University of New Mexico

Neutrosophic SuperHyperfunction Invariants for Teaching Quality in College Big Data Programs

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Abstract

College Big Data programs organize teaching and assessment through many nested layers items within modules, modules within courses, courses within tracks, and tracks within the program while being exposed to routine perturbations such as grader swaps, time-slot changes, and delivery-format shifts. This paper presents a symmetry-aware, neutrosophic framework that models the academic hierarchy with superhyperfunctions into the n-th powerset and evaluates teaching quality via a NET group action on the evaluation space. A Burnside–Neutrosophic Quality Index (BNQI) is defined by averaging fixed-set scores across the group, preserving truth (T), indeterminacy (I), and falsity (F) through superhyperaggregation. We prove existence of the aggregate on finite nests, establish bounds and invariance for BNQI, and show monotonic responses to quality-improving changes. A fully calculated case study on three tracks (Data Engineering, Data Science, Business Analytics) details the construction of the superhyperfunction, the choice of NET generators, and the computation of track- and program-level BNQI. Results reveal which signals of quality persist under admissible permutations and where uncertainty and error concentrate, offering an interpretable, stable basis for course review, track oversight, and program planning.

Keywords

neutrosophic superhyperfunction; n-th powerset; superhyperaggregation; NET group action; Burnside invariant; fixed points and orbits; teaching quality; Big Data programs; track-level rollup; symmetry robustness; truth–indeterminacy–falsity (T–I–F).

1. Introduction

In recent years, the field of Big Data has grown rapidly, becoming a key part of higher education curricula around the world. Colleges and universities are increasingly offering specialized programs in Big Data to prepare students for careers in data analysis, machine learning, and information management. These programs often include a mix of theoretical courses, practical modules, and hands-on projects, all aimed at building strong technical skills [1]. However, ensuring high teaching quality in these programs is essential for student success and program effectiveness. Teaching quality directly affects how well students learn complex topics like data processing, algorithms, and ethical data use, which are central to Big Data education [2].

Assessing teaching quality in Big Data programs presents unique challenges due to the structured nature of the curriculum. These programs typically follow a deep hierarchy: individual rubric items (such as specific learning objectives or assessment criteria) nest within modules, which in turn form courses, then tracks (like analytics or engineering specializations), and finally the overall program [3]. This multi-level structure allows for organized learning but complicates evaluation. For example, a single course might include multiple modules with varying rubric items, and performance at one level can influence others. Traditional methods, like simple averaging of scores from student feedback or exams, often overlook this nesting, leading to inaccurate or incomplete assessments [4].

Another major issue is systematic variations in how evaluations are conducted. Factors such as who performs the grading (e.g., different instructors or teaching assistants), when the assessment occurs (e.g., mid-semester versus end-of-term), and the format used (e.g., online surveys, in-person observations, or peer reviews) can introduce biases [5]. These variations create inconsistencies, making it hard to get a fair picture of true teaching quality. Classical approaches, which rely on basic statistical averages, ignore these symmetries and fail to account for the uncertainty or indeterminacy in data from diverse sources [6]. In Big Data programs, where data volumes are large and sources are varied, this can result in misleading results that do not reflect the program's real strengths or weaknesses.

To address these problems, this paper proposes a novel model based on neutrosophic superhyperfunctions. Neutrosophic sets, first introduced as a generalization of fuzzy and intuitionistic fuzzy sets, handle truth, indeterminacy, and falsity in a balanced way, making them suitable for uncertain environments like educational assessments [7, 8]. We extend this by using superhyperfunctions, which map elements into higher-order nested structures through the n-th powerset construction [9]. This allows us to represent the academic hierarchy rigorously, with "braces-within-braces" encoding that captures sub-systems at multiple levels. For instance, rubric items can be nested within modules, and so on, up to the program level, while allowing the empty set to represent indeterminacy in neutrosophic operations.

On the symmetry side, we incorporate Neutrosophic Extended Triplet (NET) groups to model permutations from graders, time slots, and formats [10]. NET groups provide the algebraic tools for orbits, stabilizers, and fixed points in a neutrosophic context, enabling us to neutralize biases through group actions [11]. By applying a Burnside-style lemma adapted to neutrosophic settings, we derive an invariant quality score, the BNQI,that remains consistent despite variations [12, 13].

This approach offers several advantages over traditional methods. It preserves neutrosophic values (truth, indeterminacy, falsity) across aggregation levels, ensures the model is robust to hierarchical nesting, and provides a fair, invariant measure of teaching quality. In Big Data education, where programs often span multiple tracks and involve large datasets from evaluations, this model can lead to more reliable insights for improvement.

The rest of the paper is organized as follows. Section 2 defines the superhyperfunction for encoding the hierarchy. Section 3 introduces neutrosophic superhyperaggregations. Section 4 models the permutations as NET group actions and derives the BNQI. Section 5 presents a case study on a three-track Big Data program, with detailed tables and analysis. Finally, Section 6 concludes with implications and future work.

2. Preliminaries and Notation

2.1 n -th powerset and superhyperfunctions

Let S be the set of atomic observations (rubric checks, quiz items, lab checkpoints). The n-th powerset is defined recursively by $P_0(S) = S, P_1(S) = P(S)$, and $P_n(S) = P(P_{n-1}(S))$ for $n \geq 2$. This braces-within-braces representation captures multiple levels of nesting e.g., module, course, track, program and reflects how complex systems are structured in reality. A SuperHyperFunction (unary form) is

$$f_{SH}: S \longrightarrow P_n(S), n \geq 1$$

and more generally $f_{SH}: P_r(S) \rightarrow P_n(S)$ for integers $r, n \geq 0$. In the neutrosophic setting, operations may legitimately produce $\emptyset$, so we use $P_n(S)$ (which may include $\emptyset$) rather than $P_n^*(S)$ (which excludes it).

<u>*Remark*</u>. The distinction between $P_n^*(S)$ (no empty set) and $P_n(S)$ (empty set allowed) separates classical superhyperalgebra from neutrosophic superhyperalgebra.

2.2 Neutrosophic superhyperoperations

We use (m, n)-superhyperoperations whose codomain is an n-th powerset. In the neutrosophic version,

$$\star: H^m \rightarrow P_n(H)\ (m, n \geq 1),$$

with the codomain allowed to include $\emptyset$ to represent indeterminacy. This lets aggregation across levels (module → course → track → program) propagate (T, I, F) components without collapsing them prematurely.

2.3 NET group actions (orbits, stabilizers, fixed points)

A NET group N acts on a set X of evaluation objects, e.g., graded items, and induces orbits, stabilizers, and fixed points in the neutrosophic sense. Denote the orbit of x by $\mathrm{Orb}(x)$ and its stabilizer by $\mathrm{Stab}(x)$; a fixed point satisfies $g \cdot x = x$ for all $g \in N$. These notions are developed explicitly for NET groups and used alongside a Burnside-type perspective [6]. Interpretation for teaching quality. We model grader permutations, time-slot swaps, and format flips (online/in-person) as generators of N. Scores or patterns that remain fixed under these actions are symmetry-robust signals of quality.

2.4 Notation used throughout

S : atomic observation set (rubric cells, items).
$P_n(S)$: n-th powerset (nests), recursive as above.
f_{SH} : superhyperfunction assigning each item to its nest(s).
$T, I, F \in [0,1]$: neutrosophic components attached to items and nests (truth, indeterminacy, falsity).
N : NET group acting on evaluation objects; Orb, Stab, Fix denote orbit, stabilizer, fixed-point sets.

3. The Proposed Model of SuperHyperFunction Construction and Neutrosophic Superhyperaggregation

3.1 Hierarchical representation of a Big Data program

Let S be the set of atomic observations (e.g., rubric items, quiz questions, lab checkpoints). Define the hierarchy of academic nests by the n-th powerset:

$$P_0(S) = S, P_1(S) = P(S), P_n(S) = P(P_{n-1}(S))(n \geq 2)$$

A SuperHyperFunction assigns each atom to its higher-level nests:

$$f_{SH}: S \longrightarrow P_n(S), s \mapsto f_{SH}(s),$$

where the braces-within-braces structure captures the embedding (Module ⊂ Course ⊂ Track ⊂ Program).
Each object x at any level carries a neutrosophic triple

$$\mathrm{v}(x) = (T(x), I(x), F(x)) \in [0,1]^3$$

with no restriction other than componentwise boundedness.

3.2 From atoms to nests: superhyperaggregation

For any nonempty finite collection $\mathcal{C} \subseteq P_r(S)$ at the same level, define the neutrosophic superhyperaggregate $\mathcal{A}(\mathcal{C}) \in [0,1]^3$ by

$$\mathcal{A}(\mathcal{C}) = (A_T(\mathcal{C}), A_I(\mathcal{C}), A_F(\mathcal{C}))$$

where, for weights $\{w_x\}_{x\in\mathcal{C}}$ with $w_x \geq 0$ and $\sum \ w_x = 1$,

$$A_T(\mathcal{C}) = \textstyle\sum_{x\in\mathcal{C}} \ w_x T(x), A_I(\mathcal{C}) = \sum_{x\in\mathcal{C}} \ w_x I(x), A_F(\mathcal{C}) = \sum_{x\in\mathcal{C}} \ w_x F(x) \qquad (3.1)$$

see Eq. (3.1) is the canonical representative of the superhyperresult: it preserves neutrality, idempotence, and convexity while keeping T, I, F independent.
Properties of $\mathcal{A}$
For any disjoint unions $\mathcal{C} = \bigsqcup_{k=1}^{m} \ \mathcal{C}_k$ and any admissible weights:

1. Boundedness: $0 \leq A_\cdot(\mathcal{C}) \leq 1$ for $\cdot \in \{T, I, F\}$.
2. Monotonicity in T, antimonotonicity in I, F : If $T(x)$ increases (resp. I, F decrease) pointwise, then A_T increases (resp. A_I, A_F decrease).
3. Idempotence: If all $x \in \mathcal{C}$ share the same triple (t, i, f), then $\mathcal{A}(\mathcal{C}) = (t, i, f)$.
4. Refinement invariance: Refining $\mathcal{C}$ into subgroups and re-aggregating yields the same result (associativity of the convex sum).

Proofs are immediate from convexity and linearity in (3.1).

3.3 Level-by-level rollup

Let $\mathcal{M}(c)$ be the modules of course c, $\mathcal{C}(t)$ the courses of track t, and $\mathcal{T}(p)$ the tracks of program p.
Define recursively:

$\mathrm{v}(c)=\mathcal{A}(\mathcal{M}(c)), \mathrm{v}(t)=\mathcal{A}(\mathcal{C}(t)), \mathrm{v}(p)=\mathcal{A}(\mathcal{T}(p)).$ (3.2)
rollups follow Eq. (3.2),Weights can encode credit hours, enrollment, or strategic priority; all proofs below hold for any nonnegative weights summing to 1 at each aggregation.

3.4 Consistency and existence
Because each aggregation step is a convex combination in a compact set $[0,1]^3$, existence and uniqueness of $\mathrm{v}(\cdot)$ at every level follow directly. Moreover, (3.2) is consistent under regrouping: any partition of a level and re-aggregation yields the same $\mathrm{v}(\cdot)$ (associativity of convex averaging).

4. BNQI: Burnside-Neutrosophic Quality Invariant
4.1 NET group action on evaluations
Let X be the set of evaluation objects (graded atomic items or their immediate nests). A NET group N acts on X :

$$\alpha: N \times X \rightarrow X,(g, x) \mapsto g \cdot x,$$

encoding grader permutations, time-slot swaps, and format flips admissible in the study. For each $x \in X$, define its orbit $\operatorname{Orb}(x)=\{g \cdot x: g \in N\}$ and stabilizer $\operatorname{Stab}(x)=\{g \in N: g \cdot x=x\}$. The fixedpoint set of $g \in N$ is

$$\operatorname{Fix}(g)=\{x \in X: g \cdot x=x\}$$

Each $x \in X$ has a neutrosophic triple $\mathrm{v}(x)=(T(x), I(x), F(x))$ (either atomic or already rolled-up via (3.2)).

Unless otherwise noted, averages over a finite set A use equal weights: $\bar{T}(A)=\frac{1}{|A|} \sum_{x \in A} T(x)$, and similarly for $\bar{I}, \bar{F}$. When enrollment or credits are used, the weights are normalized to sum to 1.

4.2 Fixed-point scoring under an action
For $g \in N$ define the fixed-point score
$Q(g)=w_T \bar{T}(\operatorname{Fix}(g))-w_I \bar{I}(\operatorname{Fix}(g))-w_F \bar{F}(\operatorname{Fix}(g))$ (4.1)
where $w_T, w_I, w_F \geq 0$ (not all zero), $\bar{\cdot}$ denotes the weighted average of the component over $\operatorname{Fix}(g)$ (if $\operatorname{Fix}(g)=\emptyset$ we set $\bar{T}=\bar{I}=\bar{F}=0$ by convention). Componentwise bounds ensure $Q(g) \in\left[-w_I-w_F, w_T\right]$.

If Fix $(g)=\emptyset$, we set $\bar{T}=\bar{I}=\bar{F}=0$; this convention is used in Section 5.4 for track-level computations.

4.3 Burnside-Neutrosophic Quality Index (BNQI)

Define the BNQI by averaging over the group:
$\mathrm{BNQI}:=\frac{1}{|N|} \sum_{g \in N} Q(g)$ (4.2)
scored by Eq. (4.1) and averaged by Eq. (4.2), Intuitively, (4.2) rewards truth that survives all admissible permutations, while penalizing indeterminacy and falsity that

persist under the same symmetries.

4.4 Fundamental properties

Theorem 4.1 (Invariance).

For any $h \in N$, let the action be relabeled by conjugation $g \mapsto hgh^{-1}$. Then BNQI is unchanged.

Proof. Conjugation is a bijection of N that permutes the summands in (4.2). Moreover $\mathrm{Fix}(hgh^{-1}) = h \cdot \mathrm{Fix}(g)$, and averaging (T, I, F) over $\mathrm{Fix}(hgh^{-1})$ equals that over $\mathrm{Fix}(g)$ because the group action merely relabels points. Thus each term $Q(g)$ is carried to another term with the same value; the average is invariant.

Theorem 4.2 (Bounds).

If $w_T + w_I + w_F = 1$, then

$$\mathrm{BNQI} \in [-(w_I + w_F), w_T] \subseteq [-1,1].$$

Proof. For any $g \in N$, by definition

$$Q(g) = w_T \bar{T}(\mathrm{Fix}(g)) - w_I \bar{I}(\mathrm{Fix}(g)) - w_F \bar{F}(\mathrm{Fix}(g))$$

and since $\bar{T}, \bar{I}, \bar{F} \in [0,1]$, we have

$$-(w_I + w_F) \leq Q(g) \leq w_T$$

Averaging $Q(g)$ over $g \in N$ in $\mathrm{BNQI} = \frac{1}{|N|} \sum_{g \in N} Q(g)$ preserves these bounds.

Theorem 4.3 (Monotonicity).

Suppose a transformation of X satisfies, for every $g \in N$ and every $x \in \mathrm{Fix}(g)$,

$$T(x) \text{ does not decrease, } I(x), F(x) \text{ do not increase.}$$

Then BNQI does not decrease. Moreover, if at least one of these inequalities is strict on a nonempty Fix (g) for some g, then BNQI strictly increases.

Proof. For each g, the averages $\bar{T}(\mathrm{Fix}(g))$ weakly increase while $\bar{I}(\mathrm{Fix}(g))$ and $\bar{F}(\mathrm{Fix}(g))$ weakly decrease, hence $Q(g)$ weakly increases. Averaging over g yields the claim; strict improvement on any nonempty Fix (g) yields a strict increase in BNQI.

Corollary 4.4 (Stability under benign relabeling).

If a change in graders, time slots, or delivery formats corresponds to applying an element of N (i.e., a symmetry of the action), then the sets $\mathrm{Fix}(g)$ and their averages are merely permuted across $g \in N$. Consequently, BNQI remains unchanged.

4.5 Program-level BNQI and rollups

Let X_c be the evaluation objects attached to course c. For a track t, set

$$X_t = \bigcup_{c \in \mathcal{C}(t)} X_c,$$

and for the program p,

$$X_p = \bigcup_{t \in \mathcal{T}(p)} X_t.$$

Applying the definition (4.2) on $X_c t X_t$, and X_p yields the course-, track-, and program-level indices

$$\mathrm{BNQI}(c), \mathrm{BNQI}(t), \mathrm{BNQI}(p).$$

Track and program rollups are obtained by convex averaging of the lower-level BNQIs using the same weights used in the superhyperaggregation (3.2). Because convex combinations preserve both bounds and invariance, the properties of Theorems 4.1-4.2 carry over to the rolled-up indices.

4.6 Computation recipe (finite groups)

1. Specify the action. Choose generators for N that reflect admissible symmetries (grader permutations, time-slot swaps, format flips).
2. Find fixed points. For each $g \in N$, compute $\mathrm{Fix}(g) \subseteq X$.
3. Aggregate components. Evaluate $\bar{T}, \bar{I}, \bar{F}$ on each $\mathrm{Fix}(g)$ using the aggregation rule (3.1).
4. Score and average. Compute $Q(g)$ via (4.1), then obtain BNQI by averaging as in (4.2).
5. Roll up. Combine $\mathrm{BNQI}(c)$ across courses to $\mathrm{BNQI}(t)$, and across tracks to $\mathrm{BNQI}(p)$, using the convex rollups in (3.2).

5. Case Study - Fully Calculated BNQI for a Three-Track Big Data Program

This case study applies the framework to a college Big Data program with three tracks: Data Engineering (DE), Data Science (DS), and Business Analytics (BA). We construct the superhyperfunction, specify a finite NET group action capturing admissible symmetries (grader permutations and format flips), compute fixedpoint averages, evaluate $Q(g)$ for each group element, and obtain BNQI at the course, track, and program levels. Every number below is explicitly calculated.

5.1 Data and superhyperfunction nests

We consider six courses: DE1, DE2, DS1, DS2, BA1, BA2. Each course's neutrosophic triple (T, I, F) is the level-wise superhyperaggregate (Eq. (3.1)) of its modules/items:

DE1: (0.78, 0.12, 0.10)
DE2: (0.82, 0.10, 0.08)
DS1: (0.88, 0.07, 0.05)
DS2: (0.85, 0.09, 0.06)
BA1: (0.80, 0.11, 0.09)
BA2: (0.76, 0.13, 0.11)

A sample of the superhyperfunction f_{SH} placements (braces-within-braces indicating nesting) is shown in Table 1; these feed the course-level triples above.

Table 1. Example superhyperfunction placements f_{SH} (subset)

Atomic item	Nest (module → course → track → program)
s_1	{{Module A (DE1)}CDE1}CDE ⊂ Program
s_2	{{Module B (DS1)}CDS1}CDSCProgram
s_3	{{Module C (BA2)}CBA2}CBACProgram
s_4	{{ Module D(DE2)} ⊂ DE2} ⊂ DE ⊂ Program

Table 1 illustrates how atomic rubric items are mapped by the superhyperfunction into n-th powerset nests. These placements induce the course-level (T, I, F) values via the convex representative in Eq. (3.1), then roll up by Eq. (3.2).

5.2 NET group, action, and fixed points

Let X be the set of course-level evaluation objects $\{\mathrm{DE1}, \mathrm{DE2}, \mathrm{DS1}, \mathrm{DS2}, \mathrm{BA1}, \mathrm{BA2}\}$.

Define a finite NET group $N=\{e, a, b, ab\}$ with generators:

a : grader permutation within DE , swapping DE1 ↔ DE2; fixes $\mathrm{DS1}, \mathrm{DS2}, \mathrm{BA1}, \mathrm{BA2}$.

b : format flip within BA , swapping BA1 ↔ BA2; fixes $\mathrm{DE1}, \mathrm{DE2}, \mathrm{DS1}, \mathrm{DS2}$.

ab : their composition (swap in DE and BA simultaneously).

e : identity.

For $g \in N, \mathrm{Fix}(g) \subseteq X$ is the set of courses unmoved by g. The fixed sets are:

a. $\mathrm{Fix}(e)=\{\mathrm{DE1}, \mathrm{DE2}, \mathrm{DS1}, \mathrm{DS2}, \mathrm{BA1}, \mathrm{BA2}\}$ (all 6)
b. $\mathrm{Fix}(a)=\{\mathrm{DS1}, \mathrm{DS2}, \mathrm{BA1}, \mathrm{BA2}\}$ (4)
c. $\mathrm{Fix}(b)=\{\mathrm{DE1}, \mathrm{DE2}, \mathrm{DS1}, \mathrm{DS2}\}$ (4)

Table 2. NET group, generators, and fixed-point structure

Element g	Action (generator)	Fixed courses Fix(g)	Size |Fix(g)|
e	Identity	DE1, DE2, DS1, BA1, BA	6
a	Swap DE1 ↔ DE2 (grader permutation in DE)	DS1, DS2, BA1, BA2	4
b	Swap BA1 ↔ BA2 (format flip in BA)	DE1, DE2, DS1, DS	4
ab	Apply both swaps (DE and BA simultaneusly)	DS1, DS2	2

The NET group models admissible symmetries of the evaluation process. For each element, we list its fixed courses, which determine the averages used in $Q(g)$ and hence in BNQI (Eqs.(4.1)-(4.2)). The structure reflects where the action preserves scores.

5.3 Fixed-point averages and program-level BNQI

Set the quality weights $(w_T, w_I, w_F)=(0.60, 0.25, 0.15)$ with $w_T+w_I+w_F=1$.

For each $\mathrm{Fix}(g)$, compute componentwise means using Eq. (3.1):

$g=e$ (all six):

$$\bar{T}=0.815, \bar{I}=0.10333, \bar{F}=0.08167$$
$$Q(e)=0.6(0.815)-0.25(0.10333)-0.15(0.08167)=0.45092$$

$g=a$ (DS1, DS2, BA1, BA2):

$$\bar{T}=0.8225, \bar{I}=0.1000, \bar{F}=0.0775$$
$$Q(a)=0.45688$$

$g=b$ (DE1, DE2, DS1, DS2):

$$\bar{T}=0.8325, \bar{I}=0.0950, \bar{F}=0.0725$$
$$Q(b)=0.46488$$

$g=ab$ (DS1, DS2):

$$\bar{T}=0.8650, \bar{I}=0.0800, \bar{F}=0.0550$$
$$Q(ab)=0.49075$$

Finally,

$$\mathrm{BNQI}(p)=\frac{1}{4}(Q(e)+Q(a)+Q(b)+Q(ab))=0.46585(\approx 0.4659).$$

5.4 Track-level BNQI and course-level BNQI

Track sets (used with the same group N):

$$X_{\mathrm{DE}}=\{\mathrm{DE1},\mathrm{DE2}\}, X_{\mathrm{DS}}=\{\mathrm{DS1},\mathrm{DS2}\}, X_{\mathrm{BA}}=\{\mathrm{BA1},\mathrm{BA2}\}.$$

We evaluate $\bar{T},\bar{I},\bar{F}$ on $\mathrm{Fix}(g)\cap X_t$ and then average $Q(g)$ over $g\in N$ (Eq. (4.2)). If $\mathrm{Fix}(g)\cap X_t=\emptyset$, we use the convention of zero contribution.

DE track (X_{DE}):

$$\mathrm{Fix}(e)\cap X_{\mathrm{DE}}=\{\mathrm{DE1},\mathrm{DE2}\}: \bar{T}=0.80, \bar{I}=0.11, \bar{F}=0.09 \Rightarrow Q(e)=0.439.$$

$$\mathrm{Fix}(a)\cap X_{\mathrm{DE}}=\emptyset \Rightarrow Q(a)=0$$

$$\mathrm{Fix}(b)\cap X_{\mathrm{DE}}=\{\mathrm{DE1},\mathrm{DE2}\} \Rightarrow Q(b)=0.439$$

$$\mathrm{Fix}(ab)\cap X_{\mathrm{DE}}=\emptyset \Rightarrow Q(ab)=0$$

$$\Rightarrow \mathrm{BNQI}(\mathrm{DE})=\frac{1}{4}(0.439+0+0.439+0)=0.2195$$

DS track (X_{DS}):

Every $g\in N$ fixes DS1 and DS2, so $\bar{T}=0.865, \bar{I}=0.08, \bar{F}=0.055 \Rightarrow Q(g)=0.49075$ for all g.

$$\Rightarrow \mathrm{BNQI}(\mathrm{DS})=0.49075.$$

BA track (X_{BA}):

$$\mathrm{Fix}(e)\cap X_{\mathrm{BA}}=\{\mathrm{BA1},\mathrm{BA2}\} \Rightarrow \bar{T}=0.78, \bar{I}=0.12, \bar{F}=0.10, Q(e)=0.423.$$

$$\mathrm{Fix}(a)\cap X_{\mathrm{BA}}=\{\mathrm{BA1},\mathrm{BA2}\} \Rightarrow Q(a)=0.423.$$

$$\mathrm{Fix}(b)\cap X_{\mathrm{BA}}=\emptyset \Rightarrow Q(b)=0.$$

$$\mathrm{Fix}(ab)\cap X_{\mathrm{BA}}=\emptyset \Rightarrow Q(ab)=0.$$

$$\Rightarrow \mathrm{BNQI}(\mathrm{BA})=\frac{1}{4}(0.423+0.423+0+0)=0.2115.$$

Course-level BNQI. For a single course c, use its stabilizer subgroup $N_c=\{g\in N: g\cdot c=c\}$ and $X_c=\{c\}$. Then $\mathrm{BNQI}(c)=\frac{1}{|N_c|}\sum_{g\in N_c}[w_T T(c)-w_I I(c)-w_F F(c)]$, which equals the scalar $w_T T(c)-w_I I(c)-w_F F(c)$ because all summands coincide on a singleton.

a. DE1: $0.6(0.78)-0.25(0.12)-0.15(0.10)=0.4230$.
b. DE2: 0.4550 .
c. DS1: 0.5030 .
d. DS2: 0.4785 .
e. BA1: 0.4390 .
f. BA2: 0.4070 .

Table 3. BNQI by course, by track, and at the program level

Level	Object	BNQI
Course	DE1	0.4230
	DE2	0.4550
	DS1	0.5030
	DS2	0.4785
	BA1	0.4390
	BA2	0.4070
Track	DE (from X_{DE})	0.2195
	DS (from X_{DS})	0.4908

	BA (from X_{BA})	0.2115
Program	p (all courses)	0.4659

As shown in Table 3, per-course BNQI equals the weighted scalar $w_T T - w_I I - w_F F$ because the stabilizer acts trivially on a singleton. Track indices reflect fixed points within the track set; the program index uses fixed points over the union of all courses, hence it can differ from a simple average of track-level values.

5.5 Sensitivity analysis (weights)

We test robustness to weights by replacing $(w_T, w_I, w_F) = (0.60, 0.25, 0.15)$ with $(0.50, 0.30, 0.20)$. Recomputing $Q(g)$ on the same fixed-point averages yields: $Q(e) = 0.3602, Q(a) = 0.3658, Q(b) = 0.3733, Q(ab) = 0.3975$, hence $\mathrm{BNQI}(p) = \frac{1}{4}(0.3602 + 0.3658 + 0.3733 + 0.3975) = 0.3742$.

Table 4. Sensitivity of program-level BNQI to weight changes

Weights (w_T, w_I, w_F)	$Q(e)$	$Q(a)$	$Q(b)$	$Q(ab)$	$\mathrm{BNQI}(p)$
(0.60, 0.25, 0.15)	0.4509	0.4569	0.4649	0.4908	0.4659
(0.50, 0.30, 0.20)	0.3602	0.3658	0.3733	0.3975	0.3742

Increasing penalties on I and F lowers $\mathrm{BNQI}(p)$ as expected, while all values stay within the theoretical bounds of Theorem 4.2. The ordering across g persists, showing stable fixed-point behavior under the NET action.

5.6 Analysis

a. Symmetry robustness. BNQI rewards quality signals that survive grader/format symmetries; DS shows the highest invariance (largest BNQI), aligning with its strong truth and low indeterminacy.
b. Track vs. program differences. Program-level $\mathrm{BNQI}(p)$ (0.4659) exceeds the simple convex average of track BNQIs because the union includes fixed points from multiple tracks for each g, raising $Q(g)$ terms that vanish within a single track.
c. Managerial takeaway. DS is the most symmetry-stable; DE and BA benefit from reducing I and F in the specific scenarios where their fixed sets vanish (under a for DE , under b for BA).

6. Discussion

The case study shows that a symmetry-aware metric can separate true instructional signal from artifacts such as grader swaps and format flips. BNQI rose where courses retained high truth and low uncertainty on the fixed sets of the NET action (notably in the DS track) and fell where quality depended on labels the action permuted (DE and BA under their respective generators). Two practical levers emerge:

a. Weights tuning. Raising penalties on I and F predictably lowers BNQI but improves caution in borderline cases. Programs can set (w_T,w_I,w_F)by policy (e.g., capstone rigor vs. early-semester exploration).

b. Action design. The choice of NET generators encodes what the institution treats as “benign relabeling.” If a perceived bias is *not* modeled as a symmetry, it will not be filtered. Departments should therefore agree on the admissible permutations before scoring.

Methodologically, superhyperfunction nesting avoids premature averaging: items roll up to modules, courses, tracks, and the program with the neutrosophic triple intact. This made the DS advantage transparent its truth stayed high and its uncertainty low across all fixed sets while revealing where DE and BA lose stability (no fixed courses under specific generators).

Limitations. BNQI depends on a finite, well-specified action and on reliable (T,I,F)inputs. Extremely small fixed sets can make estimates noisy; adding semester-level pooling or minimum-size rules helps. Also, if important differences are *not* symmetries (e.g., lab vs. project outcomes), they should be modeled as separate strata rather than folded into N. Implications for program leadership.

a. Use BNQI at course review to spot sections whose quality collapses under grader/format permutations.
b. At track review, prioritize actions that reduce I and F on fixed sets (rubric calibration, clearer submission rules, standardized lab environments).
c. For curriculum planning, align capstones and practicum experiences to preserve fixed-set strength, then verify gains via the same NET action next term.

7. Conclusion

This work introduced a mathematically grounded approach to teaching-quality analysis that is both hierarchy-aware and symmetry-robust. By encoding nested academic structures with superhyperfunctions and filtering admissible relabelings through a NET group action, the proposed BNQI reports what remains credible after grader, schedule, and format permutations. The case study showed how this perspective distinguishes durable instructional strength from artifacts, guiding departments toward interventions that reduce indeterminacy and falsity on the relevant fixed sets. The framework is simple to deploy define the nests, agree on the action, compute fixed-set averages, and report BNQI at course, track, and program levels yet it provides firm guarantees of boundedness, invariance, and sensible monotonic behavior. Future extensions include multi-semester panels, cross-institution comparisons under a common action, and integration with accreditation metrics while retaining the neutrosophic separation of T, I, and F.

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Received: March 11, 2025. Accepted: Aug 24, 2025