeterminacy/falsehood coexist in data; (ii) structural wiring matters; (iii) improvement has path-dependent costs. Two worked scenarios showed how topology modes and cross-indicator distances reshape costs and the final quality index. Sensitivity experiments demonstrated that either cheaper remediation (λ) or shorter paths $d\tau$ can substantially raise quality, providing direct, auditable guidance for policy design. Future work may incorporate staged transitions, entropic regularization, and portfolio optimization of interventions without diluting the core interpretation of quality as the inverse of minimal transport effort.

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