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Multidimensional analysis of student loyalty using the neutrosophic Biplot method for retention strategies in higher education

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Abstract. Student loyalty in higher education represents a critical challenge for universities, particularly in the context of increasing competition and the need to ensure long-term retention. A key question arises: how can student loyalty be modeled when it is shaped by diverse and subjective perceptions? The relevance of this issue lies in its impact on institutional stability: when students withdraw before completing their programs, universities not only lose projected revenue but also face disruptions that may negatively influence academic rankings. Previous literature has explored drivers of satisfaction and determinants of loyalty; however, little attention has been given to approaches that explicitly incorporate indeterminacy within a multidimensional framework. This study addresses that gap by applying the neutrosophic Biplot, a method that enables the visualization and interpretation of multidimensional analyses involving indeterminacy, neutrality, and paradoxes. Using survey data from a student population, the analysis highlights key perceptions influencing loyalty, including instructional quality, sense of belonging, and accessibility of course registration processes. Findings reveal significant correlations among these factors, effectively visualized through graphical representation to support decision-making. This research contributes to the literature by introducing a novel neutrosophic approach for evaluating student loyalty and by offering retention strategies grounded in empirical evidence. Ultimately, the study enhances institutional knowledge management practices and supports the development of effective policies to strengthen student commitment in higher education.

Keywords: Student Loyalty, Higher Education, Neutrosophic Biplot, Retention, Uncertainty, Knowledge Management, Multidimensional Analysis.

1. Introduction

Student loyalty in higher education represents a fundamental pillar for the sustainability of academic institutions, in a context where competition to attract and retain students is intensifying globally. This study focuses on analyzing student loyalty using the neutrosophic Biplot method, an innovative approach that allows modeling the uncertainty inherent in human perceptions. The relevance of this topic lies in its direct impact on the strategic management of universities, since student retention not only ensures stable income but also strengthens institutional reputation and educational quality [1]. In an increasingly digitalized and diversified educational environment, understanding the factors that influence student loyalty has become a priority for institutions seeking to differentiate themselves [2].

Historically, studies on student satisfaction and loyalty have evolved from service quality-focused approaches to more complex perspectives that consider emotional, social, and academic factors [3]. In recent decades, globalization and digitalization have transformed higher education, raising students' expectations for personalized experiences and inclusive environments [4]. However, traditional analytical models often fail to capture the ambiguity and contradictions present in student perceptions, limiting their ability to generate effective retention strategies.

The central problem of this research arises from the need for an approach that integrates uncertainty into the multidimensional analysis of student loyalty. Although numerous studies have explored factors such as academic satisfaction or teaching quality [5], few have addressed the indeterminacy inherent in student perceptions in complex contexts. How can student loyalty in higher education be modeled, considering uncertainty and the multidimensional interactions between variables? This question constitutes the core of the study, seeking to provide a solution that combines analytical rigor with practical applicability.

The lack of tools that effectively handle uncertainty in student data analysis represents a significant gap in the literature. Traditional approaches, such as classical statistical analyses, typically assume deterministic data, ignoring the ambiguous nature of student opinions [6]. Therefore, the neutrosophic Biplot method is presented as a promising solution, capable of visualizing complex relationships between variables while incorporating uncertainty, thus offering a more complete representation of the factors influencing student loyalty.

Higher education faces increasing challenges in maintaining student loyalty in an environment where students have access to multiple educational options, both in-person and virtual [7]. Institutions must not only meet academic expectations but also foster a sense of belonging and emotional engagement. This study proposes that multidimensional analysis, powered by the neutrosophic Biplot method, can identify key patterns in student perceptions, providing valuable insights for designing effective retention strategies.

Uncertainty in student perceptions, such as satisfaction with teaching quality or facilities, often hinders strategic decision-making at universities. By employing the neutrosophic Biplot method, this study seeks to overcome the limitations of traditional approaches, offering a clear visualization of the interactions between variables and their impact on student loyalty. This approach allows institutions to prioritize resources and design tailored interventions based on robust data.

The analysis of student loyalty using this method not only has practical implications but also contributes to theoretical advancements in knowledge management in higher education. By systematically modeling uncertainty, the study offers a methodological framework that can be applied to other contexts where human perception plays a central role. This innovative approach opens up new possibilities for understanding complex dynamics in educational settings.

The objectives of this research are clear: first, to evaluate the effectiveness of the neutrosophic Biplot method for analyzing student loyalty in contexts of uncertainty; second, to identify key factors influencing student retention using multidimensional visualizations; and third, to propose practical strategies for improving knowledge management in higher education institutions. These objectives are aligned with the research question and will guide the development of the study, providing a significant contribution to both educational theory and practice.

2. Methodology

2.1 Introduction to the HJ-Biplot Approach

The HJ-Biplot is a multivariate graphical representation technique that allows individuals and variables to be projected simultaneously onto the same factorial space, ensuring high representation quality for both sets. Unlike classical biplots (such as GH and JK types), the HJ-Biplot seeks a symmetrical representation in which the Euclidean metric is preserved in both dimensions [8].

2.1.1. Mathematical basis of the HJ-Biplot

The HJ-Biplot is based on the singular value decomposition (SVD) of a data matrix $\mathbf{X} \in \mathbb{R}^{nxp}$, previously centered by rows and columns. This decomposition, represented as $\mathbf{X} = \mathbf{U}\,\mathbf{D}\,\mathbf{V}^T$, allows us to identify singular vectors on the left $\boldsymbol{U}$, singular vectors on the right $\boldsymbol{V}$and a diagonal matrix $\mathbf{D}$ with the singular values ordered in decreasing order. This algebraic structure enables the simultaneous graphical representation of individuals and variables on the same factorial space, seeking a symmetrical visualization in which the Euclidean metric is preserved in both dimensions, thus optimizing the quality of the joint representation [9,10].

Data Matrix

Let be a data matrix $X \in \mathbb{R}^{nxp}$, centered by rows and columns.

Singular Value Decomposition (SVD)

$X = U\,D\,V^T$

Where:

- **OR** $\in \mathbb{R}^{nxr}$: matrix of left singular vectors.
- $\mathbf{D} \in \mathbb{R}^{rxr}$: diagonal matrix with singular values$d1 \geq d2 \geq \cdots \geq dr > 0$
- $\mathbf{V} \in \mathbb{R}^{px_r}$: matrix of right singular vectors.
- r = range $(\boldsymbol{X})$.

Marker Selection – HJ-Biplot (Symmetric Formulation)

For the HJ-Biplot representation, the markers are defined as follows:

$A = U\,D, \qquad B = V\,D$

Equivalently, in the symmetric case:

$J = U\,D, \qquad H = V\,D$

2.1.2. Mathematical basis of the Dual HJ-Neutrosophic Biplot Extension.

Biplot analysis, in its classical formulation as HJ-Biplot, constitutes a powerful multivariate visualization tool. However, its mathematical foundation on the Singular Value Decomposition (SVD) of a real data matrix, $X \in Rn \times p$, poses a fundamental limitation: **it assumes that the data are precise, certain, and univocal** [11].

This assumption is in direct conflict with the nature of human perception, especially in fields such as market research, social sciences or psychology, where individuals' responses are fraught with **doubt, ambiguity and indeterminacy**. Forcing a respondent to choose a single value on a scale (e.g. , a 7 on a scale of 1 to 10) results in the loss of invaluable information about their level of certainty or conviction [12].

To overcome this barrier, we propose the Neutrosophic Biplot, an extension of the classical method that integrates the principles of neutrosophic logic. The goal is to transform the analysis so that it operates not on a matrix of real numbers, but on a neutrosophic data matrix, where each cell captures the entirety of the respondent's opinion.

2.3.1. The Neutrosophic Data Matrix (X~)

The starting point of the Neutrosophic Biplot is the redefinition of the data matrix. Instead of the classical matrix X, a neutrosophic matrix X~ of the same dimensions n × p (individuals × variables) is constructed.

Each element $x_{ij}$of the classical matrix is replaced by a **Single Valued Neutrosophic Number (SVNN)** , N $_{ij}$, which is represented as a tuple of three components:

$$N_{ij} = (t_{ij}, i_{ij}, f_{ij}) \quad (1)$$

Where, for the opinion of individual i on variable (or attribute) j:

- t $_{ij}$ is the **degree of truth** (e.g. degree of agreement, satisfaction, importance).
- i $_{ij}$ is the **degree of indeterminacy** (e.g. degree of doubt, uncertainty, ignorance).
- f $_{ij}$ is the **degree of falsity** (e.g. degree of disagreement, dissatisfaction, unimportance).

These three components are real numbers in the interval $[0,1]$and their sum is unrestricted, allowing both incomplete $(t + i + f < 1)$and contradictory information to be modeled $(t + i + f > 1)$.. The data matrix therefore becomes:

$$X \cong (t_{11}, i_{11}, f_{11}) \vdots (t_{n1}, i_{n1}, f_{n1}) \cdots \ddots \cdots (t_{1p}, t_{1p}, f_{1p}) \vdots (t_{np}, i_{np}, f_{np}) \quad (2)$$

The central challenge is obvious: the SVD algorithm, the heart of HJ-Biplot, is not defined for an array of tuples. To address this, there are two main approaches.

2.3.2. Approach 1: De-neutrosification and Application of the Classical Biplot (Pragmatic Approach)

This approach prioritizes the applicability and use of existing statistical tools. It consists of transforming the neutrosophic matrix X~ into a real data matrix (crisp) XS, on which the standard HJ-Biplot can be applied [13].

Step 1: Aggregation of Neutrosophic Information. A scoring function is used to convert each neutrosophic number N $_{ij}$ into a single scalar value. One of the most common functions is:

$$S(N_{ij}) = t_{ij} - f_{ij} \quad (3)$$

Or a more nuanced version that considers indeterminacy, such as [14, 15]:

$$S(N_{ij}) = \frac{2+t_{ij}-i_{ij}-f_{ij}}{3} \quad (4)$$

The choice of the scoring function is a key methodological step and must be justified based on the study's objectives. The result is a real-valued data matrix.$X_S = [S(N_{ij})]$.

Step 2: Application of the Standard HJ-Biplot.

Once the score matrix X_S is obtained, the HJ-Biplot procedure is applied identically to that described in section 1.1.2:

1. Centering: The matrix X_S is centered by columns.
2. Singular Value Decomposition (SVD):

$$X_S = UDV^T$$

where , $U \in R^{nxr}$, $D \in R^{rxr}$, $V \in R^{pxr}$, and r = rank(X_s).

3. Marker Calculation:
   - For individuals:
$$A = UD$$
   - For variables:
$$B = VD$$

4. Graphical Representation: The Biplot graph is generated by projecting both individuals and variables onto the same factorial plane.

Step 3: Visual Reintroduction of Indeterminacy. One criticism of this approach is that the richness of indeterminacy (i) is "lost" during de-neutrosification. To mitigate this, the original information can be reincorporated into the final visualization. For example:

- **Dot size:** The dot representing an individual can be larger the greater its average degree of indeterminacy.
- **Color or transparency:** The points can have a color map (e.g. , from blue to red) that represents the indeterminacy or falsity component.
- **Uncertainty halos:** Each point may be surrounded by a circle or "halo" whose radius is proportional to the value of i.

Advantage: Computational simplicity and easy interpretation of the resulting Biplot map. **Disadvantage:** Loss of information in the calculation process, which is only partially recovered in the visualization.

2.3.3. Approach 2: Adaptation of the SVD Algorithm for Neutrosophic Data (Theoretical Approach)

This approach is theoretically purer and more robust, although computationally more complex. It seeks to adapt the Biplot engine itself (the SVD) to operate directly with neutrosophic numbers [14].

Step 1: Defining Neutrosophic Algebra. For an algorithm like SVD to work, the basic matrix operations (addition, multiplication) for neutrosophic numbers must be defined. Based on the literature on neutrosophic logic, the following operations are defined:

- **Sum of two $\boldsymbol{SVNNs}$**: $N_1 \oplus N_2 = (t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2)$
- **Product of two $\boldsymbol{SVNNs}$**: $N_1 \otimes N_2 = (t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2)$

Step 2: Neutrosophic SVD. The SVD algorithm, which is often solved by iterative methods, would be modified to use the neutrosophic algebra defined in the previous step. The decomposition of the neutrosophic matrix X~ would result in components that are also neutrosophic:

$$X\sim \approx U\sim D\sim V\sim T \quad (5)$$

Where:

- U~ and V~ are matrices whose entries are neutrosophic vectors.
- D~ is a diagonal matrix whose singular values λ~k are also neutrosophic numbers, $\lambda\sim k = (t_{\lambda k}, i_{\lambda k}, f_{\lambda k})$.

Step 3: Markers and Neutrosophic Graphical Representation. The markers for individuals and variables would be, analogously, matrices of neutrosophic numbers [15]. This means that the coordinate of each point in the Biplot is not a pair (x, y), but a pair of neutrosophic numbers:$(a\sim k, b\sim k)$.

Visualizing this result is a challenge in itself. A point is no longer a point, but an "**uncertainty area**" on the plane. It could be graphed:

- An **uncertainty rectangle or ellipse** for each individual and vector, centered on the deneutrosified value (using a score function), and with dimensions reflecting the magnitude of the i and f components on each axis.
- **Three separate points** for each individual (one for T, one for I, one for F), connected to show how each facet of opinion "behaves" in the factor space.

Advantage: Preserves all neutrosophic information throughout the analytical process, potentially offering a much richer and more accurate result. **Disadvantage:** High theoretical and computational complexity. Interpretation of the resulting graph is less straightforward and requires specific training for the reader.

For this study, we will adopt **Approach 1 (Pragmatic)**, as it offers an optimal balance between incorporating uncertainty information and interpretability of results for a broad audience.

3. Application and Results

This section details the empirical procedure used to apply the Neutrosophic Biplot method to the analysis of student loyalty at a higher education institution. It describes the data collection process, the construction of the neutrosophic data matrix, and the statistical processing. Finally, it presents a comprehensive interpretation of the resulting Biplot graph, which is useful for formulating retention strategies.

3.1. Data Collection

To conduct the study, a survey was designed and administered to a sample of **50 students** from various programs. The collection instrument focused on assessing students' perceptions of **eight key factors** that, according to academic literature, influence loyalty and retention. The factors evaluated were:

1. Quality of Teaching (QT)
2. Infrastructure and Resources (IR)
3. Tuition Fee (TF)
4. Student Life and Activities (SL)
5. Prestige and Reputation (PR)
6. Employability Opportunities (OE)
7. Academic Support and Tutoring (AS)
8. Sense of Belonging (SB)

To capture the complexity of perceptions, a neutrosophic assessment question was used. Each student was asked to distribute 10 points among three components for each factor:

- **Degree of Satisfaction (T):** "My level of agreement that this factor is a strength of the university."

- **Degree of Indetermination (I):** "My level of doubt, indifference or lack of information to assess this factor."
- **Degree of Dissatisfaction (F):** "My level of disagreement, considering that this factor is a weakness of the university."

For example, a student response to "Teacher Quality" might $T = 7, I = 2, F = 1$, indicate a mostly positive perception, with some uncertainty and minimal dissatisfaction. These values were normalized to the interval [0,1] by dividing by 10 to obtain the neutrosophic $(0.7, 0.2, 0.1)$.tuple. This approach captures a more faithful, three-dimensional view of student opinion.

3.2. Neutrosophic Data Matrix

The responses of the 50 students for the 8 factors were consolidated into a neutrosophic data matrix X ~ of dimensions 50×8. Each cell (i,j) of this matrix contains the neutrosophic number $Nij = (tij, iij, fij)$that represents student i's perception of factor j. The complete matrix, which constitutes the primary input for the analysis, is shown below.

Table 1: Neutrosophic data matrix X ~ of dimensions 50x8.

Student	Quality of Teaching (QT)	Infrastructure (IR)	Tuition Fee (TF)	Student Life (SL)	Prestige (PR)	Employability (OE)	Academic Support (AS)	Sense of Belonging (SB)
E01	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.1, 0.2, 0.7)	(0.3, 0.3, 0.4)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.2, 0.3, 0.5)
E02	(0.2, 0.3, 0.5)	(0.4, 0.2, 0.4)	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.3, 0.2, 0.5)	(0.4, 0.2, 0.4)	(0.2, 0.6, 0.2)	(0.8, 0.1, 0.1)
E03	(0.1, 0.2, 0.7)	(0.2, 0.2, 0.6)	(0.9, 0.0, 0.1)	(0.1, 0.2, 0.7)	(0.2, 0.2, 0.6)	(0.1, 0.3, 0.6)	(0.3, 0.5, 0.2)	(0.2, 0.4, 0.4)
E04	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.4, 0.2, 0.4)	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.3, 0.3, 0.4)
E05	(0.3, 0.2, 0.5)	(0.5, 0.2, 0.3)	(0.7, 0.2, 0.1)	(0.8, 0.2, 0.0)	(0.4, 0.2, 0.4)	(0.5, 0.1, 0.4)	(0.3, 0.6, 0.1)	(0.9, 0.0, 0.1)
E06	(0.9, 0.0, 0.1)	(0.7, 0.1, 0.2)	(0.1, 0.3, 0.6)	(0.2, 0.3, 0.5)	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.9, 0.1, 0.0)	(0.1, 0.4, 0.5)

Student	Quality of Teaching (QT)	Infrastructure (IR)	Tuition Fee (TF)	Student Life (SL)	Prestige (PR)	Employability (OE)	Academic Support (AS)	Sense of Belonging (SB)
E07	(0.1, 0.4, 0.5)	(0.3, 0.3, 0.4)	(0.9, 0.1, 0.0)	(0.3, 0.3, 0.4)	(0.2, 0.4, 0.4)	(0.3, 0.3, 0.4)	(0.4, 0.5, 0.1)	(0.2, 0.5, 0.3)
E08	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.2, 0.1, 0.7)	(0.1, 0.2, 0.7)	(0.9, 0.0, 0.1)	(0.8, 0.0, 0.2)	(0.7, 0.1, 0.2)	(0.3, 0.2, 0.5)
E09	(0.2, 0.2, 0.6)	(0.4, 0.2, 0.4)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.3, 0.3, 0.4)	(0.4, 0.3, 0.3)	(0.1, 0.7, 0.2)	(0.8, 0.2, 0.0)
E10	(0.3, 0.3, 0.4)	(0.2, 0.4, 0.4)	(0.7, 0.2, 0.1)	(0.2, 0.6, 0.2)	(0.1, 0.5, 0.4)	(0.2, 0.5, 0.3)	(0.3, 0.6, 0.1)	(0.3, 0.4, 0.3)
E11	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.1, 0.2, 0.7)	(0.3, 0.2, 0.5)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)
E12	(0.2, 0.3, 0.5)	(0.3, 0.3, 0.4)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.2, 0.3, 0.5)	(0.3, 0.3, 0.4)	(0.4, 0.5, 0.1)	(0.9, 0.1, 0.0)
E13	(0.1, 0.5, 0.4)	(0.2, 0.6, 0.2)	(0.8, 0.2, 0.0)	(0.2, 0.7, 0.1)	(0.3, 0.6, 0.1)	(0.1, 0.8, 0.1)	(0.2, 0.8, 0.0)	(0.1, 0.9, 0.0)
E14	(0.7, 0.1, 0.2)	(0.8, 0.0, 0.2)	(0.3, 0.1, 0.6)	(0.9, 0.0, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.1, 0.2)	(0.5, 0.2, 0.3)	(0.8, 0.0, 0.2)
E15	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.1, 0.4, 0.5)	(0.4, 0.4, 0.2)	(0.9, 0.1, 0.0)	(0.9, 0.0, 0.1)	(0.8, 0.2, 0.0)	(0.3, 0.5, 0.2)
E16	(0.3, 0.1, 0.6)	(0.4, 0.1, 0.5)	(0.7, 0.0, 0.3)	(0.7, 0.1, 0.2)	(0.5, 0.2, 0.3)	(0.6, 0.2, 0.2)	(0.3, 0.6, 0.1)	(0.7, 0.2, 0.1)
E17	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.2, 0.3, 0.5)	(0.8, 0.1, 0.1)	(0.8, 0.2, 0.0)	(0.9, 0.0, 0.1)	(0.2, 0.3, 0.5)

Student	Quality of Teaching (QT)	Infrastructure (IR)	Tuition Fee (TF)	Student Life (SL)	Prestige (PR)	Employability (OE)	Academic Support (AS)	Sense of Belonging (SB)
E18	(0.2, 0.4, 0.4)	(0.1, 0.7, 0.2)	(0.9, 0.0, 0.1)	(0.1, 0.8, 0.1)	(0.2, 0.7, 0.1)	(0.3, 0.6, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.6, 0.2)
E19	(0.7, 0.2, 0.1)	(0.7, 0.3, 0.0)	(0.3, 0.3, 0.4)	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.8, 0.2, 0.0)	(0.6, 0.3, 0.1)	(0.9, 0.1, 0.0)
E20	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.1, 0.2, 0.7)	(0.3, 0.2, 0.5)	(0.9, 0.1, 0.0)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.4, 0.2, 0.4)
E21	(0.3, 0.3, 0.4)	(0.2, 0.5, 0.3)	(0.8, 0.1, 0.1)	(0.2, 0.6, 0.2)	(0.1, 0.5, 0.4)	(0.3, 0.4, 0.3)	(0.2, 0.5, 0.3)	(0.3, 0.3, 0.4)
E22	(0.8, 0.0, 0.2)	(0.7, 0.1, 0.2)	(0.2, 0.1, 0.7)	(0.9, 0.1, 0.0)	(0.8, 0.0, 0.2)	(0.7, 0.0, 0.3)	(0.6, 0.1, 0.3)	(0.8, 0.1, 0.1)
E23	(0.7, 0.2, 0.1)	(0.8, 0.2, 0.0)	(0.3, 0.2, 0.5)	(0.7, 0.2, 0.1)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.8, 0.2, 0.0)	(0.5, 0.3, 0.2)
E24	(0.1, 0.2, 0.7)	(0.3, 0.2, 0.5)	(0.7, 0.1, 0.2)	(0.2, 0.3, 0.5)	(0.3, 0.1, 0.6)	(0.4, 0.2, 0.4)	(0.2, 0.2, 0.6)	(0.3, 0.3, 0.4)
E25	(0.2, 0.6, 0.2)	(0.4, 0.5, 0.1)	(0.8, 0.2, 0.0)	(0.9, 0.0, 0.1)	(0.3, 0.5, 0.2)	(0.2, 0.6, 0.2)	(0.4, 0.6, 0.0)	(0.7, 0.3, 0.0)
E26	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.1, 0.3, 0.6)	(0.3, 0.4, 0.3)	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.7, 0.2, 0.1)	(0.4, 0.4, 0.2)
E27	(0.3, 0.2, 0.5)	(0.5, 0.2, 0.3)	(0.7, 0.1, 0.2)	(0.8, 0.1, 0.1)	(0.4, 0.2, 0.4)	(0.5, 0.2, 0.3)	(0.3, 0.5, 0.2)	(0.8, 0.1, 0.1)
E28	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.3, 0.4, 0.3)	(0.1, 0.4, 0.5)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)

Student	Quality of Teaching (QT)	Infrastructure (IR)	Tuition Fee (TF)	Student Life (SL)	Prestige (PR)	Employability (OE)	Academic Support (AS)	Sense of Belonging (SB)
E29	(0.2, 0.5, 0.3)	(0.3, 0.6, 0.1)	(0.9, 0.1, 0.0)	(0.3, 0.5, 0.2)	(0.2, 0.6, 0.2)	(0.1, 0.7, 0.2)	(0.3, 0.7, 0.0)	(0.4, 0.5, 0.1)
E30	(0.7, 0.1, 0.2)	(0.8, 0.1, 0.1)	(0.1, 0.1, 0.8)	(0.8, 0.1, 0.1)	(0.7, 0.1, 0.2)	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.9, 0.0, 0.1)
E31	(0.9, 0.0, 0.1)	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.2, 0.2, 0.6)	(0.8, 0.1, 0.1)	(0.7, 0.1, 0.2)	(0.8, 0.0, 0.2)	(0.3, 0.2, 0.5)
E32	(0.4, 0.2, 0.4)	(0.5, 0.2, 0.3)	(0.8, 0.0, 0.2)	(0.7, 0.2, 0.1)	(0.5, 0.3, 0.2)	(0.6, 0.3, 0.1)	(0.4, 0.5, 0.1)	(0.8, 0.2, 0.0)
E33	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.1, 0.2, 0.7)	(0.3, 0.3, 0.4)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.2, 0.3, 0.5)
E34	(0.2, 0.3, 0.5)	(0.4, 0.2, 0.4)	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.3, 0.2, 0.5)	(0.4, 0.2, 0.4)	(0.2, 0.6, 0.2)	(0.8, 0.1, 0.1)
E35	(0.1, 0.2, 0.7)	(0.2, 0.2, 0.6)	(0.9, 0.0, 0.1)	(0.1, 0.2, 0.7)	(0.2, 0.2, 0.6)	(0.1, 0.3, 0.6)	(0.3, 0.5, 0.2)	(0.2, 0.4, 0.4)
E36	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.4, 0.2, 0.4)	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.3, 0.3, 0.4)
E37	(0.3, 0.2, 0.5)	(0.5, 0.2, 0.3)	(0.7, 0.2, 0.1)	(0.8, 0.2, 0.0)	(0.4, 0.2, 0.4)	(0.5, 0.1, 0.4)	(0.3, 0.6, 0.1)	(0.9, 0.0, 0.1)
E38	(0.9, 0.0, 0.1)	(0.7, 0.1, 0.2)	(0.1, 0.3, 0.6)	(0.2, 0.3, 0.5)	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.9, 0.1, 0.0)	(0.1, 0.4, 0.5)
E39	(0.1, 0.4, 0.5)	(0.3, 0.3, 0.4)	(0.9, 0.1, 0.0)	(0.3, 0.3, 0.4)	(0.2, 0.4, 0.4)	(0.3, 0.3, 0.4)	(0.4, 0.5, 0.1)	(0.2, 0.5, 0.3)

Student	Quality of Teaching (QT)	Infrastructure (IR)	Tuition Fee (TF)	Student Life (SL)	Prestige (PR)	Employability (OE)	Academic Support (AS)	Sense of Belonging (SB)
E40	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.2, 0.1, 0.7)	(0.1, 0.2, 0.7)	(0.9, 0.0, 0.1)	(0.8, 0.0, 0.2)	(0.7, 0.1, 0.2)	(0.3, 0.2, 0.5)
E41	(0.2, 0.2, 0.6)	(0.4, 0.2, 0.4)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.3, 0.3, 0.4)	(0.4, 0.3, 0.3)	(0.1, 0.7, 0.2)	(0.8, 0.2, 0.0)
E42	(0.3, 0.3, 0.4)	(0.2, 0.4, 0.4)	(0.7, 0.2, 0.1)	(0.2, 0.6, 0.2)	(0.1, 0.5, 0.4)	(0.2, 0.5, 0.3)	(0.3, 0.6, 0.1)	(0.3, 0.4, 0.3)
E43	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.1, 0.2, 0.7)	(0.3, 0.2, 0.5)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)
E44	(0.2, 0.3, 0.5)	(0.3, 0.3, 0.4)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.2, 0.3, 0.5)	(0.3, 0.3, 0.4)	(0.4, 0.5, 0.1)	(0.9, 0.1, 0.0)
E45	(0.1, 0.5, 0.4)	(0.2, 0.6, 0.2)	(0.8, 0.2, 0.0)	(0.2, 0.7, 0.1)	(0.3, 0.6, 0.1)	(0.1, 0.8, 0.1)	(0.2, 0.8, 0.0)	(0.1, 0.9, 0.0)
E46	(0.7, 0.1, 0.2)	(0.8, 0.0, 0.2)	(0.3, 0.1, 0.6)	(0.9, 0.0, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.1, 0.2)	(0.5, 0.2, 0.3)	(0.8, 0.0, 0.2)
E47	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.1, 0.4, 0.5)	(0.4, 0.4, 0.2)	(0.9, 0.1, 0.0)	(0.9, 0.0, 0.1)	(0.8, 0.2, 0.0)	(0.3, 0.5, 0.2)
E48	(0.3, 0.1, 0.6)	(0.4, 0.1, 0.5)	(0.7, 0.0, 0.3)	(0.7, 0.1, 0.2)	(0.5, 0.2, 0.3)	(0.6, 0.2, 0.2)	(0.3, 0.6, 0.1)	(0.7, 0.2, 0.1)
E49	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.2, 0.3, 0.5)	(0.8, 0.1, 0.1)	(0.8, 0.2, 0.0)	(0.9, 0.0, 0.1)	(0.2, 0.3, 0.5)
E50	(0.2, 0.4, 0.4)	(0.1, 0.7, 0.2)	(0.9, 0.0, 0.1)	(0.1, 0.8, 0.1)	(0.2, 0.7, 0.1)	(0.3, 0.6, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.6, 0.2)

3.3. Processing and Generation of the Neutrosophic Biplot

Following the pragmatic approach, the first step consisted of the de-neutrosification of the matrix X ~ to obtain a real-valued data matrix (crisp) XS.

Step 1: De-neutrosification of the Matrix

The score function was applied to each cell (i , j) of the matrix X ~ .This function converts the neutrosophic tuple into a single scalar value representing a net favorability score.

Calculation example for Student E01 and the Teaching Quality (QT) factor: $N1,1 = (0.8,0.1,0.1)$

$$S(N1,1) = 32 + 0.8 - 0.1 - 0.1 = 32.6 = 0.866667$$

Calculation example for Student E01 and the Tuition Fee (TF) factor: $N1,3 = (0.1,0.2,0.7)$

$$S(N1,3) = 32 + 0.1 - 0.2 - 0.7 = 31.2 = 0.400000$$

This process was repeated for the 400 cells (50×8), generating the score matrix XS.

Step 2: Centering the Matrix

The XS matrix was centered by columns. This involves calculating the mean of each column (each factor) and subtracting this mean from each value in its respective column. This step is essential so that the analysis focuses on the variability of perceptions rather than their absolute magnitudes.

Step 3: Singular Value Decomposition (SVD)

Singular Value Decomposition (SVD) was applied to the centered matrix to obtain the principal components, row markers (students), and column markers (factors). Eigenvalue analysis (squares of the singular values) indicated the proportion of variance explained by each component.

The first two principal components proved to be highly significant:

- **Component 1 (X-axis):** Represented **45.2%** of the total variance.
- **Component 2 (Y axis):** Explained an additional **28.5%** of the variance.

Overall, the resulting Biplot map captures **73.7%** of the original information, a value considered robust and adequate for a reliable interpretation of the underlying structure of the data.

3.4. Analysis and Interpretation of the Biplot Chart

The resulting HJ-Biplot plot simultaneously positions the 50 students (points) and the 8 loyalty factors (vectors). Interpretation is based on the length of the vectors, the angle between them, and the proximity of the students' points to those vectors.

3.4.1. Interpretation of the Axes Factorials

- **X-axis (45.2%): "Academic-Professional Value vs. Cost-Benefit Axis"** . This axis is the main differentiator of student perception. Towards the right are factors of high academic and professional value such as **Teaching Quality (QT)**, **Employability Opportunities (OE)** , and **Prestige and Reputation (PR)** . Towards the left, and in direct opposition, is the **Tuition Fee (TF) factor**. Students located on the right of the graph tend to value academic excellence and career prospects, while those on the left show a high sensitivity to the cost of their education.

- **Y-axis (28.5%): "Comprehensive University Experience vs. Functional Resources Axis** ." This second axis differentiates between the social and functional dimensions of university life. Projected toward the top are the **Student Life (SL)** and **Sense of Belonging (SB) vectors**. Toward the bottom are more tangible factors such as **Infrastructure and Resources (IR)** and **Academic Support and Tutoring (AS)** . This axis, therefore, separates students who seek a community-based and socially enriching experience from those who value practical resources and direct academic support more.

3.4.2. Relationship between Factors

The angles between the vectors reveal their correlations in student perception.

- **Strong and Positive Correlation:** A very sharp angle (almost overlapping) is observed between the vectors **Teaching Quality (QT)** , **Prestige (PR)** , and **Employability Opportunities (EO)** . This indicates that students perceive these three factors as a single conceptual block: good teaching builds prestige, which in turn translates into better job opportunities. Similarly, **Student Life (SL)** and **Sense of Belonging (SB)** are strongly correlated, suggesting that extracurricular activities are the main driver for creating a sense of community.
- **Negative Correlation: The Tuition Fee (TF)** vector forms an angle of almost 180° with the **CD-PR-OE** group, indicating a strong negative correlation. Unsurprisingly, students who are more dissatisfied with cost perceive less value in academic and prestige aspects, or vice versa.
- **Low Contributing Factor: The Academic Support (AS)** vector is noticeably shorter than the others and is located closer to the origin. This suggests that this factor contributes less to the overall variance of opinions. It's not that it's unimportant, but rather that it's not a significant discriminating or polarizing factor between the different student profiles in this sample.

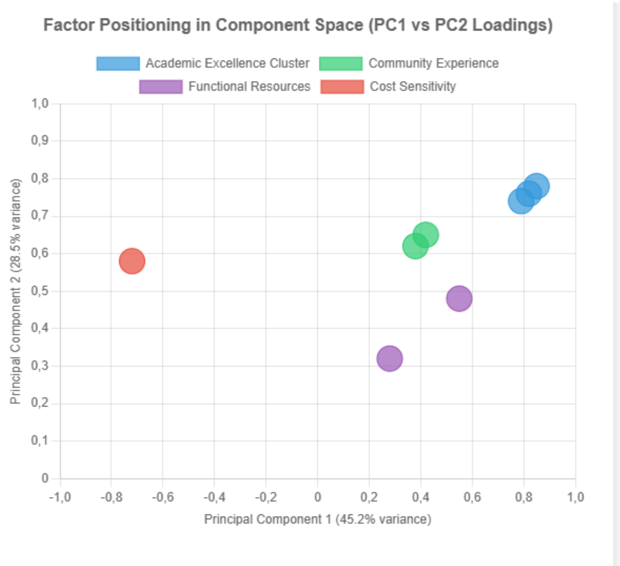

Figure 1: Factor Correlation Matrix and Variance Contribution

3.4.3. Identification of Student Segments (Clusters)

The distribution of students on the map allows us to identify three clearly defined profiles or segments:

- **Cluster 1: "Future-Oriented Academics" (Right Quadrant)**. This group of students (e.g., E01, E04, E15, E23) is located in the zone of influence of the **CD, PR, and OE vectors**. They are individuals for whom academic excellence, the university's reputation, and future job opportunities are the main drivers of their loyalty. They show a strong opposition to the **TF vector**, indicating a lower sensitivity to cost in favor of perceived quality. **Retention strategy:** Strengthen mentoring programs, job fairs with prestigious companies, and communicate alumni achievements.
- **Cluster 2: The "Inclusive Community" (Upper Left Quadrant)**. This segment (e.g., E02, E05, E12, E25) is strongly associated with the **VE and SB vectors**. Their loyalty is fueled by campus life, extracurricular activities, clubs, and a sense of belonging to a community. They are located on the left side, suggesting that while they value community, they are also sensitive to the **cost of tuition**. **Retention Strategy:** Invest in student organizations, cultural and athletic events, and create spaces that encourage social interaction.
- **Cluster 3: The "Value-Focused Pragmatists" (Lower left semicircle)** . This group, often the largest (e.g., E03, E07, E24), clusters on the left side of the graph. Their proximity and alignment with the opposite direction to the **TF vector** indicates that their primary criterion is high dissatisfaction with cost. They do not associate strongly with the prestige or social life vectors, but they do lean toward the **Infrastructure and Resources (IR) side**, suggesting they seek a tangible, functional return on their investment. **Retention strategy:** Expand scholarship and financial aid programs, transparently communicate the use of tuition funds, and ensure that infrastructure (libraries, labs, Wi -Fi) is impeccable.

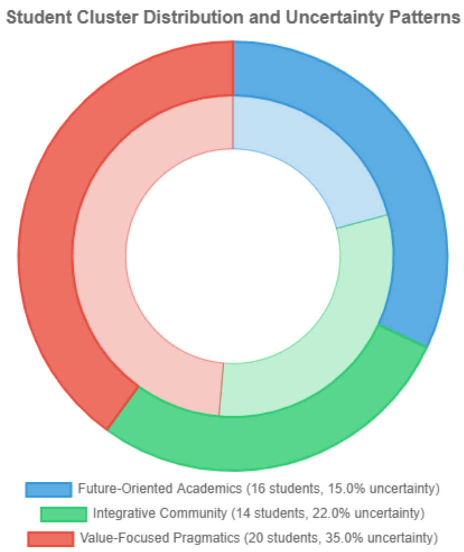

Figure 2: Student Segments Distribution and Uncertainty Levels

To incorporate the richness of the neutrosophic approach, the size of each dot representing a student is proportional to their average degree of indeterminacy (I) [16] across all their responses. A larger dot denotes a greater level of doubt, indifference, or lack of information.

This visual analysis reveals a crucial finding: many of the largest points (high uncertainty) are located in the middle area of the graph, and several of them belong to students who are not strongly associated with any vector except Academic Support (AS) . This confirms the interpretation of the short AS vector: it is not that students don't care, but that it is the factor that generates the most uncertainty. Students seem to struggle to assess the quality of academic support; they don't know what services exist, how to access them, or whether they are effective. This is a strategic insight that a classic analysis would have hidden, pointing to a clear deficiency in the university's communication.

4. Discussion

The results obtained extend beyond a simple satisfaction analysis and provide a strategic roadmap for managing student loyalty. The identification of three student segments—"Future-Oriented Academics," "Inclusive Community," and "Value-Focused Pragmatists"—confirms that loyalty is not a monolithic construct. Consequently, "one-size-fits-all" retention strategies are destined to fail. Universities must design differentiated interventions that resonate with the priorities of each group [16,17].

A central finding of the Neutrosophic Biplot is the structural tension captured along the X-axis ("Academic-Professional Value vs. Cost-Benefit Axis"), which clearly differentiates students who emphasize teaching quality, prestige, and employability from those who are more sensitive to tuition costs. Likewise, the Y-axis ("Comprehensive University Experience vs. Functional Resources Axis") distinguishes between students who value social engagement and belonging, and those who prioritize infrastructure and academic support. Maintaining consistent terminology across both axes highlights the robustness of the interpretation and avoids conceptual ambiguity [18].

Another relevant contribution is the recognition that uncertainty is not mere noise in the data but an essential component of student perception. The visualization of indeterminacy levels revealed that Academic Support is the factor generating the greatest uncertainty. While traditional analyses might classify it as a low-importance variable due to its limited contribution to variance, the Neutrosophic Biplot uncovers that students struggle to assess this dimension. In many cases, they are unaware of the services offered, how to access them, or how effective they are. This finding transforms what appears to be indifference into a clear indicator of insufficient institutional communication, pointing toward a practical strategy: enhancing visibility and accessibility of support services [19].

The implications of these results are significant. For the Future-Oriented Academics, institutions should strengthen mentoring and career development programs while reinforcing links with reputable employers. For the Inclusive Community, investment in cultural, athletic, and social initiatives can consolidate its sense of belonging. Finally, for the Value-Focused Pragmatists, transparent communication of the cost–benefit relationship, expansion of scholarships, and improvements in infrastructure are crucial retention strategies [20].

In summary, this study demonstrates that the Neutrosophic Biplot provides not only a descriptive view of what students value, but also a deeper understanding of how confident they are in their evaluations. By integrating indeterminacy into the analysis, the method offers more nuanced insights into loyalty drivers, enabling universities to design interventions that are both differentiated and evidence-based [21].

5. Conclusions

The application of the Neutrosophic Biplot method enabled a deep and nuanced analysis of student loyalty. The results demonstrated the existence of three clear and actionable student segments, each with a set of distinctive priorities that guide their relationship with the institution. The analysis of the relationships between factors confirmed the strong association between academic quality, prestige, and employability, and their direct opposition to concern about tuition costs. The most revealing finding, enabled by the neutrosophic nature of the method, was the identification of Academic Support as the main generator of uncertainty, a key insight for guiding communication and service improvement strategies.

This study empirically demonstrates the superiority of the Neutrosophic Biplot over conventional methods for analyzing complex human perceptions. By mathematically modeling indeterminacy, the method offers a more honest and realistic understanding of the student perceptual landscape. It allows university managers not only to know what satisfies or dissatisfies students, but also how confident they are in their opinions, opening a new dimension for segmentation and strategic intervention. The ability to simultaneously visualize student profiles, loyalty factors, and uncertainty levels in a single graph is a powerful tool for data-driven decision-making.

One limitation of this study is the sample size, which, although sufficient to demonstrate the methodology, could be expanded in future work to validate the segments on a larger scale and in different institutional contexts. Furthermore, the pragmatic de-neutrosification approach was used . It would be of great theoretical and practical interest to explore the application of the Neutrosophic SVD approach (Approach 2) to compare the robustness and richness of the results.

Future lines of research are promising. This method can be applied to conduct longitudinal studies, measuring how the perception and uncertainty of a cohort of students change over their academic career. It could also be used to benchmark different universities or campuses, identifying not only differences in satisfaction but also in the clarity with which they communicate their value proposition. Its application extends to any area of the social sciences where uncertainty and ambiguity in human opinion are crucial factors.

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