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Upside-Down Neutrosophic Multi-Fuzzy Ideals for IT-Enhanced College Dance Teaching Quality Assessment

Junqiao Lou*, Daoping Zhang, Youyin Mo
School of Music and Dance, Chongqing Preschool Education College, Chongqing, 404047, China
*Corresponding author, E-mail: loujunqiao2014@163.com

Abstract: This paper proposes a novel framework for evaluating the quality of college dance education in environments enhanced by information technology. The approach models dance performance data captured through motion sensors, video-based pose estimation, pressure-sensitive flooring, and audio beat-tracking as neutrosophic multi-fuzzy sequences (T,I,F) that represent, respectively, the degree of technical accuracy, indeterminacy due to sensor noise or ambiguous movements, and deviations from the intended choreography. A pedagogical near-ring structure formalizes instructional operations such as tempo adjustment, camera viewpoint changes, and choreography resampling, while rubric ideals encode the acceptable performance boundaries for specific dance styles. Building on the concept of upside-down logic, we introduce Upside-Down Polarity Morphisms that reclassify certain deviations such as stylistic off-beat timing or deliberate imbalance as positive contributions to performance quality, depending on contextual cues. The final Quality-as-Ideal-Proximity metric computes the distance between a performance's neutrosophic multi-fuzzy profile and its corresponding rubric ideal, adjusted by polarity morphisms. The framework is theoretically supported with proofs of closure, monotonicity, and stability, and its practical applicability is demonstrated through a case study using multimodal sensor data from contemporary and classical dance classes.

Keywords: Neutrosophic multi-fuzzy set; rubric ideal; pedagogical near-ring; upside-down polarity morphism; dance education analytics; motion capture; pose estimation; IT-enhanced teaching.

1. Introduction

The integration of information technology into college-level dance education has transformed the way instructors teach, assess, and refine student performance. Modern classrooms now rely on a wide array of sensing modalities, including high-speed motion capture, video-based pose estimation, wearable inertial measurement units (IMUs), pressure-sensitive flooring, and audio beat-tracking systems, to capture every detail of a dancer's movement and timing. While these technologies provide unprecedented access to quantitative performance data, they also introduce new challenges: data streams can be noisy or incomplete, stylistic deviations may be mistaken for technical errors, and traditional assessment methods often fail to account for creative interpretations of choreography [1].

Within this context, neutrosophic theory offers a compelling mathematical foundation for modeling the ambiguity and multi-dimensionality inherent in dance performance evaluation. By representing performance metrics in the triplet form (T,I,F) where T denotes truth or correctness, I represents indeterminacy, and F captures falsity or deviation, evaluators can separate genuine technical errors from uncertainties and intentional stylistic choices. Building on the concept of neutrosophic multi-fuzzy ideals in algebraic structures [2], we can embed these evaluations in a formal mathematical framework that is both expressive and adaptable. In this way, a dance performance is not just scored it is mathematically mapped into an idealized quality space defined by the expected style-specific standards.

However, dance as an art form is unique in that certain movements considered "incorrect" in one context may be celebrated as expressive innovation in another. For example, deliberate off-beat timing, exaggerated pauses, or intentional imbalance can be key stylistic elements in jazz, contemporary, or avant-garde dance. Inspired by the principles of *upside-down logic* [3], which allow for the inversion of evaluative polarities under certain conditions, this paper introduces Upside-Down Polarity Morphisms—formal transformations that reclassify some deviations as truth components when justified by choreographic context. This approach ensures that the evaluation process aligns with both technical and artistic values, rather than imposing rigid, context-insensitive scoring rules. By combining IT-driven multimodal performance capture with a mathematically rigorous neutrosophic multi-fuzzy ideal framework, and enriching it with polarity inversion mechanisms, we create an assessment method that is robust to measurement uncertainty, sensitive to artistic nuance, and adaptable to different dance styles. This research not only advances the theoretical landscape of neutrosophic applications but also offers a practical tool for educators seeking to balance objective measurement with the subjective essence of dance artistry.

2. Related Work

The mathematical foundation of neutrosophic theory was originally established to extend beyond the limitations of classical and fuzzy set theories by introducing a tripartite representation of truth (T), indeterminacy (I), and falsity (F) [1]. This framework has proven effective in modeling uncertainty, inconsistency, and incompleteness in various application domains, from engineering to social sciences. In the realm of algebraic structures, the development of neutrosophic multi-fuzzy ideals has provided a robust mechanism for handling graded membership across multiple dimensions simultaneously [2]. These ideals preserve closure properties and support logical operations while capturing nuanced variations in membership degrees, making them particularly suited for applications where performance metrics are multidimensional and influenced by context.

The concept of upside-down logic builds upon the recognition that the classification of an event or statement as true or false can be contingent on situational or contextual factors [3]. In this logic, a fact may be "truthified" or "falsified" depending on its relationship to a specified framework of interpretation. This is particularly relevant in the performing arts, where a movement that would traditionally be scored as incorrect in classical ballet might be a deliberate stylistic choice in contemporary dance. The upside-down logic approach formalizes this reclassification, offering a mathematically grounded way to adjust evaluation rules without compromising the integrity of the scoring system.
In the field of performing arts education, and specifically in dance, assessment methods have historically relied on qualitative judgments made by instructors during live performances [4]. While experienced educators can often distinguish between intentional stylistic deviations and accidental mistakes, such evaluations are inherently subjective and difficult to standardize. With the rise of information technology, automated motion capture systems, wearable sensors, and AI-driven pose estimation have enabled more objective, repeatable measurements of performance parameters [5]. However, these systems often interpret deviations purely as errors, failing to account for the artistic and stylistic nuances that define dance as a form of expression.

Efforts to integrate computational evaluation methods into dance education have typically involved techniques from fuzzy logic and multi-criteria decision making [6], but these approaches rarely incorporate mechanisms for handling both measurement uncertainty and context-based reclassification of performance features. Neutrosophic models have begun to appear in related domains such as e-learning assessment [7] and creative media evaluation [8], but there has been little to no work in applying neutrosophic multi-fuzzy ideals and upside-down logic to the structured assessment of dance teaching quality. This gap provides the motivation for our proposed framework, which unites these mathematical concepts with IT-enabled performance capture to deliver a context-aware, uncertainty-resilient, and style-sensitive evaluation method.

3. Methodology

The proposed framework evaluates IT-enabled college dance teaching quality by embedding multimodal performance data into a neutrosophic multi-fuzzy ideal structure, applying context-sensitive upside-down polarity morphisms, and computing a Quality-as-Ideal-Proximity score. This section presents the formal mathematical foundations of each component.

3.1 Neutrosophic Multi-Fuzzy Representation of Dance Performance

Let $\mathcal{D} = \{d_1, d_2, \dots, d_N\}$ represent a finite sequence of dance segments performed by a student in a given class. Each segment d_k is evaluated using data streams captured from motion capture systems, IMUs, pressure floors, and audio beat-tracking.
For each segment d_k, we define a neutrosophic multi-fuzzy membership vector:

$$\mu(d_k) = \left[\left(T_1^k, I_1^k, F_1^k\right), \left(T_2^k, I_2^k, F_2^k\right), \dots, \left(T_m^k, I_m^k, F_m^k\right)\right]$$

where m is the number of modalities (e.g., $m = 4$ for motion, audio, timing, posture).
Each triple $\left(T_j^k, I_j^k, F_j^k\right)$ satisfies:

$$0 \leq T_j^k, I_j^k, F_j^k \leq 1$$

and optionally:

$$T_j^k + I_j^k + F_j^k = 1 \text{ (normalized form)}$$

This representation captures:
T_j^k : Degree of correctness for modality j (e.g., posture accuracy for motion capture).
I_j^k : Indeterminacy due to measurement noise or ambiguous movement.
F_j^k : Degree of deviation from intended choreography for modality j.

3.2 Algebraic Structure of Instructional Operations

We formalize routine instructional operations in the Upside-Down Neutrosophic Multi-Fuzzy Ideal (UDNMFI) framework using a pedagogical near-semiring (dioid) structure. Let:

$\mathcal{M}$ be the set of all multimodal neutrosophic membership profiles μ, where $\mu = \left\{\left(T_j, I_j, F_j\right)\right\}_{j=1}^m$ and m is the number of modalities (e.g., posture, timing, synchronization, dynamics).

Θ be the set of permissible instructional transformations.

Two binary operations are defined:

1. Additive aggregation $(\mathcal{M}, \oplus)$: $(\mathcal{M}, \oplus)$ forms an idempotent, commutative monoid representing the convex aggregation of membership profiles. For two profiles $\mu_a, \mu_b \in \mathcal{M}$,

$$\mu_{a \oplus b} \stackrel{\text{def}}{=} \frac{\mu_a + \mu_b}{2}.$$

This operation preserves thresholds in the rubric ideals because it is convex: if μ_a and μ_b satisfy all thresholds, so does $\mu_{a \oplus b}$.

2. Multiplicative transformation $(\Theta, \circ)$ with left-distributive action on $\mathcal{M}$: For $\theta \in \Theta$ and $\mu_a \in \mathcal{M}$,

$$\mu_{\theta \otimes a} \stackrel{\text{def}}{=} \phi_\theta(\mu_a),$$

where ϕ_θ is a transformation function implementing a specific instructional adjustment. Examples include:

a. Scaling truth-membership in a targeted modality,
b. Reducing falsity-membership through corrective feedback,
c. Adjusting indeterminacy based on improved sensor calibration.

The pair ($\mathcal{M}, \oplus$) is an additive idempotent commutative monoid, ($\Theta, \circ$) is a monoid of transformations, and $\otimes$ is left-distributive over $\oplus$. Together, this forms a near-semiring structure without additive inverses, matching the practical requirements of instructional aggregation and transformation.

3.3 Rubric Ideals

A rubric ideal $\mathcal{I}_R$ is defined for each dance style R (e.g., classical ballet, jazz, contemporary) as the set of acceptable multi-fuzzy membership vectors satisfying:

$$\mathcal{I}_R = \{\mu \in [0,1]^{3m} \mid T_j \geq \tau_T^j, F_j \leq \tau_F^j, I_j \leq \tau_I^j \forall j\}$$

where $\tau_T^j, \tau_F^j, \tau_I^j$ are style-specific thresholds.

Rubric ideals are closed under $\oplus$ and stable under permitted instructional transformations $\otimes$, ensuring that combining two acceptable segments yields another acceptable segment for the same style.

3.4 Upside-Down Polarity Morphisms

In certain styles, a movement traditionally classified as a deviation (F) may be reclassified as a positive quality (T) if it matches a contextual rule (e.g., deliberate syncopation in jazz).

We define a polarity morphism $\pi_C: [0,1]^3 \rightarrow [0,1]^3$ for context C :

$$\pi_C(T, I, F) = \begin{cases} (T + \alpha F, I, (1-\alpha)F) & \text{if context condition satisfied} \\ (T, I, F) & \text{otherwise} \end{cases}$$

where $\alpha \in [0,1]$ is the inversion factor.

Applied to each modality's triple, π_C truthifies some falsity without altering indeterminacy, preserving the interpretability of the components.

3.5 Quality-as-Ideal-Proximity Metric

The final quality score is computed as the distance from the performance's multi-fuzzy vector to the rubric ideal, after applying polarity morphisms:

$$Q_{\text{final}}(d) = 1 - \frac{\min_{\mu_R \in \mathcal{I}_R} \|\pi_C(\mu(d)) - \mu_R\|_p}{\delta_{\max}}$$

where:

$\|\cdot\|_p$ is the L_p norm in $\mathbb{R}^{3m}$,

$\delta_{\max}$ is the maximum possible distance in the given norm, ensuring $Q_{\text{final}} \in [0,1]$.

A value close to 1 indicates high alignment with the rubric ideal, factoring in both style context and IT-based performance measurements.

Lemma (Projection distance to a rubric ideal). Let $\mathcal{I}_R$ denote the rectangular constraint set defined by thresholds $\tau_T^j, \tau_I^j, \tau_F^j$ for style R. The Euclidean distance from a membership vector μ to $\mathcal{I}_R$ after applying the polarity mapping π_C is:

$$\min_{\mu_R \in \mathcal{I}_R} \|\pi_C(\mu) - \mu_R\|_2 = \left(\sum_{j=1}^{m} \left(\left[[\tau_T^j - T_j]_+ \right]^2 + \left[I_j - \tau_I^j \right]_+ \right]^2 + \left[[F_j - \tau_F^j \right]_+ \right]^2 \right) \right)^{1/2},$$

where $[x]_+] = \max(0, x)$. This reflects the fact that deficits in T and excesses in I or F contribute positively to the distance, while overshooting T or undershooting I/F does not. The normalization constant $\delta_{\max}$ is the maximum such distance from the worst-case profile ($T=0, I=1, F=1$) to $\mathcal{J}_R$:

$$\delta_{\max} = \sqrt{\sum_{j=1}^{m} \left(\left(\tau_T^j\right)^2 + \left(1-\tau_F^j\right)^2 + \left(1-\tau_I^j\right)^2 \right)}.$$

By construction, $Q_{\text{final}} = 1 - \delta/\delta_{\max} \in [0,1]$.

3.6 Theoretical Properties

Closure: Rubric ideals are closed under $\oplus$ and stable under $\otimes$ for permitted transformations.

Monotonicity: If T increases and F decreases in all modalities without raising I, Q_{final} increases.

Contextual Adaptivity: For contexts where polarity inversion applies, Q_{final} can increase without artificially lowering falsity in other contexts, ensuring style-specific fairness.

4. Theoretical Analysis

This section provides formal proofs for the core properties of the Upside-Down Neutrosophic Multi-Fuzzy Ideal (UD-NMFI) framework, namely closure of rubric ideals under aggregation and transformation, monotonicity of the quality metric, and stability under context-specific polarity morphisms.

4.1 Closure of Rubric Ideals

Theorem 1 (Closure under $\oplus$):

Let $\mathcal{J}_R$ be the rubric ideal for style R, and let $d_a, d_b \in \mathcal{J}_R$. Then $d_a \oplus d_b \in \mathcal{J}_R$.

Proof:

From the definition in Section 3.3, each modality's triple $\left(T_j, I_j, F_j\right)$ for d_a and d_b satisfies:

$$T_j \geq \tau_T^j, I_j \leq \tau_I^j, F_j \leq \tau_F^j$$

Under $\oplus$, we take the component-wise average:

$$T_j' = \frac{T_j^a + T_j^b}{2}, I_j' = \frac{I_j^a + I_j^b}{2}, F_j' = \frac{F_j^a + F_j^b}{2}.$$

By convexity, $T_j' \geq \tau_{T'}^j I_j' \leq \tau_{I'}^j$, and $F_j' \leq \tau_{F'}^j$, hence $\mu(d_a \oplus d_b) \in \mathcal{J}_R$.

Theorem 2 (Stability under permitted $\otimes$):

If $d \in \mathcal{J}_R$ and θ is a permitted instructional transformation, then $\theta \otimes d \in \mathcal{J}_R$.

Proof:

From Section 3.2, $\theta \otimes d = \mu^{-1}(\phi_\theta(\mu(d)))$. For permitted θ, ϕ_θ is designed such that it does not decrease any T_j below $\tau_{T'}^j$ nor increase I_j or F_j above τ_I^j and τ_F^j respectively. Therefore, membership in $\mathcal{J}_R$ is preserved.

4.2 Monotonicity of the Quality Metric

Theorem 3 (Monotonicity):
If for all modalities $j, T_j' \geq T_j, F_j' \leq F_j$, and $I_j' = I_j$, then $Q_{\text{final}}' \geq Q_{\text{final}}$.
Proof:
From Section 3.5,

$$Q_{\text{final}} = 1 - \frac{\min_{\mu_R \in \mathcal{I}_R} \|\pi_C(\mu) - \mu_R\|_p}{\delta_{\max}}$$

An increase in T_j and a decrease in F_j without changing I_j reduces the distance to any $\mu_R \in \mathcal{I}_R$. Since π_C preserves non-negativity of changes, the numerator in the fraction decreases or remains constant, hence Q_{final} increases or remains the same.

4.3 Context-Gated Stability Under Polarity Morphisms
In the UD-NMFI framework, style-specific rubric ideals $\mathcal{I}_R$ are paired with authorized polarity mappings π_{C_R} that apply upside-down transformations to truth and falsity degrees when stylistically justified. To prevent cross-style exploitation, a context-gating mechanism is introduced.

Definition (Context Gate).
For each style R, a binary gate $g_R \in \{0,1\}$ controls whether π_{C_R} is active during evaluation. The gate is set to $g_R = 1$ only when:

a. The evaluation context matches style R, and
b. The rubric authorizes polarity inversion for the specific modality and pattern.

If either condition fails, $g_R = 0$, and π_{C_R} is replaced by the identity transformation.

Theorem (Context-Gated Stability).
Let R and R' be distinct styles with rubric ideals $\mathcal{I}_R$ and $\mathcal{I}_{R'}$, and polarity mappings π_{C_R} and $\pi_{C_{R'}}$. Suppose evaluation applies $\pi_{C_R}^{g_R}$ with the gate g_R as defined above. Then switching from style R to style R' cannot increase Q_{final} by applying π_{C_R} outside its context.

Proof.
When the style changes from R to R', the evaluation functional for R' uses $\pi_{C_{R'}}^{g_{R'}}$. If $R \neq R'$, then by the gate definition $g_{R'} = 0$ for the previous style's polarity mapping, forcing it to be the identity. The distance from the projected profile $\pi_{C_{R'}}^{g_{R'}}(\mu)$ to $\mathcal{I}_{R'}$ is therefore unaffected or increased compared to applying an unauthorized polarity inversion. As Q_{final} is monotonically decreasing in this distance (Section 3.5), unauthorized context-switching cannot produce artificial score inflation.

Practical Implication.
This property ensures that upside-down polarity inversion benefits are style-locked: in our case study, syncopation patterns in Contemporary dance (CT-2, CT-4) received authorized truthification, while attempts to apply the same inversion in Classical Ballet

were blocked by the context gate. This prevents score manipulation while preserving legitimate stylistic flexibility.

5. Case Study & Results

We evaluate the proposed Upside-Down Neutrosophic Multi-Fuzzy Ideals (UD-NMFI) framework on two ITenabled college dance classes-Classical Ballet and Contemporary-using multimodal streams (pose/motion, timing, posture, energy). Each dance segment is encoded as a neutrosophic multi-fuzzy vector across four modalities ($m = 4$), then scored via the Quality-as-Ideal-Proximity metric from Section 3.5. Style-specific rubric ideals set the admissible envelopes, and polarity morphisms (with inversion factor $\alpha = 0.4$) are applied only to Timing in approved contemporary syncopation segments (context gate = 1), consistent with the notion of evaluative polarity inversion in upside-down logics [3] and with neutrosophic multi-fuzzy ideal modeling [2]. For both styles, the normalization constant $\delta_{\max}$ is computed from the thresholds via

$$\delta_{\max} = \sqrt{\sum_{j=1}^{m} \left(\left(\tau_T^j\right)^2 + \left(1 - \tau_F^j\right)^2 + \left(1 - \tau_I^j\right)^2 \right)},$$

ensuring $Q_{\text{final}} \in [0,1]$[1], [2]. For Classical Ballet we obtain $\delta_{\max}^{\text{Classical}} = 3.0858$, and for Contemporary $\delta_{\max}^{\text{Contemporary}} = 2.8509$ (4 d.p.).

Rubric thresholds (style ideals)

Table 1. Classical Ballet rubric thresholds (per modality)

Modality	τ_T	τ_I	τ_F
Motion	0.85	0.10	0.10
Timing	0.90	0.07	0.08
Posture	0.88	0.07	0.08
Energy	0.80	0.10	0.15

Table 1 encodes a strict classical envelope: high truth requirements in Timing and Posture, low tolerances for Falsity and Indeterminacy. These constraints formalize the traditional focus on precision and line in classical technique [2].

Table 2. Contemporary rubric thresholds (per modality)

Modality	τ_T	τ_I	τ_F
Motion	0.80	0.12	0.20
Timing	0.75	0.12	0.25
Posture	0.80	0.12	0.20
Energy	0.82	0.12	0.18

Table 2 is more permissive for Timing and Posture, reflecting style-specific creative latitude. These ideals support contextual reclassification via polarity morphisms without breaking algebraic closure [2], [3].

Classical Ballet segments (raw multi-fuzzy vectors and scores)

Table 3. Classical Ballet - neutrosophic multi-fuzzy vectors per segment (T/I/F)

Segment	Motion T	Motion 1	Motion F	Timing T	Timing 1	Timing F	Posture T	Posture 1	Posture F	Energy T	Energy 1	Energy F
CL-1	0.82	0.08	0.10	0.88	0.05	0.07	0.86	0.06	0.08	0.78	0.10	0.12
CL-2	0.87	0.06	0.07	0.92	0.04	0.04	0.89	0.05	0.06	0.83	0.09	0.08
CL-3	0.84	0.09	0.07	0.90	0.06	0.04	0.83	0.09	0.08	0.79	0.11	0.10
CL-4	0.90	0.05	0.05	0.95	0.03	0.02	0.92	0.04	0.04	0.86	0.08	0.06

Commentary: CL-2 and CL-4 meet or exceed all classical thresholds across modalities. CL-1 narrowly misses Motion T and Energy T, while CL-3 is slightly below in Posture T and Energy T. These small shortfalls translate into modest distances from the ideal (Table 4).

Table 4. Classical Ballet — Quality-as-Ideal-Proximity results

Segment	$\boldsymbol{Q}_{\text{final}}$	Distance to Ideal
CL-1	0.9851	0.0458
CL-2	1.0000	0.0000
CL-3	0.9817	0.0566
CL-4	1.0000	0.0000

As cited in Table 4, the classical class achieves perfect proximity in CL-2 and CL-4. The classlevel mean is $\bar{Q}_{\text{Classical}} = 0.9917$. The results reflect the framework's monotonicity: improvements in T with stable or reduced F increase Q_{final} without ambiguity inflation, aligning with the theory in Sections 3-4.

Contemporary segments with upside-down polarity (timing syncopation) For Contemporary, Timing in selected segments is subject to context gate = 1 (deliberate syncopation), applying $\pi_C: T \leftarrow T + \alpha F, F \leftarrow (1 - \alpha)F$ with $\alpha = 0.4$ and I unchanged [3]. This truthifies part of the expressive deviation while preserving interpretability (indeterminacy is not repurposed) [1], [3].

Table 5. Contemporary - neutrosophic multi-fuzzy vectors after polarity (T/I/F) and gate

Segment	Motion T	Motion 1	Motion F	Timing T	Timing 1	Timing F	Posture T	Posture 1	Posture F	Energy T	Energy 1	Energy F	SyncGate(Timing)
CT-1	0.78	0.12	0.10	0.68	0.12	0.20	0.77	0.13	0.10	0.85	0.10	0.05	0
CT-2	0.80	0.12	0.08	0.72	0.13	0.15	0.79	0.12	0.09	0.88	0.08	0.04	
CT-3	0.82	0.10	0.08	0.70	0.10	0.20	0.81	0.11	0.08	0.86	0.09	0.05	
CT-4	0.83	0.10	0.07	0.748	0.12	0.132	0.82	0.10	0.08	0.89	0.08	0.03	

Where the context approves syncopation (CT-2, CT-4), Timing T increases ($0.62 \rightarrow 0.72$; $0.66 \rightarrow 0.748$) while Timing F is proportionally reduced ($0.25 \rightarrow 0.15; 0.22 \rightarrow 0.132$). This directly operationalizes upside-down polarity for expressive timing without altering indeterminacy, in line with [3].

Table 6. Contemporary: Quality-as-Ideal-Proximity results

Segment	Q_{final}	Distance to Ideal
CT-1	0.9722	0.0794
CT-2	0.9884	0.0332
CT-3	0.9825	0.0500
CT-4	0.9993	0.0020

Table 6 shows a clear boost when syncopation is context-authorized: CT-4 is nearly perfect ($Q = 0.9993$), and CT-2 improves markedly compared to CT-1 and CT-3. The contemporary class mean is $\bar{Q}_{\text{Contemporary}} = 0.9856$. Because the metric is a distance to a style-specific ideal, improvements occur only when aligned with the rubric and context, preserving fairness and stability as proven in Section 4.

Interpretation and practical carryout

Tables 1-6, taken together, illustrate how the UD-NMFI pipeline unifies strict technical accuracy (Classical) with context-aware creativity (Contemporary). In Classical, small posture/energy deficits slightly reduce Q when thresholds are not fully met (CL-1, CL-3), while segments fully within the ideal achieve $Q = 1$ (CL-2, CL-4). In Contemporary, approved syncopation shifts part of the Timing deviation into truth, reflecting styleappropriate expressivity without masking measurement noise, since I remains independent. The results validate the design choices behind rubric ideals and polarity morphisms and demonstrate how a single, interpretable score can remain sensitive to both technique and artistry.

6. Discussion

The results from the case study demonstrate that the proposed Upside-Down Neutrosophic Multi-Fuzzy Ideals (UD-NMFI) framework is capable of balancing strict technical requirements with context-dependent artistic flexibility in IT-enhanced college dance education. For Classical Ballet, where precision and alignment are paramount, the rubric ideal thresholds enforce a narrow tolerance for falsity and indeterminacy, producing scores that are highly sensitive to small deviations in motion, posture, and timing. The framework's closure property ensures that consistently high-quality segments, when aggregated, remain within the rubric ideal, reinforcing the alignment between segment-level and class-level evaluations.

For Contemporary dance, the application of Upside-Down Polarity Morphisms to approved syncopation patterns shows a clear improvement in Quality-as-Ideal-Proximity scores without artificially suppressing falsity in other, non-approved contexts. This selective inversion mechanism preserves the integrity of the evaluation process by ensuring that only stylistically justified deviations are truthified, while accidental errors remain penalized. The contextual stability property proven in Section 4 ensures that cross-style evaluations cannot be manipulated by inappropriate polarity mappings, maintaining fairness across different rubric ideals.

The framework's ability to separate truth, falsity, and indeterminacy in each modality provides deeper insight than aggregate scores alone. Instructors can pinpoint whether performance deficits are due to technical errors (high falsity), ambiguous sensor readings (high indeterminacy), or both, allowing targeted pedagogical interventions. By integrating multimodal IT-derived data streams, UD-NMFI reduces the subjectivity inherent in live, qualitative assessments while preserving the stylistic nuance essential to dance artistry.

7. Conclusion

This research presented the UD-NMFI framework as a novel and mathematically rigorous approach to evaluating college dance teaching quality in IT-enhanced environments. By representing performance as a neutrosophic multi-fuzzy vector across multiple modalities, embedding it within style-specific rubric ideals, and incorporating upside-down polarity morphisms for context-dependent reclassification, the framework successfully reconciles objective measurement with artistic expressivity.

The case study results confirmed that UD-NMFI produces scores that are both technically precise and style-aware, improving evaluation accuracy without sacrificing interpretability. In Classical Ballet, the system enforced strict technical adherence, while in Contemporary, it recognized the value of deliberate stylistic deviations when contextually justified. This dual capacity makes UD-NMFI a powerful tool for instructors seeking balanced assessments that reward both mastery and creativity.

Beyond dance, the mathematical principles underpinning UD-NMFI closure under aggregation, stability under permissible transformations, monotonicity, and contextual polarity inversion can be applied to other performance-based disciplines where IT sensors are used, such as theater, sports, and music performance. Future work will focus on integrating adaptive weighting functions, real-time feedback systems, and cross-style comparative analytics to further enhance the robustness and applicability of the framework.

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