



Reciprocal floor function is used in algebraic structures to extend the complex logarithmic neutrosophic set using averaging and geometric operators

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Abstract. This work presents a new method for creating complex logarithmic reciprocal floor function neutrosophic sets. This article will cover generalized weighted averaging, generalized weighted geometric, geometric, and complex logarithmic neutrosophic weighted averaging. The weighted average and geometric were computed using an aggregating model. To further examine a number of sets with important qualities, the algebraic techniques listed below will be applied.

Keywords: Reciprocal floor function, weighted averaging, weighted geometric, generalized weighted averaging, generalized weighted geometric.

1. Introduction

Many theories have been proposed to explain uncertainty, including fuzzy sets (FS), which have membership degrees (MD) ranging from zero to one. For $\iota, \varsigma \in [0, 1]$, an intuitionistic FS (IFS) was built with each element having two MDs: positive ι and negative ς , and $0 \preceq \iota + \varsigma \preceq 1$ by Atanassov. Developed by Yager, the Pythagorean FSs (PFS) concept is characterized by its MD and non-MD (NMD) with $\iota + \varsigma \succcurlyeq 1$ to $\iota^2 + \varsigma^2 \preceq 1$. Numerous studies have examined the use

of IFSs and PFSs in various fields. Their ability to communicate information is still restricted. Because of this, the experts were still having trouble interpreting the data in these sets and the associated data. Wang et al. [1] investigated the concept of complex IFS with DOMBI prioritized AOs and its application for trustworthy green supplier selection. Cuong et al. [2], the three primary ideas of the picture FS are positive MD (ι), neutral MD (ρ), and negative MD (ς). Additionally, it offers greater benefits than PFS and IFS. Since $\iota, \rho, \varsigma \in [0, 1]$, it has been noted that the picture FS is an upgrade of the IFS that may handle more inconsistency and $0 \preceq \iota + \rho + \varsigma \preceq 1$. According on the picture FS description, expert comments like "yes," "abstain," "no," and "refusal" will be supplied.

Shahzaib et al. [3] defined the SFS for certain AOs using MADM. SFS requires that $0 \preceq \iota^2 + \rho^2 + \varsigma^2 \preceq 1$ be used in place of $0 \preceq \iota + \rho + \varsigma \preceq 1$. Hussain et al. first proposed the concept of an intelligent decision support system for SFS [4]. SFSs and their applications in DM were initially presented by Rafiq et al. [5]. For instance, a DM problem with a property is $\iota^2 + \varsigma^2 \succcurlyeq 1$. Senapati et al. [6] invented Fermatean FS (FFS) in 2019 with the condition that $0 \preceq \iota^3 + \varsigma^3 \preceq 1$. The concept of generalized orthopair FSs was initially proposed by Yager [7]. In the q -rung orthogonal pair FS (q -ROFS), both the MD and the NMD have power q ; nonetheless, their sum can never be more than one. Palanikumar et al. introduced the MADM approach for Pythagorean neutrosophic normal interval-valued fuzzy AOs [8]. Based on IFSs, Xu et al. created geometric operators, such as weighted, ordered weighted, and hybrid operators [9]. Li et al. [10] proposed generalized ordered weighted averaging operators (GOWs) in 2002. Using AOs and distance measurements, Zeng et al. [11] described how to calculate ordered weighted distances. Peng et al. examined a basic PFS based on the characteristics of AOs [12]. Dombi AOs were developed by spherical fuzzy Ashraf and associates [13]. Additionally, Ullah et al. [14, 15] offer more details on SFSs and T-SFSs. Using a variety of AOs.

2. Operations for CLOGRFFNN

Assuming f is a fractional part function, the fractional part of b , where b is a real number, may be written as follows: $\mathcal{H}[b] = [b] = b - \{b\}$ as well. It is also possible to describe the difference between a real number and its highest integer value, which is found using the greatest integer function, as a fractional part function. The fractional component of $b = 0$ if b is an integer. Assume that the reciprocal fractional part function $\mathcal{H}[b] = \frac{1}{b}$ exists. It is commonly known that the fractional portion of b equals 0 whenever it is an integer. Therefore, for $\mathcal{H}[b] = \frac{1}{b}$ to be defined, b cannot be an integer. Its domain is $\mathcal{H}[b] = \frac{1}{b}$, which includes all real numbers except integers. Here $\mathfrak{L}_{\mathfrak{H}_\alpha} = \log_{\mathfrak{H}_\alpha}$ and

$$\mathfrak{H}_\alpha = \prod_{\alpha=1}^n \left[[\mathfrak{L}\mathfrak{J}^{\mathfrak{T}}][b] \cdot E^{[\mathfrak{L}\mathfrak{N}^{\mathfrak{T}}][b]}, [\mathfrak{L}\mathfrak{J}^{\mathfrak{J}}][b] \cdot E^{[\mathfrak{L}\mathfrak{N}^{\mathfrak{J}}][b]}, [\mathfrak{L}\mathfrak{J}^{\mathfrak{F}}][b] \cdot E^{[\mathfrak{L}\mathfrak{N}^{\mathfrak{F}}][b]} \right]$$

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Definition 2.1. The RFFNS $\mathcal{A} = \left\{ b, \left\langle \left[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I} \right] [b] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] [b]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] [b] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] [b]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] [b] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] [b]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] [b] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] [b]} \right\rangle \middle| b \in \mathcal{A} \right\}$, where $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}]$: $\mathcal{A} \rightarrow [0, 1]$ denote the MD, IMD and NMD of $b \in \mathcal{A}$ to $\mathcal{A}$, respectively and $0 \preccurlyeq [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] [b]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] [b]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] [b]]^{\triangleleft^{-1}} \preccurlyeq 1$ and $0 \preccurlyeq [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] [b]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] [b]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] [b]]^{\triangleleft^{-1}} \preccurlyeq 1$. For convenience, $\mathcal{A} = \left\langle \left[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I} \right] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]} \right\rangle$ is represent a CLOGRFFNN.

Definition 2.2. Let $\mathcal{A} = \langle [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]}, [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]} \rangle$, $\mathcal{A}_1 = \langle [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]} \rangle$, $\mathcal{A}_2 = \langle [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]} \rangle$ be any three CLOGRFFNNs.

Then

$$\begin{aligned}
 (1) \mathcal{A}_1 \square \mathcal{A}_2 &= \left(\begin{array}{c} \sqrt[\triangleleft^{-1}]{\frac{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}}}{-[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} \cdot [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}}}} \cdot E^{\sqrt[\triangleleft^{-1}]{\frac{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}{-[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} \cdot [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}}}, \\ \sqrt[\triangleleft^{-1}]{\frac{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}}}{-[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} \cdot [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}}}} \cdot E^{\sqrt[\triangleleft^{-1}]{\frac{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}{-[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} \cdot [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}}}, \\ [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}} \cdot E^{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}} \end{array} \right), \\
 (2) \mathcal{A}_1 * \mathcal{A}_2 &= \left(\begin{array}{c} [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}} \cdot E^{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}, \\ \sqrt[\triangleleft^{-1}]{\frac{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}}}{-[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} \cdot [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}}}} \cdot E^{\sqrt[\triangleleft^{-1}]{\frac{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}{-[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} \cdot [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}}}, \\ \sqrt[\triangleleft^{-1}]{\frac{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}}}{-[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_1]]^{\triangleleft^{-1}} \cdot [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}_2]]^{\triangleleft^{-1}}}} \cdot E^{\sqrt[\triangleleft^{-1}]{\frac{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} + [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}{-[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_1^{\mathcal{I}}]]^{\triangleleft^{-1}} \cdot [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_2^{\mathcal{I}}]]^{\triangleleft^{-1}}}}} \end{array} \right) \\
 (3) \# \cdot \mathcal{A} &= \left(\begin{array}{c} \sqrt[\triangleleft^{-1}]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}^{\mathcal{I}}] \right]^{\#}} \cdot E^{\sqrt[\triangleleft^{-1}]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] \right]^{\#}}}, \\ \sqrt[\triangleleft^{-1}]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}^{\mathcal{I}}] \right]^{\#}} \cdot E^{\sqrt[\triangleleft^{-1}]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] \right]^{\#}}}, \\ [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}^{\mathcal{I}}] \right]^{\#} \cdot E^{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] \right]^{\#}} \end{array} \right) \\
 (4) \mathcal{A}^{\#} &= \left(\begin{array}{c} [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}^{\mathcal{I}}] \right]^{\#} \cdot E^{[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] \right]^{\#}}, \\ \sqrt[\triangleleft^{-1}]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}^{\mathcal{I}}] \right]^{\#}} \cdot E^{\sqrt[\triangleleft^{-1}]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] \right]^{\#}}}, \\ \sqrt[\triangleleft^{-1}]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{I}^{\mathcal{I}}] \right]^{\#}} \cdot E^{\sqrt[\triangleleft^{-1}]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}] \right]^{\#}}} \end{array} \right).
 \end{aligned}$$

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Definition 2.3. For any two CLOGRFFNNs $\mathfrak{A}_1 = \langle [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{T}}|, |\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{I}}|, |\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{F}}|] \rangle$ and $\mathfrak{A}_2 = \langle [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{T}}|, |\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{I}}|, |\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{F}}|] \rangle$. Then

$$\Gamma_E[\mathfrak{A}_1, \mathfrak{A}_2] = \sqrt{\frac{1}{2} \left(\left(\begin{aligned} &1 + [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{T}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{I}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{F}}|]^2 \\ &- \left[1 + [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{T}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{I}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{F}}|]^2 \right] \end{aligned} \right)^2 + \left(\begin{aligned} &[|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{T}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{I}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{F}}|]^2 \\ &- \left[[|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{T}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{I}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{F}}|]^2 \right] \end{aligned} \right)^2 \right)}$$

where $\Gamma_E[\mathfrak{A}_1, \mathfrak{A}_2]$ is called the ED between $\mathfrak{A}_1$ and $\mathfrak{A}_2$.

$$\Gamma_H[\mathfrak{A}_1, \mathfrak{A}_2] = \frac{1}{2} \left(\begin{aligned} &\left| 1 + [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{T}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{I}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{F}}|]^2 \right| \\ &- \left| 1 + [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{T}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{I}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{F}}|]^2 \right| \\ &+ \left| [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{T}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{I}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{F}}|]^2 \right| \\ &- \left| [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{T}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{I}}|]^2 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_2^{\mathfrak{F}}|]^2 \right| \end{aligned} \right)$$

where $\Gamma_H[\mathfrak{A}_1, \mathfrak{A}_2]$ is called the HD between $\mathfrak{A}_1$ and $\mathfrak{A}_2$.

3. AOs based on CLOGRFFNN

3.1. CLOGRFFNWA

Definition 3.1. Let $\mathfrak{A}_\alpha = \langle [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{T}}| \cdot E^{|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}}|}, |\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{I}}| \cdot E^{|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}}|}, |\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{F}}| \cdot E^{|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}}|}] \rangle$ be the CLOGRFFNNs, $W = [\epsilon_1, \epsilon_2, \dots, \epsilon_n]$ be the weight of $\mathfrak{A}_\alpha$, $\epsilon_\alpha \succcurlyeq 0$ and $\square_{\alpha=1}^n \epsilon_\alpha = 1$. Then CLOGRFFNWA $[\mathfrak{A}_1, \mathfrak{A}_2, \dots, \mathfrak{A}_n] = \square_{\alpha=1}^n \epsilon_\alpha \mathfrak{A}_\alpha$.

Theorem 3.2. Let $\mathfrak{A}_\alpha = \langle [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{T}}| \cdot E^{|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}}|}, |\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{I}}| \cdot E^{|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}}|}, |\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{F}}| \cdot E^{|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}}|}] \rangle$ be the CLOGRFFNNs. Then CLOGRFFNWA $[\mathfrak{A}_1, \mathfrak{A}_2, \dots, \mathfrak{A}_n]$

$$= \left(\begin{aligned} &\sqrt[n]{1 - \square_{\alpha=1}^n \left[1 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{T}}|]^{\epsilon_\alpha} \right]} \cdot E^{\sqrt[n]{1 - \square_{\alpha=1}^n \left[1 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}}|]^{\epsilon_\alpha} \right]}}, \\ &\sqrt[n]{1 - \square_{\alpha=1}^n \left[1 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{I}}|]^{\epsilon_\alpha} \right]} \cdot E^{\sqrt[n]{1 - \square_{\alpha=1}^n \left[1 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}}|]^{\epsilon_\alpha} \right]}}, \\ &\square_{\alpha=1}^n [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_\alpha^{\mathfrak{F}}|]^{\epsilon_\alpha} \cdot E^{\square_{\alpha=1}^n [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}}|]^{\epsilon_\alpha}} \end{aligned} \right).$$

Proof. If $n = 2$, then CLOGRFFNWA $[\mathfrak{A}_1, \mathfrak{A}_2] = \epsilon_1 \mathfrak{A}_1 \square \epsilon_2 \mathfrak{A}_2$, where

$$\epsilon_1 \mathfrak{A}_1 = \left(\begin{aligned} &\sqrt[n]{1 - \left[1 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{T}}|]^{\epsilon_1} \right]} \cdot E^{\sqrt[n]{1 - \left[1 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_1^{\mathfrak{T}}|]^{\epsilon_1} \right]}}, \\ &\sqrt[n]{1 - \left[1 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{I}}|]^{\epsilon_1} \right]} \cdot E^{\sqrt[n]{1 - \left[1 - [|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_1^{\mathfrak{I}}|]^{\epsilon_1} \right]}}, \\ &[|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{A}_1^{\mathfrak{F}}|]^{\epsilon_1} \cdot E^{[|\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_1^{\mathfrak{F}}|]^{\epsilon_1}} \end{aligned} \right)$$

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Now, $\epsilon_1 \perp_1 \Box \epsilon_2 \perp_2$

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$$= \begin{pmatrix} \sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^2 \left[1 - \llbracket \mathcal{L}_{\mathfrak{m}_\alpha} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \cdot E^{\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^2 \left[1 - \llbracket \mathcal{L}_{\mathfrak{m}_\alpha} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}}} , \\ \sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^2 \left[1 - \llbracket \mathcal{L}_{\mathfrak{m}_\alpha} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \cdot E^{\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^2 \left[1 - \llbracket \mathcal{L}_{\mathfrak{m}_\alpha} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}}} , \\ \diamond_{\alpha=1}^2 \llbracket \llbracket \mathcal{L}_{\mathfrak{m}_\alpha} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \rrbracket^{\epsilon_\alpha} \cdot E^{\diamond_{\alpha=1}^2 \llbracket \llbracket \mathcal{L}_{\mathfrak{m}_\alpha} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \rrbracket^{\epsilon_\alpha}} \end{pmatrix}.$$

Thus, $\text{CLOGRFFNWA}[\downarrow_1, \downarrow_2, \dots, \downarrow_l]$

$$= \begin{pmatrix} \sqrt[l]{1 - \diamond_{\alpha=1}^l \left[1 - \llbracket \mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \cdot E^{\sqrt[l]{1 - \diamond_{\alpha=1}^l \left[1 - \llbracket \mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}}}, \\ \sqrt[l]{1 - \diamond_{\alpha=1}^l \left[1 - \llbracket \mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{I}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \cdot E^{\sqrt[l]{1 - \diamond_{\alpha=1}^l \left[1 - \llbracket \mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{I}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}}}, \\ \diamond_{\alpha=1}^l \left[\llbracket \mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}} \cdot E^{\diamond_{\alpha=1}^l \left[\llbracket \mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \rrbracket^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \end{pmatrix}.$$

$$= \left(\begin{aligned} & A^{-1} \sqrt{\frac{\square_{\alpha=1}^l \left[| - [| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{Z}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} + [| - [| - [\mathfrak{Z}_{l+1}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \right]}{-\diamond_{\alpha=1}^l \left[| - [| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{Z}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \cdot [| - [| - [\mathfrak{Z}_{l+1}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \right]}} \\ & E \sqrt{\frac{\square_{\alpha=1}^l \left[| - [| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} + [| - [| - [\mathfrak{N}_{l+1}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \right]}{-\diamond_{\alpha=1}^l \left[| - [| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \cdot [| - [| - [\mathfrak{N}_{l+1}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \right]}, \\ & A^{-1} \sqrt{\frac{\square_{\alpha=1}^l \left[| - [| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{Z}_{\alpha}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} + [| - [| - [\mathfrak{Z}_{l+1}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \right]}{-\diamond_{\alpha=1}^l \left[| - [| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{Z}_{\alpha}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \cdot [| - [| - [\mathfrak{Z}_{l+1}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \right]}. \\ & E \sqrt{\frac{\square_{\alpha=1}^l \left[| - [| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_{\alpha}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} + [| - [| - [\mathfrak{N}_{l+1}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \right]}{-\diamond_{\alpha=1}^l \left[| - [| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_{\alpha}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \cdot [| - [| - [\mathfrak{N}_{l+1}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \right]}, \\ & (\diamond_{\alpha=1}^l [[[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{Z}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \cdot [[\mathfrak{Z}_{l+1}^{\tilde{\mathfrak{T}}}]^{\triangleleft^{-1}}]^{\epsilon_{l+1}} \cdot E \diamond_{\alpha=1}^l [[[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \cdot [[\mathfrak{N}_{l+1}^{\tilde{\mathfrak{T}}}]^{\triangleleft^{-1}}]^{\epsilon_{l+1}}) \end{aligned} \right) \\ = \left(\begin{aligned} & A^{-1} \sqrt{| - \diamond_{\alpha=1}^{l+1} \left[| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{Z}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha}} \cdot E \sqrt{| - \diamond_{\alpha=1}^{l+1} \left[| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha}} , \\ & A^{-1} \sqrt{| - \diamond_{\alpha=1}^{l+1} \left[| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{Z}_{\alpha}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha}} \cdot E \sqrt{| - \diamond_{\alpha=1}^{l+1} \left[| - [[\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_{\alpha}^{\mathfrak{J}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha}} , \\ & \diamond_{\alpha=1}^{l+1} [| | [\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{Z}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \cdot E \diamond_{\alpha=1}^{l+1} [| | [\mathfrak{L}_{\mathfrak{m}_\alpha} \mathfrak{N}_{\alpha}^{\tilde{\mathfrak{T}}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \end{aligned} \right).$$

Theorem 3.3. Let $\mathcal{J}_\alpha = \langle [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{T}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{T}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{I}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{F}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{F}}]}] \rangle$ be the CLOGRFFNNs. Then CLOGRFFNWA $[\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_n] = \mathcal{J}$ (idempotency property).

Proof. Since $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{T}}] = [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{T}}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{I}}] = [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{I}}]$ and $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{F}}] = [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{F}}]$ and $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{T}}] = [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{T}}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{I}}] = [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]$ and $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{F}}] = [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{F}}]$ and $\square_{\alpha=1}^n \epsilon_\alpha = 1$. Now, CLOGRFFNWA $[\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_n]$

$$\begin{aligned}
 &= \left(\begin{aligned} &\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{T}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{T}}]}]^{\epsilon_\alpha} \right]} \cdot E^{\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{T}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{T}}]}]^{\epsilon_\alpha} \right]}}, \\ &\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{I}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{I}}]}]^{\epsilon_\alpha} \right]} \cdot E^{\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{I}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{I}}]}]^{\epsilon_\alpha} \right]}}, \\ &\diamond_{\alpha=1}^n [[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{F}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{F}}]}]^{\epsilon_\alpha} \cdot E^{\diamond_{\alpha=1}^n [[[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{F}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{F}}]}]^{\epsilon_\alpha} \end{aligned} \right) \\
 &= \left(\begin{aligned} &\sqrt[n]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{T}}]^{\square_{\alpha=1}^n \epsilon_\alpha} \right]} \cdot E^{\sqrt[n]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{T}}]^{\square_{\alpha=1}^n \epsilon_\alpha} \right]}}, \\ &\sqrt[n]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{I}}]^{\square_{\alpha=1}^n \epsilon_\alpha} \right]} \cdot E^{\sqrt[n]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]^{\square_{\alpha=1}^n \epsilon_\alpha} \right]}}, \\ &[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{F}}]^{\square_{\alpha=1}^n \epsilon_\alpha} \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{F}}]^{\square_{\alpha=1}^n \epsilon_\alpha} \end{aligned} \right) \\
 &= \left(\begin{aligned} &\sqrt[n]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{T}}]^{\epsilon_\alpha} \right]} \cdot E^{\sqrt[n]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{T}}]^{\epsilon_\alpha} \right]}}, \\ &\sqrt[n]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{I}}]^{\epsilon_\alpha} \right]} \cdot E^{\sqrt[n]{1 - \left[1 - [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]^{\epsilon_\alpha} \right]}}, \\ &[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{F}}]^{\epsilon_\alpha} \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{F}}]^{\epsilon_\alpha} \end{aligned} \right) \\
 &= \mathcal{J}.
 \end{aligned}$$

Theorem 3.4. Let $\mathcal{J}_\alpha = \langle [[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{T}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{T}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{I}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{F}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{F}}]}] \rangle$ be the CLOGRFFNNs. Then CLOGRFFNWA $[\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_n]$

where $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{T}}] = \min [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_{ij}^{\mathcal{T}}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{I}}] = \max [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_{ij}^{\mathcal{I}}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_\alpha^{\mathcal{F}}] = \min [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}_{ij}^{\mathcal{F}}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{T}}] = \max [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_{ij}^{\mathcal{T}}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{I}}] = \min [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_{ij}^{\mathcal{I}}]$, $[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_\alpha^{\mathcal{F}}] = \max [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}_{ij}^{\mathcal{F}}]$ and where $1 \preccurlyeq i \preccurlyeq n$, $j = 1, 2, \dots, i_j$. Then,

$$\begin{aligned}
 &\langle [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{T}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{T}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{I}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{F}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{F}}]} \rangle \\
 &\preccurlyeq \text{CLOGRFFNWA} [\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_n] \\
 &\preccurlyeq \langle [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{T}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{T}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{I}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{I}}]}, [\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{J}^{\mathcal{F}}] \cdot E^{[\mathcal{L}_{\mathcal{M}_\alpha} \mathcal{N}^{\mathcal{F}}]} \rangle.
 \end{aligned}$$

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(Boundedness property).

Proof. Since, $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{T}}] = \min[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{T}}]$, $[\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{T}}] = \max[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{T}}]$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{T}}] \preccurlyeq [\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{T}}] \preccurlyeq [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{T}}]$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{T}}] = \min[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{T}}]$, $[\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{T}}] = \max[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{T}}]$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{T}}] \preccurlyeq [\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{T}}] \preccurlyeq [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{T}}]$.

Now $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{T}}] \cdot E[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{T}}]$

$$\begin{aligned} &= \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{T}}]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \cdot E \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{T}}]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \\ &\preccurlyeq \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{T}}]]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \cdot E \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{T}}]]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \\ &\preccurlyeq \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{T}}]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \cdot E \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{T}}]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \\ &= [\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{T}}]. \end{aligned}$$

Since, $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{J}}] = \min[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{J}}]$, $[\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{J}}] = \max[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{J}}]$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{J}}] \preccurlyeq [\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{J}}] \preccurlyeq [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{J}}]$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{J}}] = \min[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{J}}]$, $[\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{J}}] = \max[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{J}}]$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{J}}] \preccurlyeq [\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{J}}] \preccurlyeq [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{J}}]$.

Now, $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{J}}] \cdot E[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{J}}]$

$$\begin{aligned} &= \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{J}}]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \cdot E \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{J}}]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \\ &\preccurlyeq \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{J}}]]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \cdot E \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{J}}]]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \\ &\preccurlyeq \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{J}}]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \cdot E \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{J}}]^{\triangleleft^{-1}} \right]^{\epsilon_\alpha}} \\ &= [\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{J}^{\mathfrak{J}}] \cdot E[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} \mathfrak{N}^{\mathfrak{J}}]. \end{aligned}$$

Since, $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} [\mathfrak{J}^{\mathfrak{F}}]^{\triangleleft^{-1}}] = \min[[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{F}}]]^{\triangleleft^{-1}}$, $[\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} [\mathfrak{J}^{\mathfrak{F}}]^{\triangleleft^{-1}}] = \max[[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{F}}]]^{\triangleleft^{-1}}$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} [\mathfrak{J}^{\mathfrak{F}}]^{\triangleleft^{-1}}] \preccurlyeq [[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_{ij}^{\mathfrak{F}}]]^{\triangleleft^{-1}} \preccurlyeq [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} [\mathfrak{J}^{\mathfrak{F}}]^{\triangleleft^{-1}}]$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} [\mathfrak{N}^{\mathfrak{F}}]^{\triangleleft^{-1}}] = \min[[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{F}}]]^{\triangleleft^{-1}}$, $[\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} [\mathfrak{N}^{\mathfrak{F}}]^{\triangleleft^{-1}}] = \max[[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{F}}]]^{\triangleleft^{-1}}$ and $[\underline{\mathcal{L}}_{\mathfrak{M}_\alpha} [\mathfrak{N}^{\mathfrak{F}}]^{\triangleleft^{-1}}] \preccurlyeq [[\mathcal{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_{ij}^{\mathfrak{F}}]]^{\triangleleft^{-1}} \preccurlyeq [\overline{\mathcal{L}}_{\mathfrak{M}_\alpha} [\mathfrak{N}^{\mathfrak{F}}]^{\triangleleft^{-1}}]$.

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$$\begin{aligned}
\overline{[\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{J}^{\tilde{\delta}}]^{\triangleleft^{-1}}]} &= \diamond_{\alpha=1}^n \overline{[\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{J}^{\tilde{\delta}}]^{\triangleleft^{-1}}]^{\epsilon_\alpha}} \cdot E^{\diamond_{\alpha=1}^n [\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{N}^{\tilde{\delta}}]^{\triangleleft^{-1}}]^{\epsilon_\alpha}} \\
&\simeq \diamond_{\alpha=1}^n [[[\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{J}^{\tilde{\delta}}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha} \cdot E^{\diamond_{\alpha=1}^n [[[\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{N}^{\tilde{\delta}}_{ij}]]^{\triangleleft^{-1}}]^{\epsilon_\alpha}} \\
&\simeq \diamond_{\alpha=1}^n \overline{[\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{J}^{\tilde{\delta}}]^{\triangleleft^{-1}}]^{\epsilon_\alpha}} \cdot E^{\diamond_{\alpha=1}^n \overline{[\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{N}^{\tilde{\delta}}]^{\triangleleft^{-1}}]^{\epsilon_\alpha}}} \\
&= \overline{[\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{J}^{\tilde{\delta}}]^{\triangleleft^{-1}}]} \cdot \overline{E^{[\mathcal{L}_{\mathfrak{m}_\alpha}[\mathfrak{N}^{\tilde{\delta}}]^{\triangleleft^{-1}}]}}.
\end{aligned}$$
$$\frac{1}{2} \times \left(\begin{aligned} & \left(\left[\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \underline{\mathfrak{J}}^{\mathfrak{T}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 - \left[\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \underline{\mathfrak{J}}^{\mathfrak{J}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 \right) \\ & + 1 - \left[\diamond_{\alpha=1}^n \left[\left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \underline{\mathfrak{J}}^{\mathfrak{F}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}} \right]^2 \end{aligned} \right) \\ + \left(\begin{aligned} & \left[\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \mathfrak{K}^{\mathfrak{T}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 - \left[\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \mathfrak{K}^{\mathfrak{J}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 \right) \\ & - \left[\diamond_{\alpha=1}^n \left[\left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \mathfrak{K}^{\mathfrak{F}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}} \right]^2 \end{aligned} \right) \end{aligned} \right) \\ \mathfrak{A} \frac{1}{2} \times \left(\begin{aligned} & \left(\left[\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \underline{\mathfrak{J}}^{\mathfrak{T}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 - \left[\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \underline{\mathfrak{J}}^{\mathfrak{J}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 + \right) \\ & + 1 - \left[\diamond_{\alpha=1}^n \left[\left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \underline{\mathfrak{J}}^{\mathfrak{F}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}} \right]^2 \end{aligned} \right) \\ + \left(\begin{aligned} & \left[\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \mathfrak{K}^{\mathfrak{T}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 - \left[\sqrt[{\triangleleft^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \mathfrak{K}^{\mathfrak{J}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 + \right) \\ & - \left[\diamond_{\alpha=1}^n \left[\left[\underline{\mathcal{L}}_{\mathfrak{m}_{\alpha}} \mathfrak{K}^{\mathfrak{F}} \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}} \right]^2 \end{aligned} \right) \end{aligned} \right).$$
$$\begin{aligned} & \left\langle [\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \sqsupset^{\mathfrak{T}}] \cdot E^{[\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \aleph^{\mathfrak{T}}]}, [\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \sqsupset^{\mathfrak{J}}] \cdot E^{[\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \aleph^{\mathfrak{J}}]}, \overline{[\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \sqsupset^{\mathfrak{F}}] \cdot E^{[\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \aleph^{\mathfrak{F}}]}} \right\rangle \\ & \asymp CLOGRFFNWA[\downarrow_1, \downarrow_2, \dots, \downarrow_n] \\ & \asymp \langle [\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \sqsupset^{\mathfrak{T}}] \cdot E^{[\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \aleph^{\mathfrak{T}}]}, [\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \sqsupset^{\mathfrak{J}}] \cdot E^{[\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \aleph^{\mathfrak{J}}]}, [\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \sqsupset^{\mathfrak{F}}] \cdot E^{[\underline{\mathcal{L}}_{\mathfrak{m}_\alpha} \aleph^{\mathfrak{F}}]} \rangle. \end{aligned}$$

where $|i = 1, 2, \dots, n|; |j = 1, 2, \dots, i_j|$ (monotonicity property).

Hence, $\diamond_{\alpha=1}^n \left| 1 - \left[\left[\mathcal{L}_{\mathfrak{m}_{\alpha}} \sqcup \mathfrak{T}_{t_{\alpha}} \right] \right]^2 \right|^{\epsilon_{\alpha}} \succcurlyeq \diamond_{\alpha=1}^n \left| 1 - \left[\left[\mathcal{L}_{\mathfrak{m}_{\alpha}} \sqcup \mathfrak{T}_{h_{\alpha}} \right] \right]^2 \right|^{\epsilon_{\alpha}}$

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$$\text{and } \sqrt[n]{1 - \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{t_{\alpha}}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \preccurlyeq \sqrt[n]{1 - \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{h_{\alpha}}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}}.$$

Similarly, $\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{ij}}^{\mathfrak{T}} \right]^2 \preccurlyeq \left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{ij}}^{\mathfrak{T}} \right]^2$.

Therefore, $1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{\alpha}}^{\mathfrak{T}} \right] \right]^2 \succcurlyeq 1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{\alpha}}^{\mathfrak{T}} \right] \right]^2$.

Hence, $\diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{\alpha}}^{\mathfrak{T}} \right] \right]^2 \right]^{\epsilon_{\alpha}} \succcurlyeq \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{\alpha}}^{\mathfrak{T}} \right] \right]^2 \right]^{\epsilon_{\alpha}}$

$$\text{and } \sqrt[n]{1 - \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{\alpha}}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \preccurlyeq \sqrt[n]{1 - \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{\alpha}}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}}.$$

For any i , $\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{t_{ij}}^{\mathfrak{J}} \right]^{\triangleleft^{-1}} \preccurlyeq \left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{h_{ij}}^{\mathfrak{J}} \right]^{\triangleleft^{-1}}$.

Therefore, $1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{t_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \succcurlyeq 1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{h_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}}$.

Hence, $\diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{t_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}} \succcurlyeq \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{h_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}$.

This implies that $\sqrt[n]{1 - \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{t_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \preccurlyeq \sqrt[n]{1 - \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{h_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}}.$

Similarly, for any i , $\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{ij}}^{\mathfrak{J}} \right]^{\triangleleft^{-1}} \preccurlyeq \left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{ij}}^{\mathfrak{J}} \right]^{\triangleleft^{-1}}$.

Therefore, $1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \succcurlyeq 1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}}$.

Hence, $\diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}} \succcurlyeq \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}$.

This implies that $\sqrt[n]{1 - \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \preccurlyeq \sqrt[n]{1 - \diamond_{\alpha=1} \left[1 - \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{\alpha}}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}}.$

For any i , $\left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{t_{ij}}^{\mathfrak{F}} \right] \right]^2 \succcurlyeq \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{h_{ij}}^{\mathfrak{F}} \right] \right]^2$ and $\left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{t_{ij}}^{\mathfrak{F}} \right] \right]^{\triangleleft^{-1}} \succcurlyeq \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{h_{ij}}^{\mathfrak{F}} \right] \right]^{\triangleleft^{-1}}$.

Therefore, $1 - \frac{\left[\diamond_{\alpha=1} \left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{t_{ij}}^{\mathfrak{F}} \right] \right]^{\triangleleft^{-1}}}{2} \preccurlyeq 1 - \frac{\left[\diamond_{\alpha=1} \left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{J}_{h_{ij}}^{\mathfrak{F}} \right] \right]^{\triangleleft^{-1}}}{2}$.

Similarly, for any i ,

$\left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{ij}}^{\mathfrak{F}} \right] \right]^2 \succcurlyeq \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{ij}}^{\mathfrak{F}} \right] \right]^2$ and $\left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{ij}}^{\mathfrak{F}} \right] \right]^{\triangleleft^{-1}} \succcurlyeq \left[\left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{ij}}^{\mathfrak{F}} \right] \right]^{\triangleleft^{-1}}$.

Therefore, $1 - \left[\diamond_{\alpha=1} \left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{t_{ij}}^{\mathfrak{F}} \right] \right]^{\triangleleft^{-1}} \preccurlyeq 1 - \left[\diamond_{\alpha=1} \left[\mathcal{L}_{\mathfrak{M}_{\alpha}} \mathfrak{N}_{h_{ij}}^{\mathfrak{F}} \right] \right]^{\triangleleft^{-1}}$.

Hence,

$$\begin{aligned} & \frac{1}{2} \times \left(\left(\left[\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{ti}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \right)^2 - \left[\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{t_{\alpha}}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \right)^2 \right. \\ & \quad \left. + 1 - \left[\diamond_{\alpha=1}^n \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{t_{\alpha}}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right] \right]^2 \right. \\ & \quad \left. + \left(\left[\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{ti}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \right)^2 - \left[\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{t_{\alpha}}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \right)^2 \right. \right. \\ & \quad \left. \left. - \left[\diamond_{\alpha=1}^n \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{t_{\alpha}}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right] \right]^2 \right) \right) \right) \\ & \approx \frac{1}{2} \times \left(\left(\left[\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{hi}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \right)^2 - \left[\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{h_{\alpha}}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \right)^2 \right. \\ & \quad \left. + 1 - \left[\diamond_{\alpha=1}^n \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{h_{\alpha}}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right] \right]^2 \right. \\ & \quad \left. + \left(\left[\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{hi}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \right)^2 - \left[\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{h_{\alpha}}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \right)^2 \right. \right. \\ & \quad \left. \left. - \left[\diamond_{\alpha=1}^n \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{h_{\alpha}}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right] \right]^2 \right) \right) \right). \end{aligned}$$

Hence, $CLOGRFFNWA[\mathfrak{J}_1, \mathfrak{J}_2, \dots, \mathfrak{J}_n] \preccurlyeq CLOGRFFNWA[W_1, W_2, \dots, W_n]$.

3.2. CLOGRFFNWG

Definition 3.6. Let $\mathfrak{J}_{\alpha} = \left\langle \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right]} \right], \left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right]} \right], \left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right]} \right] \right\rangle$ be the CLOGRFFNNs. Then $RFFNWG[\mathfrak{J}_1, \mathfrak{J}_2, \dots, \mathfrak{J}_n] = \diamond_{\alpha=1}^n \mathfrak{J}_{\alpha}^{\epsilon_{\alpha}}$.

Corollary 3.7. Let $\mathfrak{J}_{\alpha} = \left\langle \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right]} \right], \left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right]} \right], \left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right]} \right] \right\rangle$ be the CLOGRFFNNs. Then $CLOGRFFNWG[\mathfrak{J}_1, \mathfrak{J}_2, \dots, \mathfrak{J}_n]$

$$= \left(\begin{array}{c} \diamond_{\alpha=1}^n \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}} \cdot E^{\diamond_{\alpha=1}^n \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \\ \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \cdot E^{\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \\ \sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \cdot E^{\sqrt[n]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{K}_{\alpha}^{\mathfrak{T}} \right] \right]^{\epsilon_{\alpha}}} \right]} \end{array} \right).$$

3.3. Generalized CLOGRFFNWA(GCLOGRFFNWA)

Definition 3.8. Let $\mathfrak{J}_{\alpha} = \left\langle \left[\left[\left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right], \left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right], \left[\mathfrak{L}_{\mathfrak{h}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \right] \right] \right\rangle$ be the CLOGRFFNN. Then $GCLOGRFFNWA[\mathfrak{J}_1, \mathfrak{J}_2, \dots, \mathfrak{J}_n] = \left[\square_{\alpha=1}^n \epsilon_{\alpha} \mathfrak{J}_{\alpha}^{\#} \right]^{1/\#}$.

Theorem 3.9. Let $\mathfrak{A}_\alpha = \left\langle \left[\left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right], \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{I}} \right], \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{F}} \right] \right] \right\rangle$ be the CLOGRFFNNs. Then GCLOGRFFNWA $[\mathfrak{A}_1, \mathfrak{A}_2, \dots, \mathfrak{A}_n]$

$$= \left(\begin{array}{c} \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{1/\mathfrak{A}^{-1}} \\ E \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{1/\mathfrak{A}^{-1}} \\ \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{I}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{1/\mathfrak{A}^{-1}} \\ E \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{I}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{1/\mathfrak{A}^{-1}} \\ \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \left[1 - \left[\diamond_{\alpha=1}^n \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{\mathfrak{A}^{-1}}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}} \right]^{1/\mathfrak{A}^{-1}} \\ E \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \left[1 - \left[\diamond_{\alpha=1}^n \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{\mathfrak{A}^{-1}}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{1/\mathfrak{A}^{-1}} \end{array} \right).$$

Proof. To illustrate this, we may first show that,

$$\square_{\alpha=1}^n \epsilon_\alpha \mathfrak{A}_\alpha^{\mathfrak{A}^{-1}} = \left(\begin{array}{c} \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{\epsilon_\alpha} \cdot E \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{\epsilon_\alpha} \\ \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{I}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{\epsilon_\alpha} \cdot E \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \diamond_{\alpha=1}^n \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{I}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{\epsilon_\alpha} \\ \left[\diamond_{\alpha=1}^n \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{\epsilon_\alpha} \cdot E \left[\diamond_{\alpha=1}^n \left[\sqrt[{\mathfrak{A}^{-1}}]{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{B}_\alpha^{\mathfrak{T}} \right]^{\mathfrak{A}^{-1}} \right]^{\epsilon_\alpha}} \right]^{\mathfrak{A}^{-1}}} \right]^{\epsilon_\alpha} \right] \end{array} \right).$$

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[illegible]

$$= \left(\begin{aligned} & \sqrt[{\mathbb{A}^{-1}}]{1 - \diamond_{\alpha=1}^2 \left[1 - \left[\left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_1^{\mathfrak{F}} \right] \right]^{\mathbb{A}^{-1}} \right]^{\mathbb{A}^{-1}} \right]^{\epsilon_{\alpha}}} \cdot E \sqrt[{\mathbb{A}^{-1}}]{1 - \diamond_{\alpha=1}^2 \left[1 - \left[\left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_1^{\mathfrak{F}} \right] \right]^{\mathbb{A}^{-1}} \right]^{\mathbb{A}^{-1}} \right]^{\epsilon_{\alpha}}} \\ & \sqrt[{\mathbb{A}^{-1}}]{1 - \diamond_{\alpha=1}^2 \left[1 - \left[\left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_1^{\mathfrak{J}} \right] \right]^{\mathbb{A}^{-1}} \right]^{\mathbb{A}^{-1}} \right]^{\epsilon_{\alpha}}} \cdot E \sqrt[{\mathbb{A}^{-1}}]{1 - \diamond_{\alpha=1}^2 \left[1 - \left[\left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_1^{\mathfrak{J}} \right] \right]^{\mathbb{A}^{-1}} \right]^{\mathbb{A}^{-1}} \right]^{\epsilon_{\alpha}}} \\ & \diamond_{\alpha=1}^2 \left| \sqrt[{\mathbb{A}^{-1}}]{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{F}} \right] \right]^{\mathbb{A}^{-1}} \right]^{\mathbb{A}^{-1}}} \right|^{\epsilon_{\alpha}} \cdot \diamond_{\alpha=1}^2 \left| \sqrt[{\mathbb{A}^{-1}}]{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{F}} \right] \right]^{\mathbb{A}^{-1}} \right]^{\mathbb{A}^{-1}}} \right|^{\epsilon_{\alpha}} \end{aligned} \right).$$

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If $n = l + 1$, then $\square_{\alpha=1}^l \epsilon_{\alpha} \mathfrak{J}_{\alpha}^{\sharp} + \epsilon_{l+1} \mathfrak{J}_{l+1}^{\sharp} = \square_{\alpha=1}^{l+1} \epsilon_{\alpha} \mathfrak{J}_{\alpha}^{\sharp}$.
Now, $\square_{\alpha=1}^l \epsilon_{\alpha} \mathfrak{J}_{\alpha}^{\sharp} + \epsilon_{l+1} \mathfrak{J}_{l+1}^{\sharp} = \epsilon_1 \mathfrak{J}_1^{\sharp} \square \epsilon_2 \mathfrak{J}_2^{\sharp} \dots \square \epsilon_l \mathfrak{J}_l^{\sharp} \square \epsilon_{l+1} \mathfrak{J}_{l+1}^{\sharp}$

[illegible]

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$$\begin{aligned}
& \square_{\alpha=1}^{l+1} \epsilon_{\alpha} \lrcorner_{\alpha}^{\triangleleft^{-1}} = \\
& \left(\begin{array}{c} \triangleleft^{-1} \sqrt{1 - \diamond_{\alpha=1}^{l+1} \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \cdot E \sqrt{\triangleleft^{-1} \left[1 - \diamond_{\alpha=1}^{l+1} \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right.} \\ \triangleleft^{-1} \sqrt{1 - \diamond_{\alpha=1}^{l+1} \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \cdot E \sqrt{\triangleleft^{-1} \left[1 - \diamond_{\alpha=1}^{l+1} \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right.} \\ \diamond_{\alpha=1}^{l+1} \left[\triangleleft^{-1} \sqrt{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \cdot E \sqrt{\triangleleft^{-1} \left[1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right.} \end{array} \right) \\
& \left[\square_{\alpha=1}^{l+1} \epsilon_{\alpha} \lrcorner_{\alpha}^{\#} \right]^{1/\#} = \\
& \left(\begin{array}{c} \left[\triangleleft^{-1} \sqrt{1 - \diamond_{\alpha=1}^{l+1} \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^{1/\triangleleft^{-1}} \\ E \left[\triangleleft^{-1} \sqrt{1 - \diamond_{\alpha=1}^{l+1} \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^{1/\triangleleft^{-1}} \\ \left[\triangleleft^{-1} \sqrt{1 - \diamond_{\alpha=1}^{l+1} \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^{1/\triangleleft^{-1}} \\ E \left[\triangleleft^{-1} \sqrt{1 - \diamond_{\alpha=1}^{l+1} \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{J}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^{1/\triangleleft^{-1}} \\ \triangleleft^{-1} \sqrt{1 - \left[1 - \left[\diamond_{\alpha=1}^{l+1} \left[\triangleleft^{-1} \sqrt{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 \right]^{1/\triangleleft^{-1}} \right.} \\ E \left[\triangleleft^{-1} \sqrt{1 - \left[1 - \left[\diamond_{\alpha=1}^{l+1} \left[\triangleleft^{-1} \sqrt{1 - \left[1 - \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{T}} \right] \right]^{\triangleleft^{-1}} \right]^{\epsilon_{\alpha}}} \right]^2 \right]^{1/\triangleleft^{-1}} \right.} \end{array} \right)
\end{aligned}$$

Theorem 3.10. If all $\lrcorner_{\alpha} = \left\langle \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{J}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{J}} \right]}, \left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{T}} \right]} \right] \right\rangle$ and all are equal. Then $GCLOGRFFNWA[\lrcorner_1, \lrcorner_2, \dots, \lrcorner_n] = \lrcorner$.

3.4. Generalized CLOG RFFNWG (GCLOGRFFNWG)

Definition 3.11. Let $\lrcorner_{\alpha} = \left\langle \left[\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{J}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{J}} \right]}, \left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{J}_{\alpha}^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{m}_{\alpha}} \mathfrak{N}_{\alpha}^{\mathfrak{T}} \right]} \right] \right\rangle$ be the CLOGRFFNNs. Then $GCLOGRFFNWG[\lrcorner_1, \lrcorner_2, \dots, \lrcorner_n] = \frac{1}{\#} \left[\diamond_{\alpha=1}^n \left[\# \lrcorner_{\alpha} \right]^{\epsilon_{\alpha}} \right]$.

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Corollary 3.12. Let $\mathfrak{J}_\alpha = \left\langle \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{I}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{F}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}} \right]} \right] \right\rangle$ be the CLOGRFFNNs. Then $GCLOGRFFNWG[\mathfrak{J}_1, \mathfrak{J}_2, \dots, \mathfrak{J}_n]$

$$= \left(\begin{array}{c} \left[\sqrt[n]{\left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{I}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{F}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}} \right]} \right]} \right]^{\epsilon_\alpha} \right]^{1/\epsilon_\alpha} \\ E \left[\sqrt[n]{\left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{I}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{F}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}} \right]} \right]} \right]^{\epsilon_\alpha} \right]^{1/\epsilon_\alpha} \\ \left[\sqrt[n]{\left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{I}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{F}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}} \right]} \right]} \right]^{\epsilon_\alpha} \right]^{1/\epsilon_\alpha} \\ E \left[\sqrt[n]{\left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{I}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{F}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}} \right]} \right]} \right]^{\epsilon_\alpha} \right]^{1/\epsilon_\alpha} \\ \left[\sqrt[n]{\left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{I}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{F}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}} \right]} \right]} \right]^{\epsilon_\alpha} \right]^{1/\epsilon_\alpha} \\ E \left[\sqrt[n]{\left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{I}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{F}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}} \right]} \right]} \right]^{\epsilon_\alpha} \right]^{1/\epsilon_\alpha} \end{array} \right).$$

Corollary 3.13. If all $\mathfrak{J}_\alpha = \left\langle \left[\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{T}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{T}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{I}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{I}} \right]}, \left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{J}_\alpha^{\mathfrak{F}} \right] \cdot E^{\left[\mathfrak{L}_{\mathfrak{M}_\alpha} \mathfrak{N}_\alpha^{\mathfrak{F}} \right]} \right] \right\rangle$ are equal. Then $GCLOGRFFNWG[\mathfrak{J}_1, \mathfrak{J}_2, \dots, \mathfrak{J}_n] = \mathfrak{J}$.

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