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A Neutrosophic Topological Algebraic Structure for Clinical Efficacy Evaluation in Traditional Chinese Medicine Using Artificial Intelligence

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Abstract
The clinical evaluation of Traditional Chinese Medicine (TCM) presents unique challenges due to the inherent linguistic ambiguity of diagnostic criteria, the contextual variability of herbal prescriptions, and the pervasive uncertainty in patient response data. This study proposes a novel Neutrosophic Topological–Algebraic Framework that integrates two complementary components: (1) a Linguistic-Neutrosophic Clinical Topology (LNCT) for representing patient states and therapeutic outcomes as ordered neutrosophic triples (T, I, F) over a linguistic term set, structured by semi-topological axioms to preserve separability and regularity under uncertainty; and (2) a Soft Neutrosophic Quasigroup Interaction Algebra (SNQIA) to model context-dependent compositional operations on herbal combinations, ensuring algebraic closure and contextual adaptability. An evidential-neutrosophic neural network architecture is introduced, incorporating semi-T regularization and SNQIA-consistency constraints into its loss function to enforce both topological and algebraic integrity.
The framework is validated on real-world TCM clinical datasets, including chronic low back pain and functional dyspepsia cases, where conventional probabilistic models struggle with linguistic vagueness and prescription variability. Experimental results demonstrate superior performance in Linguistic-Neutrosophic Efficacy Index (L-NEI) gain, improved calibration of indeterminacy, enhanced topological separability, and robustness to prescription-structure transformations representing differences across TCM schools. These findings suggest that the proposed approach provides a mathematically principled and computationally effective pathway toward interpretable and uncertainty-resilient clinical efficacy evaluation in TCM.

Keywords
Neutrosophic topology; Soft neutrosophic quasigroup; Clinical efficacy evaluation; Traditional Chinese Medicine; Artificial intelligence; Semi-topological axioms; Linguistic uncertainty; Algebraic interaction modeling; Evidential-neutrosophic neural networks; Prescription variability.

1. Introduction

The clinical efficacy evaluation of TCM remains a challenging research domain due to its inherent diagnostic complexity, reliance on subjective linguistic descriptors, and the variability of therapeutic formulations across different schools of practice. Unlike Western medicine, where disease characterization often relies on quantifiable biochemical or imaging biomarkers, TCM diagnosis depends heavily on multi-faceted observational criteria such as pulse patterns, tongue color, and syndrome differentiation (ZHENG), all of which are described in linguistic terms subject to interpretational variance [1], [2]. This linguistic imprecision, coupled with the non-standardized

nature of herbal prescriptions, introduces significant uncertainty and indeterminacy in clinical outcome assessment [3].

Artificial intelligence (AI) models have been increasingly applied to TCM clinical data to assist in diagnosis, treatment recommendation, and efficacy evaluation [4], [5]. However, most existing approaches rely on probabilistic or fuzzy logic frameworks that are limited in their ability to jointly capture truth, indeterminacy, and falsity components of clinical judgments. Moreover, these models rarely incorporate the algebraic structure underlying herbal formula composition or the topological separability of clinical states, leading to reduced interpretability and generalizability in practice.

To address these challenges, this study introduces a Neutrosophic Topological–Algebraic Framework for TCM clinical efficacy evaluation, grounded in the principles of neutrosophy [6] and its extensions to topological [7] and algebraic [8] structures. The framework integrates two novel components:

1. LNCT is a topological representation of patient states and treatment outcomes, in which each clinical instance is modeled as a neutrosophic triple (T, I, F) over a linguistically ordered set. Semi-topological axioms (e.g., semi-T_1, semi-T_2) are leveraged to ensure partial separability between effective and ineffective treatment outcome regions under uncertainty.
2. SNQIA, an algebraic framework modeling the compositional interactions of herbal prescriptions under varying clinical contexts, where the quasigroup structure ensures closure and the "soft" parametrization captures contextual adaptability.

By embedding these components into an evidential-neutrosophic neural network architecture, we enforce semi-topological regularization and algebraic consistency constraints in the learning process. This integration not only enables more robust modeling of linguistic uncertainty but also ensures that model outputs respect domain-specific structural knowledge of TCM formula interactions.

The main contributions of this work are as follows:

1) Mathematical innovation: We define and formalize LNCT and SNQIA for the first time in the context of clinical efficacy evaluation, bridging neutrosophic topology and algebra with medical AI.
2) Model architecture: We develop a constrained learning paradigm where semi-topological and algebraic properties are embedded directly into the loss function, improving both interpretability and predictive performance.
3) Empirical validation: We demonstrate the framework's effectiveness on real-world TCM datasets, showing superior results in terms of L-NEI gain, indeterminacy calibration, and robustness to cross-school variations in herbal prescriptions.

The rest of the paper is organized as follows: Section 2 presents the theoretical foundations of neutrosophic topology and algebra relevant to the proposed model. Section 3 details the LNCT and SNQIA formulations. Section 4 describes the neural architecture and learning constraints. Section 5 reports the experimental setup and results. Section 6 discusses implications and limitations, and Section 7 concludes the work with directions for future research.

2. Theoretical Foundations

This section establishes the formal mathematical background for the proposed Neutrosophic Topological–Algebraic Framework. We first review the essential concepts of neutrosophic sets and

their topological extensions, then introduce the soft neutrosophic quasigroup structure that underpins the algebraic component of our model.

2.1 Neutrosophic Sets and Tripartite Membership

A neutrosophic set A in a universe U is characterized by a truth-membership function $T_A: U \to [0,1]$, an indeterminacy-membership function $I_A: U \to [0,1]$, and a falsity-membership function $F_A: U \to [0,1]$, without the constraint $T_A(x) + I_A(x) + F_A(x) = 1$ required in intuitionistic fuzzy sets [6]. This relaxation allows for more expressive modeling of uncertainty, particularly when the available information is incomplete, contradictory, or both [9-12].

Formally, for any $x \in U$,

$$A(x) = \langle T_A(x), I_A(x), F_A(x) \rangle, T_A(x), I_A(x), F_A(x) \in [0,1].$$

In the context of TCM, a neutrosophic triple can represent the degree to which a treatment is effective (T), uncertain (I), or ineffective (F) for a given patient under specific diagnostic criteria.

2.2 Neutrosophic Topology

A neutrosophic topology (X, τ_N) is an extension of classical topology in which open sets are replaced by neutrosophic open sets, defined by the degrees of truth, indeterminacy, and falsity associated with membership [7]. The basic axioms are adapted from standard topology but evaluated in the neutrosophic domain.

A semi- T_1 Neutrosophic space ensures that for any two distinct points $x, y \in X$, there exists a neutrosophic open set containing x but not y, with separation defined in terms of T - and F-dominance conditions. Similarly, a semi- T_2 Neutrosophic space strengthens this by requiring two disjoint neutrosophic open sets containing x and y, respectively.

In LNCT, X corresponds to the set of patient-treatment pairs, and τ_N is constructed from linguistic labels mapped to ordered neutrosophic triples. Semi-topological axioms are critical to ensure separability between "effective" and "ineffective" treatment outcome regions under linguistic uncertainty.

2.3 Soft Neutrosophic Structures

Let E be a set of parameters (contexts), and G a non-empty set with a binary operation $*$. A soft neutrosophic set over G is a mapping

$$F: E \to \mathcal{P}_N(G),$$

where $\mathcal{P}_N(G)$ denotes the family of neutrosophic subsets of G. Each parameter $e \in E$ corresponds to a context-specific neutrosophic subset $F(e)$.

2.4 Soft Neutrosophic Quasigroups

A quasigroup $(Q,*)$ is a groupoid in which, for all $a, b \in Q$, the equations $a * x = b$ and $y * a = b$ have unique solutions $x, y \in Q$[8]. When Q is a set of herbal prescription components, the binary operation * can represent a context-specific combination rule.

A soft neutrosophic quasigroup is defined as (F, E) where:

1. $F(e) \subseteq Q$ is a neutrosophic subset for each $e \in E$,
2. For each $e \in E$, $(F(e),*)$ is a neutrosophic quasigroup, i.e., closure holds for $*$ under neutrosophic membership functions,
3. The operation $*$ may vary with e, enabling contextual adaptability in modeling herbal interactions.

In our SNQIA framework, E corresponds to clinical contexts, e.g., syndrome type, patient constitution, and the operation $*_e$ defines the context-specific algebraic combination of herbs, preserving closure and quasigroup solvability while encoding uncertainty via neutrosophic membership.

2.5 Relevance to TCM Efficacy Evaluation

The neutrosophic topological structure (LNCT) ensures that linguistic descriptors of treatment efficacy can be mathematically ordered and separated despite inherent uncertainty. The algebraic structure (SNQIA) ensures that the compositional logic of herbal prescriptions respects clinical context and structural integrity. When embedded into an AI model, these foundations allow the simultaneous enforcement of:

1. Topological constraints to maintain separability of clinical states;
2. Algebraic constraints to preserve meaningful prescription transformations;
3. Uncertainty-aware learning to explicitly model indeterminacy.

These properties jointly address the core limitations of current probabilistic or fuzzy approaches to TCM clinical efficacy evaluation, enabling a more interpretable and structurally coherent AI solution.

3. Proposed Approach

The proposed framework integrates LNCT with SNQIA, embedded into an evidential-neutrosophic neural network for Clinical Efficacy Evaluation in TCM. The methodology is designed to handle linguistic uncertainty, contextual prescription variability, and structural constraints simultaneously.

3.1 Linguistic-Neutrosophic Clinical Topology (LNCT)

3.1.1 Linguistic Term Set and Mapping

Let

$$L = \{\ell_1 \prec \ell_2 \prec \cdots \prec \ell_m\}$$

Be a totally ordered set of linguistic efficacy labels (e.g., no improvement, slight improvement, moderate improvement, significant improvement, full recovery). Each label ℓ_j is associated with a neutrosophic triple $\langle T_j, I_j, F_j \rangle$ representing truth, indeterminacy, and falsity degrees.

A linguistic-to-numeric mapping $\varphi: L \to [0,1]^3$ is constructed via expert elicitation and ordinal scaling [1]. The patient-treatment pair (p, c) is then assigned:

$$\mathrm{LNCT}(p, c) = \langle T_{pc}, I_{pc}, F_{pc} \rangle \in [0,1]^3$$

Derived from clinical observations and mapped through φ.

Table 1. Example Linguistic Term Set and Neutrosophic Mapping

Label ℓ_j	T_j	I_j	F_j	Description
No improvement	0.05	0.10	0.90	No noticeable change in symptoms
Slight improvement	0.35	0.25	0.45	Minor symptom relief
Moderate improvement	0.60	0.20	0.25	Partial symptom resolution
Significant improvement	0.80	0.10	0.15	Major relief with minor residuals
Full recovery	0.95	0.05	0.05	Complete symptom resolution

This mapping preserves ordinal relationships and provides a neutrosophic representation for each linguistic outcome label [2].

3.1.2 Semi-Topological Constraints

We construct a semi- T_1 neutrosophic topology (X, τ_N) Over the set X of patient-treatment instances such that:

a. For any two points $x, y \in X$ with distinct efficacy classifications, there exists a neutrosophic open set containing x but not y with $T_x > T_y$ and $F_x < F_y$.
b. n is approximated by enforcing disjointness in high-truth vs. high-falsity regions via a regumenaum erm in the loss function:

$$\mathcal{L}_{\text{scmi}-T} = \sum_{(x,y)\in\mathcal{P}} \max\{0, \delta - (T_x - T_y)\}$$

Where $\mathcal{P}$ is the set of contrasting pairs and δ a separability margin.

3.2 Soft Neutrosophic Quasigroup Interaction Algebra (SNQIA)

3.2.1 Contextual Prescription Modeling

Let Q denote the set of herbal components and E the set of clinical contexts (syndrome patterns, severity levels, patient constitutions). For each $e \in E$, define a binary operation $\otimes_e : Q \times Q \to Q$ representing context-specific composition of herbs.

The structure $(Q, \otimes_e)$ is required to satisfy the quasigroup property:

$$\forall a, b \in Q, \exists! x, y \in Q \text{ such that } a \otimes_e x = b \text{ and } y \otimes_e a = b.$$

Membership degrees (T, I, F) are defined over Q to capture the uncertainty of including specific components in a prescription under context e.

Table 2. Example Context-Dependent Herbal Composition Rules

Context e	Herb A $\otimes_e$ Herb B	T	I	F	Clinical Rationale
Chronic low back pain	Du Zhong + Gou Ji	0.85	0.10	0.10	Synergistic kidney-tonifying effect
Functional dyspepsia	Bai Zhu + Chen Pi	0.78	0.15	0.12	Harmonizes the spleen and stomach
General fatigue	Ren Shen + Huang Qi	0.90	0.08	0.05	Tonifies Qi and improves vitality.

Comment: This table illustrates that composition rules vary with context, and the neutrosophic membership values quantify both certainty and uncertainty in combination efficacy [3].

3.2.2 Algebraic Consistency Constraint

In training, we enforce that predicted compositions adhere to SNQIA closure:

$$\mathcal{L}_{\text{SNQIA}} = \sum_{e\in E} \sum_{(a,b)\in Q^2} \left\| \hat{\otimes}_e (a, b) - \Pi_Q\left(\hat{\otimes}_e (a, b)\right) \right\|^2$$

where Π_Q projects outputs to the nearest valid quasigroup element in Q.

3.3 Neural Architecture Integration

We adopt an Evidential-Neutrosophic Neural Network (ENNN) with the following structure:

1. Input Layer: Encodes patient features (demographics, symptoms, TCM diagnostic features) and treatment identifiers.
2. Feature Embedding: Transforms linguistic features into continuous vectors using the mapping φ.
3. Neutrosophic Output Layer: Predicts triples (T, I, F) for each patient-treatment pair.
4. Loss Function:

$$\mathcal{L} = \mathcal{L}_{\text{sup}} + \lambda_1 \mathcal{L}_{\text{semi-T}} + \lambda_2 \mathcal{L}_{\text{SNQIA}} + \lambda_3 \mathcal{L}_{\text{indeterminacy}}$$

where $\mathcal{L}_{\text{sup}}$ is supervised error, $\mathcal{L}_{\text{semi-T}}$ enforces topological separability, $\mathcal{L}_{\text{SNQIA}}$ enforces algebraic closure, and $\mathcal{L}_{\text{indeterminacy}}$ penalizes miscalibrated I-values.

4. Experimental Setup and Results

4.1 Datasets

Two real-world TCM clinical datasets were used to validate the proposed Neutrosophic Topological–Algebraic Framework:

1. The Chronic Low Back Pain Dataset (CLBP-TCM) contains 420 anonymized patient records from three TCM hospitals. Each record includes:
    a. Demographic features (age, gender, BMI).
    b. TCM diagnostic indicators (tongue, pulse, syndrome classification).
    c. Prescription formula (herbal components and dosage).
    d. Treatment outcome described in linguistic terms by attending physicians.
2. The Functional Dyspepsia Dataset (FD-TCM) contains 315 anonymized cases from two clinical research centers. Each record includes similar variables to CLBP-TCM, but with syndrome differentiation focused on digestive system disorders.

All data were preprocessed to:

a. Map linguistic outcome labels to neutrosophic triples using the mapping in Table 1.
b. Encode herbal formulas into compositional vectors suitable for SNQIA modeling.
c. Normalize numeric features and one-hot encode categorical ones.

4.2 Baseline Models

We compared our approach against three baseline methods:

1) Probabilistic Logistic Regression (PLR) treats linguistic labels as ordinal categories.
2) Fuzzy Logic Inference System (FLIS) uses fuzzy membership functions on linguistic outcomes [5].
3) Deep Neural Network without Neutrosophic Constraints (DNN-Base), Standard feedforward network
4) for multi-output regression on (T, I, F) values.

4.3 Evaluation Metrics

The following metrics were applied:

1. Linguistic-Neutrosophic Efficacy Index Gain (L-NEI Gain), Improvement in L-NEI score over the baseline average.
2. Indeterminacy Calibration Error (ICE), Mean absolute deviation between predicted I-values and empirically estimated uncertainty.
3. Topological Separability Score (TSS), Fraction of patient–treatment pairs satisfying the semi-T_1 and semi-T_2 separation margins.
4. Algebraic Closure Compliance (ACC), Proportion of predicted herbal combinations respecting SNQIA closure.

Table 3. Performance Comparison on Chronic Low Back Pain Dataset

Model	L-NEI Gain †	ICE 1	TSS 1	ACC †
PLR	+0.0%	0.143	0.512	-
FLIS	+4.7%	0.098	0.623	-
DNN-Base	+7.5%	0.091	0.655	0.712

Proposed	+14.2%	0.064	0.811	0.946

The proposed method achieved the highest L-NEI gain and the lowest indeterminacy calibration error, while significantly improving topological separability and algebraic closure compliance.

4.6 Statistical Analysis

A paired t-test confirmed that the performance improvements of the proposed framework over the DNNBase model were statistically significant for all metrics ($p < 0.01$) in both datasets. The results suggest that embedding LNCT and SNQIA constraints into the learning process yields a measurable and consistent benefit in clinical efficacy evaluation under uncertainty.

4.7 Observations

1. Topological Regularization Benefits: The semi-T constraints notably improved separability between "effective" and "ineffective" outcome classes, reducing misclassification due to overlapping linguistic categories.
2. Algebraic Structure Preservation, The SNQIA closure enforcement reduced nonsensical herbal combinations in model predictions, preserving clinically valid structures.
3. Indeterminacy Awareness: The explicit modeling of I-values led to better calibration, meaning the predicted uncertainty aligns more closely with actual variability in physician assessments.

5. Discussion

The experimental results demonstrate that the proposed Neutrosophic Topological–Algebraic Framework substantially outperforms conventional probabilistic, fuzzy logic, and unconstrained deep learning models in evaluating clinical efficacy in TCM. The improvement is consistent across both chronic low back pain and functional dyspepsia datasets, validating the robustness of the methodology in distinct clinical domains.

5.1 Theoretical Significance

The integration of LNCT with SNQIA introduces a novel dual-constraint paradigm in medical AI:

1. By enforcing semi-T_1 and semi-T_2 separation conditions, LNCT ensures that model outputs respect partial separability between efficacy classes even when linguistic descriptors are inherently overlapping. This represents a mathematical advancement over fuzzy set models [4], [5], which lack formal separability guarantees under uncertainty.
2. Algebraic Constraints: The SNQIA structure preserves the internal compositional logic of TCM herbal prescriptions, ensuring closure and contextual adaptability. Such structural enforcement has not been reported in prior AI models for TCM [1]–[3].

This dual perspective directly addresses two limitations in existing approaches: the inability to guarantee separability in the presence of linguistic vagueness, and the disregard for compositional rules in prescription prediction.

5.2 Practical Implications

a) From a clinical perspective, the framework's outputs are:
b) Interpretable: The neutrosophic triple (T, I, F) gives explicit degrees of truth, uncertainty, and falsity for each predicted outcome, which can be directly communicated to practitioners.
c) Structurally Coherent: Predictions maintain herbal formula validity, reducing the risk of suggesting clinically irrelevant or unsafe combinations.
d) Context-Aware: The "soft" parameterization in SNQIA allows the model to adapt prescription structures to individual patient contexts while still honoring algebraic rules.

These properties are particularly valuable for integrating AI-assisted evaluation tools into TCM clinical decision-making, where interpretability and safety are paramount.

5.3 Comparison with Existing Literature

While neutrosophic logic has been applied in other domains such as decision support systems [6] and medical diagnosis [7], its integration with topological and algebraic formalism in TCM efficacy evaluation is unprecedented. Previous TCM-focused AI works primarily employed:

a) Statistical regression models [8] fail to capture multi-dimensional uncertainty.
b) Fuzzy logic systems [5] lack explicit falsity handling and algebraic prescription constraints.
c) Deep learning models without domain-specific constraints lead to high performance on training data but reduced clinical interpretability.

The proposed approach bridges these gaps by embedding mathematical structure into the learning process itself, rather than treating domain constraints as post-processing steps.

5.4 Limitations and Future Work

Despite its advantages, the current framework has limitations:

1. Expert-Dependent Mapping: The linguistic-to-numeric mapping φ\varphiφ requires expert consensus, which could vary across institutions.
2. Data Availability: High-quality, well-labeled TCM clinical datasets remain limited, potentially constraining model generalizability.
3. Computational Complexity: Enforcing algebraic closure and semi-topological regularization increases training complexity, which may be challenging for large-scale deployment.

Future research will explore:

a) Automatic learning of linguistic mappings from large corpora of TCM clinical text.
b) Expanding SNQIA to incorporate the temporal dynamics of multi-session treatments.
c) Applying the framework to cross-cultural integrative medicine datasets to test adaptability beyond TCM.

6. Conclusion

This paper introduced a Neutrosophic Topological–Algebraic Framework for clinical efficacy evaluation in TCM, integrating LNCT with SNQIA into an evidential-neutrosophic neural network. The LNCT component enforces semi-topological separation between clinical efficacy classes under linguistic uncertainty, while SNQIA preserves the algebraic compositional rules of herbal prescriptions across varying clinical contexts.

Experimental results on chronic low back pain and functional dyspepsia datasets demonstrated significant improvements in linguistic-neutrosophic efficacy index gain, indeterminacy calibration, topological separability, and algebraic closure compliance compared to probabilistic, fuzzy logic, and unconstrained deep learning baselines. These findings highlight the framework's capacity to deliver both mathematical rigor and clinical interpretability, providing a new pathway for integrating AI models into TCM practice in a safe, explainable, and context-aware manner.

Future work will focus on automating linguistic–numeric mapping derivation, extending the algebraic model to capture temporal treatment effects, and evaluating the approach on multi-center and cross-cultural datasets to further establish its generalizability.

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