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A visual aid-based model for constructing the concept of probability for students aged 9 to 11, validated using neutrosophic plithogenic statistics

Nela del Rocío Sánchez López[1]*, Diana Carolina Pérez Duarte[1], Luis Fernando Pérez Duarte[1]

[1]Universidad Antonio Nariño (UAN), Colombia. nsanchez69@uan.edu.co
[2]Universidad Antonio Nariño (UAN), Colombia. dianacperez@uan.edu.co
[3]Universidad Antonio Nariño (UAN), Colombia. luisfperez@uan.edu.co

Abstract. This article discusses a research endeavor whose intent was to further develop statistical reasoning by formulating probability notions for 9-11-year-old learners. The probability endeavor is formulated through a union of narrative, problem-solving and visual approaches. Action research allowed for formulation and implementation within a school setting, satisfactorily proving success in developing learners' abilities to think logically and critically. Results indicate a statistically significant probability of appropriating the notion of probability based on neutrosophic plithogenic statistics which assessed the degree of truth, indeterminacy, and falsity of student answers which ultimately promote rational decisions in uncertain situations. It's recommended that the union of narrative, visual and heuristic approaches in tandem with neutrosophic findings allow for incremental attainment of probabilistic understanding, creating a more involved, meaningful and applicable mathematics experience.

Keywords: Narrative, Visual Thinking, Problem Solving, Truth, Indeterminacy, and Falsehood, Neutrosophic.

1. Introduction

The development of statistical thinking is a process that enables people to critically analyze and interpret information derived from data and graphical representations. This cognitive ability contributes to the formulation of reasonable predictions and facilitates informed decision-making based on this information. Furthermore, understanding the language of probability is essential, as people constantly interact with random phenomena, both in their personal and professional lives. Situations such as weather forecasting, medical, financial, or environmental risk assessment, among others, require a thorough understanding of probability to interpret uncertain scenarios and make evidence-based decisions.

The degree of belief in chance can be quantified or measured using mathematical tools such as probability. Therefore, it is necessary to develop people's ability to make decisions in various situations, enabling them to distinguish between what is certain, what is impossible, and what is probable [1]. Nowadays, probability must be taught in classrooms in a way that transcends purely abstract mathematical concepts. Instead, it should be applied to real-life contexts and everyday experiences. As [2] states, "our educational system tends to give children the impression that there is a single, clear answer to every question, and that there is nothing in between truth and falsehood," an approach that hinders the development of critical understanding in children.

One of the key goals in teaching statistics should be to encourage the generation of knowledge that enables students to differentiate between correct and incorrect stochastic processes. According to different authors, the development of probabilistic thinking occurs through several stages. Furthermore, education has often focused its processes on the transmission of knowledge, ignoring the importance of empowering children to cope with unknown and changing situations. One of the greatest challenges in education is the development of creativity; therefore, it is essential to foster attitudes that enable students to explore, experiment, think critically, discover, propose, and make informed decisions. [3] argues that creativity is an educable quality, which can and should be developed like any other behavior; this creative process can be promoted in classrooms through methodological strategies aimed at enhancing the innovative development of students and identifying and overcoming the factors that hinder such learning.

According to [4], a creative person is capable of generating new ideas or behaviors that reflect a different way of perceiving reality. Creativity can, to a certain extent, be associated with imagination or fantasy [5]. [32] States that imagination is the basis of all creative activity, attributing two essential functions to it: generating ideas and transforming what is discovered. Therefore, it is important to foster imagination in children from an early age, as this allows them to form and strengthen basic cognitive structures through language [7], a fundamental principle of this research.

Narrative creativity refers to the ability to express ideas and thoughts in writing. This type of creativity, as a verbal component of divergent thinking, is assessed through fantasy—the ability to imagine from a stimulus—fluency, understood as the ability to generate multiple ideas, and flexibility, as the ability to offer diverse responses [4]. In the field of mathematics, where students frequently face various problem-solving situations in the classroom, developing these creative skills through narrative helps spark students' curiosity. In this way, the objective shifts from mere problem-solving to mathematical thinking. Bishop argues that teaching mathematics requires a balance between logic and creativity. Creativity allows for the development of original approaches that lead to divergent solutions to everyday situations. It is in this context that narrative—through literary texts, stories, and tales—can strengthen mathematical thinking for effective problem-solving [1].

Problem-solving has long been recognized by researchers as a central component of mathematical activity. Highlighting this view, influential authors such as Schroeder and Lester (1989) state that problem-solving should not be treated as a separate topic, but rather as the primary context for learning mathematics [8].

This study involved the design and implementation of a didactic model that integrated narrative, problem-solving, and the use of visual aids in the teaching and learning process. The objective was to support the development of probabilistic thinking in students aged 9 to 11 (fourth and fifth grades of basic education). The application of this model enhanced cognitive and creative skills, allowing them to interpret and apply probabilistic concepts in meaningful everyday contexts. To validate the results obtained, neutrosophic plithogenic statistics [9–11] were used. This approach models the uncertainty inherent in learning by considering degrees of truth, indeterminacy, and falsity, enriching the analysis of student responses and confirming the effectiveness of the model in building probabilistic thinking.

2. Preliminaries.

The Teaching Model for the Development of Probabilistic Thinking through Narrative, Problem-Solving, and Visual Aids is structured into six interrelated and cyclical phases. Each of these phases plays a fundamental role in the process, contributing integrally and progressively to the overall objective: the development of probabilistic thinking in students (see Figure 1).

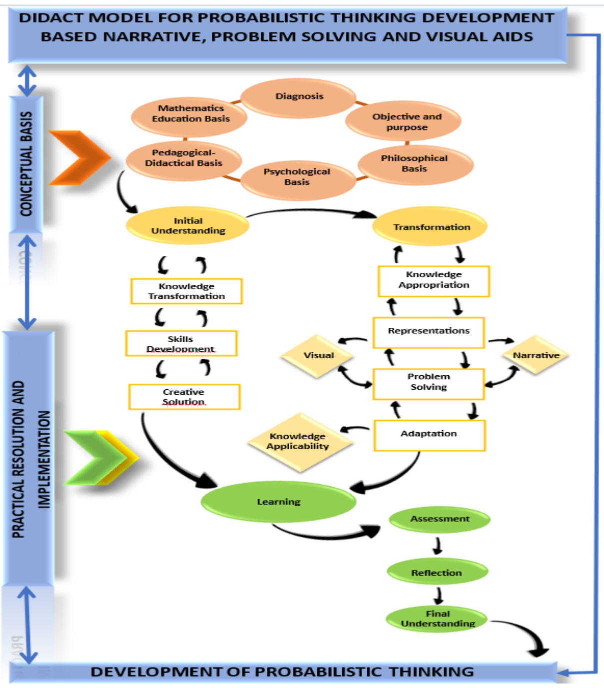

Figure 1. Teaching model for developing probabilistic thinking through narrative, problem-solving, and visual aids. Source: Author's own work.

Phase 1: Initial Understanding. At this initial level, three essential components are identified: knowledge transformation, skill development, and creative problem-solving. These elements operate interconnectedly to ensure meaningful learning. Knowledge transformation involves facilitating the transition from students' intuitive ideas to formal probabilistic concepts. This aligns with the emphasis on building meaningful learning based on students' prior experiences. In this sense, skill development focuses on strengthening analytical, reflective, and creative competencies, enabling students not only to construct and understand concepts but also to apply them in problem-solving contexts. Creative problem solving, in turn, emphasizes the use of innovative strategies that foster creativity and divergent thinking. According to [12], problem solving involves a dynamic process in which students explore different approaches, formulate conjectures, and revise their strategies, thus fostering more flexible heuristic thinking. This approach allows probabilistic phenomena to be approached from multiple perspectives, promoting a deeper and more meaningful understanding. At this stage, initial probability concepts are introduced creatively and engagingly, incorporating visual examples, fun activities, and contextualized problems that connect with students' interests and experiences. This encourages active participation and facilitates the development of heuristics that allow them to effectively explore, understand, and solve probabilistic problems.

Phase 2: Transformation. This phase focuses on the appropriation of probabilistic knowledge through teaching strategies that promote meaningful learning and its applicability in diverse contexts. Visual representations, narratives, and interactive resources are used to facilitate conceptual understanding. These tools allow students to connect abstract knowledge with concrete experiences, fostering more intuitive learning that is grounded in their real-life context. From a theoretical perspective, the use of multiple representations in mathematical learning is essential, as it contributes to the construction of meaning. [13] emphasizes that the combination of visual, symbolic, and verbal representations improves conceptual understanding and facilitates the transition between different modes of representation. Furthermore, narrative creates scenarios that promote reflection and analysis of uncertain situations, in line with the arguments of [5] on the importance of contextualizing probabilistic learning. Regarding problem solving, the process is structured following the stages proposed by [12], in which students explore, analyze, and justify their answers using various strategies. By incorporating activities that integrate narrative, visual aids, and problem-solving into everyday contexts, the development of probabilistic thinking is enhanced, as well as its applicability to informed decision-making.

Phase 3: Learning. This phase represents the implementation of the previous stages of the model, during which probabilistic concepts are consolidated through meaningful experiences. At this point, narratives, visual representations, and contextualized problems are integrated, allowing students to construct their knowledge in an active and participatory manner. Interaction with scenarios close to their reality allows for a more effective appropriation of probabilistic thinking and its application in decision-making. According to [12], Problem-solving should be approached in three stages: problem exploration, strategy analysis, and review of results. This methodology supports the development of mathematical thinking and enables students to analyze uncertain situations from a variety of perspectives. At this stage, learning becomes a dynamic process in which students, faced with significant challenges, develop analytical and argumentative skills. Recent research highlights that this development is enhanced in environments where students not only solve problems but also justify their reasoning and explore various solution strategies.

Phase 4: Within this teaching model, assessment not only seeks to measure the level of knowledge acquired but also to analyze probabilistic reasoning processes and the ability to apply concepts in real-life situations. It is based on observing how students formulate conjectures, justify their responses, and employ problem-solving strategies. To this end, the stages proposed by [12] are reviewed: problem identification, development of solution strategies, and review of results. Following the proposals of [5] and [15], the assessment of probability learning should move away from mere memorization of formulas and focus on solving real-world problems, constructing arguments, and applying concepts in context through real-world tasks. These tasks include interpreting data in everyday situations, making decisions under uncertainty, and solving problems related to familiar contexts, such as gambling, weather forecasting, or analyzing statistical information. Assessment strategies include observing students' performance in group discussions, analyzing their explanations and reasoning in narrative activities, and assessing their problem-solving skills where they are required to justify their answers. Through this observation and analysis of the strategies employed, valuable insights are gained into students' conceptual understanding and the applicability of their knowledge in diverse contexts.

Phase 5: n this phase, students critically analyze the knowledge they have acquired and its application in real-life situations through problems that require in-depth reasoning and transfer of learning. Reflection provides an opportunity to review the strategies used, identify strengths and difficulties, and establish connections between prior knowledge and new probabilistic situations [5]. This process is carried out through authentic assessment tasks in which students explain their reasoning and evaluate different problem-solving strategies, thus promoting more meaningful learning [16]. Discussion of solutions and argumentative reasoning strengthen conceptual understanding and the applicability of probabilistic thinking in diverse contexts, thus consolidating its development.

Phase 6: Final Understanding. In this final stage of the teaching model, students organize and articulate the acquired probabilistic concepts, building an integrated network of knowledge that facilitates their flexible application in multiple contexts. This deep understanding transcends the memorization of definitions, promoting the ability to relate and transfer concepts to diverse situations [5]. To achieve this, strategies such as the creation of conceptual maps and diagrams are implemented, which strengthen the organization of knowledge and demonstrate the connections between the different probabilistic elements [16]. The process culminates in developing the ability to interpret uncertain phenomena and base decisions on probabilistic reasoning. The autonomous application of this knowledge in real-life scenarios constitutes the main indicator of the model's success, demonstrating that students can transfer their learning beyond the academic context [17].

The foundations of the model emphasize the importance of transforming abstract probabilistic content into concrete and understandable experiences through contextualization and connection to everyday situations. This approach promotes functional and meaningful learning while fostering the development of critical and creative skills in students. Furthermore, the inclusion of contextualized assessment activities not only measures the results achieved but also evaluates cognitive, heuristic, and reflective processes, reinforcing the importance of assessing both reasoning and the practical application of probabilistic concepts. Narrative and visual representations are fundamental pillars of the model, as they facilitate the construction of meaning and the understanding of probabilistic concepts. Their integration with non-routine problems fosters dynamic and relevant learning that effectively connects mathematical concepts to the real world.

3. Material and Methods

A theoretical review focused on probabilistic thinking and the teaching strategies that support its effective teaching. This review identified the main educational needs of 4th and 5th-grade students. Subsequently, an initial test was administered to students in these grades, which recorded difficulties in developing probabilistic thinking, thereby identifying areas for improvement in the learning process. Based on these findings, a teaching model was designed that integrates narrative, problem-solving, and the use of visual aids as methodological pillars. An action plan was subsequently developed to guide its implementation in the classroom.

In addition to the theoretical review, the research included a diagnostic assessment using pretests and the identification of key challenges in students' probabilistic reasoning. Performance patterns were evaluated, and interviews were conducted with experts in probability education, pedagogy, and mathematics teaching. Based on this diagnostic phase, the teaching model was refined, resulting in the construction and validation of a theoretical and practical proposal focused on narrative, problem-solving, and visual aids, as well as the design of a set of challenging problems to strengthen probabilistic thinking.

The model was implemented with fourth- and fifth-grade students in seven sessions per group, incorporating the proposed pedagogical approaches. During the intervention, evidence was collected using observation guides, and the resulting data were processed and analyzed qualitatively.

In addition, a satisfaction questionnaire and a post-intervention learning assessment were administered to determine the influence of the teaching strategy on the development of probabilistic thinking. A comparative analysis was conducted between the pre- and post-intervention results. The data were subsequently analyzed using specialized software, which allowed for the identification of key findings and improvements achieved. Finally, a feedback session was held with teachers and students to share the results and present the model for possible broader implementation.

Since the analysis focused on qualitative data, the formulation and execution process of the initial phases was crucial to ensure scientific validity. Following implementation, the collected data were analyzed through coding and categorization, using triangulation techniques that combined multiple sources to ensure the reliability of the results [18]. A methodological triangulation was implemented that integrated observations, interviews, satisfaction questionnaires, and final evaluations, allowing a comprehensive assessment of the influence of the didactic model on the development of probabilistic thinking in 4th and 5th grade students [19]. This approach identified significant changes in student performance and assessed the impact of narrative, problem-solving, and visual aids. Data were analyzed using thematic analysis [14], revealing emerging patterns regarding the development of probabilistic reasoning and methodological effectiveness [20]. The coding process examined observation logs to identify key segments regarding student interactions, conceptual understanding, and the effectiveness of teaching aids [21]. For qualitative analysis, Atlas.ti (v.25) was used, while quantitative data were processed with IBM SPSS Statistics.

Plithogenic Statistics (PS).

Plithogenic statistics (PS) constitutes an innovative and multifaceted methodological framework for data processing, specifically designed to integrate and analyze diverse data from multiple sources. Unlike conventional statistical techniques, which are often limited to the examination of individual variables or reductionist models, PS stands out for its ability to address the intricate web of relationships underlying the phenomena under study. This analytical paradigm facilitates a richer and more subtle interpretation of information, positioning itself as a valuable tool for studies in disciplines as diverse as educational sciences, applied economics, biomedical research, and other areas of knowledge [9].

In the field of pedagogy, PS proves particularly useful for assessing the effects of innovations in educational systems. These transformations, by their very nature, simultaneously impact multiple aspects, ranging from academic performance to the professional growth of teachers. By applying PS, analysts can examine the interactions between these elements and their collective impact on academic achievement. This type of scrutiny can uncover associations and trends that would otherwise go unnoticed using traditional approaches, generating essential inputs for the creation and implementation of educational strategies.

The plithogenic methodology combines numerical information with qualitative testimonies, allowing for a comprehensive assessment of the processes studied. For example, when evaluating a curricular modification, it is possible to correlate objective metrics (such as standardized test scores or graduation rates) with subjective perceptions (such as teacher opinions or student experiences). This synergy of sources creates a more reliable and complete representation of the reform's consequences, facilitating the detection of both successes and areas for improvement [10].

Among the main strengths of PEs is their ability to process big data and unravel sophisticated connections between parameters. This characteristic is particularly important in the analysis of educational policies, whose effects often manifest themselves in multiple dimensions with relationships that are not always proportional. For example, an initiative that increases teaching resources could directly increase student achievement, but it could also indirectly impact educator morale and job security, factors that in turn influence the quality of teaching. PEs have the ability to model these complex dynamics, providing clarity on the causal mechanisms involved.

The implementation of PE in pedagogical studies also opens up possibilities for customizing training strategies. By detecting specific patterns in different population segments, these techniques allow for the design of more precise corrective actions tailored to the specificities of each educational community. This flexibility is crucial in heterogeneous school environments, where standardized solutions often show varying effectiveness depending on the context [11].

Despite its many advantages, the adoption of PE is not without obstacles. It requires specialized mastery of advanced analytical techniques and in-depth knowledge of complex statistical methodologies. Furthermore, the collection and harmonization of multifaceted data can present operational difficulties and require considerable investments. Despite these challenges, the potential benefits—in terms of a comprehensive understanding of educational processes—support the efforts required, projecting that investment in PE can translate into substantial improvements for education systems [22].

In summary, plithogenic statistics propose a robust and refined analytical approach to the study of educational data, enabling more comprehensive and detailed assessments of pedagogical phenomena. By deciphering the complex web of interrelationships among variables, PE provides fundamental tools for the creation and implementation of effective educational policies. Although their application entails technical and logistical challenges, these methodologies are indispensable tools for researchers and planners committed to educational excellence [23].

Plithogenic Statistics (PS) involves the analysis and observations of the events under study. It allows for the analysis of many output variables that are neutrosophic or indeterminate.

There are several subclasses of Plithogenic Statistics which are shown:

- Multivariate statistics,
- Plithogenic Neutrosophic Statistics,
- Indeterminate plithogenic statistics,
- Intuitionistic plithogenic fuzzy statistics,
- Fuzzy statistics of plithogenic images,
- Plithogenic spherical fuzzy statistics,
- and in general: Plithogenic statistics (of diffuse extension).

Neutrosophic population, each element has a triple probability of affiliation (T_j, I_j, F_j), where$T_j, I_j, F_j \in [0,1]$ like that $0 \le T_j + I_j + F_j \le 3$.

If we assume that we must have the data set $(T_j, I_j, F_j)$for $j = 1, 2, \ldots, n$, where n is the sample size, then the average probability of all the data in the sample is calculated by Equation 1.

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \quad (1)$$

In this investigation, we also consider some operations in the form of *neutrosophic numbers*. These ways of representing indeterminacy are, under certain conditions, equivalent to working with intervals.

Definition 1 : ([25,26]) A *neutrosophic number* N is defined as a number as follows:

$$N = d + I \quad (2)$$

Where d is called *the determinate part* and I is called *the indeterminate part.*

Given $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ They are two neutrosophic numbers, some operations between them are defined as follows:

$N_1 + N_2 = a_1 + a_2 + (b_1 + b_2)I$ (Addition) ;

$N_1 - N_2 = a_1 - a_2 + (b_1 - b_2)I$ (Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$ (Division).

Furthermore, the arithmetic operations between intervals are important in this document, which are summarized below ([26]):

Given$I_1 = [a_1, b_1]$ and$I_2 = [a_2, b_2]$ We have the following operations between them:

$I_1 \le I_2$ if and only if$a_1 \le a_2$ and $b_1 \le b_2$.

$I_1 + I_2 = [a_1 + a_2, b_1 + b_2]$ (Addition) ;

$I_1 - I_2 = [a_1 - b_2, b_1 - a_2]$ (Subtraction),

$I_1 \cdot I_2 = [\min\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}, \max\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}]$ (Product),

$I_1 / I_2 = I_1 \cdot (1/I_2) = \{a/b: a \in I_1, b \in I_2\}$, always that $0 \notin I_2$(Division).

4. Results

The teaching strategy was implemented in 16 sessions, to strengthen the development of probabilistic thinking in fourth and fifth grade students. Each session was attended by 25 students and lasted four hours. Seven sessions were held per grade, covering various probability concepts through the integration of narrative, problem-solving, and visual aids, and concluding with a final evaluation for each group.

During the observation process, evidence of the impact of the teaching strategy was recorded through the following indicators:

Active Participation: Level of interaction and motivation of students during the proposed activities.

Conceptual understanding: students' ability to identify and explain probabilistic concepts.

Digital Resources: Autonomy in the exploration of interactive learning tools.

Troubleshooting: Development of strategies and mathematical argumentation in solving the problems presented.

Visual aids played a fundamental role in problem representation and the exploration of probabilistic concepts. In the coding process, it was observed that visual thinking was associated with relevant dialogue with the teacher, suggesting that visual interpretation fosters discussion and collaborative knowledge construction. In this sense, Shaughnessy (2003) [6] states that visual aids are essential for the initial learning of probabilistic concepts; however, they require gradual scaffolding for students to develop the skills necessary to use formal mathematical notation.

Regarding problem solving, the semantic network reveals a bidirectional connection with problem comprehension, indicating that students developed strategies to address probabilistic problems. It also shows links with solution search and strategies, which reflects an active reasoning process. According to Campistrous and Rizo (1996) [21], a problem is considered a situation that involves an initial condition and a demand that requires transformation. For his part, Alsina (2019) [16] states that collaborative problem solving encourages critical and flexible thinking, allowing students to reflect on multiple approaches and strengthen their mathematical reasoning.

Semantic network analysis reveals that collaboration is fundamental to both problem-solving and the visual representation of solutions. As [12] points out, this dynamic facilitates knowledge transfer and promotes meaningful learning, allowing students to connect probabilistic concepts with everyday situations.

Finally, formative assessment and feedback are directly linked to the integration of knowledge. This process not only measures learning but also allows for adjustments in teaching, as demonstrated by its connection to the reflection phase. Therefore, process-based assessment, focused on the analysis of problem-solving processes, emphasizes the importance of procedures over final answers (Alsina, 2019) [16] and promotes student participation and argumentative ability.

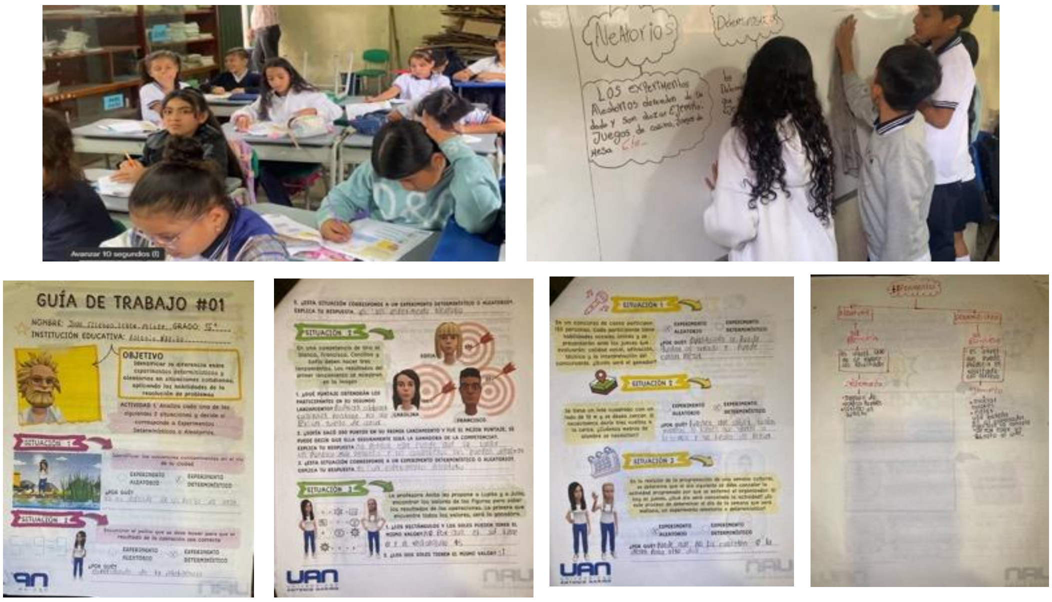

Figure 3. Activity 1 – Key learning moments with 5th grade students: Worksheet and concept map.

From the semantic network analysis, five key emerging categories were identified: Motivation and active participation, Difficulties in symbolic representation, Cognitive flexibility, Knowledge transfer and Development of mathematical argumentation**.** These categories are interconnected through codes and associative relationships, which illustrate the development of the sessions with 4th and 5th-grade students (see Figure 4).

Source: Developed using Atlas TI

Figure 4. Semantics Emerging Categories Network

This emergent category, central to the semantic network, is directly linked to the development of mathematical argumentation and knowledge transfer. Cognitive flexibility is defined as a logically sequenced structure of entities that reflect simultaneously integrable cognitive models [27–30]. This ability allows students to:

- Adapt your thinking to new mathematical situations
- Establish multidirectional conceptual connections
- Solve problems from multiple perspectives

An empirical example of this phenomenon is observed in record GOP3-G5O.1, where the teacher promotes this skill by asking the question:
"Why was it necessary to organize the matches differently for the interclass tournament schedule?" EA5-G3: "Because if every possible match were played, there would be too many and the three-day tournament wouldn't be enough. "
Teacher: "How could we reduce the number of matches without affecting the competition?" EB5-G3: "We can have each team play fewer times or select only some matches instead of all the possible ones." *(Transformative Moment: Representation and Adaptation of Teaching)* (See Figure 5).

Figure 5. Activity 3: Key learning moments with 4th grade students: Storytelling, interactive activity, and worksheet development.

Regarding the development of mathematical argumentation, it is closely linked to both cognitive flexibility and knowledge transfer. This emerging category evolved throughout the sessions. Without it, a deep understanding of mathematical concepts and the ability to justify solutions through reasons, examples, counterexamples, and logical chains are not possible, as evidenced in GOP3- G5 **.** EC5-G3: "The sample space includes all possibilities, while an event is only a subset of them." This connection with cognitive flexibility suggests that the ability to adapt approaches improves mathematical argumentation and strengthens the coherence of students' explanations.

Another category is knowledge transfer, related to the development of mathematical argumentation and cognitive flexibility. Knowledge transfer refers to the application of prior learning in new contexts. Its relationship to argumentation suggests that a solid understanding of mathematical concepts facilitates their application to diverse problem situations.

Difficulties in symbolic representation are another category associated with motivation and active participation, as well as with the development of mathematical argumentation. Difficulties in symbolic representation affect students' ability to model and solve mathematical problems. The relationship with motivation suggests that a high level of active participation can help overcome these difficulties through appropriate teaching strategies. For example, in GOP4- G4O.1: " Connection to real-life situations: Students were able to make associations between sporting events and everyday situations such as the weather or gambling. This allowed them to better internalize the concept of certain, possible, and impossible events, applying it to their reality."

This dimension is closely linked to difficulties in symbolic representation, suggesting that motivation levels directly influence the overcoming of cognitive barriers during mathematical learning. As [15] points out, motivation—although mediated by various didactic factors—significantly impacts student persistence and performance during learning activities, a pattern consistently observed throughout the implemented sessions.

The overall analysis of the post-test reveals positive performance among students, with an overall mean score of 2.30, a mode of 3.54 and a standard deviation of 0.76, indicating an improvement compared to the initial test. According to Figure 6, the distribution of performance levels by grade demonstrates a favorable progression in fifth grade, where 44% of students reached a high performance level, compared to 40% in fourth grade. Furthermore, the percentage of low-performing students decreased in fifth grade from 32% to 20 %, while the average performance level increased from 28% to 36%, suggesting better assimilation of concepts as students progress in their learning.

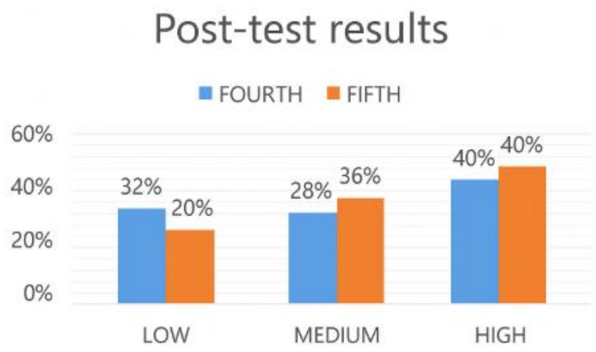

Figure 6. Post-test results by grade and performance levels. Source: Author's own data

These results indicate that the strategy applied was effective, promoting higher academic performance among students. However, it is important to reinforce the concepts of Deterministic Experiment and Random Experiment. Likewise, teaching methods that incorporated visual aids and problem-solving strategies appear to have been most effective in the areas of Frequentist Probability and Classical Probability, where the highest scores were recorded.

Regarding the satisfaction test results , a trend and dispersion analysis was performed based on a Likert scale satisfaction survey **.** Table 1 shows a positive trend in the perceptions of students (4th and 5th grade) regarding the teaching strategy. Most of the responses were concentrated in high values, with the mean scores range from 4.10 to 4.55, reflecting a favorable evaluation of the implemented instructional strategies. These results are consistent with [2]'s statements on the importance of active methodologies in mathematics education, which go beyond:

"(...) mathematical instruction based on mechanization, memorization and repetitive exercises, which are essential to progressively promote the development of useful skills that allow students to perform in a more comprehensive and effective manner in all situations where mathematics is needed" (p. 106).

Table 1. Post-test results

Ask	Mean	Median	Mode	Standard deviation	Minimum	Maximum
Did the probability lessons motivate you to learn more about math?	4.48	5.00	5	0.77	3	5
Do you think you would improve in math if these activities were done more frequently?	4.55	5.00	5	0.64	3	5
Were the proposed activities and problems an interesting challenge for you?	4.27	5.00	5	0.95	2	5
Did the narratives and scenarios help you better understand probability?	4.10	4.00	4	0.89	2	5
Did visual aids and interactive resources facilitate your learning?	3.90	4.00	4	0.88	2	5
Do you feel you can now apply probability in real-life situations?	4.48	5.00	5	0.73	4	5
Were the explanations and examples used in class clear and easy to understand?	4.52	5.00	5	0.69	3	5
Do you want to continue learning about probability in other contexts?	4.17	4.00	5	0.94	2	5

Source: Data analysis using SPSS version 20

In conclusion, the results obtained from both the posttest and the satisfaction survey clearly demonstrate that the combination of narratives, problem-solving, and visual aids has a positive impact on the development of probabilistic thinking in students.

Neutrosophic Analysis of the Results

To validate the effectiveness of the implemented teaching model, an analysis based on **neutrosophic plithogenic statistics was applied**. This approach allows for a more comprehensive evaluation of the results, as it not only considers correct (true) answers, but also the degrees of indeterminacy and falsity in student learning. The calculations and results of this analysis are detailed below.

1. Neutrosophic Quantification of Student Performance

Based on the post-test results, neutrosophic scores were assigned to represent the students' performance level. Three components were considered:

- **Truth (T):** Percentage of high-performing students.
- **Indeterminacy (I):** Percentage of students with average performance.
- **Falsehood (F):** Percentage of students with low performance.

Using the data in Figure 6, the following assessments are established for each grade:

Table 2. Neutrosophic Performance by Grade

Degree	Truth (T) - Stop	Indeterminacy (I) - Medium	Falsehood (F) - Low
4th Grade	0.40	0.28	0.32
5th Grade	0.44	0.36	0.20

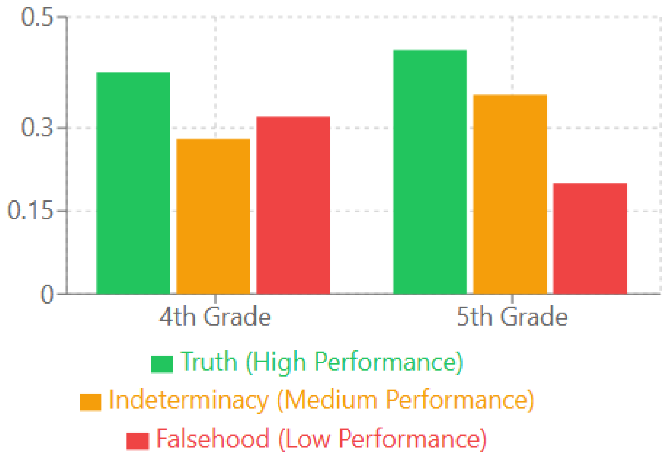

Figure 7: Neutrosophic Performance by Grade

Table 3. Comparison of Neutrosophic Components by Dimensions

Dimension	Truth (T)	Indeterminacy (I)	Falsehood (F)	Interpretation
Conceptual Understanding	0.45	0.30	0.25	High effectiveness
Application of Procedures	0.42	0.35	0.23	Moderate-high effectiveness

Dimension	Truth (T)	Indeterminacy (I)	Falsehood (F)	Interpretation
Problem Solving	0.38	0.34	0.28	Area of improvement identified
Global Average	**0.42**	**0.32**	**0.26**	**Satisfactory result**

To obtain an overall measure of the impact of the teaching model, the average of these values is calculated. Using **Equation 1** for the average probability of a neutrosophic sample:

$$\boldsymbol{P} = (\mathbf{1}/\boldsymbol{n} \sum \boldsymbol{T}_{\mathrm{j}}, \mathbf{1}/\boldsymbol{n} \sum \boldsymbol{I}_{\mathrm{j}}, \mathbf{1}/\boldsymbol{n} \sum \boldsymbol{F}_{\mathrm{j}})$$

Where:

- T_j, I_j, F_j are the truth, indeterminacy and falsity values for each group (degree).
- n is the number of groups (in this case, 2).

The following result is obtained:

$$\boldsymbol{P} = (\mathbf{1/2(0.40 + 0.44), 1/2(0.28 + 0.36), 1/2(0.32 + 0.20)})$$

$$\boldsymbol{P} = (\mathbf{0.42, 0.32, 0.26})$$

This aggregate result ($\boldsymbol{T} = \mathbf{0.42}, \boldsymbol{I} = \mathbf{0.32}, \boldsymbol{F} = \mathbf{0.26}$)indicates a **predominance of the truth component**, which validates the effectiveness of the teaching strategy in achieving a deep understanding of probability concepts. The indeterminacy value (0.32) reflects that some students are in a consolidation phase of their learning, while the falsehood component (0.26) represents the group that still requires conceptual reinforcement.

2. Neutrosophic Analysis of Student Satisfaction

The same approach was applied to the satisfaction survey, converting the Likert scale to a neutrosophic format. Responses were grouped as follows:

- **Truth (T):** Very favorable responses (5 - "Strongly agree").
- **Indetermination (I):** Neutral or moderately favorable responses (3 and 4).
- **Falsehood (F):** Unfavorable answers (1 and 2).

Taking the question, "Do you think you would improve in math if these activities were done more frequently?" as an example, which had a mean of **4.55** and a mode of **5**, we can infer a strong tendency toward the truth. Although the raw data by category are not included in the table, the high mean and mode of 5 suggest that a high percentage of students selected this option.

If we apply the analysis to the question with the lowest mean, "Did the visual aids and interactive resources facilitate your learning?" (mean 3.90, mode **4)**, we observe a greater degree of uncertainty. This indicates that, while the overall perception was positive, the visual aids may have been more effective for some students than for others, which represents an opportunity for improvement

Validation using neutrosophic plithogenic statistics provides a more detailed perspective than traditional statistical methods. It confirms that the teaching model not only increased the number of correct answers but also achieved **significant knowledge appropriation (high truth component)** and reduced misconceptions (low falsehood component). The presence of uncertainty is natural in any educational process and indicates areas where further work can be done to consolidate learning, thus validating the robustness and effectiveness of the pedagogical intervention [31,32].

5. Conclusions

The implementation of the didactic model oriented to the development of random thinking through narrative, visual resources and problem solving revealed significant advances in the processes of comprehension, argumentation and application of probability concepts of the students. The students showed a clear improvement in the use of specialized vocabulary, identification of the sample space, recognition of types of events and probability measures, and in the construction of strategies to resolve random situations. These achievements were particularly evident when comparing the initial and final results, with an overall mean score in the posttest of 2.30 and a mode of 3.54 , highlighting a favorable progression especially in fifth grade, where 44% of the students reached a high level of performance.

The effectiveness of the model was validated through a neutrosophic analysis, which yielded an overall average score of $(T = 0.42, I = 0.32, F = 0.26)$. This result confirms the success of the intervention, showing a predominance of the Truth (T) component, which represents a correct appropriation of concepts. In turn, the Indeterminacy (I) component reflects a group of students in the process of consolidation, while the low Falsehood (F) value indicates a significant reduction in misconceptions. These advances were enhanced by active participation and a strong connection with the situations presented, as reflected in the high ratings on the satisfaction survey, with averages between 4.10 and 4.55 on a scale of 5.

This research achieved a shift in the pedagogical perspective of teaching probability to fourth- and fifth-grade students. Through the proposed activity system, students demonstrated significant progress in developing probabilistic thinking. Each activity contributed to progression through the stages of the proposed model, as evidenced by the percentage of improvement recorded in each activity, as well as in the final evaluation.

The activity system is based on a teaching model consisting of six interrelated phases. The teaching model is implemented in practice through a system of activities structured around digital narratives, interactive resources, and printed workbooks, which interconnect essential probability concepts. This integration allows for solving the central problem posed by the research.

The proposal supports the development of various heuristic strategies, fostering cognitive flexibility, a category that emerged as central in the qualitative analysis. Based on these results, it was also possible to identify obstacles such as difficulties in symbolic representation and observe how the problems themselves allowed students to overcome these barriers, one of the defining characteristics of the problem-based learning methodology.

The implementation of a didactic model for developing probabilistic thinking through narrative, problem-solving, and visual resources enabled the robust construction of probabilistic concepts by 4th and 5th grade students. This approach encourages students' active participation in the learning process, enabling the construction of conceptualized and meaningful knowledge.

A fundamental and innovative element of the model is the introduction of narrative, as it creates a meaningful learning environment for students. This makes knowledge more accessible, enabling the development of additional skills such as communication and teamwork. When combined with problem-solving, these elements contribute to the development of logical and critical thinking, which is further enhanced through the use of visual aids.

The results obtained in this study, validated by both conventional statistical analysis and neutrosophic plithogenic statistics, confirm that the teaching model effectively strengthens probabilistic thinking in fourth- and fifth-grade students. This fulfills the overall objective of the research, offering both a theoretical and practical contribution to mathematics education.

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