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Neutrosophic Network Flow with Truth, Indeterminacy, and Falsity Capacities: An Innovative Mathematical Framework for Efficiency Evaluation of Achievement Transformation of College Student Innovation and Entrepreneurship Training Programs

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Abstract

This paper introduces a new mathematical framework that combines neutrosophic logic and network flow theory to improve the design of student innovation and entrepreneurship training programs. In the proposed approach, each connection in the training process is represented as a directed edge with three separate capacities: truth, indeterminacy, and falsity. These capacities describe how much confirmed learning, uncertainty, and failure can pass between stages of the program. By applying conservation rules for each component and adapting classical flow optimization methods, we can identify training paths that increase confirmed learning while reducing uncertainty and limiting the spread of failure. The framework also extends the max-flow/min-cut theorem to the neutrosophic setting, allowing rigorous analysis under incomplete or contradictory data. A fully worked case study with synthetic student data shows how the model can guide curriculum adjustments, demonstrating measurable improvements in training effectiveness. This work opens a new research direction by linking neutrosophic probability with flow networks, offering a versatile tool for decision-making in complex educational systems.

Keywords

Neutrosophic logic; Network flow; Truth–indeterminacy–falsity capacities; Optimization under uncertainty; Max-flow/min-cut extension; Innovation training; Entrepreneurship education; Neutrosophic probability; Educational network design; Flow-based decision-making.

1. Introduction

Innovation and entrepreneurship are now seen as essential skills for students in many fields. Universities, business incubators, and training centers run programs that guide learners from an initial idea to a working product or service [1]. These programs often include workshops, mentoring sessions, and project-based teamwork. However, measuring and improving the effectiveness of such training is difficult. In real situations, some learning outcomes are clearly achieved, others remain uncertain, and some fail entirely [2].

Most evaluation models in education use a single measure of success, often based on scores or completion rates. While these measures are easy to understand, they cannot capture the full range of outcomes—especially the uncertainty that appears when feedback is incomplete, contradictory, or delayed [3]. In real classrooms and training environments, this uncertainty is common: students may partially attend workshops, mentors may give conflicting advice, and project results may be unclear until much later.

Neutrosophic logic offers a way to model this complexity. It represents information in three parts: truth (T), indeterminacy (I), and falsity (F) [4]. Unlike classical probability, the three components in neutrosophic probability do not need to add up to one. This flexibility makes it possible to describe incomplete information (sum < 1), consistent information (sum = 1), and contradictory information (sum > 1) [5].

Network flow theory, on the other hand, is a mathematical tool for modeling the movement of something—such as goods, data, or resources—through a set of connected points [6]. It uses the concepts of capacity (maximum amount that can pass through a connection) and flow (actual amount that passes) to optimize performance. Network flow has been applied in engineering, logistics, and computer science, but it has not been adapted to handle the triple-valued nature of information found in neutrosophic logic.

This paper introduces a Neutrosophic Network Flow model where each connection in the network has three separate capacities: truth, indeterminacy, and falsity. In an educational setting, these capacities represent the maximum confirmed learning, uncertainty, and potential failure that can be transferred from one stage to another. We establish conservation rules for each component, propose optimization strategies to improve overall performance, and extend the classical max-flow/min-cut theorem to the neutrosophic case.

The goal is to give program designers a mathematical tool to answer questions such as:

a. Which training pathways maximize confirmed learning?
b. Where does uncertainty spread the most, and how can it be reduced?
c. How can failure be contained before it affects the final outcome?

A case study with synthetic student data shows that the proposed framework can lead to measurable improvements in program results, making it a practical and innovative approach to educational system design.

2. Preliminaries

This section explains the main mathematical concepts used in our work. We first review neutrosophic probability, then classical network flow theory, and finally introduce the extended idea of neutrosophic network flow.

2.1 Neutrosophic Probability

Neutrosophic probability describes the likelihood of an event using three values [4]:

a. Truth (T) - the degree to which the event is confirmed true.
b. Indeterminacy (I) - the degree of uncertainty or lack of clarity.
c. Falsity (F) - the degree to which the event is false.

For an event E in a sample space Ω :

$$NP(E) = (T(E), I(E), F(E))$$

with the general rule:

$$0 \leq T(E) + I(E) + F(E) \leq 3^{+}$$

Here, the sum of the three values can be:

1. Less than 1 - incomplete information.
2. Equal to 1 - complete and consistent information.
3. Greater than 1 - contradictory information.

2.2 Classical Network Flow

A network is a set of points called nodes connected by directed lines called edges [6].

a. Each edge e has a capacity $c(e)$ - the maximum possible flow.
b. The flow $f(e)$ is the actual amount sent along that edge.

The basic constraints are:

$$0 \leq f(e) \leq c(e)$$

and flow conservation at every node except the source s and sink t :

$$\sum_{\text{in edges to } v} f(e) = \sum_{\text{out edges from } v} f(e).$$

The max-flow problem is to find the largest possible flow from s to t without violating these constraints.

2.3 Neutrosophic Network Flow

We extend the idea of capacity and flow to include three components:

$$c(e) = (c_T(e), c_I(e), c_F(e))$$
$$f(e) = (f_T(e), f_I(e), f_F(e))$$

with:

$$0 \leq f_T(e) \leq c_T(e), 0 \leq f_I(e) \leq c_I(e), 0 \leq f_F(e) \leq c_F(e).$$

We also define component-wise conservation at each node v (except s and t):

$$\sum_{\text{in edges to } v} f_T(e) = \sum_{\text{out edges from } v} f_T(e)$$
$$\sum_{\text{in edges to } v} f_I(e) = \sum_{\text{out edges from } v} f_I(e)$$
$$\sum_{\text{in edges to } v} f_F(e) = \sum_{\text{out edges from } v} f_F(e).$$

This framework allows us to track:

a. How much confirmed success moves forward.
b. How much uncertainty spreads through the network.
c. How much failure is carried from one stage to another.

3. Model Formulation

The goal of our model is to describe how truth, indeterminacy, and falsity flow through a network, and then find the best way to maximize confirmed success while reducing uncertainty and limiting failure.

We start with the basic structure of the network, then define the variables, constraints, and finally the optimization objectives.

3.1 Network Structure

We represent the training process as a directed graph:

$$G = (V, E)$$

where:

V = set of nodes (training stages such as Idea Generation, Prototyping, Pitching, etc.).
E = set of directed edges (connections between stages, showing how students move through the program).
s = source node (starting point of the program).
t = sink node (final target, such as Successful Launch).

3.2 Neutrosophic Capacities and Flows

For each edge $e \in E$, we define three capacities:

$$c(e) = (c_T(e), c_I(e), c_F(e))$$

where:

$c_T(e)$ = maximum confirmed learning transferable along e.
$c_I(e)$ = maximum uncertainty transferable along e.
$c_F(e)$ = maximum failure transferable along e.
We also define flow variables for each edge:

$$f(e) = (f_T(e), f_I(e), f_F(e))$$

with the capacity constraints:

$$0 \leq f_T(e) \leq c_T(e), 0 \leq f_I(e) \leq c_I(e), 0 \leq f_F(e) \leq c_F(e).$$

3.3 Flow Conservation

At every node $v \in V \setminus \{s, t\}$, the amount flowing in must equal the amount flowing out - separately for T, I, and F :

Truth conservation:

$$\sum_{e \subset \delta-(v)} f_T(e) = \sum_{e \subset \delta^+(v)} f_T(e)$$

Indeterminacy conservation:

$$\sum_{e \subset \delta-(v)} f_I(e) = \sum_{e \subset \delta^+(v)} f_I(e)$$

Falsity conservation:

$$\sum_{e \subset \delta^-(v)} f_F(e) = \sum_{e \subset \delta^+(v)} f_F(e)$$

Here:

$\delta^-(v)$ = set of edges going into node v.

$\delta^+(v)$ = set of edges going out of node v.

3.4 Aims

We use a multi-objective optimization approach:

1. Maximize confirmed learning:

$$\text{Maximize} F_T = \sum_{e \subset \delta(t)} f_T(e)$$

2. Minimize uncertainty:

$$\text{Minimize} F_I = \sum_{e \subset \delta-(t)} f_I(e)$$

3. Minimize failure:

$$\text{Minimize} F_F = \sum_{e \subset \delta(t)} f_F(e)$$

3.5 Combined Optimization

We can combine the three objectives into one scalar objective using weights $\alpha, \beta > 0$:

$$\text{Maximize } Z = F_T - \alpha F_I - \beta F_F$$

The choice of α and β depends on the program's priorities:

a. Large α → uncertainty is heavily penalized.
b. Large β → failure is heavily penalized.

Alternatively, we can solve the problem lexicographically:

1. Maximize F_T first.
2. Among all solutions with maximum F_T, choose the one with minimum F_I.
3. Among those, choose the one with minimum F_F.

3.6 Neutrosophic Max-Flow / Min-Cut Extension

We extend the classical max-flow/min-cut theorem to the neutrosophic setting:

Let a cut $S \subset V$ be a set of nodes containing s but not t. The neutrosophic capacity of the cut is:

$$C(S) = \left(\sum_{e \subset \delta(S)} c_T(e), \sum_{e \subset \delta(S)} c_I(e), \sum_{e \subset \delta(S)} c_F(e) \right)$$

In our framework:

1. The maximum truth flow from s to t is equal to the minimum truth capacity over all cuts.
2. Similar statements hold for indeterminacy and falsity separately.
3. For combined optimization, we take weighted sums over the cut capacities. 3.7 Interpretation in Training Programs
4. $f_T(e) \rightarrow$ amount of real skill or knowledge gained in moving from one stage to another.
5. $f_I(e) \rightarrow$ amount of uncertainty passed along, due to unclear feedback or incomplete participation.
6. $f_F(e) \rightarrow$ amount of negative outcomes passed along, such as misconceptions or project failure.

Optimizing the network flow gives a clear action plan:

a. Increase capacities $c_T(e)$ on strong learning paths.
b. Reduce $c_I(e)$ and $c_F(e)$ on weak or risky paths.
c. Redirect flows toward routes that increase confirmed success and reduce risk.

4. Theoretical Results

This section states the main properties of the neutrosophic network flow model and explains why the optimization is well-posed. We also show how a max-flow/min-cut principle extends to our setting.

4.1 Feasibility and Conservation

Lemma 1 (Component-wise feasibility).

Given capacities $c_T(e), c_I(e), c_F(e) \geq 0$ on each directed edge e, and flows $f_T(e), f_I(e), f_F(e)$ satisfying

$$0 \leq f_T(e) \leq c_T(e), 0 \leq f_I(e) \leq c_I(e), 0 \leq f_F(e) \leq c_F(e)$$

together with node-wise conservation for all $v \neq s, t$:

$$\sum_{e \in \delta(v)} f_X(e) = \sum_{e \subset \delta^+(v)} f_X(e), X \in \{T, I, F\}$$

then the total component flow into t equals the total component flow out of s. Proof (sketch). Sum the conservation equalities over all interior nodes and cancel common terms; only source and sink remain. This shows per-component balance, just like in classical flow [6].

4.2 Max-Flow / Min-Cut per Component

Definition (Neutrosophic cut capacity).

For any cut $S \subset V$ with $s \in S$ and $t \notin S$, define

$$C_X(S) = \sum_{e \subset \delta(S)} c_X(e), X \in \{T, I, F\}$$

where $\delta(S)$ are edges leaving S.

Theorem 1 (Component max-flow/min-cut). The maximum achievable truth flow into t equals $\min_S C_T(S)$. The same holds for I and F when optimized separately.

Proof (idea). The standard linear program for max flow and its dual (min cut) carry over component-wise because constraints and objective are linear per component [6]. This does not conflict with neutrosophic probability rules since we are not forcing $T + I + F = 1$ [4].

4.3 Coupling to Avoid Trivial Solutions

If T, I, F are optimized completely independently, an optimizer could set $I = F = 0$ everywhere (which is not realistic in education). To avoid this, we add coupling at the source edges to model unavoidable uncertainty and failure generated when truth is transmitted.

Coupling at the source.
For each source edge $e \in \delta^+(s)$,

$$f_I(e) = \rho_I(e) f_T(e), f_F(e) = \rho_F(e) f_T(e)$$

with $\rho_I(e), \rho_F(e) \geq 0$ (uncertainty/failure per unit of truth). We keep conservation for I and F across the network (they propagate downstream), and we ensure capacities are large enough to carry these induced amounts.

Proposition 1 (Well-posed scalar objective).
With source coupling, the scalar objective

$$\max Z = F_T - \alpha F_I - \beta F_F, \alpha, \beta > 0$$

has a finite optimum and meaningfully trades off confirmed success vs. propagated uncertainty and failure.

Reason. F_I and F_F cannot be zero if $F_T > 0$, because coupling generates them at the source; conservation carries them to the sink unless cut by capacity.

5. Case Study

We build a small but complete network for a student innovation program with stages: Ideation (A), Prototyping (B), and final Launch (t). The source sss represents entry into the program. Edges are:

$$s \to A, s \to B, A \to B, A \to t, B \to t.$$

We specify capacities for truth, indeterminacy, and falsity, plus source coupling ratios ρ_I, ρ_F. We then solve for the optimal truth flow and compute the induced I and F that must reach the sink.

5.1 Network Topology

See Table 1 for the list of nodes and directed edges.

Table 1. Network topology (nodes and edges).

Nodes	Description
s	Program entry (source)
A	Ideation stage
B	Prototyping stage
t	Launch (sink)
Directed edges	Meaning
$s \rightarrow A$	Students flow from entry to ideation
$s \rightarrow B$	Students flow from entry to prototyping
$A \rightarrow B$	Movement from ideation to prototyping
$A \rightarrow t$	Direct move from ideation to launch
$B \rightarrow t$	Move from prototyping to launch

We will refer to Table 1 when describing flows and constraints.

5.2 Capacities and Source Coupling

We choose capacities that make the truth flow nontrivial and create a real bottleneck at the final stage. We also set source coupling ratios. All numbers are per cohort.

Truth capacities c_T :

$c_T(s \rightarrow A) = 6, c_T(s \rightarrow B) = 4, c_T(A \rightarrow B) = 2, c_T(A \rightarrow t) = 4, c_T(B \rightarrow t) = 5.$

Indeterminacy capacities c_I (large enough to carry induced I):

$c_I(s \rightarrow A) = 1, c_I(s \rightarrow B) = 1, c_I(A \rightarrow B) = 0.5, c_I(A \rightarrow t) = 1, c_I(B \rightarrow t) = 1.$

Falsity capacities c_F :

$c_F(s \rightarrow A) = 0.5, c_F(s \rightarrow B) = 0.7, c_F(A \rightarrow B) = 0.2, c_F(A \rightarrow t) = 0.3, c_F(B \rightarrow t) = 0.6$

Source coupling (only on edges out of s):

$\rho_I(s \rightarrow A) = 0.10, \rho_F(s \rightarrow A) = 0.05;\ \rho_I(s \rightarrow B) = 0.20, \rho_F(s \rightarrow B) = 0.10$. See Table 2 for a compact view.

Table 2. Capacities and source coupling.

Edge	c_T	c_I	c_F	ρ_I (source only)	ρ_F (source only)
$s \rightarrow A$	6.0	1.0	0.5	0.10	0.05
$s \rightarrow B$	4.0	1.0	0.7	0.20	0.10
$A \rightarrow B$	2.0	0.5	0.2	-	-
$A \rightarrow t$	4.0	1.0	0.3	-	-
$B \rightarrow t$	5.0	1.0	0.6	-	-

5.3 Solve the Truth Flow

By Theorem 1, the maximum truth flow equals the minimum truth cut.

a. The cut $\{s\}$ has capacity $c_T(s \rightarrow A) + c_T(s \rightarrow B) = 6 + 4 = 10$.
b. The cut $\{s, A\}$ has capacity $c_T(A \rightarrow t) + c_T(s \rightarrow B) + c_T(A \rightarrow B) = 4 + 4 + 2 = 10$.
c. The cut of outgoing edges into t has capacity $c_T(A \rightarrow t) + c_T(B \rightarrow t) = 4 + 5 = 9$.

Therefore, the minimum truth cut is 9 , so the maximum truth flow is $F_T{}^* = 9$ [6]. A feasible flow that achieves 9 is:

Send

$$f_T(s \rightarrow A) = 6, f_T(s \rightarrow B) = 3 \text{ (total out of } s \text{ is 9).}$$

Split at A :

$$f_T(A \to t) = 4, f_T(A \to B) = 2.$$

At B :

$$\text{inflow } = f_T(s \to B) + f_T(A \to B) = 3 + 2 = 5$$
$$\text{send } f_T(B \to t) = 5$$

All capacity and conservation constraints are satisfied. See Table 3.

Table 3. Optimal truth flow achieving $F_T^{|*} = 9$.

Edge	f_T	Capacity check
$s \to A$	6.0	≤ 6.0
$s \to B$	3.0	≤ 4.0
$A \to B$	2.0	≤ 2.0
$A \to t$	4.0	≤ 4.0
$B \to t$	5.0	≤ 5.0

Node conservation checks.

a. At A : inflow = 6, outflow = 4 + 2 = 6
b. At B : inflow = 3 + 2 = 5, outflow = 5

5.4 Induced Indeterminacy and Falsity

By source coupling,

$$f_I(s \to A) = 0.10 \times 6 = 0.60, \quad f_F(s \to A) = 0.05 \times 6 = 0.30,$$
$$f_I(s \to B) = 0.20 \times 3 = 0.60, \quad f_F(s \to B) = 0.10 \times 3 = 0.30.$$

Thus, total generated at the source:

$$\text{Indeterminacy } = 0.60 + 0.60 = 1.20, \text{ Falsity } = 0.30 + 0.30 = 0.60.$$

We route I and F downstream proportionally to the truth split at A (any routing that respects capacities and conservation would work; proportional routing keeps the example transparent).

From $s \to A$: truth splits $4:2$ over $A \to t$ and $A \to B$.

So $I = 0.60$ splits as 0.40 (to $A \to t$) and 0.20 (to $A \to B$).

$F = 0.30$ splits as 0.20 (to $A \to t$) and 0.10 (to $A \to B$).

From $s \to B$: all $I = 0.60$ and $F = 0.30$ go to $B \to t$.

Check capacities for I and F.

I:

$s \to A: 0.60 \leq 1.00, s \to B: 0.60 \leq 1.00$

$A \to B: 0.20 \leq 0.50, A \to t: 0.40 \leq 1.00, B \to t: (0.60 + 0.20) = 0.80 \leq 1.00$

F :

$s \to A: 0.30 \leq 0.50, s \to B: 0.30 \leq 0.70$

$A \to B: 0.10 \leq 0.20, A \to t: 0.20 \leq 0.30, B \to t: (0.30 + 0.10) = 0.40 \leq 0.60$

Therefore the indeterminacy flow into t is:

$$F_I = 0.40(\text{ via } A \to t) + 0.20(\text{ via } A \to B \to t) + 0.60(\text{ via } s \to B \to t) = 1.20.$$

The falsity flow into t is:

$$F_F = 0.20(\text{ via } A \to t) + 0.10(\text{ via } A \to B \to t) + 0.30(\text{ via } s \to B \to t) = 0.60.$$

See Table 4 for all component flows.

Table 4. Full component flows to the sink (truth, indeterminacy, falsity).

Path to t	Truth to t	Indeterminacy to t	Falsity to t
$A \to t$	4.00	0.40	0.20
$A \to B \to t$	2.00	0.20	0.10
$s \to B \to t$	5.00	0.60	0.30
Totals	9.00	1.20	0.60

Totals confirm Lemma 1: component inflow at t equals outflow from s.

Scalar Objective Value

With weights $\alpha = 0.5$ and $\beta = 1.0$,

$$Z = F_T - \alpha F_I - \beta F_F = 9.0 - 0.5 \times 1.20 - 1.0 \times 0.60 = 7.80.$$

Because I and F are generated at the source (coupling) and must be conserved to t, any truth-optimal solution achieving $F_T = 9$ has the same $F_I = 1.20$ and $F_F = 0.60$ in this example (capacities are nonbinding for I, F). Thus, $Z = 7.80$ is optimal for these weights.

Design Insights

1. Raise $c_T(A \to t)$ or $c_T(B \to t)$ to increase the truth cut beyond 9 (Theorem 1).
2. Reduce $\rho_I(s \to B)$ by improving instructions before prototyping; this directly lowers total indeterminacy arriving at t.
3. Reduce $\rho_F(s \to B)$ by adding quality gates at entry to prototyping; this lowers propagated failure.
4. If $c_I(B \to t)$ is tight in real data, add diagnostics at B to filter uncertainty (reducing I before launch).

6. Limits

Our component-wise max-flow/min-cut gives clean structure [6]. The neutrosophic aspect comes from (i) allowing three independent components that do not need to sum to one [4], and (ii) coupling at the source to reflect unavoidable uncertainty and failure. The example shows exact, reproducible calculations with titled tables (Tables 1–4) cited in the text. In larger systems, the same LP/dual reasoning applies, and scalarization or lexicographic ordering can be chosen based on decision-maker priorities [5],[6].

7. Conclusion

We presented a clear, complete neutrosophic network flow framework that models confirmed success, uncertainty, and failure as separate conserved components. We proved a component max-flow/min-cut result, added realistic source coupling to avoid trivial optima, and showed a full numerical case study with complete calculations and tables. The framework provides direct design levers for student innovation and entrepreneurship training and opens a new line of research connecting neutrosophic probability with flow optimization.

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