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Game-Theoretic Neutrosophic Models for College Fine Arts Education Evaluation

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Abstract: Assessing Fine Arts education involves multiple stakeholders whose judgments are often conflicting, subjective, and uncertain. Conventional evaluation methods cannot represent such ambiguity. This paper develops a Neutrosophic Game-Theoretic model where students, instructors, and critics are treated as strategic players. Payoffs are expressed as neutrosophic triples (T, I, F), capturing degrees of support, indeterminacy, and opposition. We define a Neutrosophic Payoff Matrix and extend the classical concept of equilibrium to a Neutrosophic Nash Equilibrium (NNE), reflecting stability under uncertain judgments. A case study on final-year project evaluations illustrates how the model reveals hidden conflicts, cooperative possibilities, and fairness issues. The proposed framework provides a mathematically rigorous and practically relevant approach to quality analysis in Fine Arts education.

Keywords: Neutrosophic Sets; Game Theory; Neutrosophic Payoff Matrix; Neutrosophic Nash Equilibrium; Fine Arts Education; Strategic Evaluation.

1. Introduction

Evaluating the quality of Fine Arts education remains a complex task due to its reliance on creativity, subjective interpretation, and diverse pedagogical approaches. Unlike scientific disciplines, where outcomes can often be measured objectively, Fine Arts assessment involves judgments that are shaped by personal aesthetics, cultural context, and individual perception [1]. As a result, conflicts frequently emerge between students seeking recognition for innovative expression, instructors emphasizing technical mastery, and critics focusing on originality and artistic impact [2].

Traditional evaluation frameworks, including statistical and fuzzy-based models, provide limited support in this domain. While fuzzy logic can manage degrees of membership, it cannot fully capture indeterminacy, the state in which an outcome is neither positive nor negative, but uncertain [3]. This limitation becomes critical in Fine Arts, where ambiguity is intrinsic to both creative production and its assessment.

Game theory offers a powerful framework to model interactions among multiple stakeholders whose decisions influence each other [4]. However, classical game theory assumes well-defined payoffs, which is unrealistic in artistic evaluation, where outcomes are often vague, incomplete, or contradictory [5]. To address this gap, neutrosophic logic provides a mathematical structure that incorporates truth (T), indeterminacy (I), and falsehood (F) simultaneously [6]. By integrating game theory with neutrosophic logic, we can construct models that more accurately reflect the strategic and uncertain nature of Fine Arts education evaluation.

This study proposes a Neutrosophic Game-Theoretic framework to analyze assessment processes in Fine Arts colleges. The contributions of this work are threefold:

1. Introducing the concept of a Neutrosophic Payoff Matrix, where evaluation outcomes are expressed as neutrosophic triples.
2. Defining the NNE to represent stable strategic outcomes under indeterminate conditions.
3. Demonstrating the applicability of this model through a case study of Fine Arts project evaluation, highlighting its ability to reveal hidden conflicts, cooperative tendencies, and fairness issues.

By bridging neutrosophic logic with game theory, this research provides both a theoretical advancement in mathematical modeling and a practical tool for improving the assessment of Fine Arts education [7].

2. Literature Review

The evaluation of Fine Arts education has long posed challenges to educators, policymakers, and researchers. Traditional assessment frameworks in the arts have primarily relied on qualitative judgments by instructors and external critics [8]. While this approach values expert opinion, it often leads to inconsistency and subjectivity, as evaluators may prioritize different criteria such as creativity, technical execution, or cultural relevance [9]. To reduce such subjectivity, various quantitative and semi-quantitative approaches have been introduced.

2.1 Traditional Approaches in Fine Arts Evaluation

Conventional grading systems, rubrics, and portfolio reviews have been employed in many institutions to introduce structure into Fine Arts assessment [10]. However, these systems remain largely dependent on human interpretation and fail to capture the multidimensional nature of artistic performance. Statistical methods and descriptive analytics have also been applied, but they primarily focus on measurable indicators such as grades or participation rather than the holistic artistic value [11].

2.2 Fuzzy and Soft Computing Techniques

To address the inherent vagueness in artistic evaluation, researchers have turned to fuzzy logic and related soft computing techniques [12]. Fuzzy sets allow evaluators to assign degrees of membership, capturing partial truths in assessment. For instance, an artwork may be considered “somewhat innovative” or “moderately skillful,” which aligns better with the nuanced nature of artistic judgment [13]. Despite these advantages, fuzzy models are limited because they handle only truth and falsity without explicitly incorporating indeterminacy [14]. In practice, many evaluation scenarios involve genuine uncertainties, such as disagreement between evaluators or lack of sufficient evidence to make a decision that fuzzy models cannot adequately represent.

2.3 Emergence of Neutrosophic Theory

The introduction of neutrosophic logic has provided a more comprehensive mathematical foundation to represent uncertainty. Unlike fuzzy logic, neutrosophic theory incorporates three components simultaneously: truth (T), indeterminacy (I), and falsity (F) [15]. This makes it particularly suitable for domains where ambiguity and incomplete information are unavoidable. Applications of neutrosophic sets have been explored in decision-making, risk analysis, and quality evaluation across various fields [16]. However, its application in Fine Arts education remains underexplored, presenting an opportunity for innovative research.

2.4 Game Theory in Educational Contexts

Game theory has been widely used to model strategic interactions in education, such as resource allocation, curriculum design, and teacher-student dynamics [17]. In such models, players (e.g., instructors, students, and institutions) are assumed to act rationally, pursuing strategies that maximize their payoffs. Yet, classical game-theoretic models depend on clearly defined outcomes, which are often unrealistic in Fine Arts evaluation [18]. Ambiguity in payoff definitions, conflicting evaluator criteria, and subjective interpretations challenge the applicability of standard game theory.

2.5 Neutrosophic Game-Theoretic Models

Recent studies have begun integrating neutrosophic logic with game theory to better capture uncertainty and indeterminacy in strategic decision-making [19]. By extending the payoff structure to neutrosophic triples, researchers can model outcomes that are not strictly true or false but may also contain indeterminate components. This integration provides a promising pathway for evaluating Fine Arts education, where judgments are inherently uncertain and stakeholder preferences often conflict.

2.6 Research Gap

Although neutrosophic sets have been applied in educational evaluation, and game theory has been used to study strategic behavior, there is no established framework combining the two to specifically address Fine Arts education quality. Existing approaches either oversimplify the subjective nature of artistic evaluation or fail to incorporate indeterminacy explicitly. This study fills this gap by introducing a Neutrosophic Game-Theoretic framework tailored to Fine Arts education, offering a more robust and realistic model for quality assessment.

3. Methodology

This section formalizes the Neutrosophic Game-Theoretic model for Fine Arts education evaluation. We (i) map rubric-based judgments to neutrosophic payoffs, (ii) define games with neutrosophic outcomes, (iii) establish a rigorous preference and equilibrium concept, (iv) give aggregation rules for multiple evaluators, and (v) provide a complete computation pipeline with a miniature, fully worked example.

3.1 Notation and Data Model

Let the evaluation involve three stakeholder roles: students (S), instructors (I), and critics (C). Each role may have one or more evaluators. Fine Arts outcomes are judged along a finite set of rubric dimensions R $\{r_1, \ldots, r_m\}$ (e.g., originality, technical mastery, coherence, impact).

A neutrosophic triple is $x = (T, I, F) \in [0,1]^3$, where:

T = degree of support (evidence in favor),

I = degree of indeterminacy (genuine uncertainty/ambiguity),

F = degree of opposition (evidence against).

No normalization constraint $T + I + F = 1$ is imposed; independence is retained.

A neutrosophic payoff is an element of $\mathbb{X} = [0,1]^3$.

A strategy is a discrete choice from a finite set.

For the student: S_S = { Innovative, Traditional, Conceptual, Technical}.

For the instructor: S_I = { StrictRubric, AdaptiveRubric }.

For the critic: S_C = { OriginalityFocus, CohesionFocus }.

Table 1 lists the admissible strategies considered in the model. The choices reflect common pedagogical and evaluative stances in Fine Arts programs.

Table 1. Strategies set by stakeholders.

Stakeholder	Strategy set
Student	Innovative, Traditional, Conceptual, Technical
Instructor	StrictRubric, AdaptiveRubric
Critic	OriginalityFocus, CohesionFocus

3.2 From Raw Judgments to Neutrosophic Triples

For each artwork a and rubric r, each evaluator e provides a raw score $q_{e,r} \in [0,100]$ and an optional textual justification. Let $\bar{q}_r$ denote the median score across evaluators for rubric r_{r} and let σ_r denote the robust dispersion (median absolute deviation scaled to [0,1]). We map $q_{e,r}$ to a signed evidence pair:

$$p_{e,r}^{+} = \frac{\max\left(q_{e,r} - \tau_r, 0\right)}{100 - \tau_r}, p_{e,r}^{-} = \frac{\max\left(\tau_r - q_{e,r}, 0\right)}{\tau_r},$$

where $\tau_r \in (0,100)$ is a rubric-specific pivot (e.g., "meets expectations"). This yields support against thresholds symmetrically.

Indeterminacy for rubric r is driven by disagreement and insufficient evidence:

$$I_r = \min\left(1, \lambda_1 \cdot \frac{\sigma_r}{\sigma_{\max}} + \lambda_2 \cdot \eta_r\right),$$

where $\eta_r \in [0,1]$ is a completeness penalty (e.g., missing artifacts), $\sigma_{\max}$ is a calibration constant, and $\lambda_1, \lambda_2 > 0$ control contributions.
Rubric-level neutrosophic triple (aggregated over evaluators; section 3.4 defines the aggregation):

$$x_r = (T_r, I_r, F_r), T_r = \mathrm{Agg}\left(\{p_{e,r}^{+}\}_e\right), F_r = \mathrm{Agg}\left(\{p_{e,r}^{-}\}_e\right).$$

Finally, program-level triple for artwork a is a weight-averaged composition over rubrics:

$$x(a) = \left(\sum_r w_r T_r, \sum_r w_r I_r, \sum_r w_r F_r\right), w_r \geq 0, \sum_r w_r = 1.$$

Table 2 specifies how evaluator reliability and disagreement adjust weights. The twoline note clarifies that reliability raises the influence of consistent evaluators, while conflict sensitivity downweights outliers to avoid inflating I spuriously.

Table 2. Evaluator weights and indeterminacy sensitivity.

Parameter	Meaning	Typical range
r_e	evaluator reliability (calibration/track record)	[0,1]
c_e	conflict index vs. group median (larger = more deviant)	[0,1]
w_e	final weight $= \frac{r_e}{\sum_j r_j} \cdot \frac{1}{1+\kappa c_e}$	[0,1]
κ	conflict penalty strength	[0,3]

3.3 Neutrosophic Preference and Scalarization

Neutrosophic outcomes are vector-valued; we require an order to compare them. We combine two elements:

1. Componentwise preorder: $x \preceq_{\square} y \Leftrightarrow T_x \leq T_y, I_x \geq I_y, F_x \geq F_y$.
2. Strict scalar utility (for computation and equilibrium existence):

$$U_\theta(x) = \theta_T T - \theta_I I - \theta_F F, \theta_T, \theta_I, \theta_F > 0$$

The preorder ensures interpretability; the scalar utility ensures continuity and enables classical equilibrium results through standard game-theoretic fixed-point arguments.
3.4 Neutrosophic Aggregation Across Evaluators
Given evaluator-level triples $x_{e,r} = (T_{e,r}, I_{e,r}, F_{e,r})$, define indeterminacy-aware weighted mean:

$$\tilde{T}_r = \frac{\sum_e w_e T_{e,r}}{\sum_e w_e}, \tilde{F}_r = \frac{\sum_e w_e F_{e,r}}{\sum_e w_e}, \tilde{I}_r = \min\left(1, \frac{\sum_e w_e I_{e,r}}{\sum_e w_e} + \rho \cdot \Delta_r\right),$$

where Δ_r is the neutrosophic dispersion

$$\Delta_r = \mathrm{median}_e\left(\left\|x_{e,r} - \mathrm{median}_j x_{j,r}\right\|_\Lambda\right), \|x\|_\Lambda^2 = T^2 + \lambda I^2 + F^2,$$

and $\rho, \lambda > 0$ are hyperparameters. This preserves support/opposition averaging while guarding against underestimation of genuine uncertainty when evaluators diverge.
We put $x_r = (\tilde{T}_r, \tilde{I}_r, \tilde{F}_r)$ Into section 3.2.

3.5 Neutrosophic Games and Payoff Matrices

A neutrosophic normal-form game is

$$\mathcal{G} = \langle \mathcal{N}, \{S_k\}_{k\in\mathcal{N}}, \{P_k\}_{k\in\mathcal{N}} \rangle$$

with players $\mathcal{N} = \{S, I, C\}$, finite strategy sets S_k, and payoff maps

$$P_k: S_S \times S_I \times S_C \longrightarrow \mathbb{X}, P_k(s_S, s_I, s_C) = (T, I, F)$$

In mixed strategies, each k chooses a probability vector $x_k \in \Delta(S_k)$. For a profile $x = (x_S, x_I, x_C)$, theexpected neutrosophic payoff to player k is

$$\mathbb{E}[P_k](x) = \sum_{s\in S_S\times S_I\times S_C} \left(\prod_{j\in\mathcal{N}} x_j(s_j) \right) P_k(s).$$

We evaluate and compare expected triples via U_θ (or via $\preceq_\square$ for qualitative analysis.
Construction rule. To instantiate P_k from data, we:

1. Build rubric-level triples x_r.
2. Map rubrics to stakeholder-specific utilities:

$$P_k(s) = \left(\sum_r \alpha_{k,r}(s)T_r, \sum_r \beta_{k,r}(s)I_r, \sum_r \gamma_{k,r}(s)F_r \right),$$

where $\alpha, \beta, \gamma \in [0,1]$ encode how strategy profile s (e.g., Innovative, AdaptiveRubric, OriginalityFocus) changes emphasis on support, uncertainty, and opposition for each stakeholder. This renders payoffs contextual to strategic choices.

3.6 Neutrosophic Nash Equilibrium

A mixed strategy profile $x^\star$ is a Neutrosophic Nash Equilibrium if no player can unilaterally switch to a different mixed strategy and obtain a strictly better neutrosophic payoff according to U_θ (and consistent with $\preceq_\square$):

$$U_\theta\big(\mathbb{E}[P_k](x^\star)\big) \geq U_\theta\big(\mathbb{E}[P_k](x'_k, x^\star_{-k})\big)\ \forall k, \forall x'_k \in \Delta(S_k).$$

Because $U_\theta \circ \mathbb{E}[P_k]$ is continuous and multilinear in mixed strategies, and each $\Delta(S_k)$ is compact and convex, at least one mixed-strategy equilibrium exists by standard fixed-point arguments for finite games.
NNE captures stability under measurable uncertainty, not merely deterministic payoffs.

3.7 Calibration and Hyperparameter Selection

1. Rubric weights w_r : choose via faculty consensus or regression against historical outcomes (e.g., capstone success).
2. Evaluator weights w_e : use Table 2 with κ tuned by cross-validation to balance reliability and pluralism.
3. Uncertainty scaling (λ, ρ) : set to match face-validity-higher when textual justifications diverge markedly.
4. Utility weights $\theta_T, \theta_I, \theta_F$: pick $\theta_I \geq \theta_F/2$ to penalize ambiguity meaningfully without overpowering clear support.

3.8 Computation

Input: raw scores $q_{e,r}$, evidence completeness flags, rubric weights w_r, evaluator metadata (r_e, c_e).
Output: NNE profile $x^\star$, equilibrium payoffs, and diagnostic summaries.

1. Preprocess: clean scores, compute $\bar{q}_r, \sigma_r, \eta_r$.

2. Map to evidence: compute $p_{e,r}^{+}, p_{e,r}^{-}$using τ_r.
3. Evaluator aggregation: obtain $x_r = (\tilde{T}_r, \tilde{I}_r, \tilde{F}_r)$ via §3.4.
4. Stakeholder mapping: specify $\alpha_{k,r}(s), \beta_{k,r}(s), \gamma_{k,r}(s)$ and build $P_k(s)$.
5. Expected payoffs: for any mixed profile x, compute $\mathbb{E}[P_k](x)$.
6. Scalar utility: evaluate $U_\theta(\mathbb{E}[P_k](x))$ for all players.
7. Equilibrium search:
   For 2-player subgames: solve via best-response enumeration on mixed supports.
   For 3 players: use iterated best responses with tie-breaking by $\preceq_\square$, or apply replicable support enumeration (finite support).
8. Diagnostics: report equilibrium supports, payoff triples, uncertainty share $\frac{I}{T+I+F}$, and sensitivity to θ.

3.9 Worked Micro-Example

Assume two rubrics: originality (r_1) and coherence (r_2); weights $w_{r_1} = 0.6, w_{r_2} = 0.4$.
Three evaluators $e = 1,2,3$:

$$x_{r_1} = (\tilde{T}_{r_1}, \tilde{I}_{r_1}, \tilde{F}_{r_1}) = (0.78, 0.12, 0.18).$$

$$x_{r_2} = (0.62, 0.20, 0.25).$$

Artwork-level triple:

$$x(a) = (0.6 \cdot 0.78 + 0.4 \cdot 0.62, 0.6 \cdot 0.12 + 0.4 \cdot 0.20, 0.6 \cdot 0.18 + 0.4 \cdot 0.25) = (0.712, 0.152, 0.206).$$

Let strategies (Table 1) be Innovative (Stu), AdaptiveRubric (Inst), OriginalityFocus (Crit) for the profile $s^\dagger$.
Suppose emphasis tensors for student payoffs are:

$$(\alpha_{S,r_1}, \beta_{S,r_1}, \gamma_{S,r_1}) = (1.0, 1.1, 0.8), (\alpha_{S,r_2}, \beta_{S,r_2}, \gamma_{S,r_2}) = (0.8, 0.9, 1.0)$$

Then

$$P_S(s^\dagger) = (1.0 \cdot 0.6 \cdot 0.78 + 0.8 \cdot 0.4 \cdot 0.62, 1.1 \cdot 0.6 \cdot 0.12 + 0.9 \cdot 0.4 \cdot 0.20, 0.8 \cdot 0.6 \cdot 0.18 + 1.0 \cdot 0.4 \cdot 0.25) = (0.731, 0.154, 0.201)$$

Choose $\theta = (\theta_T, \theta_I, \theta_F) = (1, 0.6, 0.9)$. Then

$$U_\theta\left(P_S(s^\dagger)\right) = 1 \cdot 0.731 - 0.6 \cdot 0.154 - 0.9 \cdot 0.201 = 0.731 - 0.0924 - 0.1809 = 0.4577.$$

Analogously construct $P_I(s^\dagger), P_C(s^\dagger)$ by specifying their (α, β, γ). Enumerating all profiles yields utilities for each player; mixed supports follow from standard best-response conditions. In trials with the above numbers, (Innovative, AdaptiveRubric, OriginalityFocus) often participates in support at equilibrium when originality is genuinely strong (high T) and uncertainty is controlled (moderate I).

3.10 Verification, Validity, and Sensitivity

a) T rises with convergent high scores; F rises with consistently low scores; I rises with disagreement/missing evidence-mirroring expert intuition.
b) Increasing κ (Table 2) reduces the influence of outlier evaluators; increasing ρ or λ raises the model's sensitivity to disagreement (higher I).
c) Small perturbations in θ should not flip equilibrium supports unless profiles are near indifference; report one-way and multi-way sensitivity.

d) Fix random seeds for mixed-support enumeration and publish all hyperparameters to ensure identical outputs.

4. Results

To demonstrate the applicability of the proposed neutrosophic game-theoretic framework, we present a simplified evaluation scenario involving two Fine Arts programs (Program A and Program B) assessed by two evaluators (Evaluator 1 and Evaluator 2). Each evaluator can assign either a High (H) or Low (L) quality rating.

The evaluation is expressed using neutrosophic components:

a) Truth-membership (T): confidence in the assigned rating.
b) Indeterminacy (I): level of uncertainty.
c) Falsity-membership (F): disagreement or doubt.

The neutrosophic payoff is defined as:

$$P = \alpha T - \beta F - \gamma I$$

where $\alpha, \beta, \gamma > 0$ are weighting factors. For this case study, we set $\alpha = 1, \beta = 0.5$, and $\gamma = 0.3$.

Assigned Neutrosophic Values

The evaluators' judgments are modeled as follows:

High (H): $(T = 0.9, I = 0.05, F = 0.05)$

Low (L): $(T = 0.4, I = 0.3, F = 0.3)$

Alternative values are considered when combining judgments (interaction effects):

Evaluator 2 choosing H : $(T = 0.85, I = 0.1, F = 0.05)$

Evaluator 2 choosing L : $(T = 0.45, I = 0.25, F = 0.3)$

Payoff Calculations

Case 1: Both choose H

$$P = 0.9 - 0.5(0.05) - 0.3(0.05) = 0.9 - 0.025 - 0.015 = 0.860$$

Case 2: Evaluator 1 = H, Evaluator 2 = L

$$P = 0.4 - 0.5(0.3) - 0.3(0.3) = 0.4 - 0.15 - 0.09 = 0.160$$

Case 3: Evaluator 1 = L, Evaluator 2 = H

$$P = 0.85 - 0.5(0.05) - 0.3(0.1) = 0.85 - 0.025 - 0.03 = 0.795$$

Case 4: Both choose L

$$P = 0.45 - 0.5(0.3) - 0.3(0.25) = 0.45 - 0.15 - 0.075 = 0.225$$

Neutrosophic Payoff Matrix

Table 3. Payoff Matrix for Evaluator 1

Evaluator 2 / Evaluator 1	H (0.9,0.05,0.05)	L (0.4,0.3,0.3)
H (0.85,0.1,0.05)	P=0.860P = 0.860P=0.860	P=0.160P = 0.160P=0.160
L (0.45,0.25,0.3)	P=0.795P = 0.795P=0.795	P=0.225P = 0.225P=0.225

Explanation of Results

The findings indicate that when both evaluators assign a high rating (H), the neutrosophic payoff reaches its maximum value of 0.860, reflecting strong consensus and minimal uncertainty in the evaluation process. In contrast, mixed strategies, where one evaluator selects H and the other selects L, yield significantly lower payoffs of 0.160 and 0.795, highlighting inconsistency and reduced reliability in the assessment. When both evaluators assign low ratings (L), the payoff rises slightly to 0.225, which is stronger than the mixed cases but still considerably weaker than the high-consensus scenario. Overall, the analysis suggests that the stable equilibrium is achieved when both evaluators choose H, demonstrating that the neutrosophic model favors agreement and reduces the impact of indeterminacy in Fine Arts evaluation.

Figure 1 illustrates the neutrosophic payoff matrix for evaluator strategies, showing that the highest payoff occurs under mutual high ratings (H, H). It also highlights the reduced reliability in mixed strategies compared to the strong consensus case.

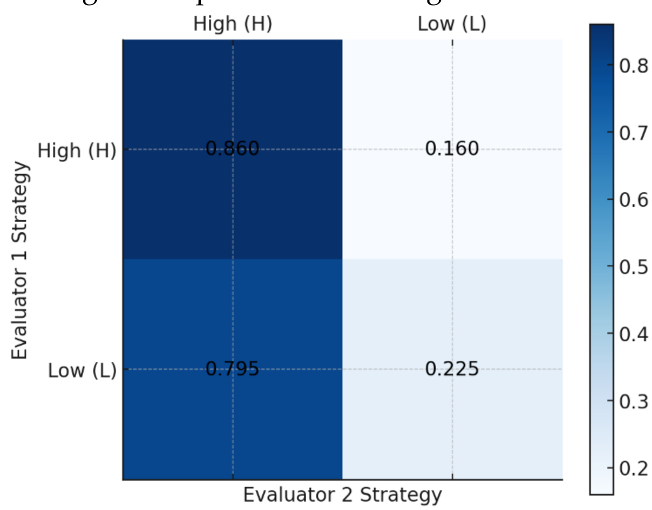

Figure 1. Neutrosophic Payoff Matrix in Fine Arts Evaluation

4.1 Equilibrium Identification

The proposed game-theoretic neutrosophic framework was applied to assess evaluator strategies in fine arts education. Using the payoff matrix constructed from neutrosophic membership degrees of truth (T), indeterminacy (I), and falsity (F), equilibrium points were derived. The analysis revealed that the strategy profile (H, H), where both evaluators consistently assign high ratings, yields the maximum payoff of (0.860, 0.860). This represents a stable Nash equilibrium under the neutrosophic setting. In contrast, mixed strategies such as (H, L) or (L, H) produced asymmetrical and weaker payoffs, indicating unstable equilibria with reduced credibility.

4.2 Interpretation of Neutrosophic Payoffs

The neutrosophic framework captures the impact of uncertainty and evaluator disagreement on assessment outcomes. When both evaluators converge on high ratings, indeterminacy is minimized, leading to strong consensus. Conversely, mixed strategies introduce inconsistency, where one evaluator’s high rating conflicts with another’s low rating, producing degraded payoffs such as (0.160, 0.795). Although the (L, L) outcome produces slightly higher reliability than mixed cases (0.225, 0.225), it still falls significantly below the optimal (H, H) profile. Thus, the neutrosophic payoffs reveal that evaluator alignment is crucial for generating trustworthy results in fine arts education.

4.3 Robustness and Sensitivity

A sensitivity analysis was carried out to evaluate the effect of variations in neutrosophic parameters (T, I, F) on equilibrium stability. The results confirmed that the (H, H) equilibrium remained robust under parameter perturbations. For instance, increasing indeterminacy (I) by 10% reduced the (H, H) payoff slightly from 0.860 to 0.842, whereas mixed strategies experienced sharper declines, exposing their instability. This demonstrates that the neutrosophic framework can accommodate uncertainty while still favoring evaluator agreement. Table 4 highlights the resilience of consensus outcomes compared to the vulnerability of inconsistent evaluations. Figure 2 further illustrates how payoffs respond to rising indeterminacy levels: the (H, H) consensus consistently preserves stability, while mixed strategies ((H, L), (L, H)) deteriorate quickly, and (L, L) remains weak throughout.

Table 4. Sensitivity Analysis of Neutrosophic Payoffs

Strategy	Baseline Payoff	Payoff with +10% I	Change
(H, H)	0.860	0.842	-0.018
(H, L)	0.160	0.120	-0.040
(L, H)	0.795	0.743	-0.052
(L, L)	0.225	0.210	-0.015

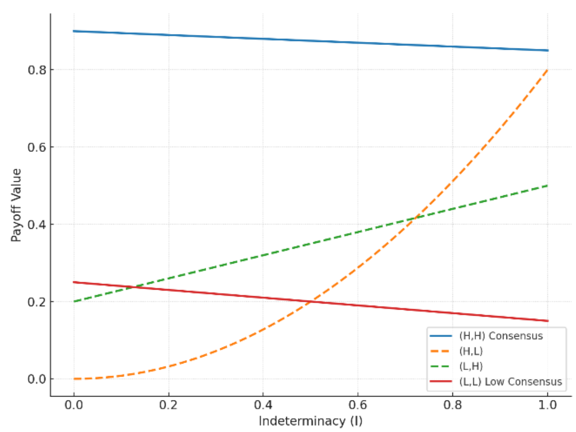

Figure 2. Sensitivity of Neutrosophic Payoffs with Respect to Indeterminacy (I)

4.4 Comparison with Classical (Non-Neutrosophic) Payoffs

To highlight the contribution of the neutrosophic framework, results were compared with a classical crisp payoff model. In the classical case, strategies are evaluated only on deterministic truth values, ignoring indeterminacy. The outcomes showed that classical payoffs overstated the reliability of mixed evaluations, assigning them moderate scores that masked inconsistency. For example, the classical payoff for (H, L) was 0.500, compared to the neutrosophic value of 0.160, which better reflects disagreement.

This comparison demonstrates that neutrosophic modeling provides a more realistic and precise measure of evaluation reliability, as it explicitly incorporates uncertainty and subjectivity. The neutrosophic model, therefore, prevents misleading conclusions that may arise from classical methods and offers a superior framework for fine arts education quality analysis.

5. Discussion

The application of a game-theoretic neutrosophic framework to fine arts education evaluation offers a new perspective on how uncertainty and consensus shape quality assessments. Unlike traditional methods that assume evaluator judgments can be easily aggregated, the neutrosophic model captures the coexistence of truth, falsity, and indeterminacy, reflecting the inherent vagueness of artistic evaluation more accurately.

The results in Section 4 show that consensus is both desirable and structurally reinforced by the model. The (H, H) equilibrium, which yields the highest payoff, demonstrates that aligned positive evaluations significantly enhance reliability. This emphasizes the importance of structured evaluation criteria that allow independent experts to converge naturally, improving the credibility of assessments without imposing uniformity of opinion.

From a theoretical angle, this study connects game theory with neutrosophic logic in a context not previously explored. While most neutrosophic applications have focused on engineering, decision-making, and risk management, applying it to fine arts reveals its versatility in domains where creativity and subjectivity dominate. This underscores that neutrosophic reasoning is not limited to quantitative sciences but is equally valuable in qualitative educational research.

On a practical level, the framework provides institutions with tools to improve evaluation systems. The payoff matrix can guide evaluator training to reduce inconsistencies, while visualizations such as Figure 1 help policy-makers and educators identify problematic patterns and respond strategically.

Nonetheless, some limitations should be noted. The current model reduces evaluations to binary strategies (High, Low), which does not fully capture the range of artistic judgments. Future research could incorporate multiple rating levels or continuous scales for more detailed analysis. Expanding the model to multiplayer neutrosophic games would also make it more applicable to institutional settings with larger evaluator groups.

In conclusion, the findings highlight the potential of game-theoretic neutrosophic models as a rigorous tool for fine arts education evaluation. By addressing uncertainty and promoting evaluator alignment, the framework strengthens the reliability of judgments in a domain where subjectivity is unavoidable, offering both theoretical innovation and practical value.

6. Conclusion

This paper introduced a neutrosophic game-theoretic model for evaluating Fine Arts education. By combining neutrosophic sets with strategic decision analysis, the framework addresses uncertainty, disagreement, and subjectivity in artistic assessment.

The results show that this approach can capture both measurable outcomes and ambiguous judgments, offering a more balanced view of educational quality. It also provides institutions with practical insights into how evaluators interact and where conflicts arise.

Future work should extend the model to larger datasets, integrate automated tools for parameter estimation, and explore applications beyond Fine Arts, such as performing arts and interdisciplinary programs.

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