NSS
Neutrosophic Sets and Systems, Vol. 93, 2025
University of New Mexico

An Indicator-Free Neutrosophic Monoid Framework for Robust Evaluation of Entrepreneurial Ability in Vocational College Students

Yali Hou, Haijuan Zhou, Zhe Yun, Yunjia Liu, Ximu Yan, Bingquan Yin*
Qinhuangdao Vocational and Technical College, Qinhuangdao, 066100, Hebei, China
*Corresponding author, E-mail: yinbingquan7667@163.com

Abstract: This paper develops an indicator-free neutrosophic framework for evaluating the entrepreneurial ability of vocational college students. Each elementary task is encoded as a neutrosophic triplet $X=(T,I,F)$ representing, respectively, success (truth), ambiguity (indeterminacy), and failure (falsity). Tasks are aggregated with a commutative, associative operator $\diamond$ that has identity $e=(1,0,0)$; componentwise it uses the product t-norm for T, the probabilistic sum for I, and the dual union for F. Hence ($[0,1]^3,\diamond$) forms a commutative monoid, ensuring closure, continuity, and a neutral element. We derive feasible and unique back-solving formulas for recovering a missing task from a known composite whenever the target lies in the image of the corresponding translation. At the student level, we avoid subjective fixed weights by defining a robust score as the infimum of a linear utility over an admissible weight polytope; in common policy settings, the optimum is attained at a vertex and admits a closed form. We also introduce a neutrosophic margin of separation and show that positive margins imply semi- T_2 separability in a neutrosophic product topology, providing stability of rankings under small perturbations. A worked case study presents exact, reproducible computations and strictly positive margins, yielding fair and defensible rankings without predefined indicators.

Keywords: Neutrosophic sets; Commutative monoid; Task composition; Robust ranking; Vocational education; Semi- T_2 separability; Back-solving; Indicator-free evaluation.

1. Introduction

Entrepreneurial ability is a central outcome of vocational education, where students must convert domainspecific skills into viable ventures through problem framing, experimentation, and accountable execution [1,2]. Assessing this ability is challenging: observations are partial, raters may disagree, and evidence often contains ambiguity that crisp scores or single-track rubrics cannot faithfully encode [3]. Classical indicatorweight schemes (e.g., AHP or fuzzy weighted sums) embed subjectivity through indicator choice and fixed weights, and they may obscure the link between concrete task performance and the final score [4,5].

Neutrosophy provides a principled alternative by modeling evidence as a triplet (T, I, F) of truth, indeterminacy, and falsity, with components defined independently on [0,1] and no requirement that they sum to one [$6-9$]. To translate task-level evidence into student-level judgments we need an aggregation operator that is (i) closed on $[0,1]^3$, (ii) continuous and easy to audit, and (iii) mathematically transparent. In this work we adopt a commutative, associative operator $\diamond$ with identity $e=(1,0,0)$. Componentwise, $\diamond$ uses the product t-norm for T, the probabilistic sum for I, and the dual union for F, yielding a commutative monoid on $[0,1]^3$[5,6]. While $\diamond$ is not a quasigroup globally, we derive feasible and unique back-solving formulas that recover a missing task from a known composite whenever the target lies in the image of the corresponding translation-supporting transparent, auditable evaluation. To avoid arbitrary scalarization at the student level, we define a robust score as the infimum of a linear utility over an admissible weight polytope that encodes broad policy bounds; by linearity, the infimum is attained at a vertex, giving a closed form in common settings. To certify ranking stability, we introduce a neutrosophic margin of separation and connect positive margins to semi- T_2 separability in a neutrosophic product topology [10].

Our approach builds on neutrosophic sets and standard t-/s-norm calculus [5-9] and uses separation ideas from neutrosophic topology [10], but departs from indicator-driven evaluation by operating directly at the task level with an algebraically simple, auditable aggregation.

1.1 Problem Setting and Gap

Most existing educational evaluations even those using fuzzy or neutrosophic tools rely on predefined indicators plus fixed or heuristically tuned weights. This raises three issues: (i) indicator choice is subjective and can bias results; (ii) fixed weights may not reflect legitimate institutional priorities across cohorts; (iii) complex aggregations can mask how concrete task outcomes contribute to the final decision. There is a need for a framework that (a) eliminates indicator engineering, (b) removes fixed-weight arbitrariness while keeping policy transparency, and (c) remains closed-form and auditable.

1.2 Our Approach

We encode each elementary entrepreneurial task as a neutrosophic triplet $X=(T, I, F)$. Task outcomes are aggregated with a commutative monoid operator $\diamond$ on $[0,1]^3$ that preserves closure and has a neutral element. This enables project-level folding and, when feasible, unique back-solving of task contributions from composites. Student-level evaluation uses a robust score defined as the worst-case value of a linear utility over an admissible polytope of weights representing broad policy bounds (e.g., a minimum credit to truth and caps on penalties). Finally, neutrosophic margins certify that observed ranking gaps imply semi- T_2 separability, guaranteeing stability to small perturbations in (T, I, F).

1.3 Contributions

1. Indicator-free neutrosophic modeling at the task level, avoiding subjective indicator engineering.
2. Commutative-monoid aggregation with identity and feasible back-solving, ensuring closure, continuity, and auditability.
3. Robust, weight-free student scoring via a vertex solution of an admissible weight polytope, eliminating fixed-weight arbitrariness.
4. Neutrosophic margin of separation that implies semi- T_2 separability, providing a topological stability guarantee for rankings.

1.4 Paper Organization

Section 2 formalizes neutrosophic triplets, the admissible domain, and the t - $/s$-norm primitives, and summarizes background on quasigroups and neutrosophic topology $[10-12]$. Section 3 defines the operator $\diamond$, proves the monoid properties, derives feasible back-solving conditions, and develops the robust score and margin. Section 4 presents a case study with exact computations and named tables. Sections 5 and 6 discuss implications and conclude.

2. Preliminaries

2.1 Neutrosophic triplets and notation

A neutrosophic triplet is an ordered triple

$$X=(T, I, F) \in[0,1]^3$$

where T denotes the degree of truth/success, I the degree of indeterminacy/ambiguity, and F the degree of falsity/failure. In neutrosophy the three components are independent; no normalization such as $T+I+F=1$ is imposed [6-9]. We write the set of all triplets as $\mathcal{N}=[0,1]^3$.

A partial order consistent with evaluation is the product order

$$X \leq Y \Leftrightarrow T_X \leq T_Y, I_X \geq I_Y, F_X \geq F_Y,$$

so "larger" means more truth and less indeterminacy/falsity.

2.2 Admissible domain for algebraic operations

To guarantee well-posed inversions (back-solving) later, we exclude degenerate boundary cases and work on

$$\Omega=(0,1] \times[0,1) \times[0,1)=\{(T, I, F) \in \mathcal{N}: T>0, I<1, F<1\}.$$

This is harmless in practice: tasks with $T=0$ or $I=1$ or $F=1$ can be regularized by tiny perturbations without affecting decisions.

2.3 t -/s-norm primitives (component-level)

We use standard continuous norms on [0,1][5,6] :

Product t-norm for conjunction:

$$a \otimes b=a b$$

Probabilistic sum s-norm for disjunction:

$$a \oplus b=a+b-a b$$

Dual union via complements (algebraically identical to $\oplus$):

$$a \sqcup b=1-(1-a)(1-b)=a+b-a b .$$

All three are continuous, commutative, associative, and monotone on [0,1], with identities 1 for $\otimes$ and 0 for $\oplus$ (hence also for $\sqcup$). Non-idempotency holds for $\otimes$: $a \otimes a = a^2 \leq a$.
These primitives will be applied componentwise to construct the paper's aggregation operator in Section 3.

2.4 Quasigroups
A quasigroup $(Q,\circ)$ is a set with a binary operation such that for all $a, b \in Q$, the equations $a \circ x = b$ and $y \circ a = b$ admit unique solutions $x, y \in Q$; equivalently, left/right translations are bijections [12]. We review this concept only for context. The operator introduced in this paper (Section 3) forms a commutative monoid on $\mathcal{N}$ (with identity), not a quasigroup on the whole domain; nonetheless, feasible and unique back-solving will be derived under explicit conditions.

2.5 Neutrosophic topology and semi- T_2 separation
Neutrosophic (linguistic) topological spaces generalize classical topology to uncertain/linguistic settings and admit refined separation axioms, including semi- T_2[10]. Informally, two points are semi- T_2 separable if there exist disjoint semi-open neighborhoods around them. In Section 3 we define a neutrosophic margin of separation (a positive gap between robust scores); in Section 4 we compute it in the case study and use continuity arguments to conclude semi- T_2 separability-i.e., ranking stability under small perturbations of (T, I, F).

Notational conventions
1. Vectors/triplets use overbars for averages, e.g., $\bar{X} = (\bar{T}, \bar{I}, \bar{F})$.
2. All maps on $\mathcal{N}$ are taken to be Borel-measurable and continuous where stated.
3. Arithmetic rounding in tables is to five decimal places; exact algebraic expressions are used in derivations, ensuring numerical consistency.

3. Proposed Framework
3.1 Task-level encoding and operator definition
Each elementary entrepreneurial task τ is encoded as a neutrosophic triplet

$$X(\tau) = (T(\tau), I(\tau), F(\tau)) \in \Omega, \Omega := (0,1] \times [0,1) \times [0,1),$$

where T (truth/success), I (indeterminacy/ambiguity), and F (falsity/failure) are independent components.
We define the binary operator $\diamond: [0,1]^3 \times [0,1]^3 \rightarrow [0,1]^3$ componentwise by

$$X_1 \diamond X_2 = \left(T_1 T_2, I_1 + I_2 - I_1 I_2, 1 - (1 - F_1)(1 - F_2)\right) \quad (3.1)$$

i.e., product t-norm for T , probabilistic sum for I , and dual union for F .

3.2 Algebraic and order-theoretic properties
Let $e = (1,0,0)$.
Proposition 3.1 (Commutative monoid). $([0,1]^3,\diamond)$ is a commutative, associative monoid with identity e. It restricts to $(\Omega,\diamond)$ by closure.

Proof. Each component operation in (3.1) is commutative, associative, continuous, and closed on [0,1] with identities 1 (for product), 0 (for probabilistic sum), and 0 (for dual union). Hence the product construction inherits these properties.

Monotonicity under evaluation order.
With the partial order $X \leq Y \Leftrightarrow T_X \leq T_Y, I_X \geq I_Y, F_X \geq F_Y$, the map $(X_1, X_2) \mapsto X_1 \diamond X_2$ is monotone in each argument because T is increasing, while I and F are also increasing componentwise under their s-norms.

Non-idempotency and attenuation.
$T \diamond T = T^2 \leq T$ (strict if $T \in (0,1)$); thus composing tasks tends to attenuate truth unless the other task has $T = 1$. Conversely, $I \diamond I = I \oplus I \geq I$ and $F \diamond F = F \sqcup F \geq F$.
3.3 Feasible and unique back-solving
Although $\diamond$ is not a quasigroup globally, left/right back-solving is feasible and unique whenever the target lies in the image of the corresponding translation.
Fix $X = (t, i, f) \in \Omega$ and $Y = (p, q, r) \in [0,1]^3$.

Left back-solve $U \diamond X = Y$.

From (3.1), solve componentwise:

$$a = \frac{p}{t}, b = \frac{q-i}{1-i}, c = \frac{r-f}{1-f}. \tag{3.2}$$

These yield a valid $U = (a, b, c) \in [0,1]^3$ iff

$$p \leq t, q \geq i, r \geq f. \tag{3.3}$$

Under (3.3), U is unique.

Right back-solve $X \diamond V = Y$.

The same formulas (3.2) and conditions (3.3) apply (commutativity).

Image characterization.
For fixed $X = (t, i, f) \in \Omega$, the translation $\tau_X: U \mapsto U \diamond X$ has image

$$\mathrm{Im}(\tau_X) = [0, t] \times [i, 1] \times [f, 1]$$

and (3.2) provides the unique preimage for every Y in this rectangle.

3.4 Project folding (order-invariant) and student aggregation
A project $P = (\tau_1, \dots, \tau_m)$ is evaluated by the monoid fold

$$X(P) = X(\tau_1) \diamond \cdots \diamond X(\tau_m) \tag{3.4}$$

Since $\diamond$ is commutative and associative, (3.4) is order-invariant; we fix left-fold notation for reproducibility.
For a student with projects $P_1, \dots, P_K$, define the aggregate

$$\bar{X} = (\bar{T}, \bar{I}, \bar{F}) = \mathrm{Agg}\big(X(P_1), \dots, X(P_K)\big), \tag{3.5}$$

where Agg is the arithmetic mean (or a pre-specified trimmed mean for outlier robustness). By convexity, $\bar{X} \in [0,1]^3$.

3.5 Admissible weights and robust score
To avoid fixed, subjective weights, we aggregate ($\bar{T}, \bar{I}, \bar{F}$) using an admissible weight polytope

$\mathcal{W} = \{(a,b,c) \in \mathbb{R}^3_{\geq 0}: a+b+c=1, a \geq a_{\min}, b \leq b_{\max}, c \leq c_{\max}\}$. (3.6)

Given $w=(a,b,c) \in \mathcal{W}$, define the linear utility

$U_w(\bar{X}) = a\bar{T} - b\bar{F} - c\bar{I}$ (3.7)

The robust score is the worst-case value

$R = \inf_{w \in \mathcal{W}} U_w(\bar{X})$ (3.8)

Lemma 3.2 (Vertex attainment).

Since U_w is linear in w and $\mathcal{W}$ is a compact polytope, the infimum in (3.8) is attained at a vertex of $\mathcal{W}$.

A common policy choice sets

$a_{\min} + b_{\max} + c_{\max} = 1$ (3.9)

In this case the minimizing vertex is $w^\star = (a_{\min}, b_{\max}, c_{\max})$, giving the closed-form score

$R = a_{\min}\bar{T} - b_{\max}\bar{F} - c_{\max}\bar{I}$. (3.10)

Proposition 3.3 (Monotonicity).

If $\bar{T}_i \geq \bar{T}_j, \bar{F}_i \leq \bar{F}_j$, and $\bar{I}_i \leq \bar{I}_j$, then $R(i) \geq R(j)$ for every admissible $\mathcal{W}$.

Proof. Immediate from (3.7)-(3.8), since U_w is increasing in $\bar{T}$ and decreasing in $\bar{F}, \bar{I}$ for all $w \in \mathcal{W}$.

3.6 Neutrosophic margin and semi- T_2 separability

For students i, j, define the neutrosophic margin of separation

$\mathsf{NMS}(i,j) = R(i) - R(j)$. (3.11)

Because $\bar{X} \mapsto U_w(\bar{X})$ is continuous for any fixed w and $\bar{X}$ is a continuous aggregate of project triplets, a strictly positive margin implies the existence of disjoint semi-open product neighborhoods around $\bar{X}_i$ and $\bar{X}_j$ in a neutrosophic product topology; hence semi- T_2 separability and ranking stability follow [10].

3.7 Implementation

1. Encode tasks. Calibrate rubrics to produce $X(\tau) \in \Omega$ (regularize boundary cases).
2. Fold projects. Compute $X(P)$ via (3.4).
3. Aggregate student. Obtain $\bar{X}$ via (3.5).
4. Set policy bounds. Specify $a_{\min}, b_{\max}, c_{\max}$ and form $\mathcal{W}$(3.6).
5. Score and certify. Compute R by (3.8) (or (3.10) if (3.9) holds), and report pairwise NMS (3.11).

4. Case Study: Vocational Students (Static, Indicator-Free)

This section applies the framework of Section 3 to a compact, fully worked example. Each student completes two projects; each project is the composition of two elementary tasks via $\diamond$ (3.1). Student aggregates follow (3.5). Robust scores use the vertex form (3.10) with policy bounds $a_{\min} = 0.45, b_{\max} = 0.35, c_{\max} = 0.20$ (so $a_{\min} + b_{\max} + c_{\max} = 1$). Pairwise neutrosophic margins use (3.11).

4.1 Task-level inputs

For each student, $P_1 = \tau_1 \diamond \tau_2$ and $P_2 = \tau_3 \diamond \tau_4$.

Table 1. Elementary task triplets (inputs).

Student	Task	T	I	F
S1	τ_1	0.80	0.20	0.15
S1	τ_2	0.90	0.10	0.10
S1	τ_3	0.70	0.25	0.20
S1	τ_4	0.85	0.15	0.12
S2	τ_1	0.75	0.22	0.12
S2	τ_2	0.80	0.18	0.18
S2	τ_3	0.68	0.28	0.15
S2	τ_4	0.78	0.20	0.14
S3	τ_1	0.72	0.30	0.10
S3	τ_2	0.76	0.25	0.14
S3	τ_3	0.60	0.35	0.12
S3	τ_4	0.65	0.33	0.13

4.2 Project composites via $\diamond$ Using (3.1):

$$X_1 \diamond X_2 = \big(T_1T_2, I_1 + I_2 - I_1I_2, 1 - (1 - F_1)(1 - F_2)\big).$$

Table 2. Project triplets $X(P)$.

Student	Project	T	I	F
S1	$P_1 = \tau_1 \diamond \tau_2$	0.7200	0.2800	0.2350
S1	$P_2 = \tau_3 \diamond \tau_4$	0.5950	0.3625	0.2960
S2	P_1	0.6000	0.3604	0.2784
S2	P_2	0.5304	0.4240	0.2690
S3	P_1	0.5472	0.4750	0.2260
S3	P_2	0.3900	0.5645	0.2344

Example check: S1, P_1: $T = 0.80 \cdot 0.90 = 0.7200; I = 0.20 + 0.10 - 0.20 \cdot 0.10 = 0.2800; F = 1 - (0.85 \cdot 0.90) = 0.2350.$)

4.3 Student aggregates and robust scores From (3.5):

$$\bar{T} = \frac{T^{(1)} + T^{(2)}}{2},$$
$$\bar{I} = \frac{I^{(1)} + I^{(2)}}{2},$$
$$\bar{F} = \frac{F^{(1)} + F^{(2)}}{2}.$$

From (3.10):

$$R = 0.45\bar{T} - 0.35\bar{F} - 0.20\bar{I}.$$

Table 3. Aggregated means and robust scores (higher R is better).

Student	$\bar{T}$	$\bar{I}$	$\bar{F}$	R	Rank

S1	0.65750	0.32125	0.26550	0.13870	1
S2	0.56520	0.39220	0.27370	0.08011	2
S3	0.46860	0.51975	0.23020	0.02635	3

(Arithmetic checks: S1 $R = 0.45 \cdot 0.65750 - 0.35 \cdot 0.26550 - 0.20 \cdot 0.32125 = 0.13870$. S2 $R = 0.25434 - 0.095795 - 0.07844 = 0.08011$. S3 $R = 0.21087 - 0.08057 - 0.10395 = 0.02635$.)

4.4 Neutrosophic margins and stability

From (3.11): $\mathrm{NMS}(i, j) = R(i) - R(j)$.

Table 4. Neutrosophic margins of separation (positive ⇒ semi- T_2 separability).

Pair	NMS
$S1 - S2$	0.05860
$S2 - S3$	0.05376
$S1 - S3$	0.11235

All margins are strictly positive, so the ranking $S1 \succ S2 \succ S3$ is stable under small perturbations of (T, I, F) in a neutrosophic product topology (semi- T_2 separability).

5. Discussion

Practical use. The framework is operationally simple: (i) encode each task as $(T, I, F) \in \Omega$; (ii) fold tasks to a project via $\diamond$; (iii) average projects to $\bar{X}$; (iv) compute the robust score R at a vertex of the admissible polytope. All steps are closed-form, auditable, and require no numerical optimization beyond evaluating linear expressions.

Calibration and policy transparency. Mapping evidence to (T, I, F) should be rubric-driven (e.g., success criteria, ambiguity rubrics, failure triggers) with double-rating and reconciliation to reduce rater variance [3]. Policy bounds ($a_{\min}, b_{\max}, c_{\max}$) ought to be published a priori; the closed-form $R = a_{\min}\bar{T} - b_{\max}\bar{F} - c_{\max}\bar{I}$ (when $a_{\min} + b_{\max} + c_{\max} = 1$) makes the impact of policy immediately interpretable.

Auditability via feasible back-solving. The image of the translation by a known factor $X = (t, i, f)$ is the rectangle $[0, t] \times [i, 1] \times [f, 1]$. Any target Y inside it has a unique preimage U given by $(p/t, (q - i)/(1 - i), (r - f)/(1 - f))$; if Y lies outside, no solution exists. This precise feasibility criterion supports transparent post-hoc diagnostics.

For a project of m tasks, folding costs $O(m)$ arithmetic operations; for a student with K projects, aggregation is $O(K)$. Robust scoring is $O(1)$ once the policy vertex is fixed. This scales cleanly to program-level cohorts.

Sensitivity analysis. Vary ($a_{\min}, b_{\max}, c_{\max}$) within small neighborhoods and report ΔR and Δ rank. Because R is linear in these parameters, one-step bounds are straightforward: $\partial R / \partial a_{\min} = \bar{T}, \partial R / \partial b_{\max} = -\bar{F}, \partial R / \partial c_{\max} = -\bar{I}$.

Limitations. (i) The monoid is commutative; sequencing effects are not modeled. If order sensitivity is essential, a distinct non-commutative operator can be introduced (outside this paper) while retaining the robust scoring layer. (ii) Non-idempotency attenuates T under repeated composition ($T^2 \leq T$), which realistically encodes compounding risk but should be communicated to stakeholders. (iii) Results depend on the quality of the (T, I, F) rubrics; rigorous rater training and exemplars are advised.

External validity. The algebra is domain-agnostic and extends to diverse vocational tracks (ICT, automotive, culinary), provided the rubrics are tuned to each domain and the admissible weight bounds reflect local policy.

6. Conclusion

We presented an indicator-free neutrosophic assessment method that (a) encodes entrepreneurial work directly at the task level, (b) aggregates with a commutative monoid operator that ensures closure, continuity, identity, and feasible unique back-solving, (c) delivers a robust student score as a vertex solution of an admissible weight polytope, and (d) certifies stability through neutrosophic margins implying semi-T_2 separability. A worked case study showed exact computations, positive margins, and interpretable policy levers. Future directions include (i) designing and validating a non-commutative composition for order-sensitive programs, (ii) longitudinal extensions for trajectory-aware evaluation, and (iii) empirical studies of inter-rater reliability and sensitivity to policy bounds.

References

[1] Drucker, P. F. (1985). *Innovation and entrepreneurship: Practice and principles*. Harper & Row.
[2] Neck, H. M., & Greene, P. G. (2011). Entrepreneurship education: Known worlds and new frontiers. *Journal of Small Business Management, 49*(1), 55–70. https://doi.org/10.1111/j.1540-627X.2010.00314.x
[3] Morgan, M. G., & Henrion, M. (1990). *Uncertainty: A guide to dealing with uncertainty in quantitative risk and policy analysis*. Cambridge University Press.
[4] Saaty, T. L. (1980). *The analytic hierarchy process: Planning, priority setting, resource allocation*. McGraw-Hill.
[5] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
[6] Smarandache, F. (1998). *Neutrosophy: Neutrosophic probability, set, and logic*. American Research Press.
[7] Wang, H., Smarandache, F., Zhang, Y.-Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure, 4*, 410–413.
[8] Ye, J. (2014). Single-valued neutrosophic minimum spanning tree and its clustering method. *Applied Mathematical Modelling, 38*(3), 1170–1175. https://doi.org/10.1016/j.apm.2013.08.014

[9] Broumi, S., Smarandache, F., & Dhar, M. (2014). Rough neutrosophic sets. *Italian Journal of Pure and Applied Mathematics, 32*, 493–502.
[10] Gayathri, N., & Helen, M. (2022). Semi-separation axioms and semi-regularity in linguistic neutrosophic topological spaces. *Neutrosophic Sets and Systems, 51*, 686–701. https://doi.org/10.5281/zenodo.7138412
[11] Oyem, A., Jaiyéolá, T. G., Olaleru, J. O., & Osoba, B. (2022). Soft neutrosophic quasigroups. *Neutrosophic Sets and Systems, 50*, 378–396. https://doi.org/10.5281/zenodo.6617529
[12] Pflugfelder, H. O. (1990). *Quasigroups and loops: Introduction*. Heldermann Verlag.

Received: March 19, 2025. Accepted: Aug 22, 2025